Thermal radiation energy density in inhomogeneous transparent media

A. G. Zagorodniĭ, A. S. Usenko, and I. P. Yakimenko

N. N. Bogolyubov Institute of Theoretical Physics, Ukrainian Academy of Sciences, Ul. Metrologicheskaya 14b, 252143 Kiev, Ukraine

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We generalize the Planck law for the spectral energy density of equilibrium thermal radiation to the case of an inhomogeneous transparent system consisting of two homogeneous media. We study in detail the effect of the inhomogeneity of the system on the temperature-dependent component of the energy density spectrum and on the spectral component corresponding to the zero-point oscillations of the field. We show that the energy density of the quasistationary field remains finite at any distance from the boundary. We generalize the results of the theory to the case of media with different temperatures. We study the features of the frequency dependence of the spectral energy density for systems which are and which are not in thermodynamic equilibrium.

INTRODUCTION

The spectral energy density of equilibrium radiation is a universal function of the frequency and the temperature. The presence of a transparent medium reduces, according to the Kirchhoff–Clausius law, simply to multiplying the energy density of equilibrium radiation in vacuo by the cube of the refractive index of the medium. One of the most wide spread methods for obtaining the energy density of equilibrium radiation is to represent the electromagnetic field as a superposition of oscillators and to use classical or quantum statistics to find the average energy of an oscillator. The Rayleigh-Jeans and Planck distributions which are then obtained are uniform and isotropic.

The spectra of the electric and magnetic fields of the equilibrium radiation in an unbounded transparent medium can also be obtained on the basis of the general correlation theory of thermal fluctuations[1] which uses the fluctuation-dissipation theorem (FDT).[1–5] When applied to the problem of the energy density of equilibrium radiation this approach is considerably more laborious than the traditional methods used for deriving the Planck formula. Nonetheless, the method based on using the FDT turns out to be very useful since it makes it possible in a single approach to obtain the statistical averages for various quadratic quantities consisting of components of the electromagnetic field taken in the general case at different points of space. One then does not require that there is no dissipation in the system. On the contrary, the transition to a transparent medium is considered to be a special case and the corresponding correlation functions are obtained from the general relations, taking in them in the correct way[6–9] the limit to the transparent medium. Moreover, one can, in principle, on the basis of this approach calculate correlation functions also in the case of an inhomogeneous equilibrium medium.

Of course, the inhomogeneity of the medium destroys the homogeneity and the isotropy of the Planck distribution. The presence of inhomogeneity in the system leads to the appearance of quasistationary components in the fluctuating electromagnetic field which are localized in a thin surface layer near the boundary between the media. In a study of heat exchanges by the fluctuating electromagnetic field between two semi-infinite heated bodies separated by a transparent plane gap[10] it was established that the contribution of the quasistationary part of the fluctuating electromagnetic field can be substantial in the case of a narrow gap.

We study in the present paper the energy density spectra of equilibrium radiation in the case of the simplest inhomogeneous system consisting of two uniform media.

In the first section of the paper we find the spectral energy density distributions of equilibrium radiation in the case of an inhomogeneous transparent medium. We show that the energy density is finite at any distance from the boundary. We obtain the total radiation energy distribution and study its deviation from the Stefan-Boltzmann distribution. In the second section we find the spectral radiation energy density distributions in the case of media with different temperatures. In the third section we give the results of a numerical analysis of the effect of the temperature and the parameters of the media which are in contact on the spectral energy density distributions.

I. EQUILIBRIUM SYSTEM

We find the energy density of equilibrium radiation which is characterized by the temperature T in a system consisting of two homogeneous, transparent, semibounded media with the $z=0$-plane as the dividing boundary. The first and the second media, which occupy respectively the $z<0$ and $z>0$ regions are characterized by the dielectric permittivities ε_1 and ε_2. The origin of the Cartesian coordinate system is taken on the boundary between the media.

To find the spectral energy densities of the equilibrium radiation,

$$U^{(n)}(\mathbf{r},\omega)=\frac{1}{8\pi^2}\{\varepsilon_n\langle|\mathbf{E}|^2\rangle_{\mathbf{r}\omega}+\langle|\mathbf{B}|^2\rangle_{\mathbf{r}\omega}\},\quad n=1,2 \tag{1}$$

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in the $z<0$ (for $n=1$) and $z>0$ (for $n=2$) regions we need know the correlation functions of the fluctuating electric and magnetic fields of the thermal radiation in the system.

Since the system is in an equilibrium state the correlation functions of the electric field taken in the general case at different points of space, $\mathbf{r}$ and $\mathbf{r}'$, can be found by using the FDT[1-5]

$$\langle E_i(\mathbf{r})E_j^*(\mathbf{r}')\rangle_\omega=-\theta(\omega,T)\{G_{ij}(\mathbf{r},\mathbf{r}',\omega)+G_{ji}^*(\mathbf{r}',\mathbf{r},\omega)\},\quad i,j=x,y,z, \tag{2}$$

where $\theta(\omega,T)$ is the average energy of a quantum harmonic oscillator of frequency ω, in a state of thermodynamic equilibrium with a thermostat at temperature T, which is measured in energy units,

$$\theta(\omega,T)=\frac{\hbar\omega}{2}+\frac{\hbar\omega}{\exp(\hbar\omega/T)-1}, \tag{3}$$

and $G_{ij}(\mathbf{r},\mathbf{r}',\omega)$ is the Green function of a single harmonic point source, $\mathbf{I}\cdot\delta(\mathbf{r}-\mathbf{r}')\cdot\exp(-i\omega t)$ positioned in either the first (when $z'<0$) or the second (when $z'>0$) medium. The correlation functions of the magnetic field are obtained by using Eq. (2) and the connection between the electric and magnetic fields given by the Maxwell equations.

Since the system is homogeneous in the transverse directions it is convenient to use the Fourier transform

$$A(\mathbf{r}_\perp)=\frac{1}{2\pi^2}\cdot\int_{-\infty}^{\infty}d\mathbf{k}_\perp\,\exp(i\mathbf{k}_\perp\mathbf{r}_\perp)A(\mathbf{k}_\perp) \tag{4}$$

to change from the spatial variables $\mathbf{r}_\perp=(x,y,0)$ to the variables $\mathbf{k}_\perp=(k_x,k_y,0)$.

Using the expression for the Green function in the given system[11] the final relations for the Fourier components of the spectral components of the correlation functions of the electric and magnetic fields can for $z,z'<0$ be written in the form

$$\langle E_i(z)E_j^*(z')\rangle_{\mathbf{k}_\perp\omega}=\langle E_i(z)E_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega}+\langle E_i(z)E_j^*(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega},$$

$$\langle B_i(z)B_j^*(z')\rangle_{\mathbf{k}_\perp\omega}=\langle B_i(z)B_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega}+\langle B_i(z)B_j^*(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega},\quad i,j=x,y,z, \tag{5}$$

where $\langle E_i(z)E_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega}$, $\langle B_i(z)B_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega}$ are the well known correlation functions of the fluctuating electric and magnetic fields in an unbounded transparent medium with a dielectric permittivity ε_1,

$$\langle E_i(z)E_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega}=\theta(\omega,T)\frac{4\pi\omega}{c^2}\left\{\mathrm{Re}\left(\left[\left(\delta_{ij}^\perp-\frac{k_{\perp i}k_{\perp j}}{k_\perp^2}\right)+\delta_{iz}\delta_{jz}\frac{k_\perp^2}{k_1^2}\right]\frac{\exp(ik_{z1}|z_-|)}{k_{z1}}\right)-i(k_{\perp i}\delta_{jz}+k_{\perp j}\delta_{iz})\times\mathrm{Im}\left(\frac{\exp(ik_{z1}|Z_-|)}{k_1^2}\right)\mathrm{sign}\,Z_-\right\},$$

$$\langle B_i(z)B_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega}=\varepsilon_1\langle E_i(z)E_j^*(z')\rangle^{(0)}_{\mathbf{k}_\perp\omega},\quad i,j=x,y,z, \tag{6}$$

while the terms $\langle E_i(z)E_j^*(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega}$, $\langle B_i^*(z)B_j(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega}$ are caused by the inhomogeneity of the system and defined as follows:

$$\langle E_i(z)E_j^*(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega}=\theta(\omega,T)\frac{4\pi\omega}{c^2}\left\{\left(\delta_{ij}^\perp-\frac{k_{\perp i}k_{\perp j}}{k_\perp^2}\right)\times\mathrm{Re}\left[R_s(k_\perp,\omega)\frac{\exp(ik_{z1}|Z_+|)}{k_{z1}}\right]+\frac{k_{\perp i}k_{\perp j}}{k_1^2}\times\mathrm{Re}\left[R_p(k_\perp,\omega)\frac{k_{z1}\exp(ik_{z1}|Z_+|)}{k_1^2}\right]-\delta_{iz}\delta_{jz}k_\perp^2\times\mathrm{Re}\left[R_p(k_\perp,\omega)\frac{\exp(ik_{z1}|Z_+|)}{k_{z1}k_1^2}\right]+i(k_{\perp i}\delta_{jz}-k_{\perp j}\delta_{iz})\times\mathrm{Im}\left[\frac{\exp(ik_{z1}|Z_+|}{k_1^2}R_p(k_\perp,\omega)\right]\mathrm{sign}\,Z_+,\right.$$

$$\langle B_i(z)B_j^*(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega}=\varepsilon_1\langle E_i(z)E_j^*(z')\rangle^{(S)}_{\mathbf{k}_\perp\omega}|_{R_{p,s}(k_\perp,\omega)\to-R_{s,p}(k_\perp,\omega)},\quad i,j=x,y,z. \tag{7}$$

The quantities $R_{p,s}(k_\perp,\omega)$,

$$R_{p,s}(k_\perp,\omega)=-\frac{1+r_{p,s}(k_\perp,\omega)}{1-r_{p,s}(k_\perp,\omega)} \tag{8}$$

have the meaning of the Fresnel coefficients for reflection from the second medium of homogeneous (for $k_\perp<k_1$) or inhomogeneous (for $k_\perp>k_1$) electromagnetic p and s polarized waves incident from the $z<0$ region. The quantities $r_{p,s}(k_\perp,\omega)$, taken with a minus sign, are the ratio of the surface impedance of the second medium to the surface impedance of the first medium for fields with p and s polarizations,

$$r_s(k_\perp,\omega)=-\frac{k_{z1}}{k_{z2}},\quad r_p(k_\perp,\omega)=-\frac{\varepsilon_1}{\varepsilon_2}\frac{k_{z2}}{k_{z1}},\tag{9}$$

$Z_\pm=z\pm z'$, $k_n=\omega\varepsilon_n^{1/2}/c$, $k_{zn}=(k_n^2-k_\perp^2)^{1/2}$, $\mathrm{Im}\,k_{zn}\geqslant 0$, $n=1$, 2, and δ_{ij} is the Kronecker symbol.

The correlation functions of the electric and magnetic fields in the second medium ($z,z'>0$) are determined by the same Eqs. (5) to (7) as in the first medium ($z,z'<0$) with the substitutions $\varepsilon_1\leftrightarrow\varepsilon_2$ and $\delta_{iz,jz}\to-\delta_{iz,jz}$ in them.

Equations (5) to (7) make it possible to study various objects of the correlation theory of equilibrium fluctuations in the case of an inhomogeneous transparent medium. Since we restrict ourselves in the present paper to studying the radiation energy density we must evaluate the field correlations in the same point in space ($\mathbf{r}'=\mathbf{r}$). In that case the spatial correlation functions of the fluctuating electromagnetic fields are reduced to diagonal form and the required spectral energy density distributions can be written as follows:

$$U^{(n)}(\mathbf{r},\omega)\equiv U^{(n)}(z,\omega,T)$$
$$=U^{(n)}(\omega,T)+U^{(n)S}(z,\omega,T),\quad n=1,2.\tag{10}$$

Here $U^{(n)}(\omega,T)$ is the energy density of equilibrium radiation with a temperature T in an unbounded transparent medium with a dielectric permittivity ε_n which can be written as the sum of two terms,

$$U^{(n)}(\omega,T)=\theta(\omega,T)\frac{\omega^2\varepsilon_n^{3/2}}{\pi^2c^3}$$
$$=\frac{\hbar\omega}{2}\frac{\omega^2\varepsilon_n^{3/2}}{\pi^2c^3}+\frac{\hbar\omega}{\exp(\hbar\omega/T)-1}\frac{\omega^2\varepsilon_n^{3/2}}{\pi^2c^3}$$
$$\equiv U^{(n)}(\omega)+U^{(n)Pl}(\omega,T),\quad n=1,2.\tag{11}$$

The first term, which is independent of the temperature,

$$U^{(n)}(\omega)=\varepsilon_n^{3/2}U_v(\omega),\quad n=1,2,\quad U_v(\omega)=\frac{\hbar\omega^3}{2\pi^2c^3},\tag{12}$$

corresponds to the energy density of the zero-point oscillations of the field in the medium while the second term, $U^{(n)Pl}(\omega,T)$, is determined by the classical Planck formula,

$$U^{(n)Pl}(\omega,T)=\varepsilon_n^{3/2}U_v^{Pl}(\omega,T),\quad n=1,2,$$

$$U_v^{Pl}(\omega,T)=\frac{\hbar\omega}{\exp(\hbar\omega/T)-1}\frac{\omega^2}{\pi^2c^3}\tag{13}$$

for the equilibrium thermal radiation with temperature T in a transparent medium characterized by the dielectric permittivity ε_n.

The deviation of the spectral energy density of equilibrium radiation, $U^{(n)S}(z,\omega,T)$, from the classical distribution $U^{(n)}(\omega,T)$ in an unbounded homogeneous medium is defined as follows:

$$U^{(n)S}(z,\omega,T)=L^{(n)}(z,\omega)\,U^{(n)}(\omega,T),\quad n=1,2,\tag{14}$$

where

$$L^{(n)}(z,\omega)=\pm\frac{1}{2k_n^3}\mathrm{Re}\int_0^\infty dk_\perp\,\frac{k_\perp^3}{k_{zn}}[R_s(k_\perp,\omega)$$
$$-R_p(k_\perp,\omega)]\exp(i2k_{zn}|z|),\quad n=1,2.\tag{15}$$

It is convenient, as in the case of a homogeneous system, to split off as a separate term in the expression for the spectral density $U^{(n)}(z,\omega,T)$ the spectral energy density corresponding to the zero-point oscillations of the field in the system,

$$U^{(n)}(z,\omega,T)=U^{(n)}(z,\omega)+U^{(n)Pl}(z,\omega,T),\quad n=1,2,\tag{16}$$

where

$$U^{(n)}(z,\omega)=[1+L^{(n)}(z,\omega)]\,U^{(n)}(\omega),\quad n=1,2,\tag{17}$$

$$U^{(n)Pl}(z,\omega,T)=[1+L^{(n)}(z,\omega)]\,U^{(n)Pl}(\omega,T),\quad n=1,2.\tag{18}$$

It follows from Eqs. (15) and (16) that when one is sufficiently far from the dividing boundary the energy density goes over into the corresponding spectral energy density distributions of equilibrium radiation $U^{(n)}(\omega,T)$ in an unbounded transparent medium with a dielectric permittivity ε_1 for $z<0$ or ε_2 in the $z>0$ case. This result is valid both for the temperature-independent component of the energy density spectrum and for the quantity determined by the temperature. One should note that the spectral energy density of the zero-point oscillations (17) of the field differs from the result obtained in Ref. 1 since in it the zero-point oscillations of the field were taken into account in only one of the media. It was found in such an approach that the boundary changes the energy density spectrum of the zero-point oscillations even at an infinite distance from it. This difficulty does not arise when we take into account, as is assumed in the present paper, that there are zero-point oscillations in the whole of space.

The energy density spectrum $U^{(n)Pl}(z,\omega,T)$ of the thermal radiation contains components corresponding to the wave ($k_\perp<k_n$) and the quasistationary ($k_\perp>k_n$) parts of the field,

$$U^{(n)Pl}(z,\omega,T)=U_r^{(n)Pl}(z,\omega,T)+U_{qs}^{(n)Pl}(z,\omega,T),\quad n=1,2,\tag{19}$$

where

$$U_r^{(n)Pl}(z,\omega,T)=U^{(n)Pl}(\omega,T)+U_{r1}^{(n)Pl}(z,\omega,T),$$

$$U_{r1}^{(n)Pl}(z,\omega,T)=L_{r1}^{(n)}(z,\omega)\,U^{(n)Pl}(\omega,T),$$

$$L_{r1}^{(n)}(z,\omega)=\pm\frac{1}{2k_n^3}\int_0^{k_n}dk_\perp\,\frac{k_\perp^3}{k_{zn}}\,\mathrm{Re}([R_s(k_\perp,\omega)-R_p(k_\perp,\omega)]\exp(i2k_{zn}|z|)),$$

$$U_{qs}^{(n)Pl}(z,\omega,T)=\theta(\varepsilon_m-\varepsilon_n)\,L_{qs}^{(n)}(z,\omega)\,U^{(n)Pl}(\omega,T),$$

$$L_{qs}^{(n)}(z,\omega)=\pm\frac{1}{2k_n^3}\int_{k_n}^{k_m}dk_\perp\,\frac{k_\perp^3}{\varkappa_n}\,\mathrm{Im}(R_s(k_\perp,\omega)-R_p(k_\perp,\omega))\exp(-2\varkappa_n|z|),\tag{20}$$

$\varkappa_n=(k_\perp^2-k_n^2)^{1/2}$, $n,m=1,2$, $m\neq n$, and $\theta(x)$ is the Heaviside function.

The spectral energy density distributions (19) of the fluctuating electromagnetic field can be considered to be a generalization of the Planck law for the energy density of equilibrium radiation to the case of an inhomogeneous transparent medium.

It follows from Eq. (20) that the spectral energy density of the quasistationary field occurs in only one of the media, namely, in the medium with the smallest value of the dielectric permittivity. This conclusion can be reached also without having recourse to calculations if we use the same approach as when describing the equilibrium radiation in an unbounded transparent medium when one assumes that this medium is in a cavity which is thermally insulated from the outside with fixed nontransparent walls maintained at a constant temperature T. In the case of a uniform medium there is equilibrium radiation in it characterized by the uniform and isotropic spectral energy density $U^{Pl}(\omega,T)$. Since the fluctuating electromagnetic field in the medium is the field of the thermal radiation of the walls of the cavity, in view of the unbounded dimensions of the system there are in it only those components of the field which correspond to propagating ($k_\perp<k$) electromagnetic waves.

In the case of an equilibrium inhomogeneous system consisting of two homogeneous transparent media with a plane dividing boundary the fluctuating electromagnetic waves propagating in it undergo reflection and refraction at the boundary. Since the reflected and transmitted fields are connected with the field incident upon the dividing boundary by the Fresnel reflection coefficients $R_{p,s}(k_\perp,\omega)$ only those coefficients should occur in the various correlation functions for the fluctuating quantities. Electromagnetic waves incident upon the dividing boundary from the medium with the largest value of the dielectric permittivity at an angle larger than the angle for total internal reflection are completely reflected from the boundary, penetrating only to some depth into the second medium. In the medium with the smaller value of the dielectric permittivity there is thus in a thin surface layer an exponentially damped quasistationary electromagnetic field which contributes to the energy density of the equilibrium radiation in the form of its quasistationary component U_{qs}^{Pl}. Since there are quasistationary fields only in one of the media, there must also be a quasistationary field energy density only in the medium with the smallest value of the dielectric permittivity and it is produced only by electromagnetic waves for which the modulus of the transverse component of the wavevector lies in the range between the absolute values of the wavevectors of the electromagnetic waves propagating in the two transparent media. These conclusions are fully in agreement with the results (20) obtained on the basis of direct calculations.

We can estimate the behavior of the spectral energy density of the thermal radiation in the system of two transparent media at large and small distances from the dividing boundary. Far from the boundary ($b_n=2k_n|z|\gg1$) the partial components $U_{r1}^{(n)Pl}(z,\omega,T)$ and $U_{qs}^{(n)Pl}(z,\omega,T)$ of the energy density differ in the nature of their behavior, depending on the point of observation: the quantity $U_{r1}^{(n)Pl}(z,\omega,T)$ decreases inversely proportional to the distance from the boundary,

$$U_{r1}^{(n)Pl}(z,\omega,T)\simeq\frac{2\cos b_n}{b_n}\,U^{(n)Pl}(\omega,T),\quad n=1,2,\tag{21}$$

whereas the energy density of the quasistationary field tends to zero, inversely proportional to the square of the distance from the boundary[1]

$$U_{qs}^{(n)Pl}(z,\omega,T)\simeq\theta(\varepsilon_m-\varepsilon_n)\,\frac{\xi_n+1}{(\xi_n-1)^{1/2}}\,\frac{1}{b_n^2}\,U^{(n)Pl}(\omega,T),$$

$$\xi_n=\frac{\varepsilon_m}{\varepsilon_n},\quad n,m=1,2,\quad n\neq m.\tag{22}$$

Near the boundary ($b_n\ll1$) the quasistationary component of the energy density may make the main contribution to the spectral energy density. We note that, in contrast to the conclusion of Ref. 1 that the energy density of the quasistationary field diverges inversely proportionally to the square of the distance of the point of observation from the boundary, the quantity $U_{qs}^{(n)Pl}(z,\omega,T)$ remains finite at any distance from the boundary, including on the boundary itself. This statement follows both from the model considered above of the radiation in an inhomogeneous transparent medium as radiation from the walls of a cavity, and is also seen directly from Eq. (20) for $U_{qs}^{(n)Pl}(z,\omega,T)$ as an integral with finite limits of integration of a function without singularities.

If the point of observation lies on the boundary the quantities $L_{r1}^{(n)}(0,\omega)$ and $L_{qs}^{(n)}(0,\omega)$ are independent of the frequency and are determined solely by the ratio of the dielectric permittivities of the media $[L_{r1}^{(n)}(0,\omega)\equiv L_{r1}^{(n)}(\xi_n), L_{qs}^{(n)}(0,\omega)\equiv L_{qs}^{(n)}(\xi_n)]$. The inhomogeneity of the system thus reduces simply to multiplying the Planck distribution by a constant quantity which is equal to $[1+L_{r1}^{(n)}(\xi_n)+\theta(\varepsilon_m-\varepsilon_n)L_{qs}^{(n)}(\xi_n)]$.

Integrating Eq. (14) over positive frequencies we obtain the following representation for the total energy density of the equilibrium radiation $U_n^S(z,T)$ caused by the inhomogeneity of the system:

$$U_n^S(z,T)=\int_0^\infty d\omega U^{(n)S}(z,\omega,T)$$

$$=30\left\{\frac{\xi_n-1}{\xi_n+1}+2\int_0^\infty dx\frac{x^3}{(x^2+1)^{1/2}}\left[\frac{(x^2+1)^{1/2}-(x^2+\xi_n)^{1/2}}{(x^2+1)^{1/2}+(x^2+\xi_n)^{1/2}}-\frac{\xi_n(x^2+1)^{1/2}-(x^2+\xi_n)^{1/2}}{\xi_n(x^2+1)^{1/2}+(x^2+\xi_n)^{1/2}}\right]\right.$$

$$\times\left.\frac{1+\exp(-b_{Tn}(x^2+1)^{1/2})+4\exp(-2b_{Tn}(x^2+1)^{1/2})}{(1-\exp(-b_{Tn}(x^2+1)^{1/2}))^4}\exp(-b_{Tn}(x^2+1)^{1/2})\right\}U_n^{Pl}(T), \tag{23}$$

where $U_n^{Pl}(T)$ is the energy density of equilibrium radiation in an unbounded transparent medium with a dielectric permittivity ε_n,

$$U_n^{Pl}(T)=\frac{\pi^2\varepsilon_n^{3/2}}{15\hbar^3c^3}T^4, \tag{24}$$

$b_{Tn}=2k_{Tn}|z|, k_{Tn}=\omega_{T}\varepsilon_n^{1/2}/c, \omega_T=2\pi_1 T/\hbar, n=1,2.$

For $b_{Tn}\gg 1$ and finite values of the quantity ξ_n the asymptotic behavior of Eq. (23) has the form

$$U_n^S(z,T)\simeq\frac{120}{\xi_n^{1/2}}\frac{\xi_n^{1/2}-1}{\xi_n^{1/2}+1}\frac{1}{b_{Tn}}U_n^{Pl}(T). \tag{25}$$

The inhomogeneity of the medium thus leads for the medium with the smaller dielectric permittivity to an increase in the energy density as compared to the energy density $U_n^{Pl}(T)$ in the unbounded system and to a decrease in it in the medium with the larger value of ε. When we go away from the boundary $U_n^S(z,T)$ decreases inversely proportional to the distance from it. In the special case when one of the media (e.g., the second) is filled with an ideal conductor Eq. (23) takes an especially simple form,

$$U_2^S(z,T)\simeq\frac{30}{b_{T2}^3}U_2^{Pl}(T). \tag{26}$$

The effect of an ideal conductor on the energy density far from the boundary is thus considerably weaker than for a dielectric.

II. NONEQUILIBRIUM SYSTEM

The results obtained for the spectral energy density of the radiation in the case of a system in thermodynamic equilibrium can be generalized to the case when each of the media is characterized by its own temperature T_n. To find the given quantities we shall assume that there is nonvanishing dissipation in the media,

$$\operatorname{Im}\varepsilon_n=\eta_n\operatorname{sign}\omega,\quad \eta_n>0,\quad n=1,2. \tag{27}$$

The fluctuating electromagnetic fields in the system considered are then the fields of the characteristic thermal radiation of the media and the difference in temperature makes it possible to identify the contribution of the thermal radiation fields of each of the media separately. To find the correlation functions of the fluctuating electromagnetic fields in the system we use the general solution[9] of the problem of the excitation of a given system by arbitrarily distributed external sources $\mathbf{J}^{(1)e}(\mathbf{r},t)$ and $\mathbf{J}^{(2)e}(\mathbf{r},t)$ which are given, respectively, in the $z<0$ and $z>0$ regions:

$$E_i(\mathbf{k}_\perp,z,\omega)$$

$$=S_i^{(1)}(\mathbf{k}_\perp,z,\omega)+\frac{1}{2}\left\{e_{zij}k_jt_s(k_\perp,\omega)\frac{e_{zln}k_ll_n(\mathbf{k}_\perp,\omega)}{k_\perp^2}\right.$$

$$\left.-\left(\delta_{iz}+\frac{k_{\perp i}k_{z1}}{k_\perp^2}\right)t_p(k_\perp,\omega)\frac{[\mathbf{k}_\perp\mathbf{l}_\perp(\mathbf{k}_\perp,\omega)]}{k_{z1}}\right\}$$

$$\times\exp(ik_{z1}|z|),$$

$$B_i(\mathbf{k}_\perp,z,\omega)=V_i^{(1)}(\mathbf{k}_\perp,z,\omega)-\frac{c}{2\omega}\left\{e_{zij}k_j\frac{k_1^2}{k_\perp^2k_{z1}}t_p(k_\perp,\omega)\right.$$

$$\times[\mathbf{k}_\perp\mathbf{l}_\perp(\mathbf{k}_\perp,\omega)]+\left(\delta_{iz}+\frac{k_{\perp i}k_{z1}}{k_\perp^2}\right)t_s(k_\perp,\omega)$$

$$\times[e_{zjn}k_jl_n(\mathbf{k}_\perp,\omega)]\}\exp(ik_{z1}|z|),$$

$$i=x,y,z \tag{28}$$

in the $z<0$ region and

$$E_i(\mathbf{k}_\perp,z,\omega)=S_i^{(2)}(\mathbf{k}_\perp,z,\omega)+\frac{1}{2}\left\{e_{zij}k_jt_s(k_\perp,\omega)r_s(k_\perp,\omega)\right.$$

$$\times\frac{e_{zln}k_ll_n(\mathbf{k}_\perp,\omega)}{k_\perp^2}+\left(\delta_{iz}-\frac{k_{\perp i}k_{z2}}{k_\perp^2}\right)t_p(k_\perp,\omega)$$

$$\left.\times r_p(k_\perp,\omega)\frac{[\mathbf{k}_\perp\mathbf{l}_\perp(\mathbf{k}_\perp,\omega)]}{k_{z2}}\right\}\exp(ik_zz),$$

$$B_i(\vec{k}_\perp,z,\omega)=V_i^{(2)}(\vec{k}_\perp,z,\omega)+\frac{c}{2\omega}\left\{e_{zij}k_j\frac{k_2^2}{k_\perp^2k_{z2}}t_p(k_\perp,\omega)\right.$$

$$\times r_p(k_\perp,\omega)(\vec{k}_\perp\vec{l}_\perp(\vec{k}_\perp,\omega))-\left(\delta_{iz}-\frac{k_{\perp i}k_{z2}}{k_\perp^2}\right)$$

$$\times t_s(k_\perp,\omega)r_s(k_\perp,\omega)(e_{zjn}k_jl_n[\mathbf{k}_\perp,\omega)]\}$$

$$\times\exp(ik_{z2}z),\quad i=x,y,z \tag{29}$$

for $z>0$.

Here

$$S_i^{(n)}(\mathbf{k}_\perp,z,\omega)$$

$$=-i\frac{2}{\omega}\int_{-\infty}^{\infty}dk_z\left(\delta_{ij}-\frac{k_ik_j}{k_n^2}\right)J_j^{(n)e}(\mathbf{k},\omega)$$

$$\times\frac{\exp(ik_zz)}{\Delta_n(k,\omega)},\quad i=x,y,z,\quad n=1,2,$$

$$\mathbf{V}^{(n)}(\mathbf{k}_\perp,z,\omega)=-i\frac{2c}{\omega^2}\int_{-\infty}^{\infty}dk_z\frac{[\mathbf{k}\mathbf{J}^{(n)e}(\mathbf{k},\omega)]}{\Delta_n(k,\omega)}\times\exp(ik_zz),\quad n=1,2,$$

$$\mathbf{l}(\mathbf{k}_\perp,\omega)=\mathbf{S}^{(1)}(\mathbf{k}_\perp,\omega)-\mathbf{S}^{(2)}(\mathbf{k}_\perp,\omega),$$

$$\mathbf{S}^{(1)}(\mathbf{k}_\perp,\omega)\equiv\mathbf{S}^{(1)}(\mathbf{k}_\perp,z=-0,\omega),$$

$$\mathbf{S}^{(2)}(\mathbf{k}_\perp,\omega)\equiv\mathbf{S}^{(2)}(\mathbf{k}_\perp,z=+0,\omega)$$

$$t_{p,s}(k_\perp,\omega)=1-R_{p,s}(k_\perp,\omega), \tag{30}$$

$\Delta_n(k,\omega)=\varepsilon_n-c^2k^2/\omega^2$, $\mathbf{J}^{(n)e}(\mathbf{k},\omega)$ is the Fourier component of the current, given in the $z<0$ region for $n=1$ and the $z>0$ region for $n=2$ and continued mirrorwise into the adjacent region, and e_{ijl} is the third rank absolutely antisymmetric unit tensor.

We note that Eqs. (28) and (29) for the fields in the first and the second media go over into one another under the substitution: $\varepsilon_1\leftrightarrow\varepsilon_2$, $\mathbf{J}^{(1)e}(\mathbf{k},\omega)\leftrightarrow\mathbf{J}^{(2)e}(\mathbf{k},\omega)$.

Assuming the external sources to be random and that the sources of the fluctuating field given in the different spatial regions ($z<0$ and $z>0$) are uncorrelated, and also taking into account the nature of the analytical continuation of the sources, we get

$$\langle J_i^{(n)e}(\mathbf{k},\omega)J_j^{(m)e*}(\mathbf{k}',\omega')\rangle=(2\pi)^3\delta(\omega-\omega')\delta(\mathbf{k}_\perp-\mathbf{k}'_\perp)\times\delta_{nm}\langle J_i(k_z)J_j^*(k'_z)\rangle_{\mathbf{k}_\perp\omega}^{(n)e},$$
$$i,j=x,y,z,\quad n,m=1,2, \tag{31}$$

$$\langle J_i(k_z)J_j^*(k'_z)\rangle_{\mathbf{k}_\perp\omega}^{(n)e}=2\pi[\delta(k_z-k'_z)+(1-2\delta_{jz})\times\delta(k_z+k'_z)]\langle J_iJ_j^*\rangle_{\mathbf{k}\omega}^{(n)e},\quad i,j=x,y,z,\quad n=1,2, \tag{32}$$

where $\langle \mathbf{J}_i\mathbf{J}_j^*\rangle_{\mathbf{k}\omega}^{(n)e}$ is the correlation function of the random sources in the case of an unbounded system characterized by a dielectric permittivity ε_n and a temperature T_n. According to the FDT[1-5] this quantity is equal to

$$\langle J_iJ_j^*\rangle_{\mathbf{k}\omega}^{(n)e}=\theta(\omega,T_n)\frac{\pi}{2\pi}\delta_{ij}\,\mathrm{Im}\,\varepsilon_n,\quad i,j=x,y,z,\quad n=1,2. \tag{33}$$

Substituting Eqs. (28) and (29) into (1), using Eqs. (31) and (32) to perform a statistical averaging, and also correctly taking the limit to a transparent medium (the limit $\lim_{\eta_n\to+0}$ is taken after integration over dk_z; see Refs. 6 to 9) we get the following final equation for the spectral energy density

$$U^{(n)}(z,\omega,T_n,T_m)=U^{(n)}(z,\omega)+U^{(n)Pl}(z,\omega,T_n,T_m),\quad n,m=1,2,\quad n\neq m, \tag{34}$$

where $U^{(n)}(z,\omega)$ is the spectral energy density of the zero-point oscillations in a medium with a dielectric permittivity ε_n determined by the same Eq. (17) as in the case of a system in thermodynamic equilibrium and calculated using the FDT. The agreement of the results for the spectral energy density of the zero-point oscillations in the cases where the system is or is not in equilibrium is completely natural since the system is always in equilibrium as regards the zero-point oscillations.

The second term in Eq. (34) depends on the temperatures of the two media and, as in the case of an equilibrium system, can be written as a sum of components corresponding to the wave and the quasistationary parts of the field,

$$U^{(n)Pl}(z,\omega,T_n,T_m)=U_r^{(n)Pl}(z,\omega,T_n,T_m)+U_{qs}^{(n)Pl}(z,\omega,T_m),$$

$$U_r^{(n)Pl}(z,\omega,T_n,T_m)=U^{(n)Pl}(\omega,T_n)+U_{r1}^{(n)Pl}(z,\omega,T_n)+U_{r2}^{(n)Pl}(\omega,T_n,T_m),$$

$$U_{r1}^{(n)Pl}(z,\omega,T_n)=L_{r1}^{(n)}(z,\omega)\,U^{(n)Pl}(\omega,T_n),$$

$$U_{r2}^{(n)Pl}(\omega,T_n,T_m)=L_{r2}^{(n)}(\xi_n)\,[U^{(n)Pl}(\omega,T_m)-U^{(n)Pl}(\omega,T_n)],$$

$$U_{qs}^{(n)Pl}(z,\omega,T_m)=\theta(\varepsilon_m-\varepsilon_n)\,L_{qs}^{(n)}(z,\omega)\,U^{(n)Pl}(\omega,T_m),$$
$$n,m=1,2,\quad n\neq m. \tag{35}$$

The quantities $L_{r1}^{(n)}(z,\omega)$ and $L_{qs}^{(n)}(z,\omega)$ are determined by the same Eqs. (20) as in the case of a system in thermodynamic equilibrium while the quantity $L_{r2}^{(n)}(\xi_n)$ depends only on the relative refractive index ξ_n and is equal to

$$L_{r2}^{(n)}(\xi_n)=\frac{1}{2k_n}\int_0^K dk_\perp\frac{k_\perp}{k_{zn}}\Gamma(k_\perp,\omega), \tag{36}$$

where

$$\Gamma(k_\perp,\omega)=1-|R(k_\perp,\omega)|^2=\tfrac{1}{2}[\Gamma_p(k_\perp,\omega)+\Gamma_s(k_\perp,\omega)]$$

is the coefficient for absorption of unpolarized light by the $z>0$ region,

$$|R(k_\perp,\omega)|^2=\tfrac{1}{2}[|R_p(k_\perp,\omega)|^2+|R_s(k_\perp,\omega)|^2] \tag{37}$$

is the energy coefficient for reflection of a plane unpolarized electromagnetic wave from the second medium, while $\Gamma_{p,s}(k_\perp,\omega)=1-|R_{p,s}(k_\perp,\omega)|^2$ and $|R_{p,s}(k_\perp,\omega)|^2$ are the coefficients for absorption and the energy coefficients for reflection of plane p- and s-polarized electromagnetic waves, and $K=\min(k_1,k_2)$ is the smallest of k_1 and k_2.

The fact that the system is not in equilibrium thus leads to the appearance of an additional term $U_{r2}^{(n)\mathrm{Pl}}(\omega,T_n,T_m)$, which is independent of the distance, in the energy density spectrum of the wave part of the field. It follows from the definition (35) of this quantity that it vanishes when we go over to a system in thermodynamic equilibrium ($T_1=T_2\equiv T$):$U_{r2}^{(n)Pl}(\omega,T,T)=0$.

We note that the expressions for the spectral energy densities in the first and the second media go over into one another if we perform the following substitution in them: $\varepsilon_n \leftrightarrow \varepsilon_m$, $T_n \leftrightarrow T_m$.

The quantity $U_{r2}^{(n)Pl}(\omega,T_n,T_m)$ determines the total spectral radiation energy flux density produced by the two media if the characteristic thermal radiation of a transparent medium is interpreted as the radiation field of infinitely removed sources, positioned at the limits of the region occupied by the transparent medium.[1,9] One can then write $U_{r2}^{(n)Pl}(\omega,T_n,T_m)$ as the difference of the spectral energy densities of unidirectional radiation fluxes, directed in mutually opposite directions, away from the boundary, $U_u^{(n)Pl}(\omega,T_n,T_m)$, and toward it, $U_d^{(n)Pl}(\omega,T_n)$, in the $z<0$ region for $n=1$ or the $z>0$ region for $n=2$:[1)]

$$U_{r2}^{(n)Pl}(\omega,T_n,T_m)=U_u^{(n)Pl}(\omega,T_n,T_m)-U_d^{(n)Pl}(\omega,T_n), \tag{39}$$

where

$$\begin{aligned} U_u^{(n)Pl}(\omega,T_n,T_m)=&\frac{U^{(n)Pl}(\omega,T_m)}{2} \\ &-\frac{U^{(n)Pl}(\omega,T_m)-U^{(n)Pl}(\omega,T_n)}{2k_n} \\ &\times\int_0^{k_n} dk_\perp \frac{k_\perp}{k_{zn}}|R(k_\perp,\omega)|^2, \\ U_d^{(n)Pl}(\omega,T_n)=&\frac{U^{(n)Pl}(\omega,T_n)}{2}. \end{aligned} \tag{40}$$

Indeed, one can easily show, by using the expressions, obtained in Ref. 9, for the intensities of unidirectional thermal radiation fluxes of the system considered away from the boundary, $I_u^{(n)}(\omega,\theta,T_n,T_m)$, and toward it, $I_d^{(n)}(\omega,T_n)$,

$$\begin{aligned} I_u^{(n)}(\omega,\theta,T_n,T_m)=&I_0^{(n)}(\omega,T_m)-[I_0^{(n)}(\omega,T_m)-I_0^{(n)} \\ &\times(\omega,T_n)]\times|R(k_\perp,\omega)|^2, \\ &k_\perp=k_n\cdot\sin\theta\ , \\ I_d^{(n)}(\omega,T_n)=&I_0^{(n)}(\omega,T_n), \end{aligned} \tag{41}$$

that the spectral energy densities $U_u^{(n)Pl}(\omega,T_n,T_m)$ and $U_d^{(n)Pl}(\omega,T_n)$ which correspond to them are given by the following expressions:

$$\begin{aligned} U_u^{(n)Pl}(\omega,T_n,T_m)=&\int_{\theta<\pi/2} d\Omega \frac{I_u^{(n)}(\omega,\theta,T_n,T_m)}{v_g^{(n)}}, \\ I_d^{(n)Pl}(\omega,T_n)=&\int_{\theta<\pi/2} d\Omega \frac{I_d^{(n)}(\omega,T_n)}{v_g^{(n)}}. \end{aligned} \tag{42}$$

Here $I_0^{(n)}(\omega,T_m)$ is the intensity of the radiation from an absolutely black body with a temperature T_m into unit solid angle $d\Omega=\sin\theta d\theta d\varphi$ in a transparent medium with dielectric permittivity ε_n,

$$I_0^{(n)}(\omega,T_m)=\frac{\omega^2\varepsilon_n}{4\pi^3c^2}\frac{\hbar\omega}{\exp(\hbar\omega/T_m)-1}, \tag{43}$$

where θ and φ are the polar and azimuthal angles giving the solid angle $d\Omega$, and $v_g^{(n)}=c/\varepsilon_n^{1/2}$ is the group velocity for the propagation of electromagnetic waves in a transparent medium with dielectric permittivity ε_n.

We note that Eq. (35) for the spectral energy density $U_{qs}^{(n)Pl}(z,\omega,T_m)$ of the quasistationary field retains the same form as in the case of an equilibrium system. However, this quantity is determined solely by the temperature T_m of the adjacent medium which agrees with the representation of the characteristic thermal field of a transparent medium as the radiation field of infinitely far away sources[1,9] which can propagate in the given medium only for $k_\perp<k_n$.

The conclusion that the energy of the quasistationary field occurring only in one of the media is determined solely by the temperature of the adjacent medium can be reached also by using qualitative considerations, assuming that the given system is placed in a cavity the walls of which are maintained at constant, but different temperatures: the walls enclosing the first medium have a temperature T_1 and the walls enclosing the second medium a temperature T_2. Since the field in the system is a superposition of the radiation fields of the walls of the cavity which are at temperatures T_1 and T_2 while the quasistationary field in one medium (say, in the first one when $\varepsilon_1<\varepsilon_2$) is the radiation field of the walls of the cavity with temperature T_2 which penetrates into that medum and is exponentially damped when one moves away from the boundary, the quasistationary part of the energy density in the first medium can depend only on the temperature of the second one which is equal to the temperature of the walls of the cavity adjacent to it.

On the other hand, the component $U_{r1}^{(n)Pl}(z,\omega,T_n)$ of the spectral energy density is determined by the temperature only from the medium in which it is calculated. This quantity which depends on the point of observation can thus be interpreted as the result of the interference of waves of the radiation from the walls of the cavity with temperature T_n incident upon the dividing boundary of the media and reflected from it. The quantity $2k_{zn}|z|$ which occurs in the $\exp(i2k_{zn}|z|)$ factor takes into account the difference in paths traversed by those waves.

The spectral component $U_{r2}^{(n)Pl}(\omega,T_n,T_m)$ of the energy density which is independent of the coordinate can be considered to be the difference of the spectral energy densities of the unidirectional radiation fluxes from opposite walls of the cavity which are heated, respectively, to temperatures T_n and T_m, and where the refraction of the radiation field at the dividing boundary is taken into account.

This model for describing the fluctuating fields of the thermal radiation in a system of two transparent media thus enables us to reach correct qualitative conclusions about the peculiarities of the energy density distribution in the system based solely on the properties of the propagation of electromagnetic waves in it. Bearing in mind the existing large amount of material about the propagation of electromagnetic waves in various inhomogeneous media it is advisable to use this approach also for constructing a correlation theory for geometrically more complicated systems.

One can consider Eqs. (34) and (35) as a generalization of the results for the spectral density of thermal radiation to the case of an inhogeneous system.

In the special case when the temperature of only one of the media is taken into account Eq. (35) for the spectral density of the thermal radiation goes over into the results of Ref. 1 for the spectral radiation energy density of a semibounded body in an external cold medium.

Far from a surface layer of thickness b_n the quantities $U_{r1}^{(n)Pl}(z,\omega,T_n)$ and $U_{qs}^{(n)Pl}(z,\omega,T_m)$ decrease (see Eqs. (21) and (22)), whence it follows that at large distances from the dividing boundary the spectral energy density of the thermal radiation reaches a constant level,

$$U^{(n)Pl}(z,\omega,T_n,T_m)\simeq U^{(n)Pl}(\omega,T_n)+U_{r2}^{(n)Pl}(\omega,T_n,T_m), \tag{44}$$

which is the same as the classical Planck value $U^{(n)Pl}(\omega,T_n)$ only in the case of a system which is in thermodynamic equilibrium.

If the temperature of one medium is much higher than the temperature of the other one (for definiteness we assume that $T_2 \gg T_1$) the energy density in the whole of space is for $\varepsilon_2>\varepsilon_1$ primarily determined solely by the temperature of the second medium,

$$\begin{aligned} U^{(1)Pl}(z,\omega,T_1,T_2) &\simeq [L_{r2}^{(1)}(\xi_1)+L_{qs}^{(1)}(z,\omega)] \\ &\quad \times U^{(1)Pl}(\omega,T_2), \\ U^{(2)Pl}(z,\omega,T_2,T_1) &\simeq [1-L_{r2}^{(2)}(\xi_2)+L_{r1}^{(2)}(z,\omega)] \\ &\quad \times U^{(2)Pl}(\omega,T_2). \end{aligned} \tag{45}$$

Far from the dividing boundary ($b_n \gg 1$) the energy density in both media is defined as the radiation energy density of an absolutely black body with a temperature T_2,

$$\begin{aligned} U^{(1)Pl}(z,\omega,T_1,T_2) &\simeq L_{r2}^{(1)}(\xi_1)\,U^{(1)Pl}(\omega,T_2), \\ U^{(2)Pl}(z,\omega,T_2,T_1) &\simeq [1-L_{r2}^{(2)}(\xi_2)]\cdot U^{(2)Pl}(\omega,T_2), \end{aligned} \tag{46}$$

differing, however, from the Planck distribution by the presence of a constant factor determined solely by the relative refraction index ξ_n of the media in contact.

III. NUMERICAL ANALYSIS OF THE SPECTRAL ENERGY DENSITY

Using the general Eqs. (35) we have calculated the frequency dependence of the spectral energy density distribution of the fluctuating electromagnetic field in an inhomogeneous transparent medium for the cases of systems which are in thermodynamic equilibrium ($T_1=T_2\equiv T$, Fig. 1) and which are not in equilibrium (Fig. 2) for which $\varepsilon_1=1$, $\varepsilon_2=10$, $T_2=10^3$ K and $T_2 \gg T_1$. The energy density was normalized to the energy density of the equilibrium thermal radiation in the corresponding unbounded transparent medium with temperature T_2. The frequency scale along the abscissa axis is logarithmic. It can be seen from the figures that in the case of the equilibrium system the quantities $U^{(1)Pl}(z,\omega,T)/U^{(1)Pl}(\omega,T)$ and $U^{(2)Pl}(z,\omega,T)/U^{(2)Pl}(\omega,T)$ at the boundary surface are finite and independent of the frequency (curves 1 in Figs. 1a and 1b) which is in complete agreement with the conclusion from the theory that for $z=0$ the effect of the inhomogeneity of the system reduces simply to multiplying the Planck distribution by a constant coefficient, the value of which, equal to $(1+L_{r1}^{(1)}(\xi_1)+L_{qs}^{(1)}(\xi_1))$ for the first and $[1+L_{r1}^{(2)}(\xi_2)]$ for the second medium, determines the extremum value (in the given case the maximum value for the first and the minimum value for the second medium) of the energy density in the system. Since the dielectric permittivity of the second medium is larger than that of the first one the excess of the energy density $U^{(1)Pl}(0,\omega,T)$ over its Planck level $U^{(1)Pl}(\omega,T)$ in the first medium (in Fig. 1a the quantity $U^{(1)Pl}(0,\omega,T)/U^{(1)Pl}(\omega,T)\simeq 6.79$) is connected with the presence of quasistationary fields in the first medium which make a basic contribution to the energy density for $z=0$. The transverse component $k_\perp$ of the wavevector can for the quasistationary fields only take on values from the range $k_1<k_\perp<k_2$. Since the point of observation is at the boundary the quasistationary fields with all possible values of $k_\perp$ will for any frequency contribute to the energy density.

When we move away from the boundary one can for the quasistationary fields conventionally distinguish three frequency ranges. In the first (long-wavelength) region all quasistationary fields in the first medium make, as in the $z=0$ case, a contribution to the quasistationary component of the field at the point of observation. In that frequency range the energy density is thus independent of the distance from the boundary and is the same as the energy density of the electromagnetic field at the boundary. The transition to the second frequency region when the frequency increases is characterized by the fact that the quasistationary field in the point considered here is determined not by all quasistationary fields which are present near the boundary but only by those which satisfy the condition $k_1<k_\perp<k_{qs}(\omega)$ where the quantity $k_{qs}(\omega)<k_2$, which depends on the distance from the boundary, decreases when the frequency increases. As a result the normalized level of the energy density is lowered when the frequency increases down to unity when in the point considered all quasistationary fields which are present in the first medium are damped exponentially. A further increase in the frequency does not change the Planck distribution $U^{(1)Pl}(\omega,T)$ which describes the energy density in the third (short-wavelength) frequency region. Moving away from the boundary is accompanied by a narrowing of the first frequency region which is well illustrated by the similar curves *2* to *4* in Fig. 1a.

In the optically denser medium (Fig. 1b) there are no quasistationary fields and, in accordance with Eq. (18), the energy density is completely determined by the quantity $L_{r1}^{(2)}(z,\omega)$. Since the quantity $L_{r1}^{(2)}$ for $z=0$ is independent of the frequency the presence of the first medium leads to a decrease in the Planck distribution $U^{(2)Pl}(\omega,T)$

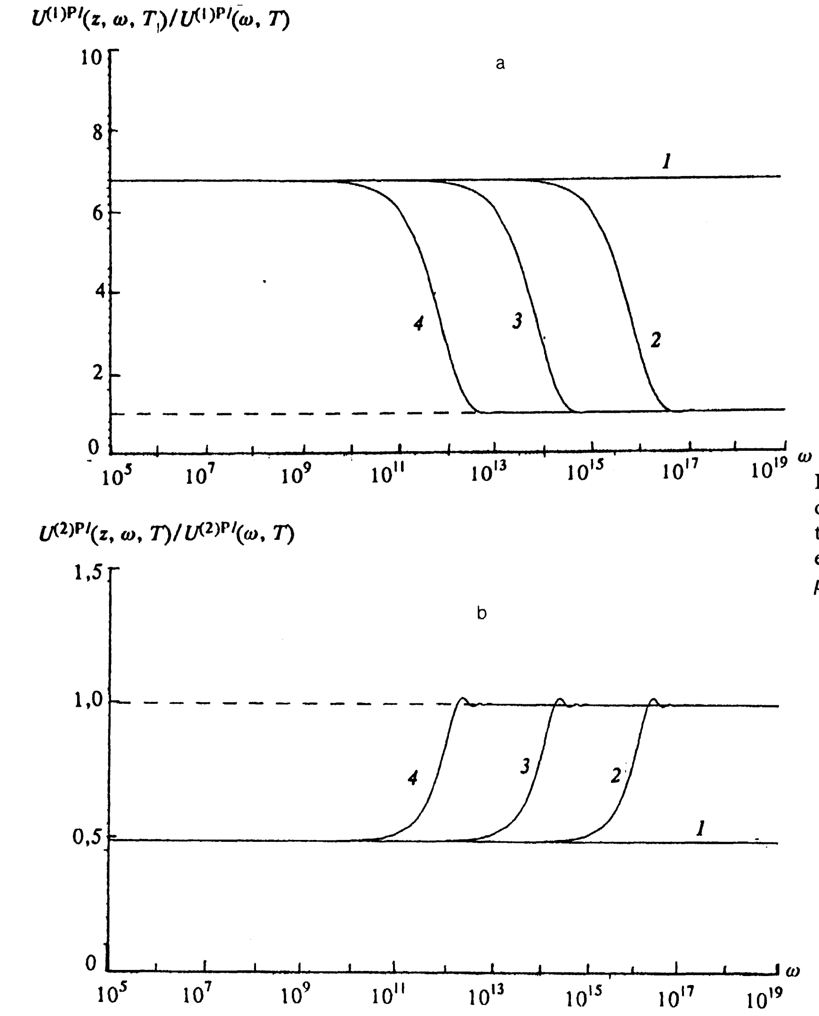

FIG. 1. Frequency distributions of normalized energy densities of the electromagnetic field in the first (a) and the second (b) transparent media for the case of a system in thermodynamic equilibrium (1): $z=0$; 2: $z=0.01$ μm; (3): $z=1$ μm; (4): $z=100$ μm).

by approximately a factor two (curve *1*). Here, as in the first medium, at non-zero distances z from the boundary one can distinguish three frequency bands: the long-wavelength one, for $b_2 \ll 1$, the short-wavelength one, for $b_2 \gg 1$, and the interval corresponding to intermediate frequencies. In the long-wavelength range, which narrows when we go away from the boundary surface (curves *2* to *4*) the ratio $U^{(2)Pl}(z,\omega,T)/U^{(2)Pl}(\omega,T) \simeq 1 + L_{r1}^{(2)}(\xi_2)$ is constant and independent of the distance from the boundary. When the frequency increases the quantity b_2 reaches values for which the quantity $|L_{r1}^{(2)}(z,\omega)|$ starts to decrease which is accompanied by an increase in $U^{(2)Pl}(z,\omega,T)/U^{(2)Pl}(\omega,T)$. The damped oscillations of the energy density near the Planck level $U^{(2)Pl}(\omega,T)$ are

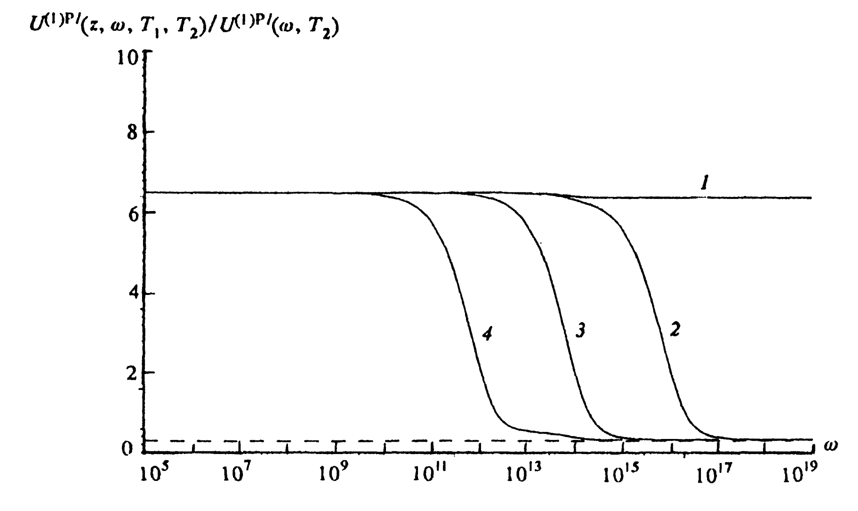

FIG. 2. Frequency distributions of the normalized energy density of the electromagnetic field in the first medium for the case of a non-equilbrium system ($T = 300$ K. The other parameters are the same as for Fig. 1).

caused by the asymptotical oscillating behavior (21) of the quantity $L_{r1}^{(2)}(z,\omega)$ in the short-wavelength region.

The calculations show that in the case of a system far from equilibrium the energy density is practically completely determined by the temperature of the second medium in complete agreement with Eqs. (45). It turns out that for the given parameters $L_{r2}^{(2)}(\xi_2)$ is very approximately 0.02 and the contribution from the term $L_{r2}^{(2)} \times(\xi_2) U_{r2}^{(2)Pl}(\omega,T_2,T_1)$, which takes into account the fact that the system is not in equilibrium, to the energy density $U^{(2)Pl}(z,\omega,T_2,T_1)$ can thus be neglected in the whole of the frequency range even for a ratio $T_1/T_2=0.3$ of the temperatures of the media. The fact that the system is not in equilibrium therefore hardly affects the energy density in the second medium and the required distributions $U^{(2)\mathrm{Pl}}(z,\omega,T_2,T_1)$ are determined by the same results as in the case of an inhomogeneous system at thermodynamic equilibrium at a temperature T_2 (Fig. 1b). In the first medium (Fig. 2) taking into account that the system is not in equilibrium is accompanied by a lowering of the maximum value of the normalized density to 6.51 from the value 6.79 in the equilibrium case. Otherwise the nature of the behavior of the quantity $U^{(1)Pl}(z,\omega,T_2,T_1)/U^{(1)Pl}(\omega,T_2)$ is similar to that of the normalized energy density in the first medium in the case of a system in thermodynamic equilibrium but with a smoother reaching of the asymptotic value which lies below the Planck level and is equal to $L_{r2}^{(1)}(\xi_1)\simeq 0.32$.

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[1)]Equation (38) never appeared in Russian original.

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