Double-dip negative magnetoresistance and quantum effects in the conductivity of compensated InSb crystals

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Unexpected behavior of the conductivity in a magnetic field was observed near the percolation metal–insulator transition. The behavior is at variance with the traditional predictions of the theory of quantum corrections. This transition is related to localization of electrons in the large-scale fluctuation potential of impurities, which in turn is determined by the high degree of compensation. In weak magnetic fields the compensation influences primarily the magnetic-field dependence of the conductivity and leads to the appearance of unusual negative magnetoresistance, which has two minima. In addition to negative magnetoresistance, arising due to weak localization, there is negative magnetoresistance associated with an additional mechanism. In particular, redistribution of electrons between two spin subbands split by a magnetic field is possible. In systems with a short electron-momentum relaxation time and long spin-relaxation time this raises the Fermi level and hence decreases the resistance. The phase diagram of the electronic system of compensated semiconductors in a magnetic field is discussed as an application of this work.

1. INTRODUCTION

At low temperatures the magnetic-field and temperature dependence of the conductivity of metallic crystals are determined by quantum interference effects. The first effect, which was first predicted by Abrahams *et al.*,[1] is due to the interference of electron waves propagating clockwise and anticlockwise (weak localization) along self-intersecting trajectories. The second effect, which was studied by Al'tshuler and Aronov,[2] is associated with renormalization of the electron–electron interaction by the scattering of electrons by defects of the crystal lattice (*e–e* interaction). The theory of quantum effects in the conductivity of metals is well developed and predicts the following relations for the temperature-dependent corrections to the conductivity (see the review in Refs. 3 and 4):

$$\Delta\sigma \approx \frac{e^2}{2\pi^2\hbar}\frac{1}{L_\varphi}, \tag{1}$$

with $L_\varphi = \sqrt{(D\tau_\varphi)}$, and

$$\Delta\sigma \approx \frac{e^2}{2\pi^2\hbar}\frac{1}{L_T}, \tag{2}$$

with $L_T = \sqrt{(D\hbar/kT)}$, where weak localization (1) and *e–e* interaction (2) play the main role. Here D is the electron diffusion coefficient and τ_φ is the phase error time for the electron wave function. Quantum effects in the conductivity are associated with the nonideal behavior or disordering of the crystal. They arise due to the small-scale potential fluctuations with characteristic sizes less than or of order the wavelength of an electron with the Fermi energy. Indeed, small-scale fluctuations of the potential lead to electron scattering, which is why self-intersecting trajectories and renormalization of the *e–e* interaction arises.

In disordered crystals macroscopic nonuniformities can be present in addition to the small-scale fluctuation potential. Large-scale fluctuations are manifested in the form of classical effects, and as the degree of disorder increases, they lead to a metal–insulator transition.[5] It is convenient to investigate the effects associated with percolation in highly compensated semiconductors. In this case the large-scale fluctuation potential forms as a result of density fluctuations of a large number of charged impurities. A percolation metal–insulator transition occurs when the percolation level ε_p lies above the Fermi level ε_F. It has been observed experimentally both as ε_p increases with increasing degree of compensation[6] and under the action of a magnetic field, which reduces ε_F.[7]

Since compensation and the large-scale fluctuation potential which it engenders change qualitatively the type of metal–insulator transition (percolation transition and not Anderson localization), it can be expected that compensation and large-scale fluctuations influence quantum effects in conductivity. In the general case, in the presence of potential fluctuations of any scale, the conductivity of the crystal is determined by the superposition of quantum and classical effects.[8,9] The role of large-scale fluctuations does not reduce to mechanical superposition. In particular, Aronov *et al.*[10] observed in experiments on granular systems a qualitative change in the way quantum effects show up in the conductivity near a percolation transition. Near a percolation metal–insulator transition in compensated semiconductors Aronzon *et al.*[11,12] observed a curve of the

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TABLE I.

	№14	№12	№2	№10	№5	№7
n, 10^{15} cm^{-3}	0.312	2.1	1.35	3.0	2.41	2.83
μ, 10^4 cm^2/V·s	0.098	0.31	0.92	0.89	1.5	9.35
μ_{loc}, 10^4 cm^2/V·s	1.4	1.4	1.66	1.5	2.0	10.0
K	0.95	0.7	0.8	0.55	0.65	—
K_{BH}	0.99	0.96	0.9	0.88	0.82	0.23
a_{cal}, $(\Omega\cdot\text{cm}\cdot\text{K}^{1/2})^{-1}$	1.5	0.6	0.4	0.3	0.25	—
a_{exp}, $(\Omega\cdot\text{cm}\cdot\text{K}^{1/2})^{-1}$	0.02	—	0.25	0.6	0.3	—
b_{cal}, $(\Omega\cdot\text{cm}\cdot\text{T}^{1/2})^{-1}$	3.0	3.0	3.0	3.0	3.0	—
b_{exp}, $(\Omega\cdot\text{cm}\cdot\text{T}^{1/2})^{-1}$	0.03	0.42	0.75	1.0	1.25	—
Form of magnetoresistance curve	DD ($T<1.3$ K) MD ($T>1.3$ K)	DD ($T<0.7$ K) MD ($T>0.7$ K)	DD ($T<0.4$ K) MD ($T>0.4$ K)	MD ($T>1.3$ K)	MD ($T>0.04$ K)	—

Note. DD–double-dip negative magnetoresistance; MD–negative magnetoresistance of the usual form with one minimum.

negative magnetoresistance with unusual form, namely, the curve has two minima. Note here that the negative magnetoresistance effect is well known as one of the most striking manifestations of quantum effects in conductivity. However, in all cases the corresponding theory predicts only one minimum in the magnetoresistance curve.

Thus our aim in the present work was to study quantum effects in the conductivity of strongly compensated semiconductors and to clarify the nature of the anomalous double-dip negative magnetoresistance.

In this work a comprehensive experimental investigation was made of the galvanomagnetic properties of a number of samples of InSb with different degrees of compensation. It is shown that the first minimum of the double-dip negative magnetoresistance is caused by weak localization. Possible reasons for the second minimum in the magnetoresistance curve are discussed. It is apparently associated with an increase of the Fermi level due to redistribution of electrons in spin subbands split by the magnetic field. It was found that the numerical parameters characterizing the conductivity of material with a high degree of compensation are different from the parameters predicted by the theory of quantum corrections to the conductivity. It is shown that the observed difference is associated with the influence of the percolation transition.

2. MEASUREMENT PROCEDURE AND SAMPLES

The measurements were performed on a series of single-crystalline samples of n-InSb with carrier concentration $n=N_D-N_A=(0.3\text{–}3)\cdot 10^{15}$ cm^{-3} and different degrees of impurity compensation $K=0.23\text{–}0.95$. Here N_D and N_A are the donor and acceptor concentrations, respectively. The characteristic dimensions of the sample are $0.05\times 0.1\times 0.5$ cm^3. The parameters of the experimental samples are given in Table I. Note that, with the exception of sample No. 7, all samples were cut out of the same crystal which was doped uniformly with donors to $N_D=7\cdot 10^{15}$ cm^{-3} and in which the degree of acceptor doping varied continuously along the axis of growth. This enabled us to determine the degree $K=N_A/N_D$ of impurity compensation in the samples. In addition, we estimated the Brooks–Herring compensation K_{BH} under the assumption that the carrier mobility is determined by scattering by ionized impurities. This estimate gives too large a value for N_i and K for strongly doped compensated samples, since it ignores the effect of the large-scale fluctuation potential of the impurities. It does make sense, however, since it conveys correctly the tendency for K to vary in the series of the experimental samples.

Data from previous measurements[13] performed on the same samples showed that in these crystals a percolation metal–insulator transition occurs under the action of a magnetic field. This transition occurs when the Fermi level ε_F lies below the percolation threshold ε_p. In the case at hand the transition occurs because the Fermi energy decreases as $\varepsilon_F\propto B^{-2}$ with increasing magnetic field at the quantum limit, i.e., when all electrons are in a single lower Landau level. The transition field B_T is given by the relation[7]

$$B_T=B_p\equiv\frac{\hbar c}{e}a_B n\frac{(1-K)}{(1+K)},$$

where a_B is the Bohr radius. In accordance with the relation presented, the field B_T in the experimental samples

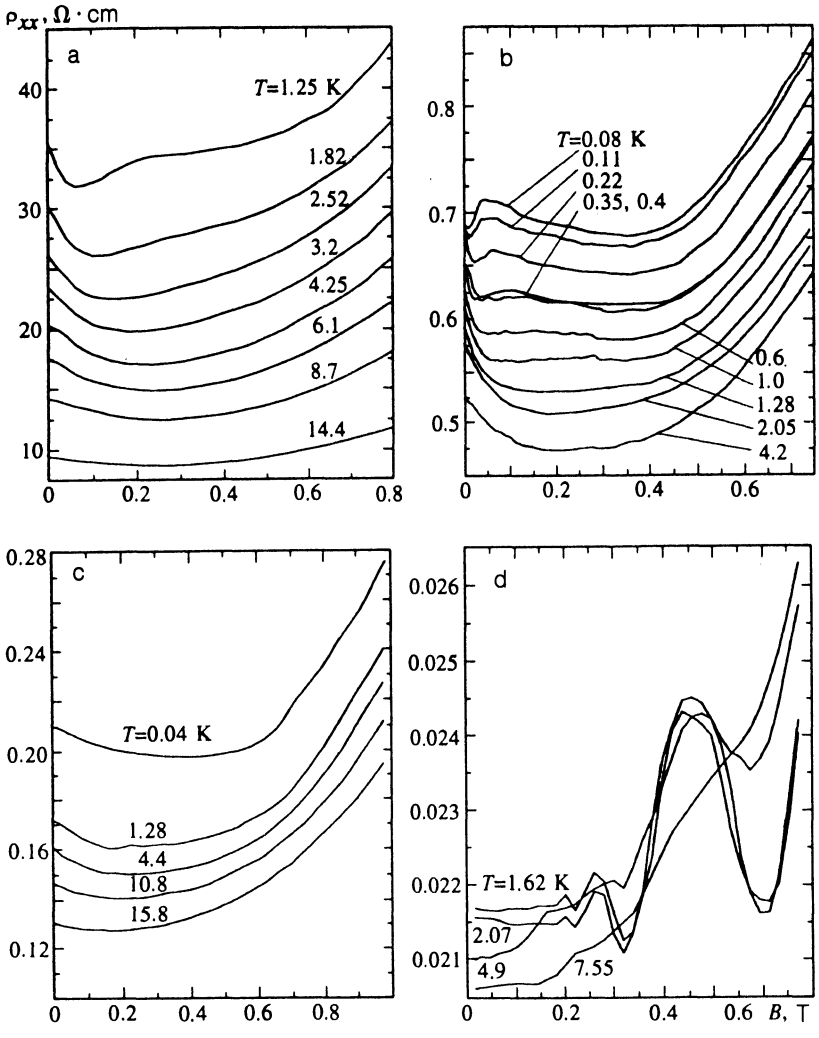

FIG. 1. Transverse resistance as a function of magnetic field for InSb samples No. 14 (a), No. 2 (b), No. 5 (c), and No. 7 (d), at different temperatures.

decreases with increasing compensation and reaches for the most highly compensated sample (No. 14) the minimum possible value $B = B_q$, where B_q is the field at which the quantum limit is established. (In weaker fields the Fermi level is virtually independent of the magnetic field and from the standpoint of the metal–insulator transition under discussion the case $B < B_q$ is equivalent to zero magnetic field.) Hence it can be concluded that even in zero magnetic field the sample No. 14 is very close to the percolation limit. At the same time, in the sample No. 7 with the lowest degree of compensation there is no percolation transition and the metal–insulator transition is the standard localization of electrons due to compression of the electron wave function in a magnetic field.[14]

Thus the collection of samples, presented in Table I, with different degrees of proximity to the percolation metal–insulator transition is entirely sufficient for solving the problem posed. As the degree of compensation increases, in addition to the approach to the percolation limit, the parameter $k_F l$ characterizing the proximity to the Anderson transition (k_F is the Fermi momentum and l is the electron mean free path length) also decreases. The quantity $k_F l$ calculated from the effective values of the parameters for the experimental samples lies in the interval 0.03–30. In such a macroscopically nonuniform medium, however, the effective values are not an adequate characteristic and, as will be shown below, the quantity $k_F l$, found from the local values of the parameters equals 1–30 for the same samples.

In the present paper we present the results of measurements of the components ρ_{xx}, ρ_{xy}, and ρ_{zz} of the resistivity tensor (**B**|| **z**) at temperatures of 50 mK–20 K and in magnetic fields up to 1 T. The measurements were performed using low-frequency (<300 Hz) ac current with the help of a selective voltmeter as well as with synchronous detection. Three different cryogenic systems were used in order to reach low temperatures: an ^{4}He cryostat, a system with evacuation of ^{3}He, and a dilution cryostat.

3. EXPERIMENTAL RESULTS

The results of magnetoresistance measurements for several samples with different degrees of compensation of the impurities are presented in Fig. 1. At low temperatures the magnetoresistance curve has an anomalous form: It has two minima—double-dip negative magnetoresistance. This effect is manifested in the same way in both longitudinal magnetoresistance $\rho_{zz}(B)$ and transverse magnetoresis-

tance $\rho_{xx}(B)$. At the same time no peculiarities were found in the field dependences $\rho_{xy}(B)$ in weak magnetic fields.

The double-dip negative magnetoresistance is most strongly manifested in sample No. 14 with the highest degree of compensation of impurities. In this sample this effect is noticeable even at $T\approx 1.3$ K. Analysis of the experimental results shows that the effect increases with K. As K decreases, the region where the effect is observed shifts in the direction of lower temperatures (see Table I). In sample No. 2 ($K=0.8$) double-dip negative magnetoresistance is observed at temperatures $T<0.4$ K. In sample No. 5, having a lower degree of compensation, the effect is not observed, even at a temperature of 0.04 K; only a very small distortion of the magnetoresistance curve with a single minimum is noticeable.

Since the percolation transition point is approached with increasing K, it can be inferred that the effect under discussion is associated with the proximity of the electronic system to the metal–insulator transition. At the same time, the parameter k_Fl decreases with increasing degree of compensation. The magnitude of the quantum corrections to conductivity changes simultaneously with k_Fl in accordance with the relation $\Delta\sigma\approx 1/k_F^2l^2$. In particular, in the uncompensated sample No. 7, in which the electron density is approximately the same as in sample No. 5, the magnitude of the negative magnetoresistance is found to be significantly lower and falls within the limits of error of the measurements.

In order to analyze the nature of the double-dip negative magnetoresistance and quantum effects in conductivity it is first necessary to clarify the character of the conductivity in the experimental crystals: Are we dealing with a metal or insulator? As we have already pointed, data from preceding measurements indicate that in zero magnetic field the conductivity of all experimental samples is metallic, and a metal–insulator transition occurs with increasing magnetic field.

The question of the character of the conductivity can be answered directly by analyzing the temperature dependence, with whose help the conductivity σ_0 at zero temperature can be determined. If $\sigma_0>0$, the substance is a metal and in the opposite case the substance is an insulator. The temperature dependence of the conductivity likewise gives an answer to the question of the nature of the quantum effects. In the absence of a magnetic field the temperature dependence of the conductivity is given by the relation

$$\sigma(T)=\sigma_0+a\sqrt{T}, \tag{3}$$

predicted for a metal by the theory of quantum corrections due to renormalization of the electron–electron interaction. The temperature dependence $\sigma(T)$ of sample No. 2 is displayed in Fig. 2. Extrapolating σ to zero temperature gives metallic conductivity $\sigma_0>0$. The conductivity satisfies $\sigma_0>0$ in zero magnetic field ($B=0$) and likewise in the range of magnetic fields where the double-dip negative magnetoresistance is observed, in other words, under these conditions the material is a metal. The relatively large change in the conductivity with decreasing temperature proves that the quantum corrections play a significant role,

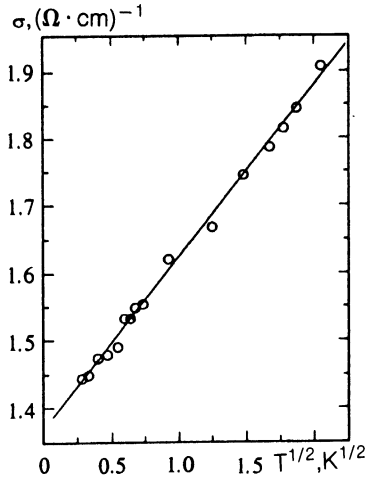

FIG. 2. Temperature dependence of the conductivity of InSb sample No. 2 in the absence of a magnetic field.

and they are determined by the e–e interaction. Analogous results are obtained for all samples with $K\approx 0.6$–0.9, which are also metallic ($\sigma_0>0$) in weak fields. When the magnetic field is increased, however, σ_0 decreases and a metal–insulator transition occurs. Electrons are localized in wells of the fluctuation potential of the charged impurities. At low temperatures sample No. 14, which has the highest degree of compensation $K\approx 0.95$, is close to the insulator state, even in the absence of a magnetic field, though it remains a metal under these conditions, $\sigma_0\approx 0.003$ $(\Omega\cdot\text{cm})^{-1}$.

The coefficient a, determined from the slope of the experimental curve $\sigma(T^{1/2})$, for weakly compensated samples agrees with the estimate obtained on the basis of the theory of quantum corrections (see Table I). The computed values a_{cal} were determined from the relations for the quantum corrections arising in the conductivity due to the interelectronic interaction in the diffusion channel:[4]

$$\sigma(T)-\sigma_0=0.915\frac{e^2}{2\pi\hbar}\left(\frac{2}{3}-F\right)\left(\frac{kT}{\hbar D}\right), \tag{4}$$

where $F=(1/x)\ln(1+x)$, $x=(2k_Fr_s)^2$, and r_s is the Thomas–Fermi screening radius. As the degree of compensation of the impurities increases, the agreement between the experimental and computed values of this coefficient breaks down, $a_{\text{exp}}\neq a_{\text{cal}}$, but the qualitative form of the temperature dependence $\sigma\propto\sqrt{T}$ remains the same.

We now examine in greater detail the magnetic-field-dependent part of the conductivity $\Delta\sigma(B)$. In order to make it easier to compare with the theory it is convenient to represent the experimental data in the form of curves $\Delta\sigma(B)$. Such curves are displayed in Figs. 3 and 4 for samples No. 2 and No. 14, respectively. The second in the magnetoresistance curve (and correspondingly the maximum in the conductivity curve) is manifested only in compensated crystals at low temperatures. When this minimum is absent the curve $\Delta\sigma(B)$ has the typical form for the conductivity owing to quantum corrections, and the

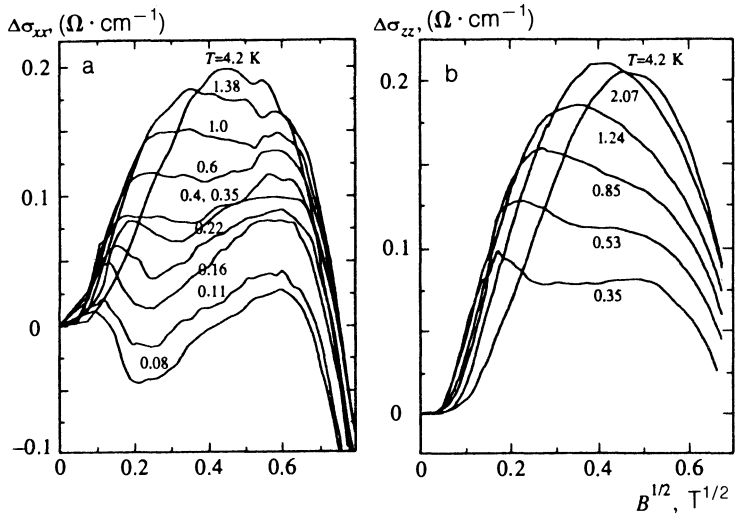

FIG. 3. Transverse σ_{xx} (a) and longitudinal σ_{zz} (b) components of the conductivity versus the magnetic field for sample No. 2 at different temperatures.

corresponding theory describes the curve $\Delta\sigma(B)$ well. An analogous assertion is likewise valid for the curves $\Delta\sigma(B)$ in which two maxima are observed, but only for the part of this curve that refers to weak fields, namely, fields less than the field in which the conductivity is minimum, $B < B_{\min}$. Here the field $B_{\min}$ is the field in which a minimum of $\Delta\sigma(B)$ is observed. For sample No. 2 at low temperatures $B_{\min} \approx 0.05$ T.

The agreement between the observed field dependence $\Delta\sigma(B)$ and the theoretical field dependence in the region $B \ll B_{\min}$ is confirmed by the following analysis. According to the theory of weak localization the field dependence $\Delta\sigma(B)$ is described by the relation[4]

$$\Delta\sigma(B) = \frac{e^2}{2\pi^2\hbar}\frac{1}{L_H} f\left(4\frac{D\tau_\varphi}{L_H^2}\right), \tag{5}$$

where

$$f(x) = x^{3/2}/48 \quad \text{for } x \ll 1 = 0.605 - L_H/D\tau_\varphi \quad \text{for } x \gg 1,$$

and $L_H = \hbar c/eB$, and according to the theory of renormalization of the electron–electron interaction we have

$$\Delta\sigma(B) = -0.915\frac{e^2\Gamma}{2\pi^2\hbar}\sqrt{\frac{g^*\mu_B B}{D}}, \tag{6}$$

where μ_B is the Bohr magnetron and g^* is the effective electron g-factor. In accordance with Eqs. (5) and (6) we observe the following field dependences $\Delta\sigma(B)$, which replace one another as the magnetic field increases.

1) In magnetic fields which are weak for weak localization and electron-electron interaction ($D\tau_\varphi/L_H^2 < 1$) a quadratic field dependence is observed: $\Delta\sigma \propto B^2$ (see Fig. 5).

2) In fields which are strong for weak localization ($D\tau_\varphi/L_H^2 > 1$) but weak for electron–electron interaction ($g^*\mu_B B < \pi kT$) the conductivity increases as the square root of the field: $\Delta\sigma \propto \sqrt{B}$, as one can see from Figs. 3 and 4.

3) In fields which are strong for weak localization and which satisfy the condition $g^*\mu_B H > \pi kT$, the conductivity decreases with increasing field as $\Delta\sigma \propto -\sqrt{B}$. This assertion is likewise illustrated in Figs. 3 and 4, whence one can

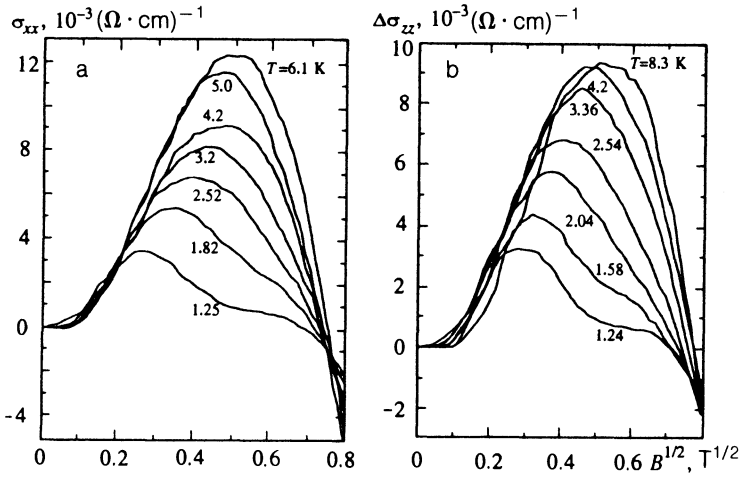

FIG. 4. Transverse σ_{xx} (a) and longitudinal σ_{zz} (b) conductivity versus the magnetic field for sample No. 14 at different temperatures.

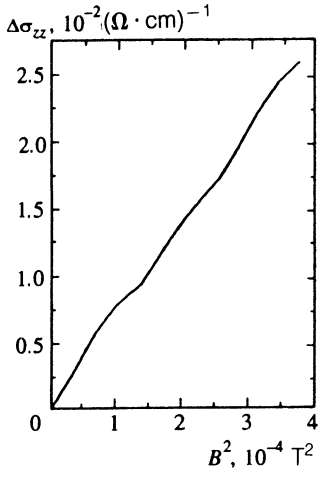

FIG. 5. Corrections to the longitudinal conductivity versus the magnetic field for sample No. 2 in the region of weak magnetic fields.

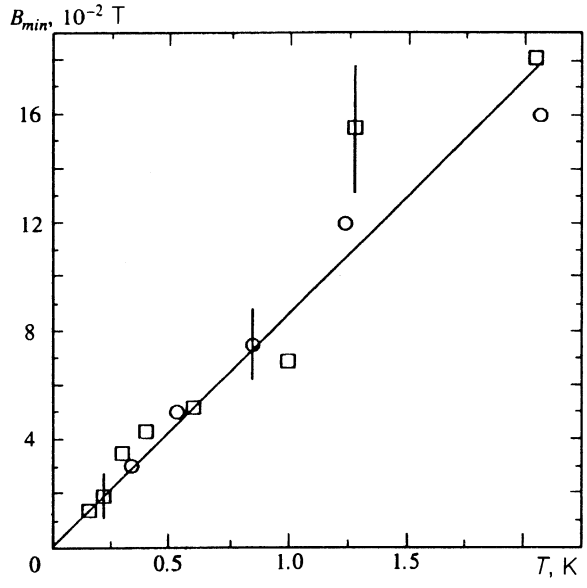

FIG. 7. Temperature dependence of the magnetic field in which a minimum of the magnetoresistance (maximum of conductivity) is observed for sample No. 2: ρ_{zz}—circles; ρ_{xx}—squares.

see that after the maximum the field dependence $\Delta\sigma(\sqrt{B})$ is nearly linear.

The field in which $4D\tau_\varphi/L_H^2=2$ holds and the quadratic dependence $\Delta\sigma\propto B^2$ is replaced by a square-root dependence makes it possible to estimate the average inelastic scattering time τ_ε, if it is assumed that $\tau_\varphi=\tau_\varepsilon$. The quantity τ_ε, and also its temperature dependence, which is displayed in Fig. 6, for sample No. 2 agrees with the results of Ref. 15 and the theoretical estimate obtained assuming that the electron energy relaxes on the deformation potential of acoustic phonons.

The contribution of interaction effects to the magnetoresistance becomes significant in fields $g\mu_B B\geqslant kT$. Correspondingly, in the field such that $g\mu_B B_{\min}=kT$ holds a minimum of $\rho(B)$ [maximum of the conductivity $\Delta\sigma(B)$] should be observed. The temperature dependence of $B_{\min}$, presented in Fig. 7, makes it possible to determine g^*. The obtained value $g^*=53$ agrees very well with the well-known value $g^*=51$ reported in Ref. 16.

The value of $B_{\min}$ will increase with the temperature until the classical and not the quantum mechanism of positive magnetoresistance, associated with the curvature of the electron trajectory in a magnetic field, dominate. Indeed, for $T\geqslant$(3–4) K the position of the maximum on the curve $\Delta\sigma(B)$ is virtually temperature independent, and the value of $B_{\min}\approx$0.2–0.3 T agrees with the estimate of the field in which the classical effect becomes significant.[1)]

The compensation dependence of $\Delta\sigma(B)$ is likewise understandable. As the degree of compensation increases, the absolute magnitude of the conductivity decreases and the relative magnitude of the negative magnetoresistance increases.

As we have already mentioned, the character of the field dependence $\Delta\sigma(B)=b\sqrt{B}$ corresponds to the predictions of the theory of quantum corrections to the conductivity, but the coefficient b varies from sample to sample (see Table I), while the theory gives a constant value $b_{\text{cal}}\approx 3\ (\Omega\cdot\text{cm}\cdot\text{T}^{1/2})^{-1}$. The values of b_{exp} shown in Table I were obtained from the slope of the curves $\Delta\sigma(B^{1/2})$. Experiment gives a significantly lower value of b for samples with a high degree of compensation of the impurities. As the compensation decreases, the coefficient b increases and approaches the theoretical value $b_{\text{exp}}=1.5\ (\Omega\cdot\text{cm}\cdot\text{T}^{1/2})^{-1}$ for the relatively weakly compensated sample No. 5.

Thus we can assert that the first minimum is associated with quantum interference effects, but in strongly compensated nonuniform material near the metal–insulated transition a quantitative discrepancy is observed between the observed values of the coefficients a and b and the values given by the theory of quantum corrections.[4]

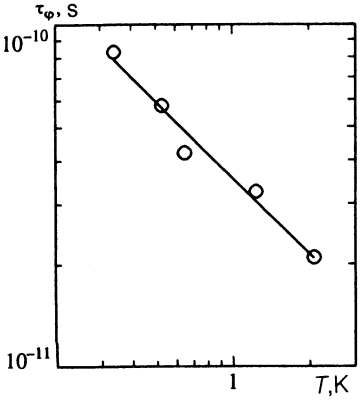

FIG. 6. Temperature dependence of the lifetime of the phase of the electron wave function of sample No. 2.

4. DISCUSSION

What are the reasons for the disagreement between the theoretical and experimentally observed values of a and b? In particular, for sample No. 14 we have

$$a_{cal}=1.5(\Omega\cdot cm\cdot K^{1/2})^{-1},\quad a_{exp}=0.02(\Omega\cdot cm\cdot K^{1/2})^{-1},$$

$$b_{cal}=3(\Omega\cdot cm\cdot T^{1/2})^{-1},\quad b_{exp}=0.03(\Omega\cdot cm\cdot T^{1/2})^{-1}.$$

The value of the parameter k_Fl for strongly compensated samples is likewise significantly less than expected. If it is assumed that the quantity l found from the expressions for σ_{xx} and σ_{xy} according to standard formulas is indeed the average mean free path length and k_F is the Fermi momentum, then $k_Fl=0.03$ and the sample would be a typical Anderson insulator. At the same time, as already mentioned above, this sample manifests metallic conductivity, even though it is close to the metal–insulator transition point.

Strongly compensated InSb near the percolation transition is a very nonuniform medium with conductivity along percolation channels. We infer that the observed contradiction is associated with the fact that the local values of the conductivity and electron density in the percolation channel are different from the effective values calculated under the assumption that the medium is uniform. Since the percolation channels occupy only a part of the volume of the sample, the local electron density in a channel is a higher and hence the real local value of the Fermi momentum $k_F\approx n^{1/3}$ is larger than the value found from the average electron density over the volume of the sample.

The carrier mobility in the temperature range considered is determined by scattering by ionized impurities. Since all samples, with the exception of sample No. 7, were cut from the same uniformly donor-doped crystal and the degree of compensation is high, the total concentration of scattering centers in the series of samples investigated is approximately the same and equals $\sim 2N_D$. In accordance with the Brooks–Herring formula, this means that the local values of the carrier mobility should also be close in all samples. Thus in strongly compensated samples the local mobility should be close to that in sample No. 5. The fact that the local carrier mobility μ_{loc} is approximately the same in all samples is confirmed by experiment. As follows from the classical theory of conductivity in a magnetic field, the field dependence $\sigma_{xy}(B)$ has a maximum at $\omega_c\tau=1$ or, which is the same thing, $\mu B=1$ (μ is the electron mobility). Indeed, in all experimental samples the maximum of the conductivity is observed for the same value of the magnetic field, corresponding to the value of the mobility found for the weakly compensated sample No. 5, $\mu_{loc}\approx\mu$ (No. 5). The geometric factor, associated with the percolation character of the conductivity, reduces the effective value of the mobility. For the form factor reducing the mobility in sample No. 14 we have $\mu_{loc}/\mu=15$.

As we have already pointed out, sample No. 14 is near the percolation transition. At this point the volume of a percolation cluster is 1/3 of the volume of the sample, whence it follows that the local electron density n_{loc} in the percolation channels for sample No. 14 is three times higher than the average value over the sample. Using the ratio $\mu_{loc}/\mu=15$ we obtain a local value of the parameter $(k_Fl)_{loc}$ close to 1:

$$(k_Fl)_{loc}=k_FV_F\tau=\left(\frac{n_{loc}}{n}\right)^{2/3}\frac{\mu_{loc}}{\mu}(k_Fl)_{eff}\approx 1,$$

where $(k_Fl)_{eff}$ is the effective value of the parameter k_Fl, found from the average values of the electron density and mobility over the sample. It is this value that should be expected for sample No. 14, where $\sigma_0\approx 0$.

The relations predicted in the theory of quantum corrections should hold for local values of the conductivity. Any deviation of the effective values of the parameters μ and n from the local values likewise results in underestimation of the effective value of the diffusion coefficient $D=V_F^2\tau/3$: $D_{loc}/D_{eff}\approx 30$. Therefore, the estimated coefficient $a_{cal}=\Delta\sigma/T^{1/2}\propto D_{eff}^{-1/2}$ presented in Table I is too high by a factor of $(D_{loc}/D_{eff})^{1/2}\approx 5$ compared to the local value a_{loc}. Moreover, it is also necessary to take into account the fact that the average conductivity σ_{eff} over the sample is much lower than the local value

$$\sigma_{loc}=\sigma_{eff}\frac{n_{loc}}{n}\frac{\mu_{loc}}{\mu}\approx 45\sigma_{eff}$$

in the percolation channel because both the conductivity ($\mu_{loc}/\mu\approx 15$) and carrier density ($n_{loc}/n\approx 3$) are underestimated. Then the local value of the coefficient a_{exp}, calculated from the experimental data, should be 45 times higher than the value presented in Table I: $a=0.02\cdot 45\approx 0.9$, which agrees with the theoretical estimate $a_{loc}\approx 0.3$.

Now that we know that the local conductivity in the percolation channels is different from the average conductivity over the volume of the sample, we can also understand why in compensated samples the value of the coefficient b in the magnetic-field dependence of the conductivity differs from the theoretical value. The value of b calculated for the local conductivity $b_{loc}=45b_{exp}\approx 1.4$ is close to the computed value $b_{cal}=3.0$.

Aronov *et al.*[10] studied the effect of the percolation transition on the character of the manifestation of quantum effects in the conductivity. They focused their attention on the case $\xi > L_\varphi, L_T$. Here ξ is the correlation length. Under these conditions the spatial dispersion of the diffusion coefficient leads to a qualitative change in the field and temperature dependence of σ. Our data refer to the case $\xi \ll L_\varphi$ (sample No. 14 has $\xi\approx 3\cdot 10^{-5}$ cm and $L_\varphi\approx 10^{-4}$ cm). The results obtained indicate that even in this case percolation influences the quantum effects and leads to different numerical values of the coefficients a and b.

We now discuss the nature of the double-dip negative magnetoresistance. We note first that such a magnetic-field dependence of the resistance was also observed previously[17,18] in compensated InP crystals. It should be noted that the experimental data and the experimental conditions in Refs. 17 and 18 are very close to those presented here. First, in all cases two minima are observed on the magnetoresistance curve. Second, the effect always occurs in compensated materials near the metal–insulator transition. Third, the effect is always observed in the region $\omega_c\tau < 1$, and only at quite low temperatures.

In Refs. 17 and 18 the observed effect was attributed to Shubnikov–de Haas oscillations. However, we feel that this explanation is unsatisfactory for the following reasons:

1. The effect is observed for $\omega_c\tau \ll 1$, i.e., outside the region of existence of the Shubnikov–de Haas effect. Attempting to save the situation, Finlayson has suggested[18] (for $\omega_c\tau \cong 0.1$) that when the average electron free-flight time differs by a large amount from the lifetime of the electron in a cyclotron orbit, it is possible to observe oscillations for $\omega_c\tau < 1$. Without analyzing this point of view, we nonetheless refrain from discussing this question, since there are much more serious grounds for not considering the observed features of the magnetoresistance to be Shubnikov–de Haas oscillations. These considerations are presented in Secs. 2 and 3.

2. The field at which a maximum of σ (minimum of ρ) is observed depends strongly on the temperature (see Fig. 7), and this is not characteristic of the Shubnikov–de Haas effect.

3. As the degree of compensation increases, $\omega_c\tau$ decreases, and the insulator state is approached the effect intensifies (see Table I), in contrast to what should happen for Shubnikov–de Haas oscillations. Indeed, oscillations, clearly observed in sample No. 7 (see Fig. 1d) disappear with increasing compensation.

The double-dip negative magnetoresistance was also observed by Raikh *et al.*[19] in two-dimensional structures in the hopping-conduction regime. In our case this effect is associated with three-dimensional and not two-dimensional conduction. First, the effect is identically manifested in a magnetic field both parallel and perpendicular to the current (see Fig. 2). Second, the three-dimensional character of the effect is confirmed by the experimental results on absorption of ultrasound at 100–1000 MHz. The magnetic-field dependence of the ultrasonic absorption coefficient in these crystals is determined by the conductivity of the free carriers, and as shown in Fig. 8 its form is similar to that of the curve for the case of double-dip negative magnetoresistance.

In Ref. 19 the minimum of resistance (maximum of conductivity) in weak fields is explained by the overwhelming effect of the magnetic field on the interference amplitudes of electrons propagating along different paths between prescribed initial and final states. For the hopping-conduction regime this mechanism is analogous to weak localization in a metal and was predicted in Ref. 20.

In Ref. 21, in order to explain the second maximum of the conductivity, Raikh proposed a mechanism that takes into account the change in the density of states in the impurity energy band in a magnetic field. The magnetic field decreases the overlapping of the wave functions of electrons on neighboring donors. This narrows the impurity energy band in a magnetic field and decreases the density of states in the band. Conservation of total electron number leads to the conclusion that as the field increases, the Fermi level moves toward the center of the impurity energy band.

Raikh's model was developed for an insulator and cannot explain the origin of the second maximum of conductivity in our experiment, since on the metallic side of the transition we observe conduction not along the impurity energy band but rather in the conduction band. The impurity energy band in InSb with $N_D \geqslant 2 \cdot 10^{14}$ cm^{-3} is not separated from the conduction band. Analysis of the results of the investigation of hopping conductivity in sample No. 14 in fields $B > B_T$ shows that in this sample such a transition occurs only in fields ≈ 1.5 T. It can be inferred that some restructuring of the combined band under the action of the magnetic field can also occur in the metallic state, and for this reason Raikh's mechanism as a reason for the double-dip negative resistance apparently cannot be completely excluded in our case.

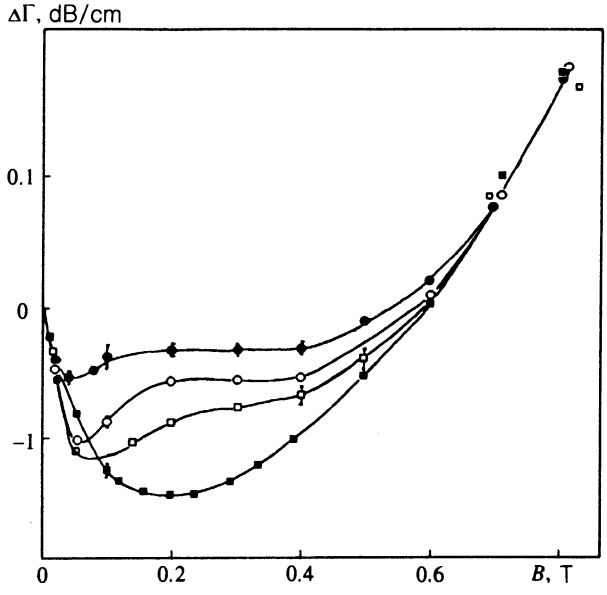

FIG. 8. Magnetic-field dependence of the ultrasonic absorption coefficient for sample No. 2 at different temperatures: $T = 0.46$ K (filled circles), 0.8 K (open circles), 1.2 K (open squares), and 4.2 K (filled squares).

The second maximum of conductivity (minimum of magnetoresistance) could be associated with the effect studied by Shapira and Kautz in Ref. 22 and Fukuyama and Yosida in Ref. 23. This effect is caused by redistribution of electrons between two spin subbands, split in a magnetic field. In systems with short electron-momentum relaxation time τ and long spin-relaxation time τ_{s_0} this increases the Fermi energy in a magnetic field and decreases the resistance. For not too strong magnetic fields this effect can exceed the diamagnetic decrease of the Fermi level. Such a model was previously considered in order to explain negative magnetoresistance in semimagnetic semiconductors.[24]

Indeed, let $\omega_c\tau \ll 1$ and $\omega_c\tau_{s_0} \gg 1$. Then the classical positive magnetoresistance effect is very weak and the change in the electron density of states due to the formation of Landau levels in a magnetic field does not occur. At the same time the two spin subbands will be split by an amount $g\mu_B B$. The electron density is determined by summing over the two spin subbands:

$$n \propto \left\{ \left[\varepsilon_F \left(1+\delta-\frac{g\mu_B B}{2\varepsilon_F} \right) \right]^{3/2} + \left[\varepsilon_F \left(1+\delta+\frac{g\mu_B B}{2\varepsilon_F} \right) \right]^{3/2} \right\},$$

where δ is the relative displacement of the Fermi level. From the condition that the electron density is constant we obtain

$$\delta = -\frac{1}{16} \left(\frac{g\mu_B B}{\varepsilon_F} \right)^2.$$

The Fermi level decreases relative to its position in zero magnetic field, but the Fermi energy, measured from the bottom of the lower spin subband, increases. Since the mobility depends on the energy, this leads to an increase of the conductivity. For $\mu \propto \varepsilon^r$, then

$$\sigma \propto \left\{ \left[\varepsilon_F \left(1+\delta-\frac{g\mu_B B}{2\varepsilon_F} \right) \right]^{r+2/3} + \left[\varepsilon_F \left(1+\delta+\frac{g\mu_B B}{2\varepsilon_F} \right) \right]^{r+3/2} \right\}. \quad (7)$$

The main mechanism of electron scattering at low temperatures is interaction of the electrons with charged impurities. For this mechanism $\mu \propto \varepsilon^{3/2}$. Hence we obtain for the negative magnetoresistance due to the Shapira–Kautz mechanism[22]

$$\frac{\Delta\sigma}{\sigma} = \frac{9}{16} \left(\frac{g\mu_B B}{\varepsilon_F} \right)^2. \quad (8)$$

Using the experimental results and local values of ε_F, we obtain the computed values $(\Delta\sigma/\sigma)_{\text{cal}} = 4.5 \cdot 10^{-2}$, $1.5 \cdot 10^{-2}$, and $0.5 \cdot 10^{-2}$ for samples Nos. 14, 2, and 5, respectively. These values agree with the experimental values $(\Delta\sigma/\sigma)_{\text{exp}} = 5 \cdot 10^{-2}$ and $3 \cdot 10^{-2}$ for samples Nos. 14 and 2. Some distortion of the magnetoresistance curve at $T = 0.04$ K in a field ≈ 0.6 T is also noticeable for sample No. 5.

According to the relation (8), the magnitude of the change in the conductivity $\Delta\sigma$ at the second maximum should not depend strongly on temperature for $g\mu_B B \ll kT$, since the quantity σ itself for the metallic state is virtually independent of the temperature. These arguments agree with the data displayed in Fig. 3, whence one can see that the increment to the conductivity at the second maximum at temperatures $T \leqslant 0.4$ K is virtually temperature independent. The second maximum in the magnetoconductivity curve should vanish at temperatures exceeding the spin-splitting energy, i.e., for $kT \geqslant g\mu_B B/2$. The corresponding estimate of the temperature at which the effect vanishes gives the value 3 K, which is not so far from the experimental value ≈ 1 K.

The model presented explains the compensation dependence of the effect. As K increases, the electron density and the Fermi energy decrease. As one can see from Eq. (8), a decrease of the electron density results in an increase of $\Delta\sigma$. The effect becomes stronger as the metal–insulator transition is approached. In our case the compensation dependence of the effect is not associated with the change in the ratio of the spin relaxation time τ_{s_0} and the momentum relaxation time τ, since the local mobility is virtually the same in the samples Nos. 5–14.

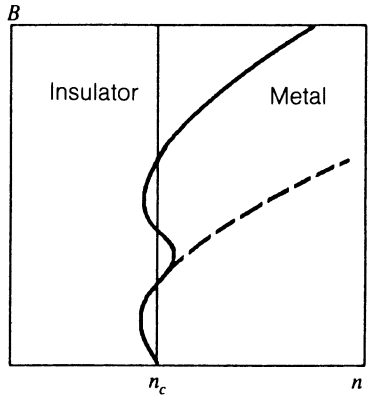

FIG. 9. Proposed phase diagram of the state of the electronic system in a magnetic field. The dashed line is the Shapira phase diagram in the absence of any effects associated with renormalization of the e–e interaction and negative magnetoresistance in the Shapira–Kautz model.[22]

The effect apparently will remain in the insulator region until τ_{s_0} is too short. In this case, it will be determined by the splitting of the impurity energy band and the increase of the Fermi energy in it. As the electron energy increases, the hopping conductivity increases, as happens in Raikh's model.

These considerations prove that the description of the experimental data is consistent with the Shapira–Kautz model.[22] However, this is not enough to finalize the model. The amplitude of the second magnetoconductivity peak which we observed increases sharply as the degree of compensation of the samples increases and the metal–insulator transition is approached. This suggests that this effect could be related to the recently discovered and as yet unexplained giant negative magnetoresistance in the region of hopping conductivity.[25]

If a metal is defined as a substance in which the conductivity at $T = 0$ is different from zero and an insulator is a substance in which $\sigma = 0$ holds at $T = 0$, then the increase in conductivity in a magnetic field could be responsible for the insulator–metal transition predicted by Khmel'nitskii and Larkin.[26] This possibility leads to a phase diagram, shown by the dashed line in Fig. 9 and first constructed by Shapira,[27] for the electronic system of matter in a magnetic field. Shapira considered only one mechanism for negative magnetoresistance, namely, weak localization. The existence of a second independent mechanism, observed in a different range of magnetic fields, could lead to a distortion of the phase diagram.

The existence of two mechanisms of negative magnetoresistance in different ranges of magnetic fields could be the reason why the residual conductivity $\sigma_0(B)$ also has two maxima. This should lead to the phase diagram shown by the solid line in Fig. 9. To what extent does this phase diagram correspond to reality? In order to answer this question it is necessary to clarify the form of the curve

$\sigma_0(B)$. But the determination of $\sigma_0(B)$ depends on the validity of the extrapolation of $\sigma(T)$ to zero temperature. The difficulty lies in the fact that as $T \to 0$ the region where the e–e interaction is significant and the metal–insulator transition is observed shifts to zero magnetic field. This shift can cause the maximum of the function $\sigma_0(B)$ to vanish in the region of weak localization. Thus the question of the reality of the double-hump phase diagram reduces to the question of the existence of the region of weak localization as $T \to 0$ in crystals where the e–e interaction effect does not vanish, and it is equally connected to the standard Shapira phase diagram.[27]

5. CONCLUSIONS

The experimental results presented in this paper indicate that in compensated semiconductors, even away from the percolation transition, i.e., for $\xi < L_\varphi$, quantum effects in conductivity are influenced by the percolation transition. A percolation cluster, formed by the large-scale fluctuation potential, decreases the effective volume of the sample accessible to electrons. Ultimately, the values of the numerical coefficients in the magnetic-field and the temperature dependences of the conductivity change. At the same time, in contrast to the case $\xi > L_\varphi$, the character of the manifestation of quantum corrections to the conductivity remains qualitatively the same.

The most important result of our investigation is the discovery of a new quantum effect in compensated material. This effect is manifested in the fact that the magnetic-field dependence of the resistivity $\rho(B)$ has two minima while the traditional theory predicts only one minimum. The first minimum (occurring in a weak field) is associated with the effect of weak localization. The second minimum is apparently due to an increase of the Fermi level as a result of the splitting of two spin subbands in a magnetic field. In weak magnetic fields, for $\tau_{s_0} \gg \tau$, this effect may turn out to be stronger than the usual diamagnetic decrease of the Fermi energy in a magnetic field and could be an additional mechanism of negative magnetoresistance.

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[1)]This happens when the classical magnetoresistance becomes greater than the quantum magnetoresistance, i.e., for $d(\Delta\sigma/\sigma)/dB)_{cl} \simeq (\omega_c\tau)^2/B > (d(\Delta\sigma/\sigma)/dB)_q = e^2 f/2\pi^2\hbar \; dL_H/dB$, and not for $\omega_c\tau > 1$ (ω_c is the cyclotron frequency and τ is the electron-momentum relaxation time).

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