Thermoelectric convection in a horizontal fluid layer

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We analyze a new thermoelectric mechanism for the excitation of motion during heating. The action of the new mechanism is systematically compared with the excitation of the motion by the Archimedean force and the thermocapillary effect. In our considerations we use the example of liquid semiconductors (semimetals). Heating is possible both along and across the layer. We show that heating along the layer leads to a motion sharply (resonantly) increasing under the same conditions under which cellular motion is excited during preheating across the layer. This result is obtained on the basis of an exact solution of the appropriate problem. We consider the conditions for the excitation of an instability in the fluid film simultaneously heated from both directions. The thickness of the film changes during the heating. We analyze the effect of the new mechanism on the thickness. We give a numerical solution of the problem of the simultaneous, self-consistent determination of the temperature and the thickness of the film under such motion conditions. We compare existing experiments with proposed new ones.

1. If the thermoelectric effect is possible in a fluid, a temperature gradient $A=|\nabla T|$ leads to the appearance of a force which is electric in character. The volume force density is

$$f=enE=\varepsilon\gamma^2|\nabla T|\Delta T,$$

where $E=\gamma A$ is the thermoelectric field produced by heating, $en=\varepsilon\gamma\Delta T$ is the volume charge density produced by this field in a fluid with permittivity ε, γ is the thermo-emf coefficient, and n is the density of carriers with charge e. The simplest example of such a fluid is a liquid semiconductor (semimetal).

It is just this force which competes with the Archimedean force produced by thermal expansion[1,2] and is equal to $\rho\beta gT$ (ρ is the density of the fluid, β the thermal expansion coefficient, and g the free-fall acceleration) and with the thermocapillary force[3,4] produced by the temperature dependence of the surface tension coefficient α and equal to $\sigma\Delta T$ (σ is the thermocapillarity coefficient). The thermoelectric force leads to motion of the fluid. This motion is one variety of the electroconvection structures[5] and the thermoelectric mechanism leads to an instability also when there is heating from above or from a free surface when Archimedean (Rayleigh) or thermocapillary (Pearson) convection is impossible.

The action of the thermoelectric force is characterized by the dimensionless number

$$\mathscr{E}=\frac{\varepsilon\gamma^2A^2h^2}{\rho\varkappa\nu}. \qquad (1)$$

This number shows by how much the thermoelectric force exceeds the dissipation force due to viscosity and thermal conductivity (ν and $\varkappa$ are the kinematic viscosity and thermal conductivity coefficients). The characteristic dimension of the motion in the fluid is h, the thickness of the layer; if we use this most suitable model.

The number $\mathscr{E}$ has the same meaning as the Rayleigh number R or the Marangoni number M:

$$R=\frac{\beta gAh^4}{\varkappa\nu},\quad M=\frac{\sigma Ah^2}{\rho\nu\varkappa}, \qquad (2)$$

which characterize the action of the Archimedean[2] and thermocapillary[4] forces, respectively.

It is clear from Eqs. (1) and (2) that the numbers R and M are proportional to the heating A to the first power, and therefore in the case of heating from above or from a free surface, i.e., when $A=A_z$ (the z-axis is perpendicular to the layer), the instabilities corresponding to them will not occur. The number $\mathscr{E}$ is independent of the direction of the heating and it is thus important to take thermoelectricity into account when analyzing experiments about heating from above.[6–10]

Experiments on the melting of samples by a laser beam are carried out under conditions when the heated part of the surface is much smaller than the total area. Experiments on local heating require for their explanation a consideration of the motion occurring under the action of heating in the horizontal direction, $A=A_x$ (the x-axis is directed along the layer).

Of course, Rayleigh convection and the thermocapillary effect have a stabilizing effect[11] on the excitation of the instability when there is heating from above (or from a free surface), but in actually observed melts with a thickness up to several tens of μm[7,8] it is just the thermoelectricity which should be the main cause of the motion. Since the dimensions of the fluid in the longitudinal direction are much larger than in the transverse direction, we can use the "film" approximation. Its principal premise is that the

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velocity along the layer, $v_x = v$, can be assumed to be much larger than the velocity of the same fluid element in the direction perpendicular to the film.

The rest of the paper is constructed as follows: in §2 we analyze an exact solution of a model problem. The exact solution of the equations of free convection with a constant longitudinal temperature gradient is given for a plane horizontal fluid layer in the Appendix. In reality this is motion in the central part of the heated zone. The motion under such conditions[12] occurs at arbitrarily small values of $\mathscr{E}$. It turns out that for $\mathscr{E} = \mathscr{E}_* = 4\pi^2$, i.e., when the number $\mathscr{E}$ reaches the value necessary for the excitation of thermoelectric convection under transverse heating,[11] the velocity in the longitudinal direction increases steeply (resonantly). In § 3 we solve the case when the temperature in the heating pulse varies both along the x-axis and along the z-axis, i.e., when the temperature in the film varies both along and across the layer. Up to then we consider a layer of constant thickness. In § 4 we analyze how the thermoelectric effect affects the thickness of the layer. It turns out that the film thickness is the smaller the larger the heating, and the layer outside the heating region consists of "hills" and "dales," i.e., its surface is wavelike. In § 5 we consider the self-consistent problem of the simultaneous calculation of the film thickness and the fluid velocity in it. § 6 is devoted to an analysis of experimental data which, however, are scanty.

2. We consider the solution of the problem of convection for a plane horizontal layer of a liquid semiconductor. Motion of the kind considered arises, for instance, in the central part of a wide rectangular vessel with a plane horizontal bottom. If one of the vertical walls ($x=0$) is cold and the opposite one hot, it is impossible to have equilibrium for the fluid and motion occurs for an arbitrarily small temperature difference. Such problems have been solved in Refs. 4 and 12 to 14.

We can write the boundary condition for the temperature in the form

$$T = Ax \quad \text{for } z=0 \text{ and } z=h. \tag{3}$$

The z-axis is directed vertically upwards and the x-axis from the cold to the hot wall. The motion is independent of the y-coordinate. Here and henceforth we reckon the temperature T from the value $T(0)$ on the cold wall and A is the horizontal temperature gradient, $A \equiv A_x$. In the thin film approximation we can assume that the flow occurs only along the bottom and the surface so that $v_y = 0$, $v_z = 0$, $v_x = v(z)$. The temperature T, the pressure p, and the carrier density n depend on both coordinates.

The equations of motion (the x- and z-components of the Navier-Stokes equation) take the form

$$\nu \frac{\partial^2 v}{\partial z^2} - \frac{1}{\rho} \frac{\partial p}{\partial x} = -\frac{en\gamma}{\rho} \frac{\partial T}{\partial x}, \tag{4}$$

$$-\frac{1}{\rho} \frac{\partial p}{\partial z} + g\beta T = -\frac{en\gamma}{\rho} \frac{\partial T}{\partial z}. \tag{5}$$

Using the fact that the electric field strength is caused solely by thermoelectricity, as in the equations above, i.e., that the field strength is equal to $\gamma \nabla T$, we can easily eliminate n by using the electrostatic equation. We have

$$en = \gamma\varepsilon \operatorname{div} \nabla T = \gamma\varepsilon \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2} \right). \tag{6}$$

It is possible to find an exact but very cumbersome solution of the problem posed here (see Appendix).

We turn to an analysis of the results. Of course, for $\gamma = 0$ ($\mathscr{E} = 0$) Eqs. (5) and (6) change into the exact solutions for the flow of the kind considered for the case when only the thermocapillary and the thermal expansion effects are operating; these solutions were found in Refs. 12 to 14. It is also natural that there occurs at the boundaries (for $\xi = z/h = 0$ and $\xi = 1$) a surface charge density which is constant, $\sigma = \varepsilon E_z(0) = \varepsilon E_z(1)$, but varies under different boundary conditions. The velocity profile in the case of two rigid boundaries is antisymmetric and for $\xi = 0.5$ we have $v = 0$. In the lower part of the film in the case of heating along the layer the velocity under such boundary conditions has the same direction as the heating gradient (up to $\xi = 0.5$) and reaches its maximum for $\xi = 0.25$. In the upper part ($0.5 < \xi < 1$) the velocity is in the opposite direction and reaches the same maximum value for $\xi = 0.75$. We can similarly describe the motion when there is a free boundary present (see below in §6), as will be done for an analysis of the experiments.

However, it is more important that the exact solution enables us to reveal also solution properties which appear just because of the action of the thermoelectric effect and are not present in a fluid in which this effect is not important. We are referring to the steep (resonant) increase of all the characteristics of the motion when the number $\mathscr{E}$ approaches a value $\mathscr{E}_*$ $4\pi^2$ for the motion between rigid boundaries and given by

$$\sqrt{\mathscr{E}_*} = \frac{3\pi}{2} - \frac{2}{3\pi} - \frac{2}{5}\left(\frac{2}{3\pi}\right)^3 - \frac{13}{15}\left(\frac{2}{3\pi}\right)^5 - \dots$$
$$\simeq 4.5, \quad \mathscr{E}_* \simeq 20$$

when there is a free boundary present. This formula is the result of solving the equation $\tan \sqrt{\mathscr{E}_*} = \sqrt{\mathscr{E}_*}$.

The quantities v, T (in any x-cross-section), n, E_z, the heat flow Q_z, and $F_x = enE_x$ increase near $\mathscr{E}_*$ as $(\sqrt{\mathscr{E}_*} - \sqrt{\mathscr{E}})^{-1}$ and the heat flows in the horizontal direction,

$$Q_x = \rho C_p \left[-\varkappa \int_0^h \frac{\partial T}{\partial x} dz + \int_0^h vT dz \right]$$

(C_p is the heat capacity of the fluid at constant pressure) and the force per unit volume acting in the fluid in the direction perpendicular to the layer, $F_z = enE_z$, even as $(\sqrt{\mathscr{E}_*} - \sqrt{\mathscr{E}})^{-2}$. This effect is characteristic just for thermoelectricity and is not present when $\gamma = 0$.

Such a resonant increase when one heats from the side is a manifestation of the electric nature of the thermoelectric force. Indeed, both the Archimedean force produced by thermal expansion and the surface force produced by the thermocapillary effect depend significantly on the direction of the heating. In contrast, the thermoelectric force is independent of the direction of the heating and excites an instability both for heating in a direction perpendicular to the layer[11] and for heating along the layer. This is just the reason why the resonant values of $\mathscr{E}$, which are approximately equal to 40 and 20, are close to the values necessary for the excitation of cellular motion.

CONCLUSION

In sufficiently thin layers heating from the side can reach a value such that in fluids in which a thermoelectric effect is possible, instability sets in when the dimensionless number $\mathscr{E}$, which characterizes the action of this effect, approaches the value $\mathscr{E}_*$. There occurs then a deviation from a one-dimensional motion and cellular motion is excited.[11] It is not possible to excite instability when one heats from the side by normal convection or by the thermocapillary effect.

3. A sample can be heated by laser radiation in a melt both along the layer $A=A_x$, and at right angles $A=A_z$. It is well known that, since the heating is from above (from a free surface), in a fluid with $\beta>0$ and $\sigma<0$ excitation of the cellular-motion instability is impossible either by Archimedean forces[1,2] or by the thermocapillary effect[3,4] but it is possible, for instance, by thermoelectricity[11] both for heating along and heating across the layer. In the case of heating across the layer the Archimedean force and thermocapillarity turn out to be stabilizing effects, but not in the case of heating along the layer.

We consider therefore a liquid semiconductor (semimetal) layer acted upon solely by thermoelectric forces, with the aim of elucidating the conditions for the excitation of the instability. We shall discuss the effect of other forces later.

The linearized equations of motion (two projections) of electrostatics and thermal conductivity, in the same film approximation in the central part of the melt, i.e., for $v=v_x \gg v_z$, relate the deviations of the velocity, v, of the temperature, $T_1=T-T_0$, of the pressure, $p_1=p-p_0$, and of the electric charge carrier density. We have

$$\frac{\partial p_1}{\partial z}=e\gamma n_1 A_z, \quad \frac{\partial p_1}{\partial x}=\rho\nu\frac{\partial^2 v}{\partial z^2}, \tag{7}$$

$$en_1=\varepsilon\gamma\frac{\partial^2 T_1}{\partial z^2}, \quad \varkappa\frac{\partial^2 T_1}{\partial z^2}=vA_x. \tag{8}$$

The incompressibility equation $\partial v/\partial x+\partial v_z/\partial z=0$ enables us to determine the vertical velocity component which does not enter in the remaining equations in the approximation used. We assume the field to be thermoelectric. Eliminating all variables except v we find

$$\frac{\partial^4 v}{\partial z^4}=\frac{\varepsilon\gamma^2 A_z A_x}{\rho\nu\varkappa}\frac{\partial^2 v}{\partial x\partial z}.$$

If we introduce the natural length unit, the layer thickness h, it is clear that the start of the instability is characterized by the dimensionless "film" number

$$\mathscr{E}^f=\frac{\varepsilon\gamma^2 A_x A_z h^2}{\rho\nu\varkappa},$$

which is completely analogous to the number $\mathscr{E}$ by which we characterized earlier the action of thermoelectricity.

The equation itself is solved by the method of separation of variables. It is clear that

$$v=v(z)\exp\left(-k_x\frac{x}{h}\right),$$

where the separation constant $k_x>0$ since the heating is the largest at the center of the layer (for $x=0$). In fact, k_x determines the size of the region of the motion in the longitudinal direction, λ, according to the law $k_x=2\sqrt{2}\pi h/\lambda$, if we assume that a rectangularly shaped cell is excited.

As usual, the problem of the excitation of free convection[2] is a dual eigenvalue problem. Indeed, the homogeneous system (7) and (8) must be supplemented by homogeneous boundary conditions. The lower boundary ($z=0$) can under actual conditions always be assumed to be rigid, $v=0$, and isothermal, $T_1=0$. This is a boundary with a substrate. Numerical calculations show that if the upper boundary is the same, motion appears when the film number which characterizes the action of thermoelectricity reaches the value

$$\mathscr{E}^f_*=5.64/k_x.$$

On the other hand, assuming that the excitation of the instability occurs for values of $\mathscr{E}$ close to the resonant one (see §2), i.e., for $\mathscr{E}^f\simeq 40$, we find that $\lambda\simeq 70h$, i.e., the cell is strongly elongated in the longitudinal direction. This result may serve as an indirect confirmation of the validity of the chosen film model. For estimates we must put $A_x\simeq A_z$.

The effect of the Archimedean force $\rho\beta g T_1$ which we can take into account in Eq. (7) means that we must in all calculations replace $\mathscr{E}^f$ by $\mathscr{E}^f b-R$ where b is a number of the order of 100. Using Eqs. (1) and (2) we find that the thermoelectric effect dominates for film thicknesses

$$h\lesssim\left(b\frac{\varepsilon\gamma^2\varkappa\nu}{4\beta^2 q^2\rho}\right)^{1/6}, \tag{9}$$

although the Archimedean force may exert a stabilizing action when we heat across the layer.

4. So far we have considered a layer of constant thickness. If the film has a free surface it may change its thickness under the action of the heating.

The condition determining the thickness of the layer (film) is the condition that the pressure have no discontinuity on the free surface. This condition must be written down with the field taken into account.[15] The field is due to the action of the thermoelectric effect. Therefore we have

$$p-p_0=\rho g(h-z)-\tfrac{1}{2}\varepsilon\gamma^2 A^2. \tag{10}$$

We shall assume that the temperature varies only along the layer, i.e., that, as in §2, $A=A_x$. We shall consider the general case in §5. Here, on the other hand, we shall assume that the temperature is a given function of the coordinate along the layer, $A=A(x)$. We shall assume that the thickness variation is sufficiently smooth so that we can neglect the change in pressure due to the twisting of the surface. This approximation is in complete agreement with the film approximation, $\lambda \gg h$ and $v = v_x \gg v_z$, which we assume to be satisfied. All forces inside the fluid are unimportant under such conditions and there remains solely the viscous friction force which prevents spreading by the pressure. The connection between these forces is given by Eq. (7).

Integrating this equation twice after using (10) to substitute the pressure into it we find

$$\rho \nu v = \left(\frac{z^2}{2} - zh\right)\left[g\frac{d(\rho h)}{dx} - \varepsilon\gamma^2 A\frac{dA}{dx}\right] - \frac{g}{2}\left(\frac{z^3}{3} - zh^2\right) \times \frac{d\rho}{dx} + z\frac{d\alpha}{dx}.$$

To determine the integration constants we used the condition of "rigidity" at the bottom, $z=0$, and the presence of thermocapillarity at the free surface, $\nu\rho dv/dz = d\alpha/dx = -\sigma dTdx$. We recall that α is the surface-tension coefficient. We now use the condition that the flow is closed. We find that in the more realistic case when $Ah = T$ is independent of h that we can write the equation in the form

$$h^2 = h_0^2\left(\frac{\rho_0}{\rho}\right)^{3/4} - \frac{3\sigma}{g\rho^{3/4}}\int \rho^{-1/4} dT + \frac{2\varepsilon\gamma^2}{g\rho^{3/4}}\int \rho^{-1/4} T d\left(\frac{T}{h}\right), \tag{11}$$

where h_0 is the thickness of the layer at the place where $\rho = \rho_0$, $T = T_0$, and so on. If the temperature changes little along the layer we find

$$h^2 = h_0^2\left(\frac{\rho_0}{\rho}\right)^{3/4} + \frac{3}{g\rho}(\alpha - \alpha_0) + \frac{2\varepsilon\gamma^2 T^2}{g\rho h_0}\left[1 - \left(\frac{\rho}{\rho_0}\right)^{3/8}\right].$$

This formula changes into the relation given in Ref. 1 when there is no thermoelectric effect.

5. When evaluating the conditions for excitation (§3) we assumed that the thickness of the fluid layer even with a free boundary was given. The temperature distribution was determined by the heating conditions. On the other hand, when calculating the thickness we assumed that the temperature distribution in the layer was given. In reality[9,10] neither the one nor the other quantity is given independently, but they must be determined simultaneously, i.e., self-consistently, from the theory. We can assume that the characteristics of the external heating pulse the given parameters may assume, as a very rough approximation, that A_x and A_z are such parameters.

Thus, let us have $T - T_0 = A_x x$ at the boundaries of the layer (one can solve the problem also for different horizontal temperature gradients on the boundaries) while inside the temperature is determined by the vertical gradient and can be written as $A_z z + T$. The lower boundary $z=0$ of the layer is rigid: $v_x = 0$, $v_z = 0$, and at the upper one the viscous tension is balanced by the thermocapillarity.

Under appropriate conditions, the problem posed here goes over into the problem solved above.

Nonlinear terms such as $v_x \partial v_x/\partial x$ and $v_z \partial v_x/\partial z$ were dropped from the equations since the velocities of the convective motions are always small as compared to the characteristic velocities $\varkappa/h$ and ν/h.

One can, of course, solve the problem posed here only numerically. In the first approximation we always applied the layer model. Taking onto account the actual thickness of melted layers[7,8] we assumed the main force to be the thermocapillary one, in the second approximation we took the thermoelectric force into account in thin layers, and in thicker ones the Archimedean force. The thickness for which the main force is the thermoelectric one,

$$h < \left(\frac{\rho\varkappa\nu\varepsilon\gamma^2}{\sigma^2}\right)^{1/2}, \tag{12}$$

can be reached only when the layer becomes thinner under the action of the heating.

The calculation of the excitation conditions shows that a consistent change in the layer thickness leads only to small changes in the results obtained in §3. Cellular motion does not occur in first approximation, and in second approximation cells appear under the action of thermoelectricity, with $\lambda/h \simeq 70$ for $\mathscr{E}_* \simeq 30$ to 40. The results of the calculations in the first approximation are close to those obtained in Ref. 13. The horizontal velocity component has a parabolic profile and is determined by the value of A_x for $z=h$. The vertical component v_z has a cubic profile in z and is determined by the quantity $\partial A_x/\partial x$ for $z=h$. Of course, both velocity components are proportional to the number M which characterizes the action of thermocapillarity.

Taking thermoelectricity into account thus leads to an increase of the longitudinal dimension of the cell by an order of magnitude. This result is due just to the action of the thermoelectric field produced by the longitudinal temperature gradient. When there is no thermoelectric field, cellular motion occurs only under the action of a transverse gradient A_z and leads to cells with $\lambda/h \approx 3$ to 5. One can approximate the numerical solutions obtained also by harmonic functions. This is principally important for the x-dependence of the calculated quantities.

Using the relation

$$\beta T = a\cos\left(\frac{2\pi x}{\lambda}\right)$$

which is the usual one in the theory of convection in a layer,[2] and the fact that

$$\rho = \rho_0(1 - \beta T)$$

one can determine from the formula which in these calculations replaces Eq. (11) how the layer thickness depends on the coordinate along the layer in a form which is close to repeating the results of the numerical calculations in the self-consistent problem. For $\mathscr{E}^f/R \lesssim 1$ the largest terms in the relative change in the layer thickness in the $x/h_0 \gg 1$ region will be

$$\delta h = \frac{h - h_0}{h_0} = \frac{5}{16}a\left[\frac{\pi\mathscr{E}^f}{R}\frac{2\pi x}{\lambda} - \frac{6}{5}\cos\left(\frac{2\pi x}{\lambda}\right)\right].$$

Hence it is clear that the layer thickness for motion along the layer from hot to cold regions increases gradually, executing oscillations around the line of this increase. The slope of the line is determined by the quantity $\mathscr{E}^f/R$ and is small in accord with the assumptions of the model used. The coordinates of the points in which the deviation from the initial even surface is extremal are equal to

$$x_{\rm ex}=\frac{\lambda}{2\pi}\left[\arccos\left(\frac{5\pi}{6}\frac{\mathscr{E}^f}{R}\right)-\frac{\pi}{2}\right].$$

The largest difference in layer thickness will be

$$\Delta h=\frac{3h_0a}{8}\sqrt{1-\left(\frac{5\pi\mathscr{E}^f}{R}\right)^2}.$$

The thermocapillary effect, which is the main effect producing motion, leads to a total increase in the level (thickness) in the "cold" region considered by $2Mh_0/R$.

The simultaneous determination of the characteristics of the motion (convection) and the thickness of the fluid layer thus shows that it is necessary to take the temperature distribution which appears under the action of heating into account when calculating the thickness. However, the change in the thickness affects the characteristics of the motion but little, at any rate, in the problem considered here. On the other hand, the model considered is applicable for small relative changes in the thickness. For instance, the calculation of the coordinates of the points in which the layer, so to speak, disappears, $h=0$, goes beyond the present framework. In the general case, however, when the pressure produced by the thermoelectric field enters into the discontinuity of the pressure at the free surface (10) as

$$\varepsilon\gamma^2\left(A_x\frac{\partial T}{\partial x}+A_z\frac{\partial T}{\partial z}\right),$$

one must determine the film thickness from the formula

$$\frac{h}{h_0}=1+\frac{3}{8}\beta T+\frac{2M}{R}-\frac{11}{13}\frac{\mathscr{E}B}{R}\beta T-\frac{\mathscr{E}^f}{R}h_0\int_0^x \times(1+\beta T)\frac{\partial^2(\beta T)}{\partial x^2}dx.$$

This formula has been written down taking into account all three effects produced by the motion and taking into account the conditions for heat transfer at the free surface. So far the solution has been carried out only either for the case of thermal insulation, $\partial T_1/\partial z=0$, or for the isothermal case, $T_1=0$. The thermocapillary effect is impossible in the case where the free surface is isothermal. In the general case we have

$$\frac{\partial T_1}{\partial z}=-\frac{BT_1}{h_0},$$

where $B=\eta h_0/\rho C_p\varkappa$ is the Biot number characterizing the conditions of heat transfer from the free surface,[3] where η is the heat transfer coefficient. In the region where we carried out the numerical calculations, i.e., for $x=\lambda/4$, we can assume that the quantity $(2M/R)h_0$ is enclosed by the layer thickness if we solve the problem for a layer with constant thickness.

6. The kind of experiments, for the interpretation of which we can use the results of the theoretical analysis given here, are the experiments mentioned already in §1 on the heating from above by laser light.[6–10] A detailed analysis of the table of ratios of the sizes in the alloyed zone,[8] i.e., the zone in which motion took place, shows that the best fit occurs for the ratio of the transverse and longitudinal dimensions, which is close to $\lambda/h\approx 40$, i.e., the ratio λ/h occurring when the thermocapillary and thermoelectric effects act simultaneously in the case of heating along the layer. This does not exclude the possibility that heating in the perpendicular direction there can also produce cells inside with a much smaller ratio λ/h.

Unfortunately, the experiments described in Refs. 6 to 10 were performed for particularly practical purposes and far from the conditions of the models we considered. Also the parameters of the heating which occurs when the laser radiation is absorbed are badly known. For the melts we take the fluid parameters:[10]

$$\beta=(9\text{–}6)\cdot 10^{-4}\text{K}^{-1},\quad \rho=1\text{–}10\ \text{g/cm}^3,$$

$$\nu\approx\varkappa\approx 5\cdot 10^{-2}\text{–}1\ \text{mm}^2/\text{s},$$

$$\sigma=(1\text{–}10)\cdot 10^{-5}\ \text{G/m}\cdot\text{K}.$$

We can write the thermoelectric-power coefficient in the form $\gamma=dk_B/e$ where k_B is the Boltzmann constant, e the electron charge, and we can take the number d to be equal to 10 to 100. We can assume the heating, the difference in the temperatures acting on the layer, to be equal to $\Delta T\simeq 10^3$ to 10^4 K. Using Eqs. (9) and (12) to estimate the thickness we see then that for actual melt thicknesses up to tens of μm the main effect causing motion is the thermocapillary one and the thermoelectric effect on the background of the latter must be taken into account as a correction.

Numerical calculations of the current lines in the fluid, $dx/v_x=dz/v_z$ with velocities v_x and v_z actually coinciding with those found in Ref. 13, show that one can obtain lines resembling the "tungsten filaments" shown in the photographs in Ref. 6. It is unclear, however, in how far these filaments can be identified with current lines.

Unfortunately, it was completely impossible to find data about the temperature distribution in the melt. The crucial quantity, the constant a in the expression for βT, therefore remains unknown. However, it is a known fact that additional material is carried away to the edge of the alloyed zone (crater) for powerful pulses (more than 5 J per cubic mm of the melt); it was observed and also occurs in calculations.[8] The crater itself is, of course, the result of simple evaporation, but the fact of the raising of the edge lies within the confines of the theory developed here.

We were also unable to find a description of experiments on heating, from above or from the side, of a fluid between rigid plates, for instance, in a microwave oven, so that we indicate only effects expected for heating from a free surface. The velocity of the fluid is largest at the surface and after that it vanishes for $\xi=0.6$. Even closer to the bottom the flow is in the opposite direction. The velocity is maximal for $\xi=0.3$.

The result that the velocity of the fluid along the surface must be from "hot" to cold" corresponds to the motion from the center of the heating pulse to the periphery.[6–8] Appendix Exact solutions of Eqs. (4) to (6)

Eliminating the density and also the pressure we get the equation

$$\frac{\partial^3 v}{\partial z^3}-\frac{\beta g}{\nu}\frac{\partial T}{\partial x}=-\frac{\gamma^2 \varepsilon}{\rho \varkappa \nu}\frac{\partial v}{\partial z}\left(\frac{\partial T}{\partial x}\right)^2,$$

which must be solved together with the heat conduction equation

$$v\frac{\partial T}{\partial x}-\varkappa\left(\frac{\partial^2 T}{\partial z^2}+\frac{\partial^2 T}{\partial x^2}\right)=0.$$

The continuity equation is satisfied identically. One can solve the problem exactly when it is stated in this way.

Of course, for a complete solution one must supplement the system (4) to (6) by boundary conditions. On the bottom the boundary is always assumed to be "rigid" and on it the sticking conditions are satisfied: $v=0$ for $z=0$. The other boundary may be "rigid": $v=0$ for $z=h$, or free.[2] On the free boundary thermocapillarity can occur and then the boundary condition will be

$$\rho\nu\frac{\partial v}{\partial z}=-\sigma\frac{\partial T}{\partial x}\quad \text{for } z=h.$$

These conditions, Eq. (3), and also the condition that the current be closed,

$$\int_0^h v dz=0,$$

are sufficient to obtain a solution.

It is convenient to write down the solution, introducing a dimensionless variable in the direction perpendicular to the layer, $\xi=z/h$. We note, first of all, some general properties: firstly, it turns out that the temperature depends linearly on the x coordinate along the layer so that $\partial T/\partial x=A$. This relation had already been used when deriving Eq. (8). Secondly, we have for the density

$$en=\frac{\gamma\varepsilon}{\varkappa}v\frac{\partial T}{\partial x}=\frac{\gamma\varepsilon A}{\varkappa}v,$$

i.e., the charges are completely frozen in. Finally, $E_x=\gamma A$.

The actual dependence of the velocity, the temperature, and the field on the coordinates is determined by the boundary conditions. In the case when the boundaries of the fluid are rigid surfaces we get

$$v=\frac{\varkappa}{h}\frac{R}{2\mathscr{C}}\left[2\xi-1+\frac{\cos(\sqrt{\mathscr{C}}\xi)-\cos[\sqrt{\mathscr{C}}(1-\xi)]}{1-\cos\sqrt{\mathscr{C}}}\right],$$

$$T=Ah\left\{\frac{x}{h}+\frac{R}{2\mathscr{C}}\left(\frac{1}{3}\xi^3-\frac{1}{2}\xi^2+\frac{1}{6}\xi\right)-\frac{1}{\mathscr{C}}\frac{v}{\varkappa/h}\right\},$$

$$E_z=\gamma A\left\{\frac{R}{2\mathscr{C}}\left(\xi^2-\xi+\frac{1}{6}\right)-\frac{R}{2\mathscr{C}^2}\times\left[2-\frac{\sqrt{\mathscr{C}}\{\sin(\sqrt{\mathscr{C}}\xi)+\sin[\sqrt{\mathscr{C}}(1-\xi)]\}}{1-\cos\sqrt{\mathscr{C}}}\right]\right\}.$$

When the fluid has a free surface and all three effects which cause motion (thermal expansion, thermocapillarity, and thermoelectricity) operate it is also possible to get an exact solution of Eqs. (4) to (6) but the formulas become cumbersome.

It is well known that in thin layers the action of the thermocapillary effect is stronger than that of the Archimedean force.[3] A comparison of $\mathscr{C}$ from (1) with R and M from (2) shows that thermoelectricity is important only in yet thinner films. We therefore give the solutions obtained taking into account only thermocapillarity and thermoelectricity. Thus, assuming that $R=0$ ($\beta=0$) we have

$$v=-\frac{\varkappa}{h}\frac{M}{\sqrt{\mathscr{C}}}\frac{(1-\cos\sqrt{\mathscr{C}})[\cos(\sqrt{\mathscr{C}}\xi)-1]+(\sqrt{\mathscr{C}}-\sin\sqrt{\mathscr{C}})\sin(\sqrt{\mathscr{C}}\xi)}{\sqrt{\mathscr{C}}\cos\sqrt{\mathscr{C}}-\sin\sqrt{\mathscr{C}}},$$

$$T=Ah\left\{\frac{x}{h}+\frac{M}{\sqrt{\mathscr{C}}}\frac{(1-\cos\sqrt{\mathscr{C}})[\mathscr{C}\xi^2+(4-\mathscr{C})\xi]-2\sqrt{\mathscr{C}}\xi\sin\sqrt{\mathscr{C}}}{\sqrt{\mathscr{C}}\cos\sqrt{\mathscr{C}}-\sin\sqrt{\mathscr{C}}}-\frac{1}{\mathscr{C}}\frac{v}{\varkappa/h}\right\}.$$

The formula for $E_z=\gamma\partial T/\partial z$ is too complicated also in this approximation.

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