Spin-wave solitons in a sound "lattice"

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We solve the problem of the occurrence of self-localized spin-wave excitations of parametrically coupled magnetostatic waves when they are inelastically scattered by a traveling acoustic wave. Using nonlinear coupled equations in the dissipationless limit we obtain soliton solutions describing self-localized stationary spin waves inside the nontransmission band of the frequency spectrum of the linear waves. We use perturbation-theory methods to introduce equations for the adiabatic change of the soliton parameters in a dissipative medium and analyze the conditions for its stabilization when a source for the generation of one of the parametrically interacting waves is switched on.

INTRODUCTION

Nonlinear spin-waves in a ferrite may undergo automodulation and self-focusing similar to a light beam in a transparent nonlinear crystal.[1–6] The modulational instability of a uniform nonlinear wave of constant amplitude is connected with the tendency to form spatially localized spin-wave packets—envelope solitons caused by the competition between dispersive and nonlinear changes in the phase velocities of the harmonics making up the wave packet.[7] For waves with a strong dispersion for which the frequency ω does not vanish at the same time as the wave number k, i.e., $\omega(k=0)\neq 0$, the condition for the occurrence of the instability is the Lighthill criterion[8] $(\partial\omega/\partial|\varphi|^2)(\partial^2\omega/\partial k^2)<0$, where φ is the wave amplitude. This criterion makes it possible to predict the formation of small-amplitude envelope solitons for waves with a known dispersion law and nonlinearity. In particular, this refers to cases where the frequency branches of interacting waves intersect. The intersection of frequency branches is very common in the spectra of magnetostatic waves in thin magnetic films in which there exist at the same frequencies, together with nonexchange magnetostatic waves (MSW), modes of spin-wave resonant oscillations which are strongly nonuniform along the thickness of the film.[9,10] Another example of magnetic systems with an intersection of frequency branches in the spectrum of spin-wave oscillations are the magnetic layer structures.[11,12] Very typical, besides the unidirectional resonant interaction of waves, is the situation with the formation of intersecting frequency branches when one uses diffraction lattices for the control of the wave flux.[13,14] In the region where the spectral curves of the interacting waves intersect, $\omega_0(k)=\omega_0(K-k)$ where K is the wave number of the lattice, a nontransmission band, $|\omega-\omega_0|>\omega_B$, is formed, at the edges of which the dispersion $\partial^2\omega/\partial k^2$ of the hybridized wave changes sign. The nonlinear MSW can thus, in accordance with the Lighthill criterion, form solitons only near one edge of the gap. However, the behavior of the nonlinear coupled waves inside the nontransmission band itself, where linear waves transform into one another, is unclear. To solve such a problem we consider Bragg scattering of a nonlinear MSW by a traveling sound wave, $\varepsilon=\varepsilon_0\cos(\Omega t-Ky)$, which plays the role of the reflecting lattice.[15–17] Some effects of the scattering of nonlinear MSW by a sound pulse, connected especially with the occurrence of hysteresis effects when they pass through or are reflected, were studied in Refs. 16 and 18. However, the self-localization effects of nonlinear magnetostatic waves in an unbounded sound "lattice" has not been discussed before. The problems indicated above are rather general in nature and their solution is of interest also for acoustics and nonlinear optics (see, e.g., Refs. 19 to 21).

We shall analyze in the present paper the conditions for the formation and stabilization of envelope solitons of magnetostatic waves which are parametrically coupled by a pumping sound wave far from magnetoacoustic resonance. In the first part of the paper we discuss the basic coupling equations for nonlinear waves and the approximations used. On the basis of these equations and the conservation laws we find a soliton solution for a dissipationless medium. After that we derive equations for a brief description for a "gap" soliton at rest, taking into account dissipation and electromagnetic pumping, and we analyze the conditions for its stabilization.

1. BASIC EQUATIONS

A sound wave in a ferrite, $\varepsilon=\varepsilon_0\cos(\Omega t-Ky)$ with ε_0 the amplitude of the elastic strain of the crystal in the acoustic wave, y the running coordinate, and t the time, produces a spatially periodic change in the effective magnetic field, $h\sim(B\varepsilon_0/M)\cos(\Omega t-Ky)$ with B the magnetostriction coefficient and M the magnetization. This field acts on the spin subsystem of the magnetic crystal. The result is a stable parametric interaction of spin-wave Floquet harmonics, φ_n $(n=\pm 1,\pm 2,...)$, arising when a magnetostatic signal wave propagates in a moving sound "lattice"; they suffer a multiple shift in frequency and wave number by $n(\Omega,k)$ and interact with one another.[14,22] When the coupling parameter is small, $\eta=B\varepsilon_0/M^2\ll 1$, the strongest of their interactions occur in a narrow frequency

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and wave-number region, $|\omega-\omega_0|/\omega_0$, $|k-k_0|/k_0 \sim \eta \ll 1$, near the point of intersection of the reference curves $\omega_n(k)$ of neighboring harmonics $n=0, \pm 1$ for which the conditions are satisfied for phase synchronism, $\omega_n(K-k) = \omega_0(k) + n\Omega$ of the incident, φ_+ ($n=0$), and reflected, φ_- ($n=\pm 1$ for Stokes or anti-Stokes scattering), waves. The Bragg scattering of linear MSW by a traveling acoustic wave is then satisfactorily described by the coupled equations for two waves which are obtained by the reduction of the Landau–Lifshitz equations of motion for the magnetic moment and the Maxwell equations in the magnetostatic approximation, taking the appropriate boundary conditions into account.[15,23] These equations have the form

$$i(\partial_t + v_+\partial_y + \delta\omega_+)\varphi_+ + \omega_B^+\varphi_- = 0,$$
$$i(\partial_t - v_-\partial_y + \delta\omega_-)\varphi_- + \omega_B^-\varphi_+ = 0, \qquad (1)$$

where the $v_\pm$ are the group velocities, the $\omega_B^\mp$ the coupling coefficients, and the $\delta\omega_\pm$ the line widths of the interacting MSW. These parameters depend on the geometry of the magnetization of the film and the polarization of the acoustic wave. In particular, for the case of scattering of a surface MSW by a surface acoustic wave with small wave numbers, $Kd, kd \ll 1$, when the magnetic moment lies in the (zy) plane of the film and the MSW and SAW propagate at right angles to it along the y-axis, these parameters are equal to[2,15]

$$v_\pm = v_g = \partial\omega/\partial k|_{k=0} = \omega_M^2 d/4\omega_0, \quad \text{with } \omega_M = \gamma 4\pi M,$$

$$\omega_0^2 = \omega_H(\omega_H + \omega_M), \quad \omega_H = \gamma H,$$

γ is the gyromagnetic ratio,

$$\omega_B^\mp = \omega_B = \frac{\gamma B}{M}\left\{\left[\frac{(\chi_2^2-\chi_1^2)}{(1-\chi_1)}\frac{v_g k}{\omega_M} + \frac{(\chi_2^2+\chi_1^2-2\chi_1)}{\chi_2}\right](\varepsilon_{xx}-\varepsilon_{yy}) + (\varepsilon_{xx}+\varepsilon_{yy}) \times\left[\frac{(\chi_1^2+\chi_2^2)(\chi_1^2+\chi_2^2-2\chi_1)-4\chi_1\chi_2^2(1-\chi_1)}{2\chi_2(\chi_2^2-\chi_1^2)}\right]\right\},$$

with $\chi_1 = \omega_H\omega_M/(\omega^2-\omega_H^2)$, $\chi_2 = \omega\omega_M/(\omega^2-\omega_H^2)$, while ε_{xx} and ε_{yy} are components of the elastic deformation tensor in the acoustic wave near the surface.

We have neglected in the coupled Eqs. (1) the waveguide dispersion of the interacting waves which in the case of surface MSW is equal to $\partial^2\omega/\partial k^2 = -(\omega_M^4 d^2/16\omega_0^3)(1+8\omega_M^2/\omega_0^2)$. This is valid only for not too strongly localized wavepackets. Indeed, if we recognize that the coupled Eqs. (1) give for the frequency mismatch, $\Delta\omega = \omega - \omega_0$, and for the deviation, $\Delta k = k - k_0$, of the wave number from the point of phase synchronism the dispersion relation $(\Delta\omega)^2 = \omega_B^2 + (v_g\Delta k)^2$, we get from the condition $\partial^2\Delta\omega/\partial(\Delta k)^2 \gg \partial^2\omega/\partial k^2$ a restriction on the width of the wavepacket, $\Delta k \ll (\omega_B/v_g)[|v_g^2/\omega_B\partial_k^2\omega|^{2/3} - 1]^{1/2}$. This condition is satisfied for surface MSW provided that

$$\Delta k \ll (4\omega_0\omega_B/\omega_M^2 d)[(\omega_0/\omega_B)(1+8\omega_0^2/\omega_M^2)]^{1/3}. \qquad (2)$$

In the opposite case one must take the waveguide dispersion into account.

If there is no acoustic pumping, when $\omega_B^\mp = 0$, the equations of the system (1) split up and describe the evolution of the envelope of each MSW in the dispersionless approximation. Including the waveguide dispersion and the nonlinear frequency shift makes it possible to describe the evolution of nonlinear MSW in a ferrite. For the long-wavelength spin oscillations ($kd \ll 1$) the main mechanism for the nonlinear frequency shift is then the decrease in the average magnetization $\langle M_z\rangle = M(1-\langle m_x^2+m_y^2\rangle/2M^2)$ which is connected with the precession of the magnetic moment and the conservation of its total magnitude. Taking this fact into account we can find the corresponding nonlinear frequency shift, $\omega(k,\varphi) = \omega(k,0) + (\partial\omega/\partial|\varphi|^2)|\varphi|^2$, where $|\varphi|^2 = \langle m_x^2+m_y^2\rangle/2M^2$ is the square of the wave amplitude. For instance, for surface MSW we have[2] $\partial\omega/\partial|\varphi|^2 = -(\omega_H\omega_M/4\omega_0)(1+\omega_H^2/\omega_0^2)$.

When there is an acoustic wave present in the ferrite there appear parametrically coupled coherent oscillations of the signal and the scattered MSW. The decrease in the average magnitude of the saturation magnetization is thus determined by the amplitudes of both waves. After averaging over time and taking the Doppler shift in the frequencies of the interacting harmonics of the MSW into account we shall have $\langle(m_{x+}+m_{x-})^2+(m_{y+}+m_{y-})^2\rangle = \langle(m_{x+})^2+(m_{y+})^2\rangle + \langle(m_{x-})^2+(m_{y-})^2\rangle$ and thus[1)]

$$\omega_\pm(k,\varphi_\pm) = \omega_\pm(k,0) + (\partial\omega/\partial|\varphi|^2) \times(|\varphi_+ + \varphi_+^* + \varphi_- + \varphi_-^*|^2).$$

This change in the saturation magnetization will produce also a nonlinear change in the group velocities $v_\pm$ of the interacting waves and in the coupling coefficients $\omega_B^\mp$. However, for wavepackets with a weak localization in space [see condition (2)] we can neglect the nonlinear distortion of the group velocities. Moreover, the nonlinear frequency shift of the hybridized wave, caused by the nonlinearity of the coupling coefficients, will be of the order $\sim\omega_B|\varphi|^2$ which is considerably smaller than the contribution to the nonlinear frequency shift of each harmonic, which is approximately equal to $\sim(\omega_H\omega_M/4\omega_0)|\varphi|^2$. Taking all this into account we can write for the nonlinear MSW in a ferrite which are parametrically coupled with a traveling acoustic wave the following equations

$$[i\partial_t + iv_g\partial_y - \alpha(|\varphi_+|^2 + 2|\varphi_-|^2)\varphi_+ + \omega_B^+\varphi_- = -i\delta\omega\varphi_+,$$

$$[i\partial_t - iv_g\partial_y - \alpha(2|\varphi_+|^2 + |\varphi_-|^2)\varphi_- + \omega_B\varphi_+ = -i\delta\omega\varphi_-, \qquad (3)$$

where $\alpha = \partial\omega/\partial|\varphi|^2$.

If we normalize the variables $[t] = \omega_B t$, $[y] = y\omega_B/v_g$, $[\varphi] = \varphi|\alpha/\omega_B|^{1/2}$, then Eqs. (3), together with the complex conjugate pair, take the form

$$(i\partial_t+i\partial_y+sH_-)\varphi_++\varphi_-=-i\Gamma\varphi_+,$$

$$-(i\partial_t+i\partial_y-sH_-)\varphi_+^*+\varphi_-^*=i\Gamma\varphi_+^*,$$

$$(i\partial_t-i\partial_y+sH_+)\varphi_-+\varphi_+=-i\Gamma\varphi_-, \qquad (4)$$

$$-(i\partial_t-i\partial_y-sH_+)\varphi_-^*+\varphi_+^*=i\Gamma\varphi_-^*,$$

where $H_\pm=H+|\varphi_\pm|^2$, $H=|\varphi_+|^2+|\varphi_-|^2$, $s=\pm 1$ is the sign of the nonlinearity, and $\Gamma=\delta\omega/\omega_B$.

The system of equations we have obtained have for $\Gamma=0$ two integrals, the evolution of which at $\Gamma\neq 0$ describes the change in the energy and the momentum of the wavepackets. Multiplying (4) by the vector $\mathbf{g}_1=(-\varphi_+^*,\ \varphi_+,\ -\varphi_-^*,\varphi_-)$ and adding the equations together we get after some straightforward transformations

$$\partial_t H+\partial y(|\varphi_+|^2-|\varphi_-|^2)=-2\Gamma H. \qquad (5)$$

For $\Gamma=0$ this is the energy conservation law. Similarly, multiplying (4) with the vector $\mathbf{g}_2=\partial_y(\varphi_+^*,\varphi_+,\varphi_-^*,\varphi_-)$, adding the equations, and transforming the sum to the form of a divergence, we get

$$\begin{aligned} &i\partial_t[(\varphi_+\partial_y\varphi_+^*-\varphi_+^*\partial_y\varphi_+)+(\varphi_-\partial_y\varphi_-^*-\varphi_-^*\partial_y\varphi_-)]\\ &\quad+i\partial_y[(\varphi_+\partial_t\varphi_+^*-\varphi_+^*\partial_t\varphi_+)\\ &\quad+(\varphi_-\partial_t\varphi_-^*-\varphi_-^*\partial_t\varphi_-)+s(|\varphi_+|^2+|\varphi_-|^2)\\ &\quad+2|\varphi_+\varphi_-|^2+2(\varphi_+\varphi_-^*+\varphi_+^*\varphi_-)]\\ &=-2i\Gamma[(\varphi_+\partial_y\varphi_+^*-\varphi_+^*\partial_y\varphi_+)\\ &\quad+(\varphi_-\partial_y\varphi_-^*-\varphi_-^*\partial_y\varphi_-)]. \end{aligned} \qquad (6)$$

For $\Gamma=0$ this equation is the momentum conservation law.

2. SOLITONS OF PARAMETRICALLY COUPLED MSW IN A DISSIPATIONLESS MEDIUM

We first of all consider the conditions for the existence of solitons of coupled magnetostatic waves in a dissipationless medium, $\Gamma=0$, which is the limiting case of a weakly dissipative medium with a high level of acoustic pumping, $\omega_B\gg\delta\omega$ ($\Gamma\ll 1$). We seek localized solutions in the following form,

$$\varphi_\pm=R_\pm\exp[i(\tilde{k}y-\tilde{\omega}t+\chi_\pm)], \qquad (7)$$

where $\tilde{\omega}=\omega-\omega_0$ and $\tilde{k}=k-k_0$ are, respectively, the mismatch frequency and wave number. We shall then consider the amplitude and the phase of the soliton to be self-similar functions of the coordinate and the time, i.e., $R_\pm,\chi_\pm=f(y-vt)$ where v is the soliton velocity normalized by the group velocity v_g of the signal MSW. The normalization of the other variables was given above. The required solutions must be localized in space so that we have $R_\pm(|y|=\infty)=0$.

We can now obtain two integrals from (5) and (6). The first, which is essentially the analog of the Manley–Rowe relations, has the form

$$(1-v)|\varphi_+|^2=(1+v)|\varphi_-|^2 \qquad (8)$$

if we take the boundary conditions $\varphi(|y|=\infty)=0$ into account. The second integral which follows from (6) and corresponds to the momentum flux conservation law gives, if we use (8),

$$\tilde{\omega}-\tilde{k}v+\frac{3-v^2}{4}sH+(1-v^2)^{1/2}\cos\chi=0, \qquad (9)$$

where $\chi=\chi_+-\chi_-$. It follows from (8) that $R_\pm=[(1\pm v)H/2]^{1/2}$. Separating the imaginary and real parts in Eqs. (4) and using the self-similarity of the required solution we can now obtain the following equations

$$\partial_\xi H=\frac{2H}{(1-v^2)^{1/2}}\sin\chi,$$

$$\partial_\xi\chi_+=\frac{\tilde{\omega}-\tilde{k}+sH}{1-v}+\frac{\cos\chi}{(1-v^2)^{1/2}}, \qquad (10)$$

$$\partial_\xi\chi_-=-\frac{\tilde{\omega}-\tilde{k}+sH_+}{1+v}-\frac{\cos\chi}{(1-v^2)^{1/2}},$$

where $\xi=y-vt$, $H_\pm=(3\pm v)H/2$. Using (9) and integrating these equations enables us to find the required soliton solution

$$H=\frac{4[1-v^2-(\tilde{\omega}-\tilde{k}v)]^2/(3-v^2)}{s(\tilde{\omega}-\tilde{k}v)+\sqrt{1-v^2}\,\mathrm{ch}[2(y-vt)\sqrt{1-v^2-(\tilde{\omega}-\tilde{k}v)^2}/\sqrt{1-v^2}]}, \qquad (11)$$

$$\chi_\pm=\chi_0+\frac{(\tilde{\omega}v-\tilde{k})}{1-v^2}(y-vt)+\frac{2v\chi}{3-v^2}\pm\frac{\chi}{2}, \qquad (12)$$

where χ is given by Eqs. (9) and (11) and χ_0 is an arbitrary constant.

For $\tilde{\omega},\tilde{k}=0$ the solution we have found is the same as the soliton solution describing self-localized stationary waves in a periodic structure, which were obtained in Ref. 19 (see also Ref. 20). For $\tilde{\omega},\tilde{k}\neq 0$ it follows from (11) that the condition for localization of the solution obtained is not violated provided the soliton velocity lies in the range $v_+>v>v_-$, where $v_\pm=(\tilde{\omega}\tilde{k}\pm\sqrt{1+\tilde{k}^2-\tilde{\omega}^2})/(1+\tilde{k}^2)$, $|v_\pm|<1$. Hence it follows also that the soliton frequency $\tilde{\omega}$ and its wave number $\tilde{k}$ must lie between the branches of the dispersion curves of the linear MSW, i.e., $|\tilde{\omega}|<(1+\tilde{k}^2)^{1/2}$. The soliton can thus not move with a velocity exceeding the group velocity of the coupled waves which are its components, and its frequency for a given wave number of the phase mismatch cannot exceed the frequency of the linear MSW.

In terms of dimensional variables the maximum soliton amplitude is equal to

$$\varphi_{\max}=\left|\frac{4\omega_B}{\partial\omega/\partial|\varphi|^2}\right|^{1/2}, \tag{13}$$

and its characteristic width is

$$\delta y=(v_g/\omega_B)\frac{\sqrt{1-(v/v_g)^2}}{2\sqrt{1-(v/v_g)^2-(\tilde{\omega}-v_g\tilde{k})^2/\omega_B^2}}. \tag{14}$$

The soliton amplitude thus depends on the gap width ω_B, which is determined by the level ε_0 of the acoustic pumping, and on the nonlinear frequency-shift coefficient. The soliton width also depends on the magnitude of the acoustic pumping and, moreover, on the soliton frequency and velocity.

For $\tilde{k}=0$ solitons of coupled MSW exist only inside the nontransmission band, $|\omega|<\omega_B$, and they can thus be called conventionally "gap" solitons. The amplitude of a "gap" soliton at rest tends to zero as $s\tilde{\omega}\to+\infty$, i.e., close to one of the edges of the nontransmission band, near which the Lighthill criterion for the formation of small-amplitude solitons is satisfied, and reaches a maximum near the other edge, when $s\tilde{\omega}\to-1$, where this criterion is violated.

We now consider the evolution of a soliton in a weakly dissipative medium when $\delta\omega\neq 0$, but $\omega_B\gg\delta\omega$. For simplicity we restrict our analysis to the case of a soliton at rest ($v=0$) with $\tilde{k}=0$. Assuming the amplitudes and phases of the "gap" soliton, which are determined by the evolution of the mismatch frequency $\tilde{\omega}=\tilde{\omega}(\eta t)$ with η a small parameter ($\eta\ll 1$), to vary slowly with time we have, after integrating Eq. (5) over y and using the boundary conditions,

$$\langle H\rangle=\langle H\rangle|_{t=0}\exp(-2\Gamma t), \tag{15}$$

where $\langle H\rangle=\int H dy$. After integrating and using (11) we get from (15) after some transformations

$$\tilde{\omega}=s\cos[\varkappa\exp(-2\Gamma t)], \tag{16}$$

where

$$\varkappa=2\left[\operatorname{arctg}\frac{1+s\tilde{\omega}(0)}{(1-\tilde{\omega}^2)^{1/2}}-\operatorname{arctg}\frac{s\tilde{\omega}(0)}{(1-\tilde{\omega}^2)^{1/2}}\right]$$

$$=\arccos[s\omega(0)].$$

The formulas obtained show that the presence of dissipation in the absence of added pumping of energy leads not only to a decrease in the soliton amplitude, but also to its broadening and also to a shift of the mismatch frequency to the edge of the nontransmission band near which the soliton vanishes.

3. STABILIZATION OF COUPLED SELF-LOCALIZED MSW STATES IN A DISSIPATIVE MEDIUM

To stabilize the soliton we can use additional electromagnetic pumping of the energy into one of the interacting magnetostatic waves. To take this into account it is necessary to introduce into the right-hand side of the equations of the set (4), which describe the evolution of the amplitude of one of the coupled waves, an additional term describing its source. The coupled equations can then be written in the following form

$$\hat{L}\boldsymbol{\varphi}=\mathbf{f}, \tag{17}$$

where $\boldsymbol{\varphi}=(\varphi_+^*,\varphi_+,\varphi_-^*,\varphi_-)$ is the vector describing the solution and $\hat{L}$ a nonlinear operator acting on it and equal to

$$\hat{L}=\left|\begin{matrix} i\partial_t+i\partial_y+sH_- & 0 & 1 & 0\\ 0 & -i\partial_t-i\partial_y+sH_- & 0 & 1\\ 1 & 0 & i\partial_t-i\partial_y+sH_+ & 0\\ 0 & 1 & 0 & -i\partial_t+i\partial_y+sH_+ \end{matrix}\right|. \tag{18}$$

The vector $\mathbf{f}$ includes dissipation and the source $h(y,t)$ for the excitation of one of the coupled MSW. It is equal to

$$\mathbf{f}=\begin{pmatrix} -i\Gamma\varphi_++h\\ i\Gamma\varphi_+^*+h^*\\ -i\Gamma\varphi_-\\ i\Gamma\varphi_-^* \end{pmatrix}. \tag{19}$$

We shall assume in what follows that the source for the electromagnetic generation of MSW is much more strongly localized than the soliton. We can then put $h=h_0\delta(y)\exp(\chi_h-i\tilde{\omega}_1 t)$, where h_0 is the amplitude, χ_h the phase, $\tilde{\omega}_1=\omega_1-\omega_0$ the frequency of the source in normalized units, and $\delta(y)$ is the Dirac delta-function. We can carry out the analysis of the conditions for the stabilization of the soliton in a weakly dissipative medium, $\Gamma\ll 1$, using a perturbation theory for solitons, similar to the one of Ref. 24.

In zeroth approximation, when $\mathbf{f}=0$, the unperturbed set of equations $\hat{L}\boldsymbol{\varphi}=0$ has a soliton solution described by Eqs. (7), (9), (11), and (12) which we denote by the vector $\boldsymbol{\varphi}_0$. We shall look for the solution of the perturbed system (17) in the form $\boldsymbol{\varphi}=\boldsymbol{\varphi}_0+\boldsymbol{\varphi}_1$, where $\boldsymbol{\varphi}_1$ is a small correction to the zeroth approximation ($|\boldsymbol{\varphi}_1|\ll|\boldsymbol{\varphi}_0|$) which in first approximation satisfies the linearized set of equations (17). In order that the correction to the zeroth solution retain its smallness for large times $t\sim\Gamma^{-1}$ it is necessary to stipulate a slow (adiabatic) change in the soliton parameters $p_i(t)$—the frequency $\tilde{\omega}(\tau)$, the wave number $\tilde{k}(\tau)$, the position $y_0(\tau)$ of the center of the soliton, the phase $\chi_0(\tau)$, and the velocity $v(\tau)$, where $\tau\sim\Gamma t$ is the "slow" time. In that case the first approximation correction $\boldsymbol{\varphi}_1$ must satisfy the equation

$$\hat{L}_1\boldsymbol{\varphi}_1=\mathbf{f}-\sum_i \mathbf{f}_i\partial_\tau p_i(\tau), \tag{20}$$

where

$$\hat{L}_1=\begin{vmatrix} i\partial_t+i\partial_y+sH+|\varphi_+|^2 & \varphi_+^2 & 2\varphi_+\varphi_-^* & 2\varphi_+\varphi_- \\ (\varphi_+^*)^2 & -i\partial_t-i\partial_y+sH+|\varphi_+|^2 & 2\varphi_+^*\varphi_-^* & 2\varphi_+^*\varphi_- \\ 2\varphi_+^*\varphi_- & 2\varphi_+\varphi_- & i\partial_t-i\partial_y+sH+|\varphi_{0-}|^2 & \varphi_-^2 \\ 2\varphi_+^*\varphi_-^* & 2\varphi_+\varphi_-^* & (\varphi_-^*)^2 & -i\partial_t+i\partial_y+sH+|\varphi_-|^2 \end{vmatrix}, \tag{21}$$

while $\mathbf{f}_i=\partial_{p_i}(\varphi_+,-\varphi_+^*,\varphi_-,\varphi_-^*)_0$, where the derivatives are with respect to the parameters of the soliton solution $\boldsymbol{\varphi}_0$. To eliminate the secular terms in the correction $\boldsymbol{\varphi}_1(t)$ to the zeroth solution, which cause it to grow rapidly with time, it is necessary to satisfy the condition that the right-hand side of the linearized system of equations (19) be orthogonal to the solutions of the homogeneous adjoint system $\hat{L}_1^\times\mathbf{g}_i=0$. This yields an equation for the evolutionary change of the soliton parameters

$$\partial_\tau p_j\left\langle\sum_i f_i\mathbf{g}_j\right\rangle=\langle f\mathbf{g}_j\rangle,\quad j=1,2\dots, \tag{22}$$

where $\langle\rangle$ denotes integration over the whole of the y-axis.

One checks easily that the vectors $\partial_{pi}\boldsymbol{\varphi}_0$ ($i=1$ to 5) are solutions of the homogeneous adjoint set and enable us to find the corresponding equations for an abbreviated description of the soliton.

For simplicity we consider the case of a soliton at rest, $v,k=0$, which in zeroth approximation is described by the following formulas

$$\varphi_\pm^0=\left(\frac{H}{2}\right)^{1/2}\exp\left[\iota\left(\chi_0\pm\frac{\chi}{2}-\tilde{\omega}t\right)\right], \tag{23}$$

where

$$H=\frac{(4/3)(1-\tilde{\omega}^2)}{s\tilde{\omega}+\mathrm{ch}[2(y-y_0)\sqrt{1-\tilde{\omega}^2}]},\quad \cos\chi=-\tilde{\omega}-\frac{3}{4}sH.$$

Let us analyze the conditions for its stabilization when we switch on a point source generating at the point $y=0$ a wave φ_- at the synchronization frequency, when $\tilde{\omega}_1=0$. It is clear from symmetry considerations that the stabilization of the soliton will take place in the point $y_0=0$ and it is thus sufficient to consider the evolutionary equations describing the change in $\omega(t)$ and $\chi_0(t)$. It follows from (19), (20) and (21) that the required equations have the form

$$\frac{2s\partial_t\tilde{\omega}}{\sqrt{1-\tilde{\omega}^2}}=4\Gamma\arccos(s\tilde{\omega})+\sqrt{6}h_0\sqrt{1-s\tilde{\omega}}\sin\beta,$$

$$\partial_t\beta=-\tilde{\omega}-\frac{\sqrt{3}h_0}{2}\sqrt{1+s\tilde{\omega}}\cos\beta, \tag{24}$$

where $\beta=-\tilde{\omega}t+\chi_0(t)+\chi_h+\chi(0)/2$.

When there is no additional pumping of energy, when $h_0=0$, the first equation of the set (24) describes the relaxation of the mismatch frequency $\tilde{\omega}$ to the edge of the nontransmission band, $\tilde{\omega}\to s$, near which the soliton vanishes, in complete agreement with Eq. (16).

In the general case equilibrium states of the set (24), $\partial_t\tilde{\omega}=\partial_t\beta=0$, are reached for values of the frequency $\tilde{\omega}$ satisfying the equation

$$s\tilde{\omega}=\cos\left\{\frac{h_0}{2\Gamma}\sqrt{(1-s\tilde{\omega})\left[\frac{3}{2}-\frac{4\tilde{\omega}^2}{h_0^2(1+s\tilde{\omega})}\right]}\right\}. \tag{25}$$

This equation always has a solution $s\omega=1$, corresponding to a stable state with a zero soliton amplitude near the corresponding edge of the nontransmission band. For $h_0<\pi(2/3)^{1/2}\Gamma$ this equilibrium state is the only one. However, when the threshold value $h_0=\pi(2/3)^{1/2}\Gamma$ is reached a new equilibrium position appears for $\tilde{\omega}=0$, $\beta=3\pi/2$ corresponding to a node in the phase plane of the system (24). When the amplitude of the source changes into the range of values $\pi(2/3)^{1/2}\Gamma<h_0<3\pi(2/3)^{1/2}\Gamma$ there appear two equilibrium points with nonvanishing soliton amplitudes of a saddle-point and focal point type. A numerical analysis of Eqs. (24) shows that there appear

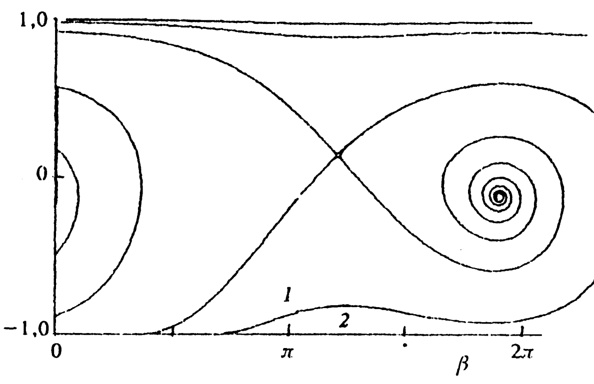

FIG. 1. Phase portrait of the set of equations (24) obtained by integrating them numerically for $h_0(3/2)^{1/2}=2\pi\Gamma=0.3$.

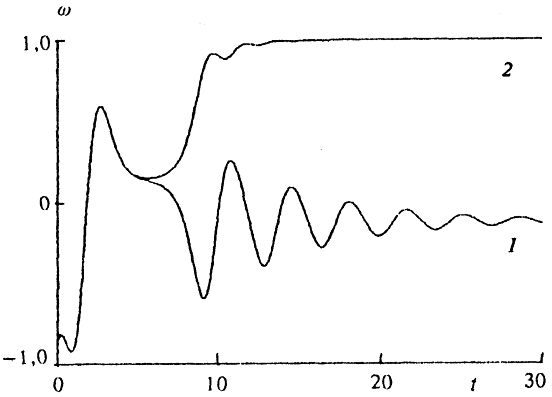

FIG. 2. Evolution of the mismatch frequency of a self-localized state of coupled waves for two different initial conditions: *1*—Initially the trajectory is chosen in the region of the phase portrait in Fig. 1 where the soliton solution is stabilized. *2*—The initial state is chosen outside the region of attraction to the equilibrium point with nonvanishing amplitude.

then in the phase portrait of the system (Fig. 1) regions where the phase trajectories are attracted to a stable equilibrium point. Outside these regions the frequency $\tilde{\omega}$ tends to the edge of the gap, $\tilde{\omega} \to s$, as for h_0. This is clear from a comparison of the two functions $\tilde{\omega}(t)$ in Fig. 2 obtained for different boundary conditions (curves *1* and *2*).

The stabilization of the soliton thus depends on its initial amplitude and phase agreement with the source. When the amplitude of the stabilizing source is increased further new pairs of stable equilibrium positions will appear. The fact that notwithstanding the nonlinear nature of the excitation of one of the coupled waves the stabilization process of coupled waves has characteristic features of the parametric stabilization of solitons[25] deserves attention.

CONCLUSIONS

The calculations performed here show thus that when nonlinear magnetostatic waves are excited in a magnetic film by a traveling sound-wave envelope solitons of parametrically coupled waves may appear in the region of the nontransmission band. Their amplitude depends on the phase mismatch frequency of the interacting waves and the soliton velocity. The solitons are "softly" produced near the edge of the nontransmission band in accordance with the Lighthill self-localization criterion and "rigidly" (with a finite amplitude) produced near the opposite edge. Outside the nontransmission band there are no self-localized coupled waves with a uniform phase shift. The soliton velocity can vary from zero to the group velocity of the signal wave when the conditions for phase synchronism of the interacting waves are satisfied.

Dissipation causes a change not only in the amplitude, but also in the width and mismatch frequency of the coupled waves in the solitonlike packet. When there is a source present for the generation of one of the coupled waves a self-localized state of parametrically coupled waves may be stabilized provided the power of the generation source exceeds a threshold value and the necessary conditions for the agreement of the initial phase of the soliton and of the generation source are satisfied.[26]

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[1)] A similar kind of nonlinearity was considered in Ref. 22 in an analysis of the effects of the propagation of nonlinear electromagnetic waves under conditions of Bragg reflection by a static periodic lattice. In that case, however, the absence of cross-terms of the $\varphi_+\varphi_-^*$ and $\varphi_+^*\varphi_-$ kind in the nonlinear frequency shift requires a justification.

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