Instabilities and catastrophes in reorientation of a liquid crystal director in quasistatic fields

R. S. Akopyan and S. R. Nersisyan

Erevan State University 375049 Erevan, Armenia
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We study instabilities, bistabilities, and jump-like changes in the orientation states of nematic liquid crystals for smooth variation of such external parameters as fields, the pitch of the cholesteric helix, chiral additives, etc. Considering these processes with the help of the conservation laws and integrals of the equilibrium equations for liquid crystals in terms of the theory of bifurcations and catastrophes made it possible to find specific results for several problems: a cell with a twisted nematic, a planar-oriented cholesteric, and a homeotropic nematic in the presence of chiral molecules. Numerical estimates show that the predicted instabilities and hysteresis can be observed experimentally.

The interaction of a liquid crystal (LC) director with static electric and magnetic fields has a special feature which creates difficulties in LC physics. The problem is that the stable equilibrium state of the LC director orientation under continuous variation of external parameters (e.g., field intensities) can become unstable, and the smooth behavior of the system can suffer discontinuities. In cholesteric liquid crystals (CLC), temperature can also be such a parameter, since the pitch of the cholesteric helix is temperature-dependent. Studies of bistable states in chiral and twisted LC are therefore very promising. For example, threshold and hysteresis effects in reorientation of homeotropically oriented CLC in external static electric and magnetic fields and also under changes in temperature have been investigated both theoretically and experimentally.[1–4]

It has recently been shown[5,6] that the theory of bifurcations and catastrophes relating possible types of instabilities to the number of control parameters makes it possible to predict and interpret new instabilities in light-induced orientational processes in nematic LC (NLC). A similar approach was used when the interaction of static fields and nematics was investigated.[7,8] In the present study, on the basis of conservation laws and integrals of the LC equilibrium equations,[9] a generalized theoretical approach to LC instabilities is developed, and it is shown that all of the studies of threshold and hysteresis phenomena carried out so far are in fact special cases of the theory of bifurcations and catastrophes. It is also shown that the latter enables one to predict new types of instabilities.

The orientational effects of LC interaction with external fields are characterized by a large number of parameters: the magnitude and direction of quasistatic fields, the concentration of chiral additives, temperature, material LC parameters, the initial orientation of molecules, etc. This circumstance leads us to expect various types of discontinuous and hysteresis behavior of the LC director orientation.

2. EQUILIBRIUM EQUATIONS IN A PLANE-PARALLEL CELL WITH AN LC

According to Noether's theorem, invariance of the LC free energy under translations results in the conservation of the flux of momentum p, while invariance under the rotation group leads to conservation of the flux of angular momentum m. Applying these conservation laws to a plane-parallel cell with an LC, we can find the equilibrium equations in their most general form:[9]

$$p=-\tfrac{1}{2}K_2q^2+\frac{1}{2}\chi_aH^2n_z^2-\frac{D^2}{8\pi}\frac{1}{\varepsilon_\perp+\varepsilon_a n_z^2}+\tfrac{1}{2}(1-n_z^2)(K_2-K_{32}n_z^2)\left(\frac{d\varphi}{dz}\right)^2+\tfrac{1}{2}\frac{K_1+K_{31}n_z^2}{1-n_z^2}\left(\frac{dn_z}{dz}\right)^2, \quad (1)$$

$$m=(1-n_z^2)(K_2+K_{32}n_z^2)\frac{d\varphi}{dz}-K_2q(1-n_z^2). \quad (2)$$

We have restricted ourselves to the case when the LC deformed state is homogeneous under translations in the xy plane of the cell, i.e. the director is the unit vector in the direction of the dominant orientation of molecules, $\mathbf{n}=\mathbf{n}(z)$. The z axis is normal to the cell walls corresponding to the planes $z=0$ and $z=L$. The function $\varphi(z)$ is the angle between the director and the x axis. Here we have assumed that the cell is subject to an external magnetic field directed, for the sake of simplicity, along the z axis, and an electric field $\mathbf{E}$ directed along the z axis, with $D=\varepsilon_{zz}E$. In (1) and (2) we have used the following notation: χ_a is the anisotropy of the LC magnetic polarizability per unit volume, $\varepsilon_{ik}=\varepsilon_\perp\delta_{ik}+\varepsilon_a n_i n_k$ is the tensor of the static dielectric constant, $\varepsilon_a=\varepsilon_\parallel-\varepsilon_\perp$, q is the "wave number" of the free CLC twisting, $q=2\pi/h$, and h is the pitch of a free cholesteric helix. Note that due to the smallness of χ_a ($\chi_a\lesssim10^{-5}$), the magnetic field H can be considered uniform even after LC deformation. For brevity, we have also introduced the notation $K_{32}=K_3-K_2$ and $K_{31}=K_3-K_1$.

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It is clear that the parameters p and m in (1) and (2) are to be found from the boundary conditions at $z=0$ and $z=L$.

Equations (1) and (2) are the most general ones determining the equilibrium configuration of the deformed LC state in a plane-parallel cell. These conservation laws can be applied to find analytic solutions of some problems related to the equilibrium of LCs with essentially nonplanar director distribution: CLCs in longitudinal electric and magnetic fields with homeotropic orientation at the walls, a planar-homeotropic CLC-cell, a cell with a twisted nematic, etc. Below we consider threshold processes for which external effects (e.g., quasistatic fields) can be either stabilizing or destabilizing for the initial equilibrium LC state.

As is well known,[9] whenever the unperturbed state of the LC director corresponds to molecules parallel to the cell walls and external destabilizing parameters tend to force the molecules out of the planes where they lie, the threshold is given by the condition

$$A=-\chi_a H^2-\frac{\varepsilon_a U^2}{4\pi L}+2K_2 q\frac{\Delta\varphi}{L}+(K_3-2K_2)\left(\frac{\Delta\varphi}{L}\right)^2+K_1\left(\frac{\pi}{L}\right)^2=0. \quad (3)$$

Here $\Delta\varphi=\varphi(L)-\varphi(0)$, and U is the potential difference between the walls of the LC cell. In the particular cases of a twisted nematic in electric ($H=q=0$) and magnetic ($D=q=0$) fields, Eq. (3) coincides with the expressions well known from the literature (see Refs. 10 and 11). For a small excess above the threshold, i.e., for $0<|A|L^2/K_1\pi^2\ll 1$, we can find a stationary structure for a small deviation $\delta\mathbf{n}=\mathbf{n}-\mathbf{n}_0$ of the director from the unperturbed equilibrium state $\mathbf{n}_0$:

$$|\delta\mathbf{n}|=\eta\sin\frac{\pi z}{L},\quad \eta=\pm\left(-\frac{A}{B}\right)^{1/2}, \quad (4)$$

$$2B=\frac{\varepsilon_a^2}{4\pi\varepsilon_\perp}\left(\frac{U}{L}\right)^2+K_3\left(\frac{\pi}{L}\right)^2-K_2 q^2-2(K_3-2K_2)q\frac{\Delta\varphi}{L}-(K_2^{-1}K_3^2-K_3+K_2)\left(\frac{\Delta\varphi}{L}\right)^2. \quad (5)$$

Equations (3) and (5) will be used below to investigate the equilibrium states of a twisted nematic and a planar cholesteric.

When the walls of the LC cell produce a homeotropic orientation of the director, $\mathbf{n}_0=\mathbf{e}_z$, the angular momentum is identically zero, $m\equiv 0$. Here $\mathbf{e}_z$ is the unit vector in the z direction. Expanding Eq. (1) for the momentum flux in a small perturbation $\delta\mathbf{n}=[\mathbf{n}\mathbf{e}_z]$, we find for A and B,

$$A=\chi_a H^2+\frac{\varepsilon_a U^2}{4\pi L^2}-K_2^2K_3^{-1}q^2+K_3\left(\frac{\pi}{L}\right)^2, \quad (6)$$

$$2B=K_1\left(\frac{\pi}{L}\right)^2+\frac{\varepsilon_a^2U^2}{4\pi\varepsilon_\parallel L^2}-3K_2^2K_3^{-2}K_{32}q^2. \quad (7)$$

Thus, when the parameter A decreases, a second-order phase transition of the Fréedericksz type takes place from a stable equilibrium unperturbed state with $\eta=0$ (for $A>0$) to a bistable state (for $A<0$), either with $\eta=(-A/B)^{1/2}$, or with $\eta=-(-A/B)^{1/2}$.

Expressions (5) and (7) for the coefficient B can become negative for some values of LC and field parameters. Then, to find the reorientation η, it is necessary to retain higher-order terms in the expansion of Eqs. (1) and (2). Retaining sixth-order terms in η, we find the LC equilibrium equation in the form

$$\eta^2(C\eta^4+B\eta^2+A)=0. \quad (8)$$

For the most frequently used LCs, the coefficient C is positive, and in the absence of fields is about 0.1.

We have considered one-dimensional deformations. Instability thresholds with respect to two-dimensional deformations can be estimated by adding the terms K_3l^2 and K_1l^2 to Eqs. (3) and (6), respectively. Here l is the mean "wave vector" of the deformation in the plane of the LC layer. These thresholds are therefore much higher than the thresholds of one-dimensional deformations. This means that the present approximation is valid if we are slightly higher than the thresholds of one-dimensional instabilities ($|A|L^2/K\pi^2\ll 1$).

3. FREE ENERGY AND EQUILIBRIUM SURFACE

In terms of free energy, we can derive the LC equilibrium equation using the Euler–Lagrange equations and demanding that the energy be stationary under small variations in η.[12] A simplified expression for the free energy of an LC state has the form

$$\Phi=\Phi_0+\tfrac{1}{2}A\eta^2+\tfrac{1}{4}B\eta^4+\tfrac{1}{6}C\eta^6. \quad (9)$$

From the extremum condition $d\Phi/d\eta=0$, we find the equation of equilibrium LC states which is equivalent to Eq. (6). In Fig. 1a we have shown the η and A dependence of the free energy Φ in three dimensions, for $B>0$. For $A>0$, i.e., below the threshold of external destabilizing influence, it clearly has a minimum for $\eta_0=0$, which corresponds to the stable equilibrium unperturbed state of the LC director orientation. Above the threshold ($A<0$), the free energy as a function of η has a local maximum at $\eta_0=0$, and the unperturbed state becomes unstable. It can also be seen in Fig. 1a that the states with antisymmetric reorientations

$$\eta_1=\left[\frac{-B+(B^2-4AC)^{1/2}}{2C}\right]^{1/2},\quad \eta_1'=-\eta_1, \quad (10)$$

which coincide with (4) in the approximation of small perturbations can be stable.

The situation is significantly different for negative B (see Fig. 1b). Above the threshold ($A<0$), the same pair of antisymmetric solutions (10) is stable, whereas the unperturbed state is unstable. For $B<0$ and below the threshold ($A>0$), all five equilibrium states are feasible, as long as $A\leqslant B^2/4C$. However, among them the state with $\eta=0$ and a new pair with antisymmetric reorientations,

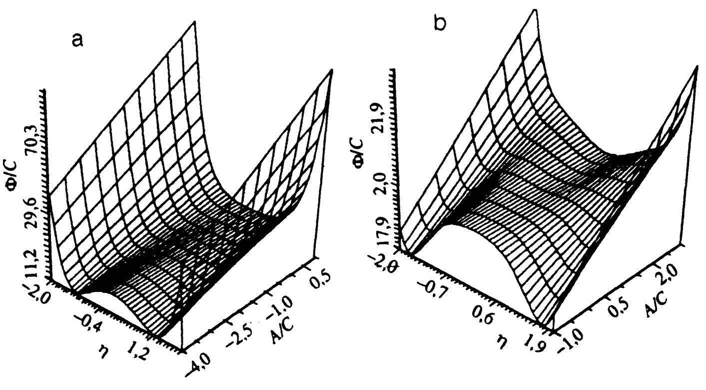

FIG. 1. Free energy ϕ versus the reorientation η and parameter A for $B>0$ (a) and $B<0$ (b).

$$\eta_2=\left[\frac{-B-(B^2-4AC)^{1/2}}{2C}\right]^{1/2},\quad \eta_2'=-\eta_2, \tag{11}$$

are stable. The states given by the reorientations (10) are unstable. Finally, for $B<0$, below the threshold but for $A>B^2/4C$, a single state with $\eta=0$ is realized which is stable.

Thus, for smoothly varying parameters A and B, the system experiences jumps from one stable equilibrium state to another. The jumps occur at the inflection points $A=0$ and $A=B^2/4C$ of the function $\Phi(\eta,A)$, where the first and second derivatives of Φ vanish simultaneously. The catastrophe described by the free energy shown in Figs. 1a and 1b is a special case of the "butterfly"-type catastrophe.[13]

To fully understand all possible transitions, we have plotted the equilibrium surface $\eta=\eta(A,B)$ in Fig. 2. Note that for arbitrary B and above the threshold ($A<0$), the state with $\eta=0$ is unstable. For $B<0$ and $0<A<B^2/4C$, the states with reorientations (10) are also unstable (inner surface in Fig. 2 adjacent to the plane $\eta=0$). Therefore, if A decreases, i.e., if the external destabilizing factors grow, η jumps at the point $A=0$ from the plane $\eta=0$ either onto the left-hand or the right-hand equilibrium surface corresponding to the reorientations (11). If the external destabilizing factors become smaller, the jump from the bistable state to the stable one with $\eta=0$ occurs for $A=B^2/4C$. Therefore, along with discontinuities, the reorientation η also has a hysteresis for increasing or decreasing "intensity" of external parameters, such as magnetic and electric fields, the concentration of chiral additives, temperature, the cell thickness, etc. The hysteresis width is $\Delta A=B^2/4C$.

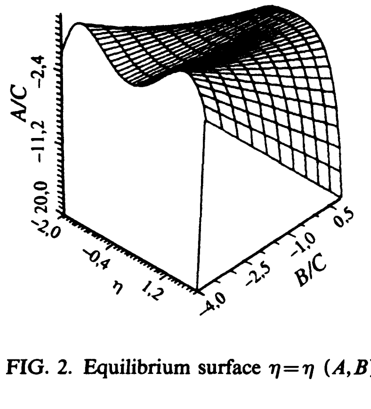

FIG. 2. Equilibrium surface $\eta=\eta\ (A,B)$.

It is easy to show that the ratio of the "intensities" corresponding to switching the hysteresis off and on is $1-\Delta A$.

The growth of the parameter B results in the maximum A values coming closer to the local minimum (for $\eta=0$), until they finally merge at the point $B=0$ (see Fig. 2). Therefore, for positive B in the transition from the unperturbed to the bistable state no jumps or hysteresis of the parameter η are observed.

Thus, consideration of director reorientation for twisted nematics and CLCs in static fields in terms of catastrophe theory makes it possible to draw generalized conclusions regarding stability and bistability of various states of LC orientation.

4. CELL WITH A TWISTED NEMATIC

Consider a cell with an NLC whose walls give a planar orientation of molecules. In the unperturbed state, we have a homogeneous director distribution parallel to the walls: $n_z^0=0$ and $\Delta\varphi\neq 0$. The applied static electric field $\mathbf{E}=E\mathbf{e}_z$ can either stabilize (for $\varepsilon_a<0$) or destabilize (for $\varepsilon_a>0$) the unperturbed state of molecule orientation.

As control parameters, we choose the field destabilizing the initial director orientation and the NLC twist $\Delta\varphi$. The other parameters characterizing the electric field and NLC are considered fixed. Then, putting $H=0$ and $q=0$ in Eq. (3) and taking $A=0$ and $A=B^2/4C$ to determine the relation between the voltage U and twist $\Delta\varphi$, we find that the bifurcation region is bounded by the curves

$$U_1^2=\frac{4\pi^3K_1}{\varepsilon_a}\left[1+\frac{K_3-2K_2}{K_1\pi^2}(\Delta\varphi)^2\right], \tag{12a}$$

$$U_2^2=\frac{4\pi^3K_1}{\varepsilon_a}\left[1-\frac{B^2}{4C}+\frac{K_3-2K_2}{K_1\pi^2}(\Delta\varphi)^2\right] \tag{12b}$$

for all $\Delta\varphi\geqslant\Delta\varphi_k$, where

$$\Delta\varphi_k=\pi\left(K_2\frac{\varepsilon_\perp K_3+\varepsilon_aK_1}{\varepsilon_\perp K_3^2-K_2K_{32}\varepsilon_\parallel+K_2^2\varepsilon_a}\right)^{1/2}. \tag{13}$$

This region is, in fact, a set of points in the control parameter plane $(U,\Delta\varphi)$ with five pre-images on the equilibrium surface $\eta=\eta(U,\Delta\varphi)$. The quantity $\Delta\varphi_k$, which is the intersection point of the curves (12), determines the value of NLC twist above which the reorientation η versus the voltage U exhibits discontinuities and hysteresis. Equation

(13) is found by requiring that (5) be negative for the coefficient B, which in the case considered takes the form

$$B(\Delta\varphi)=\frac{1}{2}\left[\frac{K_3}{K_1}+\frac{\varepsilon_a}{\varepsilon_\perp}-\left(\frac{K_3^2}{K_2}-K_{32}\frac{\varepsilon_\parallel}{\varepsilon_\perp}+K_2\frac{\varepsilon_a}{\varepsilon_\perp}\right)\frac{(\Delta\varphi)^2}{K_1\pi^2}\right]. \quad (14)$$

Thus, if $\Delta\varphi>\Delta\varphi_k$ and is fixed, then as U increases to U_1 defined by Eq. (12a), there is a jump from the state $\eta=0$ to the states corresponding to the reorientations η_2 and η_2'. When U decreases, a jump into the state $\eta=0$ occurs for the value U_2 defined by Eq. (12b), which is smaller than U_1.

Apart from the discontinuity and hysteresis of the function $\eta=\eta(U)$ for a fixed $\Delta\varphi$ described above, another type of discontinuity is possible. Here we mean the situation when the voltage is fixed and the parameter $\Delta\varphi$ is varied. In fact, if $U\leqslant U_k$, where

$$U_k=2\pi\left(\frac{\pi K_1}{\varepsilon_a}\right)^{1/2}\times\left[1+\frac{K_2(K_3-2K_2)(\varepsilon_\perp K_3+\varepsilon_a K_1)}{K_1(\varepsilon_\perp K_3^2-K_2K_{32}\varepsilon_\parallel+K_2^2\varepsilon_a)}\right]^{1/2}, \quad (15)$$

then by increasing $\Delta\varphi$ to the value

$$(\Delta\varphi)_1=\left(\frac{K_1}{K_3-2K_2}\right)^{1/2}\left(\frac{\varepsilon_a U^2}{4\pi^3K_1-1}\right)^{1/2},$$

we smoothly reach the state with $\eta=0$. If, however, $U>U_k$, then increasing $\Delta\varphi$ to the value $(\Delta\varphi)_2$ defined by (12b) yields a discontinuous change in η from the states η_2 either to the state η_2' or to $\eta=0$. For decreasing $\Delta\varphi$, the state with $\eta=0$ becomes unstable for $(\Delta\varphi)_1<(\Delta\varphi)_2$. For the NLC 5 CB, $\varepsilon_\parallel=17.3$, $\varepsilon_\perp=7.3$, $K_1=6\cdot10^{-7}$ dyn, $K_2=3\cdot10^{-7}$ dyn, $K_3=8.5\cdot10^{-7}$ dyn, and the critical twist value is $\Delta\varphi=2.04$ rad. Estimates of the coefficient C (the relevant formula is too cumbersome to be given here) yield $C\approx0.1$ for $\Delta\varphi=2.5$ rad. The value of B is about 0.56, the hysteresis width $\Delta A\approx0.78$, and the voltage ratio for on/off hysteresis is $1-\Delta A\approx0.22$.

To conclude this section, we note that similar jumps and hysteresis behavior can be induced by a static magnetic field applied normal to the plane of the NLC director orientation.

5. PLANAR-ORIENTED CHOLESTERIC

For a planar-oriented CLC, $n_z^0=0$. The control parameters in this case are the destabilizing electric field (for $\varepsilon_a>0$), the pitch of a free cholesteric helix, the cell width, etc.

As in the previous section, we find that the bifurcation region for the control parameters (U,qL) has the form

$$\left[\frac{\varepsilon_a U^2}{4\pi K_3}-\frac{K_1\pi^2}{K_3}\left(1-\frac{B^2}{4C}\right)\right]^{1/2}\leqslant qL\leqslant\left(\frac{\varepsilon_a U^2}{4\pi K_3}-\frac{K_1\pi^2}{K_3}\right)^{1/2}. \quad (16)$$

For $H=0$ and $\Delta\varphi=qL$, we find from (5) the following expression for the coefficient B:

$$B(qL)=\frac{1}{2}\left[\frac{K_1}{K_3}+\frac{\varepsilon_a}{\varepsilon_\perp}-\left(1-\frac{2K_2}{K_3}+\frac{K_3}{K_2}-\frac{\varepsilon_a}{\varepsilon_\perp}\right)\frac{K_3}{K_1\pi^2}(qL)^2\right]. \quad (17)$$

It is clear that the condition $B<0$ for the onset of hysteresis and a jump with changing electric field holds if $qL>(qL)_k$. The LC parameters must satisfy an additional condition,

$$1-2\frac{K_2}{K_3}+\frac{K_3}{K_2}-\frac{\varepsilon_a}{\varepsilon_\perp}>0.$$

This holds, for example, for the NLC 5 CB, and we also have $(qL)_k\approx3.75$.

As the parameter qL changes (for a fixed U), the jumps and hysteresis of η occur if $U>U_k$, where

$$U_k=2\pi\left[\frac{\pi K_1}{\varepsilon_a}+\frac{K_3}{\pi\varepsilon_a}(qL)_k^2\right]^{1/2}$$

defines, in fact, the threshold voltage of the Fréedericksz transition in a planar-oriented CLC with the twist "wave number" q_k, or in a cell of thickness L_k.[10]

6. HOMEOTROPIC NLC IN THE PRESENCE OF CLC MOLECULES

Consider now an NLC cell whose walls orient the molecules strictly along the normal to the boundaries. Then the unperturbed director state corresponds to $n_z^0=1$ and $n_x^0=n_y^0=0$. Assume that a small number of CLC molecules is added to the nematic. As shown in Ref. 14, when $qL>(qL)_{\rm thr}=\pi K_3/K_2$, the so-called fieldless Fréederickcz transition from the stable homogeneous distribution of molecules to the stable twisted distribution occurs. Such a destabilizing role can be also played by the external static electric field $\mathbf{E}=E\mathbf{e}_z$ for an NLC with $\varepsilon_a<0$. For an NLC with $\varepsilon_a>0$, the electric field, as well as the magnetic field $\mathbf{H}=H\mathbf{e}_z$, stabilizes the initial homeotropic orientation of the NLC molecules. In this case, the equation of equilibrium states for the perturbation $\delta\mathbf{n}=\mathbf{n}-\mathbf{n}_0=[\mathbf{n}\mathbf{e}_z]$, $|\delta\mathbf{n}|=\eta\sin(\pi z/L)$ has the same form (8), with coefficients (6) and (7). The coefficient C is given by

$$C=\frac{1}{32K_3}\left[3K_1(4K_3-K_1)\left(\frac{\pi}{L}\right)^2+14K_1\frac{\varepsilon_a^2U^2}{4\pi\varepsilon_\parallel L^2}-2K_3^{-2}K_2^2K_{32}(7K_1+30K_{32})q^2+57\left(\frac{L}{\pi}\right)^2\left(\frac{\varepsilon_a}{\varepsilon_\parallel}\right)^2\times\left(\frac{\varepsilon_aU^2}{4\pi L^2}\right)^2-144K_3^{-2}K_2^2K_{32}q\right]. \quad (18)$$

Consider a situation in which the electric field stabilizes the initial homeotropic orientation of the director ($\varepsilon_a > 0$). Then the bifurcation region in the control parameter plane (U, qL) takes the form

$$\left[\frac{\varepsilon_a K_3 U^2}{4\pi K_2^2} + \frac{\pi^2 K_3^2}{K_2^2}\left(1 - \frac{B^2}{4C}\right)\right]^{1/2} \leqslant qL \leqslant \left(\frac{\varepsilon_a K_3 U^2}{4\pi K_2^2} + \frac{\pi^2 K_3^2}{K_2^2}\right)^{1/2}. \quad (19)$$

From (6) and (7) we find the conditions on the parameters U and qL under which hysteresis and jumps occur, with one parameter varying and the other fixed:

$$U \geqslant U_k = 2\pi\left(\frac{\pi}{\varepsilon_a} \frac{3K_{32} - K_1}{\varepsilon_a/\varepsilon_\parallel - 3K_{32}/K_3}\right)^{1/2}, \quad (20a)$$

$$qL \geqslant (qL)_k = \frac{\pi K_3}{K_2}\left(\frac{\varepsilon_a/\varepsilon_\parallel - K_1/K_3}{\varepsilon_a/\varepsilon_\parallel - 3K_{32}/K_3}\right)^{1/2}. \quad (20b)$$

In particular, for $U=0$ we find the condition under which the fieldless Fréedericksz transition for a given NLC exhibits hysteresis.[14] Thus, the presence of a stabilizing electric field increases the hysteresis width for the function $\eta = \eta(qL)$ if $\varepsilon_a/\varepsilon_\parallel - 3K_{32}/K_3 < 0$. This is the case, for example, in the NLC 5 CB. If for a given NLC the fieldless Fréedericksz transition shows no hysteresis, the electric field can induce it. The critical voltage is determined by Eq. (20a). If on the contrary $\varepsilon_a/\varepsilon_\parallel - 3K_{32}/K_3 > 0$, the electric field reduces the hysteresis width to the point of complete disappearance.

When the voltage U is the control parameter, the jumps and hysteresis of the dependence $\eta = \eta(U)$ occur if the condition (20b) holds.

Let us now discuss the case in which the electric field destabilizes the initial molecule orientation ($\varepsilon_a < 0$). In the absence of chiral additives ($q=0$), an ordinary Fréedericksz transition does not have any hysteresis. The presence of right–left asymmetric molecules induces hysteresis in the Fréedericksz transition in an electric field if $qL > (qL)_k$, where $(qL)_k$ is given by Eq. (20b) with $\varepsilon_a < 0$.

Finally, taking into account Eqs. (6) and (7) for the coefficients A and B, one can show that a destabilizing electric field and gradual voltage growth result, first, in a decreasing and for $U > U_k$, a vanishing hysteresis width for a fieldless Fréedericksz transition. The value of U_k is given by (20a), again with $\varepsilon_a < 0$.

For the NLC MBBA, with $\varepsilon_\parallel = 4.7$, $\varepsilon_\perp = 5.4$, $K_1 = 6 \cdot 10^{-7}$ dyn, $K_2 = 4 \cdot 10^{-7}$ dyn, and $K_3 = 7.5 \cdot 10^{-7}$ dyn, the fieldless Fréedericksz transition occurs for $(qL)_{\text{thr}} \approx 6$, while $(qL)_k \approx 4.7$. For $qL = 5.4$ and voltage above the threshold ($\varepsilon_a U^2/2\pi^3 K_3 = 1.1$), the hysteresis width $\Delta A \approx 0.17$ and $U_{\text{off}}/U_{\text{on}} = 1 - \Delta A \approx 0.83$.

CONCLUSION

Using Noether's theorem to obtain the equilibrium equations and catastrophe theory to analyze them, we can better understand qualitatively the stable states of LC director deformations. The approach developed above enables one to predict new discontinuities and hysteresis of the director reorientation as the "intensities" of external parameters are varied. It is important that with the help of stabilizing effects, we can control the hysteresis width and, if there is no hysteresis, induce it. The control parameters ("intensities" of external effects) can include the values and directions of external quasistatic fields, the liquid crystal constants (the Frank elastic constants, the dielectric constant anisotropy, etc.), the state of the initial unperturbed director orientation, the cell width, and, in the presence of chiral additives, their concentration, the ambient temperature, etc.

The treatment presented above and numerical estimates allow us to hope that the predicted jumps and hysteresis will be observed in the future. We believe that the study of these effects must give important information about the molecular dynamics of the liquid crystal mesophase.

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