Resonant tunneling of electrons through a semimagnetic barrier

Yu. G. Rubo

Institute of Semiconductor Physics Ukrainian Academy of Sciences, 252650 Kiev, The Ukraine
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This paper studies the resonant tunneling of electrons in a heterostructure with a semimagnetic barrier with allowance for temporal fluctuations in the exchange field created by magnetic impurities in the barriers and acting on the spin of a tunneling electron. The barrier transparency is shown to depend substantially on the relationship between the correlation time for the fluctuations and the tunneling time (i.e., the electron lifetime in the resonant state). In the case of slow fluctuations, tunneling is quasielastic, and the dependence of the barrier transparency on the electron energy exhibits a double-peaked profile related to spin splitting of the resonant level in the exchange field. Fast fluctuations of the exchange field give rise to inelastic processes, and the transparency exhibits one Lorentzian peak, whose half-width, however, is greater than that given by the Breit–Wigner formula. An explicit expression is derived for the tunneling amplitude with an arbitrary value of the ratio of the tunneling time to the correlation time, provided that the decay of exchange-field correlator is exponential.

1. INTRODUCTION

Semimagnetic semiconductors, that is, solid $A^2_{1-x}M_xB^6$ solutions, where M stands for an ion of a transition metal with an incomplete $3d$ shell and a nonzero spin s (e.g., $s=5/2$ for $M=$Mn and $s=3/2$ for $M=$Co), possess some remarkable physical properties, which emerge as a result of the exchange interaction between carriers and magnetic impurities.[1–3] This interaction leads, among other things, to spin splitting of the levels of an electron localized at a center with a large radius in the absence of an external magnetic field.[4–9] Splitting is due to the total exchange field of the magnetic ions occurring within an electron orbit acting on the electron spin. In wide-gap semimagnetic semiconductors with a composition $x\sim 0.01$–0.1, where, on the one hand, the ion concentration $n=z\Omega^{-1}$ is fairly high (Ω is the unit-cell volume) and an electron effectively interacts with a large number of ions (i.e., $N=na^3\gg 1$, where a is the characteristic size of the electron wave function), and on the other, the spin states of different magnetic ions are still weakly correlated, the subsystem of magnetic ions is in the paramagnetic phase even at low temperatures, and the action of the magnetic ions on the electron spin can be described as that of a classical fluctuating exchange field.[8] In what follows this field is denoted by **f**, so that the spin Hamiltonian of the electron has the form $\boldsymbol{\sigma}\cdot\mathbf{f}$, where σ_x, σ_y, and σ_z are the Pauli matrices.

In the case of moderately low temperatures T, where the characteristic interaction energy of an electron with a single ion, αSa^{-3}, is much lower than the temperature (α is the exchange constant) and the electron has practically no effect on the orientation of the ion spin, the exchange field **f** is generated by a large number N of independently and randomly oriented ion spins and has a Gaussian distribution. In the absence of an external magnetic field, the probability of occurrence $P(\mathbf{f})$ is proportional to $\exp(-f^2/2f_0^2)$, with $f_0^2=\langle 1/3\, \mathbf{f}^2\rangle$, and is order $N(\alpha Sa^{-3})^2$. Since the spin splitting of an electron level is determined solely by the absolute value of the magnetic field, the density of states $\rho(\varepsilon)=\langle \mathrm{Tr}\{\delta(\varepsilon-\boldsymbol{\sigma}\cdot\mathbf{f})\}\rangle$ is proportional to $\varepsilon^2\exp(-\varepsilon^2/2f_0^2)$ and exhibits a double-peaked structure. Such spin splitting of a donor-electron level in a semimagnetic semiconductor was discovered in Raman-scattering experiments involving spin flip.[4,5] The frequency dependence of the Raman peaks $\mathscr{R}(\omega)\propto\rho(\hbar\omega/2)\exp(\hbar\omega/2T)$ (cf. Ref. 8) is in good agreement with the experimentally observed dependence[8,9] (the exponential factor multiplying $\rho(\hbar\omega/2)$ in the above formula emerges as the probability of the lower ($\omega>0$) and upper ($\omega<0$) spin levels of the electron become filled).

The aim of this paper is to study theoretically the resonant tunneling of electrons through such a state in a fluctuating exchange field. For instance, this may be tunneling through a doped semimagnetic barrier in the ZnSe–$Zn_{1-x}Mn_xSe$–ZnSe heterostructure (Ref. 2). Or the barrier may be formed by the compound $Cd_{1-y}Mn_yTe$ with a large y, so that the subsystem of magnetic moments is in the antiferromagnetic spin-glass phase and the localized state is formed by a quantum point $Cd_{1-x}Mn_xTe$ with $x\ll 1$ (Ref. 10); the fluctuating exchange field in this case exists only within the quantum point.

The noteworthy feature of this problem is that tunneling is a coherent quantum process with a characteristic duration of the order of the electron lifetime τ_0 in a quasistationary resonant state,[11–17] and the temporal fluctuations of the exchange field must be taken into account. Two principal mechanisms determine the characteristic scale t_c of fluctuations of the exchange field **f**. The first originates in the direct interaction of the magnetic-ion spins with phonons; the second is related to the precession of the spin of each ion in the total fluctuating magnetic field generated by the other ions. Although the second mechanism does not alter the total spin of all the magnetic ions, it too leads

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to fluctuations of **f** in both magnitude and direction, since an electron effectively interacts only with the magnetic ions that are close to its localization center (at distances of order or less than a; the magnitude of the exchange interaction of the electron with an ion is proportional to the square of the absolute value of the electron's wave function at the point where the ion is situated). As a result the correlation time t_c strongly depends on the nature of the magnetic ions, their concentration, and the temperature and can acquire values from 10^{-2} to 10^{-10} s (Ref. 18). On the other hand, the tunneling time τ_0 is of a quite different nature and is determined by the width and height of the barrier. The question, therefore, is how the resonant tunneling transparency of the barrier depends on the t_0-to-t_c ratio.

This paper considers a model in which the exchange field is assumed to fluctuate independently of the spin state of the electron at a center. Here the Schrödinger equation for the electron spin in a field $\mathbf{f}(t)$ that randomly depends on time is solved exactly. Effects of correlation between the directions of field and electron spin, important in the low-temperature range $T<f_0$, are discussed qualitatively. It is shown that at large correlation times, $t_c \gg \tau_0$, that is, when the electron tunnels in an exchange field that remains practically constant in time, the average barrier transparency, as a function of the electron energy, has a double-peaked structure; for $f_0 \gg \hbar/\tau_0$ the barrier transparency behaves like the density of states $\rho(\varepsilon)$. As t_c decreases, the situation changes radically, the double-peaked structure disappears, and in the limit $t_c \ll \tau_0$ the transparency vs electron energy dependence has the shape of a single Lorentzian peak, whose width, however, exceeds the Breit–Wigner width $\Gamma=\hbar/2\tau_0$.

In addition to limiting cases, an explicit expression is obtained for the tunneling amplitude in the intermediate range on the assumption that the exchange-field correlator decays exponentially.

2. GENERAL EXPRESSION FOR THE RESONANT TUNNELING TRANSPARENCY OF A SEMIMAGNETIC BARRIER

The problem stated in Sec. 1 is most conveniently solved by starting with the tunneling Hamiltonian introduced in Refs. 16 and 17, which in our case can be written in the form

$$H=\sum_{\mathbf{k}} E_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}+H_c+\sum_{\mathbf{p}} E_{\mathbf{p}} b_{\mathbf{p}}^{\dagger} b_{\mathbf{p}} + \sum_{\mathbf{k}} g_{l\mathbf{k}}(a_{\mathbf{k}}^{\dagger} c+c^{\dagger} a_{\mathbf{k}})+\sum_{\mathbf{p}} g_{r\mathbf{p}}(b_{\mathbf{p}}^{\dagger} c+c^{\dagger} b_{\mathbf{p}}). \quad (1)$$

Here $a^{\mathbf{k}}$ and $a_{\mathbf{k}}$, and $b_{\mathbf{p}}^{\dagger}$ and b are the electron creation and annihilation operators on the left and right banks, respectively, and $c^{\dagger}$ and c the electron creation and annihilation operators in the resonant state at a center in the barrier. The last two terms of this Hamiltonian describe tunneling from the state at the center to the states of the left and right banks (and back). The center Hamiltonian

$$H_c=(E_0+\boldsymbol{\sigma}\cdot\mathbf{f}(t))c^{\dagger} c \quad (2)$$

allows for the temporally fluctuating exchange field $\mathbf{f}(t)$. An electron in the left or right bank interacts with magnetic ions only owing to the tail of its wave function in the barrier, and this interaction can be ignored.

The wave function of a tunneling electron is

$$|\psi(t)\rangle=\sum_{\mathbf{k}} \chi_{\mathbf{k}}(t)\exp(-iE_{\mathbf{k}}t)a_{\mathbf{k}}^{\dagger}|\mathrm{vac}\rangle +\eta(t)\exp(-iE_0 t)c^{\dagger}|\mathrm{vac}\rangle + \sum_{\mathbf{p}} \xi_{\mathbf{p}}(t)\exp(-iE_{\mathbf{p}}t)b_{\mathbf{p}}^{\dagger}|\mathrm{vac}\rangle, \quad (3)$$

where $\chi_{\mathbf{k}}$, $\xi_{\mathbf{p}}$, and η are spinors describing the states of the electron spin in the left and right banks and at the center (from now on $\hbar=1$). Substituting the wave function (3) into the Schrödinger equation, we obtain a system of equations for these spinors:

$$i\frac{\partial}{\partial t}\chi_{\mathbf{k}}=\exp(i\varepsilon_{\mathbf{k}}t)\eta, \quad (4a)$$

$$i\frac{\partial}{\partial t}\eta=V(t)\eta+\sum_{\mathbf{k}} g_{l\mathbf{k}}\exp(-i\varepsilon_{\mathbf{k}}t)\chi_{\mathbf{k}}+\sum_{\mathbf{p}} g_{r\mathbf{p}}\exp(-i\varepsilon_{\mathbf{p}}t)\xi_{\mathbf{p}}, \quad (4b)$$

$$i\frac{\partial}{\partial t}\xi_{\mathbf{p}}=g_{r\mathbf{p}}\exp(i\varepsilon_{\mathbf{p}}t)\eta, \quad (4c)$$

where

$$\varepsilon_{\mathbf{k}}=E_{\mathbf{k}}-E_0, \quad \varepsilon_{\mathbf{p}}=E_{\mathbf{p}}-E_0, \quad V(t)=\boldsymbol{\sigma}\cdot\mathbf{f}(t). \quad (5)$$

The solution to the system of equations (4a)–(4c) with the initial conditions

$$\chi_{\mathbf{k}'}(0)=\chi_0\delta_{\mathbf{k}\mathbf{k}'}, \quad \eta(0)=\xi_{\mathbf{p}}(0)=0, \quad (6a)$$

$$\chi_0^{\dagger}\chi_0=1, \quad (6b)$$

corresponding to the electron initially ($t=0$) being in a state with the wave vector **k** in the left bank, can be found by a method similar to that used in Ref. 17, where the effect of electron–phonon coupling on resonant tunneling is studied. For one thing, for the spinor $\xi(t)$ we will shortly need we obtain

$$\xi_{\mathbf{p}}(t)=-g_{l\mathbf{k}}g_{r\mathbf{p}}\int_0^t dt_2\int_0^{t_2} dt_1 \exp\{i(\varepsilon_{\mathbf{p}}+i\Gamma)t_2 -i(\varepsilon_{\mathbf{k}}+i\Gamma)t_1\}G(t_2,t_1)\chi_0. \quad (7)$$

Here

$$\Gamma=\Gamma_l+\Gamma_r, \quad \Gamma_l=\pi\sum_{\mathbf{k}} g^2{}_{l\mathbf{k}}\delta(E-E_0),$$

$$\Gamma_r=\pi\sum_{\mathbf{p}} g^2{}_{r\mathbf{p}}\delta(E_{\mathbf{p}}-E_0); \quad (8)$$

in the absence of an exchange field Γ determines the half-width of the resonant-tunneling peak, where Γ_l and Γ_r are the partial half-widths corresponding to the electron escaping to the left and right banks, respectively. The electron's lifetime in the resonant state, which coincides with the tunneling time, is $\tau_0=(2\Gamma)^{-1}$ (see, e.g., Ref. 16).

The operator $G(t_2,t_1)$ in (7), which is the temporal evolution operator for $V(t)$ from (5),

$$i\frac{\partial}{\partial t_2}G(t_2,t_1)=V(t_2)G(t_2,t_1), \quad G(t_2,t_1)=1, \tag{9}$$

can be written using the Feynman ordering index on the electron-spin operator:

$$G(t_2,t_1)=\exp\left\{-i\int_{t_1}^{t_2}d\tau\, \boldsymbol{\sigma}_\tau\cdot\mathbf{f}(\tau)\right\}. \tag{10}$$

Introducing the index τ into the Pauli matrices allows $\boldsymbol{\sigma}_\tau$ to be treated as an ordinary vector function and (10) as an ordinary exponential function.[19] However, to obtain the final result explicitly we must again order $\boldsymbol{\sigma}_\tau$ so that the index decreases from left to right (for the sake of definiteness we deal with the case $t_1>t_1$), after which the index τ can be dropped.

Basing our reasoning on the expression (7) for $\varepsilon_{\mathbf{p}}(t)$, we can calculate the average tunneling rate from the state $a^\dagger|\text{vac}\rangle$ of the left bank to the state $b_{\mathbf{p}}^\dagger|\text{vac}\rangle$ of the right bank,

$$H_{\mathbf{kp}}=\lim_{t\to\infty}\frac{1}{t}\langle\xi_{\mathbf{p}}^\dagger\xi_{\mathbf{p}}\rangle=\lim_{t\to\infty}\left\langle\xi_{\mathbf{p}}^\dagger\frac{d\xi_{\mathbf{p}}}{dt}+\text{c.c.}\right\rangle. \tag{11}$$

The angle brackets stand for averaging over the realizations of the random exchange field $\mathbf{f}(t)$. In accordance with what was stated in Sec. 1, this is a Gaussian random function for which the higher correlators break up into products of pair correlators, and the averages emerging below are determined solely by the correlator

$$\langle f_i(t)f_j(t')\rangle=\Phi(t-t')\delta_{ij}, \quad \Phi(0)=f_0^2, \quad i,j=x,y,z \tag{12}$$

(here we have allowed for the homogeneity in time and the absence of a preferred direction in the spin space). The function $\Phi(t)$ has a characteristic time scale t_c and tends to zero for $t\gg t_c$.

Along with the tunneling rate $H_{\mathbf{kp}}$ specified by Eq. (11), it is expedient to introduce the barrier's tunneling transparency (the transmission coefficient) $T(\varepsilon,\varepsilon')$, which we define in the following manner. We decompose the set of quantum numbers of the initial state, $\mathbf{k}$, into k_z and κ, where κ is the wave vector of motion in the barrier plane. We divide the number of electrons that have tunneled per unit time with the final energy ε', that is, $\Sigma_{\mathbf{p}}H_{\mathbf{k}}\delta(\varepsilon_{\mathbf{p}}-\varepsilon')$, by the flux $j_z/2L=(1/2L)(d\varepsilon_{\mathbf{k}}/dk_z)$ impinging in state $a_{\mathbf{k}}^\dagger|\text{vac}\rangle$ onto the barrier, where L is the normalized length in the z direction, and then sum the obtained expression over all κ with the condition $\varepsilon_{\mathbf{k}}\equiv\varepsilon(\kappa,k_z)=\varepsilon$:

$$T(\varepsilon,\varepsilon')=\Sigma'_\kappa\frac{1}{j_z}\sum_{\mathbf{p}}H_{\mathbf{kp}}\delta(\varepsilon_{\mathbf{p}}-\varepsilon')$$

$$=2\pi\sum_{\mathbf{kp}}H_{\mathbf{kp}}\delta(\varepsilon_{\mathbf{k}}-\varepsilon)\delta(\varepsilon_{\mathbf{p}}-\varepsilon'). \tag{13}$$

Combining (7), (8), (11), (13), and (6b), we arrive at the following expression for $T(\varepsilon,\varepsilon')$:

$$T(\varepsilon,\varepsilon')=\frac{2\Gamma_l\Gamma_r\gg}{\pi\int_{-\infty}^{\infty}dt}\iint_0^\infty d\tau d\tau'$$

$$\times\exp\{i(\varepsilon-\varepsilon')t-i\varepsilon\tau+i\varepsilon'\tau'-\Gamma(\tau+\tau')\}$$

$$\times\langle G(\tau,0)G(t,t-\tau')^\dagger\rangle. \tag{14}$$

Here we have allowed for the absence of a preferred direction in the spin space of the problem considered; this leads to a situation in which the average $\langle G(\tau,0)G(t,t-\tau')^\dagger\rangle$ is an ordinary function rather than an operator. As a result the average barrier transparency is independent of the orientation of the electron spin in the initial state.

The fact that the expectation value of the product of two evolution operators in (14) is time-dependent leads to the presence of inelastic tunneling, that is, $T(\varepsilon,\varepsilon')\neq0$ for $\varepsilon\neq\varepsilon'$. At large values of t these operators can be averaged independently, and it proves expedient to introduce the definition

$$\langle\langle G(\tau,0)G(t,t-\tau')^\dagger\rangle\rangle$$

$$=\langle G(\tau,0)G(t,t-\tau)^\dagger\rangle-\langle G(\tau,0)\rangle\langle G(t,t-\tau')\rangle; \tag{15}$$

the average $\langle\langle\cdots\rangle\rangle$ in (15) tending to zero as $t\to\infty$. As a result we can write $T(\varepsilon,\varepsilon')$ as the sum of the elastic and inelastic parts:

$$T(\varepsilon,\varepsilon')=T_{\text{el}}(\varepsilon)\delta(\varepsilon-\varepsilon')+T_{\text{inel}}(\varepsilon,\varepsilon'), \tag{16}$$

where the inelastic part $T_{\text{inel}}(\varepsilon,\varepsilon')$ is obtained from (14) by replacing $\langle\cdots\rangle$ with $\langle\langle\cdots\rangle\rangle$, and

$$T_{\text{el}}(\varepsilon)=4\Gamma_l\Gamma_r|u(\Gamma+i\varepsilon)|^2, \tag{17}$$

$$u(s)=\int_0^\infty dt\exp(-st)\langle G(t,0)\rangle. \tag{18}$$

The tunneling amplitude $u(s)$ specified in (18) also determines the total barrier transparency for an electron tunneling with an initial energy ε:

$$T_{\text{tot}}(\varepsilon)=\int d\varepsilon'\,T(\varepsilon,\varepsilon')=\frac{4\Gamma_l\Gamma_r}{\Gamma}\text{Re}\,\{u(\Gamma+i\varepsilon). \tag{19}$$

In the absence of the fluctuating exchange field, when we have $G(t_2,t_1)\equiv1$, (14) implies the Breit–Wigner formula for barrier transparency in resonant tunneling:

$$T_{\text{BW}}(\varepsilon,\varepsilon')=\frac{4\Gamma_l\Gamma_r}{\varepsilon^2+\Gamma^2}\delta(\varepsilon-\varepsilon'). \tag{20}$$

An exchange field so strong that $\Phi(0)=f_0^2>\Gamma^2$ acting on an electron in the resonant state changes Eq. (20) radically, with the result strongly dependent on the ratio of the tunneling time $\tau_0=(2\Gamma)^{-1}$ to the correlation time of the fluctuating exchange field. We begin examining this effect with two limiting cases of large and short correlation times (in relation to τ_0).

3. BARRIER TRANSPARENCY IN THE LIMITING CASES OF RAPID AND SLOW EXCHANGE-FIELD FLUCTUATIONS

Employing the Feynman ordering index in writing the evolution operator in (10) makes it possible to easily obtain formal expressions of the expectation value of this operator (which enters into (18)) and for the expectation value of the product of such operators [which enters into (14)]. For one thing, the expectation value $\langle G(t,0)\rangle$ coincides with the generating functional for a Gaussian random field, whose form is well known (see, e.g., Ref. 20). We have

$$\langle G(t,0)\rangle=\exp\left\{-\frac{1}{2}\int_0^t d\tau\, d\tau'\, \Phi(\tau-\tau')\boldsymbol{\sigma}_\tau\cdot\boldsymbol{\sigma}_{\tau'}\right\}. \tag{21}$$

The problem now is to find the explicit form of Eq. (21), that is, to order the Pauli matrices in accordance with what was stated after Eq. (10).

In the case of slow exchange-field fluctuations it is sufficient to restrict our discussion to calculating $\langle G(t,0)\rangle$ for times $t\sim\tau_0\ll t_c$. Here the correlator $\Phi(\tau-\tau')$ in (21) can be replaced with $\Phi(0)=f_0^2$ and $\langle G(t,0)\rangle$ can be represented in the form

$$\langle G(t,0)\rangle=\frac{1}{(2\pi f_0^2)^{3/2}}\int d^3v\,\exp\left(-\frac{\mathbf{v}^2}{2f_0^2}\right)\times\exp\left\{i\mathbf{v}\cdot\int_0^t d\tau\,\boldsymbol{\sigma}_\tau\right\}. \tag{22}$$

In this expression the index τ can be dropped and the last exponential function can be replaced with $\exp(it\mathbf{v}\cdot\boldsymbol{\sigma})$. Integrating, we get

$$\langle G(t,0)\rangle=(1-f_0^2t^2)\exp\{-\tfrac{1}{2}f_0^2t^2\},\quad t\ll t_c. \tag{23}$$

Similarly, for $\tau,\tau'\ll t_c$ we can calculate the expectation value $\langle G(\tau,0)G(t,t-\tau')^\dagger\rangle$ entering into (14), but the result is too cumbersome to give here. Instead, we qualitatively describe the structure of $T(\varepsilon,\varepsilon')$ at large, but finite, values of t_c. The integrated value of the inelastic part, $\int d\varepsilon'\, T_{\rm inel}(\varepsilon,\varepsilon')$, proves to be of the same order of magnitude as the elastic part $T_{\rm el}(\varepsilon)$ [see Eq. (16)], and the characteristic value of electron-energy variation in inelastic tunneling, $|\varepsilon'-\varepsilon|$, is of the order of t_c^{-1}. In the limit $t_c\to\infty$ we get

$$\langle G(\tau,0)G(t,t-\tau')^\dagger\rangle=\langle G(\tau-\tau',0)\rangle, \tag{24}$$

which, when combined with (23) and (14), yields

$$T(\varepsilon,\varepsilon')=T(\varepsilon)\delta(\varepsilon-\varepsilon'),$$
$$T(\varepsilon)=\frac{4\Gamma_l\Gamma_r\gg}{\Gamma}\int_0^\infty dt\,\exp-\Gamma t\{1-f_0^2t^2\}\times\exp\{\tfrac{1}{2}f_0^2t^2\}\cos\varepsilon t. \tag{25a}$$

If the characteristic exchange spin splitting of the resonant level is large compared to Γ, we can replace $\exp(-\Gamma t)$ in (25a) with unity and write

$$T(\varepsilon)=\frac{2\pi\Gamma_l\Gamma_r}{\Gamma}\rho(\varepsilon), \tag{25b}$$

where

$$\rho(\varepsilon)=\frac{4\pi}{(2\pi f_0^2)^{3/2}}\varepsilon^2\exp\left\{-\frac{\varepsilon^2}{2f_0^2}\right\} \tag{26}$$

is the density of states of a localized electron in a semimagnetic semiconductor (see the definition in Sec. 1). Thus, the barrier transparency in this limiting case has a double-peaked structure, which, as the numerical evaluation of the integral in (25a) shows, is retained for values of Γ smaller than or approximately $2.2f_0$; the transparency has a minimum at $\varepsilon=0$, but, in contrast to $\rho(\varepsilon)$, does not vanish at this point.

In the opposite limiting case of rapid exchange-field fluctuations, when the correlation time t_c is much shorter than Γ^{-1}, we can put

$$\Phi(\tau-\tau')=2\gamma\delta(\tau-\tau'),\quad \gamma=\int_0^\infty dt\,\Phi(t). \tag{27}$$

After we substitute (27) into (21), the two spin operators in (21) are found to have the same ordering index, and since $\boldsymbol{\sigma}_\tau^2=\boldsymbol{\sigma}^2=3$, we obtain

$$\langle G(t,0)\rangle=\exp(-3\gamma|t|),\quad |t|\gg t_c. \tag{28}$$

An expression for the expectation value of the product of two evolution operators can be obtained for this limiting case in a similar way:

$$\langle G(\tau,0)G(t,t-\tau')^\dagger\rangle=\exp\{-3\gamma[|\tau|+|\tau'|+|t-\tau|+|t-\tau'|-|t|-|t-\tau-\tau'|]\}. \tag{29}$$

Substitution into (14) followed by integration yields the following expression for the barrier transparency in the limit $t_c\ll\Gamma^{-1}$:

$$T(\varepsilon,\varepsilon')=\frac{4\Gamma_l\Gamma_r}{\Gamma^{*2}+\varepsilon^2}\left[\delta(\varepsilon-\varepsilon')+\frac{(\Gamma^*-\Gamma)\Gamma^*}{\pi\Gamma(\Gamma^{*2}+\varepsilon'^2)}\right], \tag{30}$$

where

$$\Gamma^*=\Gamma+3\gamma. \tag{31}$$

Equation (30) shows that in this limiting case the barrier transparency loses its double-peaked structure and tunneling becomes inelastic. As for the elastic part of transparency, $T_{\rm el}(\varepsilon)$ [see Eq. (16)], for $t_c\ll\tau_0$ it has the form of a Lorentzian profile, with a half-width Γ^* [Eq. (31)] greater than the Breit–Wigner value Γ [cf. Eqs. (20) and (30)]. The same result was obtained by Stone and Lee,[15] who studied phenomenologically the effect of inelastic scattering on resonant tunneling. However, within the scope of their work, Stone and Lee were unable to obtain the inelastic part [the second term in (30)]. In this connection it must be noted that Eq. (30) is of a more general nature and, in contrast to the expressions for the opposite limiting case [Eqs. (25a) and (25b)], is not specific to the tunneling Hamiltonian [Eqs. (1) and (2)]. We would have obtained the same expression for $T(\varepsilon, \varepsilon')$ by simply allowing for rapid fluctuations of the resonant-level energy, that is, by selecting the center Hamiltonian H_c in Eq. (1) in the form $H_c=[E_0+\Delta E(t)]c^\dagger c$, where $\Delta E(t)$ is that part of the level's energy that fluctuates like white noise: $\langle\Delta E(t)\Delta E(t')\rangle=6\gamma\delta(t-t')$.

4. THE TUNNELING AMPLITUDE IN THE INTERMEDIATE RANGE

It is interesting to study the way in which Eqs. (25a) and (25b) transform into (30) as the correlation time t_c becomes shorter. It is natural to assume that the exchange-field fluctuations decay exponentially and accordingly, that the correlator $\Phi(\tau-\tau')$ specified in Eq. (12) has the form

$$\Phi(\tau-\tau')=f_0^2\exp(-\omega_0|\tau-\tau'|), \quad \omega_0=t_c^{-1}. \tag{32}$$

(In what follows instead of the correlation time t_c it is convenient to use its inverse ω_0.) An argument that can serve as justification for selecting the correlator in the form (32) goes as follows. The fluctuations of the exchange field are due to random variations of the projection of the magnetic ion spins. Hence, a satisfactory model is to represent $\mathbf{f}(t)$ as a sum of a large number (see Sec. 1) of terms of the form $\mathbf{w}_n\vartheta_n(t)$, where $\mathbf{w}_n$ are randomly distributed over the directions and are of the same order of magnitude, and each ϑ_n independently experiences telegraphic noise (i.e., takes on, at random, values ± 1 with an average frequency ω_0). The function $\mathbf{f}(t)$ then proves to be a Gaussian random function with a pair correlator that has, as one can easily see, the form (32).

In this case it is possible to calculate explicitly the tunneling amplitude $u(s)$ [see Eq. (18), where $s=\Gamma+i\varepsilon$], which determines the elastic part of the barrier transparency, $T_{\rm el}(\varepsilon)$ [Eq. (17)], and the total barrier transparency $T_{\rm tot}(\varepsilon)$ [Eq. (19)]. The present section is devoted to this method.

According to Eq. (18), the tunneling amplitude is a result of a Laplace transformation of the averaged evolution operator (21), which in the case at hand can be written as

$$\langle G(t,0)\rangle=\exp\left\{-f_0^2\int_0^t d\tau\exp\{-\omega_0\tau\}\int_0^\tau d\tau' \times\exp\{\omega_0\tau'\}\boldsymbol{\sigma}_\tau\cdot\boldsymbol{\sigma}_{\tau'}\right\}. \tag{33}$$

To calculate this quantity for an arbitrary ω_0, we consider the auxiliary operator

$$F=\omega_0\mathbf{d}^\dagger\cdot\mathbf{d}+if_0\boldsymbol{\sigma}\cdot(\mathbf{d}^\dagger+\mathbf{d}), \tag{34}$$

where d_x, d_y, and d_z are ordinary Bose operators obeying the following commutation relations:

$$[d_i,d_j^\dagger]=\delta_{ij}, \quad [d_i,d_j]=0, \quad i,j=x,y,z. \tag{35}$$

If we once more introduce the ordering index into the Pauli matrices, the following identity holds true:

$$e^{-Ft}=\langle G(t,0)\rangle e^{-F_0t}\exp\left\{-if_0\mathbf{d}^\dagger\cdot\int_0^t d\tau\, e^{\omega_0\tau}\boldsymbol{\sigma}_\tau\right\} \times\exp\left\{-if_0\mathbf{d}\cdot\int_0^t d\tau\, e^{-i\omega_0\tau}\right\}, \tag{36}$$

where $\langle G(t,0)\rangle$ is the same as in (33), and $F_0=\omega_0\mathbf{d}^\dagger\cdot\mathbf{d}$. The validity of the identity can be proved by differentiating with respect to time t, with allowance for the identities that follow from (35):

$$e^{-F_0t}\mathbf{d}e^{F_0t}=\mathbf{d}e^{\omega_0t}, \quad \exp\{-i\mathbf{q}\cdot\mathbf{d}^\dagger\}\mathbf{d}\exp\{i\mathbf{q}\cdot\mathbf{q}^\dagger\}=bfd+i\mathbf{q} \tag{37}$$

($\mathbf{q}$ is an arbitrary vector). Thus, the desired quantity $\langle G(t,0)\rangle$ is equal to $(0|\exp\{-Ft\}|0)$, where $\mathbf{d}|0)=0$, and, accordingly, the tunneling amplitude coincides with the matrix element of the resolvent of operator F (Eq. (34)) on the vacuum state for d_x, d_y, and d_z:

$$u(s)=\left(0\left|\frac{1}{F+s}\right|0\right). \tag{38}$$

Thus, the problem of calculating the expectation value of the evolution operator for the spin in a temporally fluctuating exchange field$+\mathbf{f}(t)$ with a correlator given by Eqs. (12) and (32) has been reduced to another problem that mathematically is more common. Note that the operator F is indeed purely auxiliary and in no way related to the tunneling Hamiltonian (1). To avoid any misunderstanding we employ the notation $|\cdots)$ rather than $|\cdots\rangle$ for the ket vectors of the space where the operators $\mathbf{d}$ and $\mathbf{d}^\dagger$ act. Note also that after we have formally replaced f_0 by a purely imaginary quantity, the operator F acquires the form of the Hamiltonian of a three-dimensional oscillator with a spin–orbit coupling proportional to $\boldsymbol{\sigma}\cdot\mathbf{p}$, with $\mathbf{p}$ the momentum (such spin–orbit coupling is possible in a gyrotropic medium).

Since σ_x, σ_y, and σ_z do not commute, it is not as simple to diagonalize the operator F in (37) as it might appear (e.g., this cannot be done by a translation operation applied to $\mathbf{d}$). To calculate the resolvent (38) we solve the equation

$$(F+s)|R)=|0), \tag{39}$$

for the ket vector $|R)$ in the coherent-states representation[22] (Eqs. (38) and (39) imply $u(s)=(0|R)$). More precisely, we introduce the quantity

$$R(\mathbf{q})=(0|\exp\{\mathbf{q}\cdot(\mathbf{d}+\mathbf{d}^\dagger)\}|R), \tag{40}$$

for which from Eq. (39) and relations of the type (37) we arrive at the equation

$$\omega_0\left(\mathbf{q}\cdot\frac{dR}{d\mathbf{q}}-q^2R\right)+if_0\boldsymbol{\sigma}\cdot\frac{dR}{d\mathbf{q}}+sR=\exp\{\tfrac{1}{2}q^2\}. \tag{41}$$

An important aspect is that $\mathbf{q}=0$ is a singular point of Eq. (41), whereby all solutions regular at $\mathbf{q}=0$ have the same desired value $R(0)$. The quantity $R(\mathbf{q})$ is an operator in the spin space, that is, a 2-by-2 matrix, and the general regular solution of the matrix equation (41) can be written as

$$R(\mathbf{q})=\exp\{\tfrac{1}{2}q^2\}\sum_{n=0}^{\infty}A_n(\boldsymbol{\sigma}\cdot\mathbf{q})^n. \quad (42)$$

[Note that $R(0)$ is proportional to the identity matrix, in accordance with what was stated after Eq. (14).] Substitution of (42) into (41) leads to trinomial recurrence formulas for the A_n coefficients:

$$\begin{aligned} &if_0^2A_{n-1}+f_0b_nA_n+ia_{n+1}A_{n+1}=0 \qquad \text{for} \quad n>0,\\ &f_0b_0A_0+ia_1A_1=f_0, \end{aligned} \quad (43)$$

where

$$b_n=s+n\omega_0,\quad a_n=\begin{cases}(n+2)f_0^2 & \text{for } n \text{ odd},\\ nf_0^2 & \text{for } n \text{ even}.\end{cases} \quad (44a)$$

The recurrence formulas (43) make it possible to write the tunneling amplitude $u(s)=A_0$ as a continued fraction:

$$u(s)=\cfrac{1}{b_0+\cfrac{a_1}{b_1+\cfrac{a_2}{b_2+\cfrac{a_3}{b_3+\cdots}}}} \quad (44b)$$

The validity of (44b) is most simply proved by truncating the series (42) at the $(2k)$th term, that is, setting $A_{2k}=0$. Then from (43) there follows a system of $2k$ linear algebraic equations for $A_0,A_1,...,A_{2k-1}$, which can easily be solved. This gives the A_0 coefficient in the form of an appropriate fraction of the $2k$th order for the continued fraction (Eqs. (44a) and (44b)).

This continued fraction is convergent according to Stolz's test (see Ref. 22, p. 289) in the entire physically important range of parameters, that is, for $\omega_0>0$ and $\Gamma=\text{Re}\,(s)>0$. Note also that the results obtained from (44a) and (44b) and from (25a) and (30) are the same in the cases of small and large values of ω_0 (for $\omega\to 0$ this can be shown numerically).

Figure 1 depicts the dependence of the total barrier transparency $T_{\text{tot}}(\varepsilon)$ on the correlation time $t_c=\omega_0^{-1}$ calculated on the basis of Eqs. (19), (44a), and (44b) at fixed values of f_0^2 and $\tau_0=(2\Gamma)^{-1}$. An interesting feature of this dependence is a plateau with a width of order f_0 that appears at a certain intermediate value of the correlation time.

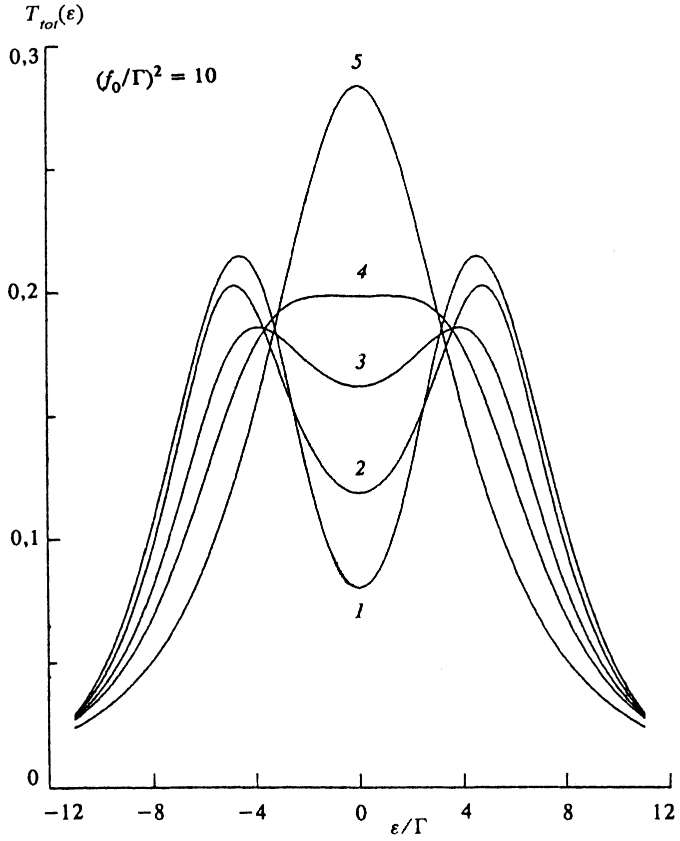

FIG. 1. Total resonant barrier transparency vs tunneling-electron energy for five values of correlation time: $\omega_0/\Gamma=0.1$, 1.0, 3.0, 5.0, and 10.0 for curves *1, 2, 3, 4*, and *5*, respectively. For the sake of definiteness we take the case of $\Gamma_l=\Gamma_r=\Gamma/2$.

5. DISCUSSION

It has been shown that temporal fluctuations of the exchange field have a pronounced effect on resonant tunneling of electrons through a semimagnetic barrier. The resonant barrier transparency depends qualitatively on the characteristic "frequency" of the fluctuations of the exchange field with respect to the unperturbed half-width Γ of the resonant peak. In the case of slow perturbations, an electron tunnels in an exchange field that remains practically constant in time, tunneling is quasielastic, the electron-energy dependence of the tunneling probability is determined by the probability of the respective resonant-level shift, and, as a result, the barrier transparency has a double-peaked structure determined by the density of states of a localized electron in the semimagnetic semiconductor. But when the tunneling time is comparable with or less than, the exchange-field correlation time, tunneling becomes essentially inelastic, its double-peaked structure gets fuzzy or completely disappears, and the transparency acquires the shape of a single peak with a Lorentzian profile. Qualitatively this result can be interpreted in the following manner. Exchange-field fluctuations primarily lead to random motion in the position of the resonant level, which results in the level being in and out of resonance. In the case of rapid fluctuations it is this motion rather than the probability of realization of the level energy that determines the position and shape of the resonant peak. This phenomenon is similar to what is known as the spectral diffusion effect in spin resonance theory[23] and nonlinear high-frequency hopping conductivity.[24] In our case, however, on the one hand, the problem of calculating the lineshape simplifies considerably because of the Gaussian nature of the fluctuations but, on the other, requires quantum mechanical treatment of the motion of spin in a time-dependent exchange field (and not simply allowance for level-energy fluctuations).

As noted in Sec. 1, at low temperatures and at times of order t_c, after a center has captured an electron a correlation appears between the direction of the electron spin and that of the exchange field, and the lower the temperature the stronger the correlation. The model considered in this paper assumes, however, that the exchange field fluctuates independently, and here we qualitatively discuss how allowing for this correlation affects the tunneling transparency $T(\varepsilon,\varepsilon')$. Naturally, at large t_c an electron still tunnels in an exchange field that is random in both direction and magnitude, a field that has no time to change substantially in the course of tunneling, $\tau_0 \ll t_c$. Hence the result specified by (25a) and (25b) remains valid. In the opposite limiting case of long tunneling times, the emerging correlation between the directions of exchange field and electron spin leads to formation of a bound magnetic polaron [2,6–8] and to reduction of the average electron energy at the center by $\varepsilon_p \sim f_0^2/T$ (here T is the temperature). At the same time, temporal exchange-field fluctuations exist and lead to dynamic smearing of resonance. Hence we can expect that for $t_c \ll \tau_0$ Eq. (30) for $T(\varepsilon, \varepsilon')$ remains valid, but the transparency peak is shifted toward lower energies by a quantity of the order of ε_p.

We have investigated a microscopic quantity, the tunneling transparency, in the event of resonant tunneling through a single resonant state. Of course, from the experimental viewpoint it is important to know the current passing through the barrier. In the case of a heterostructure with a semimagnetic barrier of a fairly large surface area, the total current is determined by the tunneling of electrons through a large number of resonant states, and its magnitude can be expressed in terms of $T(\varepsilon,\varepsilon')$ with some averaging over the energy of the resonant levels and over their position in the barrier, that is, over the partial halfwidths Γ_l and Γ_r (see, e.g., Ref. 17). Of course, experimentally the transmission coefficient can be studied in "pure" form if one investigates the quasi-one-dimensional electron transport through a microconstriction in the semimagnetic barrier, created by a voltage on the gate of a planar structure. Here, by changing the gate voltage the tunneling time can be varied from $\tau_0 \ll t_c$ to $\tau_0 \gg t_c$. Note that in the case $\tau_0 \ll t_c$ the current passing through the microconstriction fluctuates in time with a characteristic time scale t_c.

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