Stimulated Raman scattering in an extended medium

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A model of the stimulated Raman scattering of a laser beam from a chain of atoms represented by point harmonic oscillators is examined. The effect of damping in the atomic subsystem on the formation and propagation of the Stokes pulse is discussed. Results are compared with those from the simplest possible classical model of stimulated Raman scattering in an extended medium.

1. One of the most interesting questions of the modern theory of stimulated cooperative Raman scattering is the generation and propagation of the Stokes soliton in an extended medium.[1–3]

The problem of the stimulated scattering of a laser pulse by a system of damped point harmonic oscillators can be solved exactly.[1)] However, the point-interaction assumption cannot be applied to the study of light propagation effects in a medium and is valid only for the following two geometries of an atomic subsystem:

1) all atoms are located in a small volume of radius $r<\lambda$, where λ is the characteristic wavelength of the radiation;

2) all atoms are located in a rectangular box of length $L\gg\lambda$ [but $L\ll l_p$, the characteristic width of the laser (Stokes) pulse].

We investigate here the generation and propagation of a Stokes pulse in an extended medium within the following simplified model: We visualize the extended medium as a chain of successive point "atoms." One can solve the scattering problem exactly for a field containing both a laser and a Stokes component, for each one of the point oscillators, and then use the field of the wave scattered from one atom as a field incident on the next, and so on.

Needless to say, our model leaves out of account many effects of physical importance, for example backscattering effects. However, the model makes it possible to study the influence of dissipation in an atomic system on the formation and propagation of the Stokes soliton. We hope therefore that our results will be useful for understanding the physics of stimulated Raman scattering in extended media.

2. In the point interaction model, the "slow envelope" operators of the laser (L) and Stokes (S) fields combine into the two-component spinor

$$\varepsilon_\sigma=\begin{pmatrix}\varepsilon_L\\ \varepsilon_S\end{pmatrix},\quad [\varepsilon_\sigma(x),\varepsilon^+_\mu(x)]=\delta_{\sigma\mu}\delta(x-y), \tag{1}$$

and the system of M point atoms is described by the operator

$$A^+_k=\sum_{i=1}^{M} a^+_i(x_k),\quad [A_k,A^+_k]=M_k, \tag{2}$$

which is the sum of the atom operators a_i located at the kth site of the chain.

The model Hamiltonian is written in the form

$$\varkappa=\int_{-\infty}^{\infty}dx\left\{-i\varepsilon^+_\sigma(x)\frac{\partial}{\partial x}\varepsilon_\sigma(x)-J\sum_k\delta(x-x_k)\right.$$
$$\left.\times\varepsilon^+_\sigma(x)[\sigma^+_{\sigma\sigma'}A_k+\sigma^-_{\sigma\sigma'}A^+_k]\varepsilon_{\sigma'}(x)\right\}, \tag{3}$$

where J is the two-photon interaction constant and $\sigma^\pm=\sigma^x\pm i\sigma^y$ are the Pauli matrices. Summation over repeated spin indices $\sigma=L,S$ is implied.

Let us consider the one-site scattering problem (the subscript k of the operators may be omitted from this point on). The initial state of the problem can be written in the form

$$|\mathrm{in}\rangle=\left(\frac{1}{m!M^m}\right)^{1/2}\left\{\prod_{j=1}^{N}[\alpha_j\varepsilon^+_L(x_j)+\beta_j\varepsilon^+_S(x_j)]\right\}$$
$$\times(A^+)^m|0\rangle. \tag{4}$$

Here $|\alpha_j|^2$ ($|\beta_j|^2$) is the intensity of the laser (Stokes) component of the incident field.

Using the scattering matrix of the problem in the form obtained in Ref. 4,

$$S=\exp[iJ(\sigma^+A+\sigma^-A^+)],$$

we obtain the following expression for the final state of the oscillator+field system:

$$|\mathrm{out}\rangle=\left(\frac{1}{m!M^m}\right)^{1/2}\left(\prod_{j=1}^{N}d^+_j\right)(A^+)^m|0\rangle, \tag{5}$$

where

$$d^+_j=\left[\alpha_j\cos(J\sqrt{AA^+})+i\beta_jA\frac{\sin(J\sqrt{AA^+})}{\sqrt{AA^+}}\right]\varepsilon^+_L(x_j)$$
$$+\left[\beta_j\cos(J\sqrt{A^+A})+i\alpha_jA^+\frac{\sin(J\sqrt{A^+A})}{\sqrt{A^+A}}\right]\varepsilon^+_S(x_j). \tag{6}$$

To calculate the physically observable quantities of some site operator $\hat q_n$ (we take hereafter $\hat q$ to be the point-atom population operator $\hat m=A^+A$, or the laser intensity $\hat I_L=\varepsilon^+_L\varepsilon$, or the Stokes component $\hat I_S=\varepsilon^+_S\varepsilon$, of light) we must consider the nondiagonal correlation function

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$$Q_{n+1}^{pm}=\left(\frac{1}{m!p!M^{m+p}}\right)^{1/2}$$
$$\times\left\langle 0\left|(A)^p\left(\prod_{j=1}^{n+1}d_j\right)\hat{q}_n\left(\prod_{j=1}^{n+1}d_j^+\right)(A^+)^m\right|0\right\rangle, \quad (7)$$

This correlation function is not anomalous because the operators d_j contain the operators A and A^+ and hence the correlation function is nonzero for $|p-m|<n$.

Expanding the operators d in (7) and acting on the vacuum states we get the following recursion relations for the quantities Q_n^{pm}:

$$\begin{aligned}Q_{n+1}^{pm}={}&|\alpha_{n+1}|^2\cos[J\sqrt{M(m+1)}]\cos[J\sqrt{M(p+1)}]Q_n^{pm}+i\alpha_{n+1}^*\beta_{n+1}\sin(J\sqrt{Mm})\cos[J\sqrt{M(p+1)}]Q_n^{p\ m-1}\\&-i\alpha_{n+1}\beta_{n+1}^*\sin(J\sqrt{Mp})\cos[J\sqrt{M(m+1)}]Q_n^{p-1\ m}+|\beta_{n+1}|^2\sin(J\sqrt{Mp})\sin(J\sqrt{Mm})Q_n^{p-1\ m-1}\\&+|\alpha_{n+1}|^2\sin[J\sqrt{M(m+1)}]\sin[J\sqrt{M(p+1)}]Q_n^{p+1\ m+1}-i\alpha_{n+1}^*\beta_{n+1}\cos(J\sqrt{Mm})\sin[J\sqrt{M(p+1)}]Q_n^{p+1\ m}\\&+i\alpha_{n+1}\beta_{n+1}^*\cos(J\sqrt{Mp})\sin[J\sqrt{M(m+1)}]Q_n^{pm}+|\beta_{n+1}|^2\cos(J\sqrt{Mp})\cos(J\sqrt{Mm})Q_n^{p\ m}. \qquad (8)\end{aligned}$$

These equations, together wth appropriate boundary conditions obtainable from the solution of the one-particle problem, solve formally the scattering problem completely.

One can simplify the recursion relations (8) by changing from the variables p and m to $\tilde{m}=(p+m)/2$ and $\xi=(p-m)/2$ and going over to differential equations in the classical approximation. Note that the contribution given by the derivative $\partial Q/\partial\xi$ (and indicative of the ξ-dependences) is of order $1/m$ and so can be neglected in the classical limit-the limit in which a macroscopic number of photons is scattered.

Thus we obtain the following equations to describe the behavior of the population of the atomic subsystem $m=\langle A^+A\rangle$:

$$\frac{dm}{dt}=\sin^2[g(m)][I_L^{\rm inc}(t)-I_S^{\rm inc}(t)]-\sin[2g(m)]\times\left[\frac{i}{2}(\varepsilon_L^*\varepsilon_S-\varepsilon_L\varepsilon_S^*)\right]^{\rm inc}-\gamma m, \quad (9)$$

where $g(m)=J\sqrt{Mm}$; γ is the phenomenological damping constant;[4] $I_L^{\rm inc}(I_S^{\rm inc})$ is the intensity of the incident laser (Stokes) component; and $i/2(\varepsilon_L^*\varepsilon_S-\varepsilon_L\varepsilon_S^*)$ is the (rather bizarre) "interference" term.

3. We can calculate explicitly the averages of the necessary operators for final (out) one-photon scattering states

$$|{\rm out}\rangle=\left(\frac{1}{m!M^m}\right)^{1/2}d^+(A^+)^m|0\rangle,$$

in the classical approximation. Thus, the scattered light components are given by

$$I_L^{\rm scat}(t)=\cos^2[g(m)]I_L^{\rm inc}(t)+\sin^2[g(m)]I_S^{\rm inc}(t)+\sin[2g(m)]\left[\frac{i}{2}(\varepsilon_L^*\varepsilon_S-\varepsilon_L\varepsilon_S^*)\right]^{\rm inc}, \quad (10a)$$

$$I_S^{\rm scat}(t)=\cos^2[g(m)]I_S^{\rm inc}(t)+\sin^2[g(m)]I_L^{\rm inc}(t)-\sin[2g(m)]\left[\frac{i}{2}(\varepsilon_L^*\varepsilon_S-\varepsilon_L\varepsilon_S^*)\right]^{\rm inc}, \quad (10b)$$

$$\left[\frac{i}{2}(\varepsilon_L^*\varepsilon_S-\varepsilon_L\varepsilon_S^*)\right]^{\rm scat}=-\frac{1}{2}\sin[2g(m)](I_L^{\rm inc}-I_S^{\rm inc})+\cos[2g(m)]\times\left[\frac{i}{2}(\varepsilon_L^*\varepsilon_S-\varepsilon_L\varepsilon_S^*)\right]^{\rm inc}, \quad (10c)$$

where $m(t)$ obeys Eq. (9).

Unfortunately, the system (9)–(10) can be solved only numerically. As before, we consider the atomic chain model and take the field of the wave scattered on the preceding atom as that incident on the next. Numerical results for the atomic-subsystem population evolution are presented in Fig. 1.

It turns out that the "soliton" pulse forms at a very short distance from, indeed at, the boundary of the sample (the first oscillator) and that the pulse changes its front shape only slightly as it travels through the sample. The presence of dissipation in the atomic subsystem has the effect of accelerating its saturation, but the Stokes component of the scattered wave field deviates from a pure pulse-like shape. Thus dissipation does not play a significant role in Stokes pulse formation and propagation processes. The most important term in securing the stability of the pulse propagation in the medium is the "interference" term in Eq. (9).

4. In this section we consider the simplest possible classical model of stimulated Raman scattering from an extended system of harmonic oscillators.

Consider a model Hamiltonian analogous to (3), in which the system of oscillators is described by a classical field $u(x)$ in the interval $0<x<\infty$:

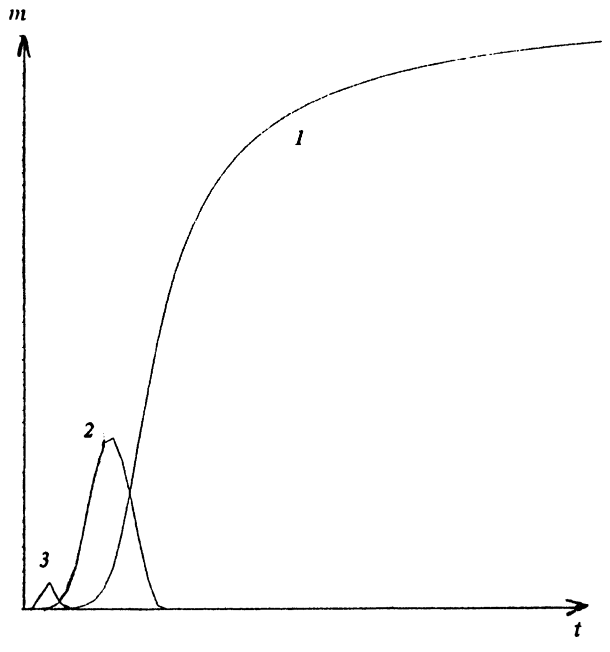

FIG. 1. Population behavior for the first (*1*), second (*2*), and fifth (*3*) atoms in the chain. It is seen that each of the atoms is less excited than its predecessor and hence exerts a diminishing effect on the propagation of the pulse.

$$\varkappa' = \int_{-\infty}^{\infty} dx \left\{ -i\varepsilon_\sigma^+(x) \frac{\partial}{\partial x} \varepsilon_\sigma(x) - Ju(x)\varepsilon_\sigma^+(x) \times [\sigma_{\sigma\sigma'}^+ + \sigma_{\sigma\sigma'}^-]\varepsilon_{\sigma'}(x) \right\}. \tag{11}$$

A model as simplified as this ignores the commutation relations and the phase of the oscillator field, both of which factors may be important.[4]

The equations of motion for the problem (11) are written in the form

$$\frac{\partial}{\partial t} u(t,x) = \{u,\varkappa'\},$$

$$\frac{\partial}{\partial t} \varepsilon_\sigma(t,x) = \{\varepsilon_\sigma,\varkappa'\}. \tag{12}$$

Using the general solution for the scattering problem,

$$\varepsilon_\sigma(t,x) = \exp\left\{ iJ \int_0^\infty dy [\sigma_{\sigma\mu}^+ + \sigma_{\sigma\mu}^-] u(t,y) \right\} \zeta_\mu(t-x), \tag{13}$$

where ζ is an arbitrary function, we obtain an equation analogous to (9) ($m \propto u^2$):

$$\frac{\partial}{\partial t} u(t,x) = \sin\left[J \int_0^x dy u(t,y) \right]. \tag{14}$$

It is evident that the asymptotic behavior of the solution to Eq. (14) is

$$u(t,x)|_{t\to\infty} = \frac{2\pi}{J} \delta(x) \tag{15}$$

in full agreement with the result of the preceding section.

The results of the numerical study of Eq. (15) are in qualitative agreement with those of Eqs. (9) and (10).

In summary, then, we have considered two simplified models to account for the formation and propagation of a Stokes soliton in an extended medium. We have obtained the following qualitative results:

1) the "soliton" pulse is formed, basically, at the boundary of the sample;

2) damping is of little significance;

3) the "interference" effect plays a crucial role in soliton pulse propagation by preventing the pulse from broadening. Hence the formation of the soliton should be extremely sensitive to the phase slip mechanisms acting in the medium.

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