Condition under which a reconnecting current sheet is evolutionary

S. A. Markovskiĭ and B. V. Somov

P. K. Shternberg State Astronomical Institute, Moscow State University, 119899, Moscow
(Submitted 12 May 1992)
Zh. Eksp. Teor. Fiz. **104**, 2736–2753 (August 1993)

The MHD approximation is used to analyze whether a reconnecting current sheet near a null point of the magnetic field is evolutionary. There exist perturbations such that boundary conditions at the current sheet are equivalent to those at a discontinuity surface. The requirement that the sheet be evolutionary with respect to these perturbations limits the velocity at which the medium flows across the sheet.

INTRODUCTION

An ideally conducting medium cannot undergo a continuous MHD motion at a magnetic-field null point, at which the electric field is nonzero. Near such a point, the frozen-in condition is violated, and a reconnecting current sheet may form. This sheet is a discontinuity separating magnetic fields opposite in direction.[1] A splitting of a sheet into other discontinuities has also been observed in numerical simulations.[2,3] Those simulations show that two slow magnetosonic shock waves attach to each end of the sheet. The reason for this splitting may be that the current sheet is not of an evolutionary nature as a discontinuity.

The $1D$ equations of ideal MHD allow discontinuous solutions: shock waves, tangential shocks, contact shocks, and rotational shocks.[4] If a steady-state discontinuity is to exist in a real medium, it must be stable with respect to decay into other discontinuities and with respect to a transition to a time-varying flow. Let us assume that the MHD properties are initially subjected to an infinitely small perturbation. Linear waves propagating away from the discontinuity surface then arise. If the amplitudes of these waves and the displacement of the discontinuity as a whole can be determined unambiguously from the linearized boundary conditions, the problem of the time evolution of an initial perturbation has a unique solution, and the discontinuity is by definition evolutionary.[5,6] If, on the other hand, the problem does not have a unique solution, then it is not legitimate to make the assumption that the initial perturbation is small. In this case an infinitely small perturbation leads to a change in the initial flow which is instantaneous (in the approximation of an ideal medium) and not small, e.g., to a decay of a nonevolutionary discontinuity into evolutionary discontinuities.

The evolutionarity requirement leads to restrictions on the unperturbed MHD properties on the different sides of the discontinuity surface. These restrictions are found by comparing the number of unknown parameters (the amplitudes of the outgoing waves and the displacement of the discontinuity as a whole) which describe an infinitely small perturbation, with the number of independent equations—the number of boundary conditions from which the parameter values are determined. If these two numbers are equal, the discontinuity qualifies as evolutionary.

In the case of a finite conductivity, a current sheet does not reduce to a $1D$ flow and is characterized by two length scales, since the velocity variations within the sheet are two-dimensional. The thickness of the sheet, by which we mean the distance between the reconnecting magnetic fluxes, determines the rate of dissipation of the magnetic field in the sheet. The width of the sheet determines the magnetic energy present in the region in which the fluxes interact. We derive below conditions under which a current sheet in a medium of fairly high conductivity interacts with small perturbations as a $1D$ discontinuity. We resolve the question of whether the sheet is evolutionary with respect to such perturbations.

1. PROPERTIES OF A RECONNECTING CURRENT SHEET

Assume that the motion of the medium satisfies the MHD approximation. We consider a current sheet which arises near a null point of a magnetic field $\mathbf{B}_0=(hy,hx,0)$, at which the electric field $\mathbf{E}=(0,0,E)$ is nonzero. The field lines of the magnetic field $\mathbf{B}$, which is frozen in the medium, are carried along the y axis into the sheet, in which this frozen-in condition is disrupted; the field lines are reconnected in the sheet, and they are carried out along the x axis. The variation of $\mathbf{B}$ along the coordinates outside the sheet can be written in complex form under the assumption that the half-thickness of the sheet, a (the dimension along the y axis), is zero:[1]

$$B_y+iB_x=h\frac{\zeta^2-b^2/2-2I/ch}{\sqrt{\zeta^2-b^2}}. \tag{1.1}$$

Here $\zeta=x+iy$, b is the half-width of the sheet (its dimension along the x axis), and I is the total current in the sheet. The current I varies over the interval $0\leqslant I\leqslant chb^2/4$. At the points

$$x^*=\pm\sqrt{\frac{1}{2}b^2+\frac{2I}{ch}} \tag{1.2}$$

the magnetic field changes sign [see (1.1)]. At $|x|<|x^*|$, the direction of the current is the same as that of the electric field (this is the forward current), while at $|x^*|<|x|<b$ it is in the opposite direction (this is the reverse current). If $x^*\sim b$ and $b-|x^*|\sim b$, the reverse current in the sheet is comparable to the forward current. Let us assume that this is the actual distribution. In this case all the MHD properties outside the sheet can be as-

 1063-7761/93/080253-09$10.00

sumed to be quasiuniform everywhere except near certain points $x=x^*$ and $x=\pm b$, which we will discuss no further.

If the medium has an infinite conductivity σ, the quantity b increases without bound as time elapses.[1] If σ is instead finite, then a finite width $2b$ is established over a finite time,[7] and we have $a/b\neq 0$ although $a\ll b$. In this case we have $B_y\neq 0$ at the surface of the sheet, in contrast with (1.1). If σ is sufficiently high, however, we have $B_x\gg B_y$ except near point (1.2) x^*. Below we set B_y equal to zero.

If $a\ll b$, all quantities except the velocity $\mathbf{v}$ are quasiuniform along the x axis within the sheet. The variation of the velocity is two-dimensional, since the continuity equation shows that the following equality holds by virtue of the symmetry properties of the flow at the point $x=0$, $y=0$:

$$\frac{\partial v_x}{\partial x}=-\frac{\partial v_y}{\partial y}.$$

If the conductivity is finite, the current sheet thus does not reduce to a 1D flow. If the conductivity is infinite, the sheet transforms into a tangential shock in the limit $t\to\infty$.

Let us assume that the sheet is in a steady state. The electric field $\mathbf{E}$ is then independent of the time. The ratio a/b can therefore be evaluated from the steady-state Ohm's law:[7]

$$\frac{a}{b}\sim\frac{\nu_m h}{cE}, \tag{1.3}$$

where $\nu_m=c^2/4\pi\sigma$ is the magnetic viscosity. The electric field is furthermore independent of the coordinates. Accordingly, medium flows into the sheet in the region of the forward current, while in the region of the reverse current there is an outflow along the y axis.

To simplify the discussion below we assume that all kinetic coefficients other than ν_m are zero, and ν_m is so small that the following conditions holds:

$$\frac{cE}{hb}\ll\frac{hb}{\sqrt{4\pi p}}. \tag{1.4}$$

On the left side of this inequality is the scale value v_y of the drift velocity of the medium along the sheet, while on the right side is the Alfvén velocity V_A.

We also assume

$$\rho^{\text{in}}\sim\rho^{\text{ex}}. \tag{1.5}$$

The superscripts "in" and "ex" label properties respectively inside and outside the sheet. This density distribution has been observed in a numerical simulation.[2]

At the surface of the current sheet, the magnetic field increases without bound as time elapses, while the drift velocity approaches zero if the conductivity is infinite. The gas pressure p outside the sheet, on the other hand, is close to its boundary value at $\zeta=\infty$, and it does not become infinite regardless of the value of σ. It can thus be assumed that if the conductivity is sufficiently high, and if we are not within a certain vicinity point (1.2), the sound velocity V_s satisfies the relation

$$v_y^{\text{ex}}\ll V_s^{\text{ex}}\ll V_A^{\text{ex}}. \tag{1.6}$$

The velocity component v_x increases in absolute value inside the sheet, from zero at $x=0$ to

$$v_x^{\text{in}}\sim hb/\sqrt{4\pi\rho} \tag{1.7}$$

at $x=x^*$ (Ref. 8), and then falls off, vanishing at $|x|=b$. Outside, the component v_x is always much smaller than the typical Alfvén velocity.

We will now make use of these properties of the flow to analyze infinitely small perturbations of the current sheet.

2. SMALL PERTURBATIONS OUTSIDE THE CURRENT SHEET

We assume that the set of MHD properties Q is subjected to an infinitely small perturbation δQ. We assume that $\delta v_z\equiv 0$ and $\delta B_z\equiv 0$, and that the perturbation outside the sheet satisfies the WKB approximation. The perturbation wave vector $\mathbf{k}$ can then be determined from the following dispersion relation in the zeroth approximation in the small parameter $1/kb$:

$$\omega_0[ik^2V_s^2(\mathbf{k}\mathbf{V}_A)^2-V_s^2k^2\omega_0(i\omega_0-\nu_mk^2)-ik^2V_A^2V\omega_0^2$$
$$+\omega_0^3(i\omega_0-\nu_mk^2)]=0, \tag{2.1}$$

where $\omega_0=\omega-(\mathbf{k}\mathbf{v})$.

We impose limits on the frequency ω:

$$\frac{v_y}{a}\ll\omega_\parallel\ll\frac{V_s}{a}, \tag{2.2}$$

where

$$\omega_\parallel=\omega-k_xv_x. \tag{2.3}$$

To simplify the calculations we make the further assumption

$$v_y\sim V_s^3/V_A^2. \tag{2.4}$$

As we will see below, this is the velocity which appears in the nonevolutionarity condition.

We first consider the case in which the perturbations propagate along the normal to the sheet ($k_x=0$). In the zeroth approximation in the small parameters given by inequality (2.2), solutions of Eqs. (2.1) are

$$k_y^d=-i\frac{v_y}{\nu_m}\frac{V_A^2}{V_s^2}, \tag{2.5}$$

$$k_y^0=\omega/v_y, \tag{2.6}$$

$$k_y^-=\omega/v_y, \tag{2.7}$$

$$k_y^+=\pm\omega/V_A. \tag{2.8}$$

The root in (2.7) is a double root.

The WKB approximation is a valid for such perturbations if $1/k_y^+b\ll 1$, since $|k_y^+|$ is the smallest wave number. This condition is equivalent to the following condition on the frequency ω:

$$\omega\gg h/\sqrt{4\pi\rho}. \tag{2.9}$$

Under condition (2.9), the derivatives with respect to the coordinates of the unperturbed quantities in the linearized MHD equations are negligible, and dispersion relation (2.1) holds.

To find the condition for evolutionarity we need to classify the perturbations as either arriving at the sheet or departing from it. In general, this classification should be based on the sign of the sum of the projections of the velocity of the medium and of the group velocity onto the normal to the surface. For normal propagation, however, it is sufficient to determine the sign of the phase velocity, since in the absence of a frequency dispersion this velocity is the same as the projection of the group velocity onto **k** in the coordinate system in which the medium is at rest.[9]

A perturbation with a vector vector k_y^0 from (2.6) corresponds to an entropy wave, while one with k_y^- from (2.7) corresponds to slow magnetosonic waves which are propagating across the magnetic field. In the coordinate system in which the medium is at rest, the phase velocity of these perturbations is zero, while in the laboratory coordinate system it is equal to **v**. When medium is flowing into the sheet, the perturbations of k_y^0 and k_y^- are thus arriving at the sheet, while if the medium is flowing out of the sheet these perturbations are instead leaving the sheet (the interaction of $1D$ discontinuities with perturbations which are at rest with respect to the medium was studied in Ref. 10). By virtue of the left side of inequality (2.2) we have $k_y^0,\ k_y^- \gg 1/a$. The current sheet is thus not a discontinuity for perturbations (2.6) and (2.7).

Perturbations with the wave vector k_y^+ from (2.8) correspond to fast magnetosonic waves. One of them is always an incoming wave, while the other is an outgoing wave, regardless of the sign of v_y. This can be seen from the circumstance that their phase velocities ω/k_y^+ satisfy the condition $V_{\rm ph}^+ \gg v_y$ [see (1.6) and (2.8)] and are directed along the normal to the current sheet, opposite each other. In contrast with k_y^0 and k_y^- we have $k_y^+ \ll 1/a$, and the waves in (2.8) interact with the sheet as if it were a discontinuity.

Finally, the perturbation of k_y^d from (2.5) is a dissipative wave and is damped over a distance much smaller than a. As a result, the amplitude of this wave cannot appear in the boundary conditions at the discontinuity surface.[11]

We turn now to oblique propagation. To determine whether the current sheet is evolutionary as a discontinuity, we need to find solutions of Eq. (2.1) with identical ω and k_x. According to Ref. 9, the number of waves coming in toward the x axis and propagating away from it with given values of k_x and ω is independent of k_x for this flow; i.e., it is independent of the propagation angle. It is therefore sufficient to determine the number of these waves in the case $k_x=0$. It follows that a single outgoing wave propagating away from each side of the surface of the sheet is possible when medium is flowing into the sheet (the region of forward current). In the case of an outflow (in the region of the reverse current), in contrast, there are four such waves.

For a current sheet under condition (2.2), however, the number of perturbations with $k_y \ll 1/a$ varies with k_x. In other words, there is a variation in the number of perturbations whose amplitudes are discontinuous across the sheet. If $k_x=0$, then there are two such perturbations, which are determined by the wave vector k_y^+ from (2.8). As we will show below, there may be three such perturbations in the case of oblique incidence. This fact is important to the discussion below.

The wave vector of a slow magnetosonic wave is given by

$$|\mathbf{k}^-| = \frac{\omega}{v_y \sin\theta + v_y \cos\theta \pm |V_{\rm ph}^-|}, \tag{2.10}$$

where $V_{\rm ph}^-$ is the phase velocity of the wave, and θ is the angle between $\mathbf{k}^-$ and the x axis. The scalar product $(\mathbf{kv})$ here can be written in the form $|\mathbf{k}|(v_y \sin\theta + v_x \cos\theta)$. Under the condition $V_s \ll V_A$ we can use the following expansion for $|V_{\rm ph}^-|$:

$$|V_{\rm ph}^-| = \frac{V_A V_s}{V_\perp}|\cos\theta| \left[1 + \frac{1}{2}\frac{V_A^2 V_s^2}{V_\perp^4}\cos^2\theta + o\left(\frac{V_A^2 V_s^2}{V_\perp^4}\right)\right], \tag{2.11}$$

where $V_\perp^2 = V_A^2 + V_s^2$.

We choose θ_0 such that $|V_{\rm ph}^-| \sim V_s$, i.e., such that $|\cos\theta_0|$ is not small. We find solutions of Eq. (2.1) at fixed values of ω and

$$k_x = |\mathbf{k}^-|\cos\theta_0. \tag{2.12}$$

For this purpose we single out the unknown variable k_y in Eq. (2.1):

$$\begin{aligned}(\omega_\parallel - k_y v_y)\{(\nu_m v_y V_s^2)k_y^5 + (i v_y^2 V_\perp^2 - \nu_m \omega_\parallel V_s^2)k_y^4 \\ - (2i\omega_\parallel v_y V_\perp^2)k_y^3 + (iA)k_y^2 + [2i\omega_\parallel v_y (V_\perp^2 k_x^2 \\ - 2\omega_\perp^2 k_y + i k_x^2 A - i\omega_\parallel^4 = 0.\end{aligned} \tag{2.13}$$

Here we have $A = \omega_\parallel^2 V_\perp^2 - k_x^2 V_A^2 V_s^2$, and we have used condition (2.2). This equation has the following solutions in the zeroth approximation in the small parameters specified by inequality (2.2), viz., (2.5) and

$$k_y^0 = \omega_\parallel / v_y, \tag{2.14}$$

$$k_y^{1-} = 2\omega_\parallel / v_y, \tag{2.15}$$

$$k_y^{2-} = k_x \mathrm{tg}\theta_0, \tag{2.16}$$

$$k_y^s = \frac{1}{2}\left[\frac{\omega_\parallel V_s^2 \cos^2\theta_0}{2 v_y V_A^2} \pm \left(-\frac{4\omega_\parallel^2}{V_s^2} + \frac{\omega_\parallel^2 V_s^4 \cos^4\theta_0}{4 v_y^2 V_A^4} \pm 2\sin\theta_0 |\cos\theta_0| \frac{\omega_\parallel^2 V_s}{v_y V_A^2}\right)^{1/2}\right]. \tag{2.17}$$

The "±" inside the square root in (2.17) has the same meaning as that of $|V_{\rm ph}^-|$ in (2.10); those in front of the square root determine two distinct solutions of Eq. (2.13). It follows from inequality (2.2) that we have $k_y \gg 1/a$ for perturbations (2.14) and (2.15), while for (2.16) and (2.17) we have instead $k_y \ll 1/a$.

The waves k_y^{1-} and k_y^2 are slow magnetosonic waves. For the wave k_y^{2-}, for k_x as in (2.12), the angle between **k** and the x axis is θ_0. The waves k_y^s may be either slow magnetosonic waves or surface waves, depending on the value of $v_y V_A^2/V_s^3$. We recall that if the perturbations are characterized by identical values of θ, rather than k_x, as in the case at hand, then there are always two slow waves, while the two others are fast magnetosonic waves.

If the expression in the square root in (2.17) is negative, k_y^s has an imaginary part, and the corresponding perturbations experience an exponential damping or growth with distance from the surface over a characteristic distance much greater than a.

Analysis of the quadratic trinomial in v_y in the square root in (2.17) shows that it vanishes at the points

$$v_y=\frac{V_s^3}{4V_A^2}|\cos\theta_0|(\mp\sin\theta_0\pm 1). \qquad (2.18)$$

The sign of $\sin\theta_0$ here is determined by the sign in (2.10). The two signs on the 1 in (2.18) determine the two ends of a segment along the v_y axis on which perturbations (2.17) are slow magnetosonic waves. Off this segment, the perturbations k_y^s become surface perturbations. That perturbation which increases with distance from the x axis must be discarded, since it does not satisfy the boundary condition at infinity. The damped perturbation should be included among the outgoing waves.[9]

Below we make use of the circumstance that when the velocity v_y is sufficiently large the waves in (2.17) are surface waves, regardless of θ_0. It can be shown that the function $v_y(\theta_0)$, given by (2.18), is bounded in absolute value by

$$v_y^{\max}=\frac{3\sqrt{3}}{16}\frac{V_s^3}{V_A^2}. \qquad (2.19)$$

The maximum value, (2.19), is reached at $\theta_0=\pi/6$. If

$$|v_y|>v_y^{\max}, \qquad (2.20)$$

the waves in (2.17) are surface waves for any value of θ_0.

Note that $v_y^{\max}$ is equal to the maximum value of the projection of the group velocity of the slow magnetosonic wave onto the y axis. In the approximation $V_s \ll V_A$, this projection is

$$(V_{\rm gr}^-)_y=\frac{V_s^3}{V_A^2}\sin\theta\cos^3\theta. \qquad (2.21)$$

In addition, this value is also reached at $\theta=\pi/6$. Inequality (2.20) thus means that all the slow waves are coming into the sheet or going out of it in the cases of an inflow or outflow of medium.

3. SMALL PERTURBATIONS INSIDE THE CURRENT SHEET

Here we wish to derive some equations for the perturbations of the MHD properties, δQ, inside the sheet, i.e., for $Q\sim Q^{\rm in}$. In this case we have $y\lesssim a$.

We first linearize the MHD equations. We set $Q_z\equiv 0$, $\partial\delta Q/\partial z\equiv 0$. The equations for δv_z and δB_z, which are set equal to zero, can then be separated from the equations for the other small quantities. In the approximation $a\ll b$, we can ignore the derivatives $\partial p/\partial x$, $\partial\mathbf{B}/\partial x$, and $\partial\rho/\partial x$ in the latter equations. The left side of inequality (2.2) shows that we can also ignore the derivative $\partial v_x/\partial_x$.

Let us consider, for example, the linearized continuity equation

$$\frac{\partial\delta\rho}{\partial t}+\delta\rho\frac{\partial v_x}{\partial x}+\rho\frac{\partial\delta v_x}{\partial_x}+\delta v_x\frac{\partial\rho}{\partial x}+v_x\frac{\partial\delta\rho}{\partial x}+v_y\frac{\partial\delta\rho}{\partial y}+\delta\rho\frac{\partial v_y}{\partial y}$$
$$+\delta v_y\frac{\partial\rho}{\partial y}+\rho\frac{\partial\delta v_y}{\partial_y}=0. \qquad (3.1)$$

Since the velocity variations inside the sheet are two-dimensional, we must ignore the terms with $\partial v_y/\partial y$ in addition to the terms containing the derivative $\partial v_x/\partial_x$. We choose the sign in (2.10) to be the same as the sign of v_x. Inside the sheet, $|v_x|$ is a decreasing function of $|y|$, while k_x is constant. It thus follows from (2.3) and (2.10)–(2.12) that $|\omega_\parallel|$ increases with decreasing $|y|$ and it satisfies the condition

$$|\omega_\parallel|>|\omega_{\parallel\,\rm ex}|. \qquad (3.2)$$

Evaluating

$$\frac{\partial\delta\rho}{\partial t}+v_x\frac{\partial\delta\rho}{\partial x}\sim\omega_\parallel\,\delta\rho,\quad \frac{\partial v_y}{\partial y}\sim\frac{v_y^{\rm ex}}{a},$$

we find the following from (3.2) and the left side of (2.2):

$$\frac{\partial\delta\rho}{\partial t}+v_x\frac{\partial\delta\rho}{\partial x}\ll\delta\rho\frac{\partial v_y}{\partial y}.$$

If we choose instead the other sign in (2.10), then there exists a y such that $\omega_\parallel=0$, and this inequality does not hold. Similar arguments hold for the other equations. In the zeroth approximation in the small parameters given by (2.2) we thus have $\partial Q/\partial x=0$. In addition, we set $\partial Q/\partial t=0$ in all the equations.

We replace $\partial\delta Q/\partial t$ by

$$-i\omega\left(\delta Q-\xi\frac{\partial Q}{\partial y}\right)\equiv -i\omega\hat{\mathscr{D}}Q, \qquad (3.3)$$

and $\partial\delta Q/\partial x$ by $ik_x\hat{\mathscr{D}}Q$, where ξ is the amplitude of the displacement of the current sheet as a whole.[4] We find a system of linear ordinary differential equations with respect to y:

$$i\omega_\parallel\hat{\mathscr{D}}\rho=ik_x\rho\hat{\mathscr{D}}v_x+(\rho\delta v_y)'+v_y\delta\rho, \qquad (3.4)$$

$$ik_x\hat{\mathscr{D}}B_x+\delta B_y'=0, \qquad (3.5)$$

$$i\omega_\parallel\rho\hat{\mathscr{D}}v_x=ik_x\hat{\mathscr{D}}p+\rho v_y\delta v_x'-\frac{B_x'\delta B_y}{4\pi}+v_x'\rho\delta v_y, \qquad (3.6)$$

$$i\omega_\parallel\rho\delta v_y=\delta\left(p+\frac{B_x^2}{8\pi}\right)'+\rho v_y\delta v_y'-ik_x\frac{B_x\delta B_y}{4\pi}, \qquad (3.7)$$

$$i\omega_\parallel\hat{\mathscr{D}}p=ik_x\gamma p\hat{\mathscr{D}}v_x+\gamma p\delta v_y'+\delta(p'v_y)$$
$$-\frac{\gamma-1}{2\pi}\nu_m B_x'\delta B_x', \qquad (3.8)$$

$$i\omega_\parallel\hat{\mathscr{D}}B_x=(B_x\delta v_y)'+v_y\delta B_x'-v_x'\delta B_y-\nu_m\delta B_x'', \qquad (3.9)$$

where the prime means the derivative with respect to y. Here we have used

$$p+B_x^2/8\pi=\text{const}, \tag{3.10}$$

which follows from the y component of the unperturbed momentum equation under approximation (1.6).

When the unperturbed MHD properties Q and the frequency ω from system (3.4)–(3.9) satisfy certain relations, we can derive boundary conditions (conservation laws) which relate the amplitudes of the perturbations on different sides of the current sheet.

For a $1D$ discontinuity these conditions are found by integrating the linearized equations over the thickness of the region in which the unperturbed quantities vary and by letting this thickness go to zero.

For example, we integrate induction equation (3.9), substituting in $v_x'=-\omega_\parallel' k_x$ [see (2.3)] and δB_y from (3.5):

$$i\omega_\parallel^{\text{ex}}\int_{-a}^{a}\delta B_x dy=\{B_x(\delta v_y+i\omega_\parallel \xi)\}+\int_{-a}^{a}v_y\delta B_x' dy -\nu_m\{\delta B_x'\}. \tag{3.11}$$

Here and below, the curly brackets mean the jump in a quantity across the discontinuity. We assume that δQ varies only slightly within the discontinuity if the condition $k_y^{\text{ex}}a \ll 1$ holds in the exterior. In particular, we can evaluate the integral proportional to $\omega_\parallel$ *supex*:

$$\omega_\parallel^{\text{ex}}\int_{-a}^{a}\delta B_x dy\sim\omega_\parallel^{\text{ex}}\delta B_x^{\text{ex}}a.$$

Let us compare this expression with the jump:

$$\{B_x\delta v_y\}\sim B_x^{\text{ex}}\delta b_y^{\text{ex}}.$$

The requirement $k_y^{\text{ex}}a \ll 1$ is satisfied in our case by waves (2.16) and (2.17). The relation between the perturbations δQ in such waves under approximation (1.6), (2,2) is given by

$$\delta p\sim V_s^2\delta\rho,\quad \delta v_x\sim V_s\frac{\delta\rho}{\rho},\quad \delta B_x\sim B_x\frac{V_s^2}{V_A^2}\frac{\delta\rho}{\rho},$$
$$\delta v_y\sim V_s\left(\frac{V_s}{V_A}\right)^2\frac{\delta\rho}{\rho},\quad \delta B_y\sim B_x\frac{V_s^2}{V_A^2}\frac{\delta\rho}{\rho}. \tag{3.12}$$

Using (3.12), we find that the condition

$$\omega_\parallel^{\text{ex}}\int_{-a}^{a}\delta B_x dy\ll\{B_x\delta v_y\}$$

is the same as the inequality $k_y^{\text{ex}}a \ll 1$, i.e., the right side of (2.2). Similar arguments for the other terms in (3.9) lead to the boundary condition

$$\{B_x(\delta v_y+i\omega_\parallel \xi)\}=0. \tag{3.13}$$

The use of the same procedure for Eq. (3.5) yields

$$\{\delta B_y-ik_xB_x\xi\}=0. \tag{3.14}$$

Since we have

$$\delta v_y=-\frac{\omega_\parallel \delta B_y}{k_xB_x}, \tag{3.15}$$

in magnetosonic waves under approximation (2.2), Eqs. (3.13) and (3.14) are satisfied if

$$\delta B_y=ik_x\xi B_x^{\text{ex}}, \tag{3.16}$$

and thus

$$\delta v_y=-i\omega_\parallel^{\text{ex}}\xi. \tag{3.17}$$

In contrast with a $1D$ discontinuity, δQ varies substantially in the current sheet. We show below that the perturbations of a medium with $k_y^{\text{ex}} \ll 1/a$ outside the sheet may lead to perturbations inside the sheet of such a nature that we have $k_y^{\text{in}} \gg 1/a$, where k_y^{in} has an imaginary part. Such perturbations undergo an exponential damping or growth with a length scale much smaller than a, and the estimates of the terms in Eq. (3.9) found above for the general case are not valid.

To derive boundary conditions at the current sheet as a discontinuity surface, we seek solutions of system (3.4)–(3.9) inside the sheet for given values of ω and k_x. We assume that only those waves for which the condition $k_y^{\text{ex}} \ll 1/a$ holds have nonzero amplitudes outside the sheet. We can make Eqs. (3.4)–(3.9) dimensionless by the following changes in the variable and in the unknown functions:

$$y=a\tilde{y}, \tag{3.18}$$

$$Q=Q^{\text{ex}}\tilde{Q}, \tag{3.19}$$

$$\delta Q=\delta Q^{\text{ex}}\delta\tilde{Q}, \tag{3.20}$$

$$\xi=(\delta v_y^{\text{ex}}/\omega_\parallel^{\text{ex}})\tilde{\xi}, \tag{3.21}$$

$$k_x=(\omega_\parallel^{\text{ex}}/V_s^{\text{ex}})\tilde{k}_x, \tag{3.22}$$

$$\delta v_y=-i\xi\omega_\parallel+\frac{a\omega_\parallel^{\text{ex}}}{V_s^{\text{ex}}}\delta v_y^{\text{ex}}\tilde{\omega}_\parallel\delta\tilde{v}_y, \tag{3.23}$$

$$\delta B_y=ik_x\xi B_x+\frac{a\omega_\parallel^{\text{ex}}}{V_s^{\text{ex}}}\delta B_y^{\text{ex}}\delta\tilde{B}_y. \tag{3.24}$$

The quantities δQ^{ex} here are related by (3.12); the expressions for δv_y and δB_y contain boundary conditions (3.16) and (3.17) explicitly.

We substitute expressions (3.18)–(3.24) into (3.4)–(3.9) and introduce the following small parameters in accordance with (2.2) and (2.4):

$$\varepsilon_0=\frac{v_y^{\text{ex}}}{a\omega_\parallel^{\text{ex}}},\quad \varepsilon_1=\frac{a\omega_\parallel^{\text{ex}}}{V_s^{\text{ex}}},\quad \varepsilon_2=\frac{v_y^{\text{ex}}}{V_s^{\text{ex}}},\quad \varepsilon_3=\left(\frac{V_s^{\text{ex}}}{V_A^{\text{ex}}}\right)^2. \tag{3.25}$$

As a result we find equations which determine dimensionless functions:

$$i\tilde{\omega}_\parallel\delta\tilde{\rho}=i\tilde{k}_x\tilde{\rho}\delta\tilde{v}_x+\varepsilon_3(\tilde{\rho}\tilde{\omega}_\parallel\delta\tilde{v}_y)'+\varepsilon_0\tilde{v}_y\delta\tilde{\rho}', \tag{3.26}$$

$$i\tilde{k}_x\delta\tilde{B}_x+\delta\tilde{B}_y'=0, \tag{3.27}$$

$$i\tilde{\omega}_\parallel\tilde{\rho}\delta\tilde{v}_x=i\tilde{k}_x\delta\tilde{p}-\frac{1}{\tilde{k}_x}\varepsilon_3\tilde{\omega}_\parallel\tilde{\omega}_\parallel'\tilde{\rho}\delta\tilde{v}_y-\tilde{B}_x'\delta\tilde{B}_y+\varepsilon_0\tilde{v}_y\tilde{\rho}\delta\tilde{v}_x', \tag{3.28}$$

$$(\delta\tilde{p}+\tilde{B}_x\delta\tilde{B}_x)'=\varepsilon_2\varepsilon_3\tilde{\rho}\tilde{v}_y[i\tilde{\xi}\tilde{\omega}_\parallel' -\varepsilon_1(\tilde{\omega}_\parallel \delta\tilde{v}_y)'] +\varepsilon_1\varepsilon_3\tilde{\omega}_\parallel^2 \tilde{\rho}(\tilde{\xi}+i\varepsilon_1\delta\tilde{v}_y) -\varepsilon_1\tilde{k}_x\tilde{B}_x(\tilde{k}_x\tilde{\xi}\tilde{B}_x-i\varepsilon_1\delta\tilde{B}_y), \quad (3.29)$$

$$i\tilde{\omega}_\parallel \delta\tilde{p}=i\tilde{k}_x\tilde{p}\delta\tilde{v}_x+\varepsilon_3\left[\tilde{p}(\tilde{\omega}_\parallel \delta\tilde{v}_y)'+\frac{1}{\gamma}\tilde{\omega}_\parallel \tilde{p}'\delta\tilde{v}_y\right] +\varepsilon_0[\tilde{v}_y\delta\tilde{p}'-2(\gamma-1)\tilde{B}_x'\delta\tilde{B}_x'], \quad (3.30)$$

$$i\tilde{\omega}_\parallel \delta\tilde{B}_x=(\tilde{B}_x\tilde{\omega}_\parallel \delta\tilde{v}_y)'+\frac{1}{\tilde{k}_x}\tilde{\omega}_\parallel' \delta\tilde{B}_y+\varepsilon_0(\tilde{v}_y\delta\tilde{B}_x'-\delta\tilde{B}_x''). \quad (3.31)$$

Since we are interested in solutions of system (3.26)–(3.31) in the approximation (2.2), we let ε_i go to zero. As a result, the equations reduce to

$$i\tilde{\omega}_\parallel \delta\tilde{\rho}=i\tilde{\rho}\delta\tilde{v}_x, \quad (3.32)$$

$$i\delta\tilde{B}_x+\delta\tilde{B}_y'=0, \quad (3.33)$$

$$i\tilde{\omega}_\parallel \tilde{\rho}\delta\tilde{v}_x=i\delta\tilde{p}-\varepsilon_3\tilde{\omega}_\parallel \tilde{\omega}_\parallel' \tilde{\rho}\delta\tilde{v}_y-\tilde{B}_x'\delta\tilde{B}_y, \quad (3.34)$$

$$(\delta\tilde{p}+\tilde{B}_x\delta\tilde{B}_x)'=0, \quad (3.35)$$

$$i\tilde{\omega}_\parallel \delta\tilde{p}=i\tilde{p}\delta\tilde{v}_x+\varepsilon_3\left[\tilde{p}(\tilde{\omega}_\parallel \delta\tilde{v}_y)'+\frac{1}{\gamma}\tilde{\omega}_\parallel \tilde{p}'\delta\tilde{v}_y\right], \quad (3.36)$$

$$i\tilde{\omega}_\parallel \delta\tilde{B}_x=(\tilde{B}_x\tilde{\omega}_\parallel \delta\tilde{v}_y)'+\tilde{\omega}_\parallel' \delta\tilde{B}_y. \quad (3.37)$$

Terms proportional to ε_3 have been retained in (3.34) and (3.36), since inside the sheet we have $\tilde{\omega}_\parallel', \tilde{\omega}_\parallel \lesssim 1/\sqrt{\varepsilon_3}$ [see (1.7)] and $\tilde{p},\tilde{p}' \sim 1/\varepsilon_3$ [see (3.10)]. In addition, we have used an expression for $\tilde{k}_x$ which follows from (2.11) and (2.12):

$$\tilde{k}_x=1+O(\varepsilon_2)+O(\varepsilon_3). \quad (3.38)$$

In system (3.32)–(3.37), Eqs. (3.32) and (3.34) are not differential but are algebraic definitions of the functions $\delta\tilde{\rho}$ and $\delta\tilde{v}_x$. After $\delta\tilde{B}_x$ from (3.33) is substituted into (3.37), the latter becomes a total derivative with respect to y, and its integration leads to

$$\delta\tilde{B}_y+\tilde{B}_x\delta\tilde{v}_y=0. \quad (3.39)$$

We have set the constant of integration in this equation equal to zero, since the perturbation outside the sheet is a superposition of magnetosonic waves, for which (3.15) holds. Integration of (3.35) leads to

$$\delta\tilde{p}+\tilde{B}_x\delta\tilde{B}_x=C_0. \quad (3.40)$$

Substitution of (3.34), (3.39), and (3.40) into Eq. (3.36) reduces the latter to an inhomogeneous first-order equation:

$$\left[\varepsilon_3\tilde{p}+\tilde{B}_x^2\left(1-\frac{\tilde{p}}{\tilde{\rho}\tilde{\omega}_{|\text{back}|0_2}}\right)\right]\delta\tilde{v}_y'+\left(\frac{1}{\gamma}\varepsilon_3\tilde{p}'+\tilde{B}_x\tilde{B}_x'\right)\delta\tilde{v}_y =iC_0\left(1-\frac{\tilde{p}}{\tilde{\rho}\tilde{\omega}_{|\text{back}|0_2}}\right). \quad (3.41)$$

Expressing the dimensionless quantities in the coefficient of $\delta\tilde{v}_y$ in terms of dimensional quantities, we find that this coefficient is [see (3.10)]

$$\left(p+\frac{B_x^2}{8\pi}\right)'\frac{4\pi a}{(B_x^{\text{ex}})^2}=0. \quad (3.42)$$

Equation (3.41) can be integrated in quadrature. The solution of system (3.33), (3.35)–(3.37) is

$$\delta\tilde{v}_y=iC_0\int\frac{(1-\tilde{p}/\tilde{\rho}\tilde{\omega}_\parallel^2)d\tilde{y}}{\varepsilon_3\tilde{p}+\tilde{B}_x^2(1-\tilde{p}/\tilde{\rho}\tilde{\omega}_\parallel^2)}+C, \quad (3.43)$$

$$\delta\tilde{B}_y=-\tilde{B}_x\delta\tilde{v}_y, \quad (3.44)$$

$$\delta\tilde{B}_x=-i(\tilde{B}_x\delta\tilde{v}_y)', \quad (3.45)$$

$$\delta\tilde{p}=C_0-\tilde{B}_x\delta\tilde{B}_x. \quad (3.46)$$

Solution (3.43)–(3.46) has a singularity at the point $\tilde{y}_0$, at which we have

$$\tilde{A}\equiv\varepsilon_3\tilde{p}+\tilde{B}_x^2\left(1-\frac{\tilde{p}}{\tilde{\rho}\tilde{\omega}_{|\text{back}|0_2}}\right)=0, \quad (3.47)$$

and the integrand in (3.43) becomes infinite. By expressing $\delta Q'$ in terms of δQ in the system (3.4)–(3.9), one can show that it has a singularity only at $y=0$, where $v_y=0$. This result means that in some neighborhood of $\tilde{y}_0$ we cannot ignore the small parameters in (3.26)–(3.31) and switch to (3.32)–(3.37). The neighborhood of the point $\tilde{y}_0$ will be discussed below.

We now wish to find the other solutions of system (3.26)–(3.31) in the region in which (3.43)–(3.46) hold. For definiteness we assume that $v_x^{\text{in}} \sim V_A^{\text{ex}}$ [see (1.7)], i.e., $\tilde{\omega}_{|\text{back}|0_2} \sim \varepsilon_3$. This relation holds if x is not close to 0 or $\pm b$. Solution (3.43)–(3.46) is valid when the integrand in (3.43) is on the order of one. Since we have $\tilde{B}_x \lesssim 1, \tilde{p}\sim 1/\varepsilon_3$ inside the sheet, we find from (3.43) and (3.47) that in this case we have

$$\tilde{A}\sim 1. \quad (3.48)$$

The other solutions of system (3.26)–(3.31) satisfy then the WKB approximation in the sheet and can be found from dispersion relation (2.13). Expressing the dimensionless quantities in $\tilde{A}$ in terms of dimensional quantities, and noting that we have $k_x = \omega_\parallel^{\text{ex}}/V_s^{\text{ex}}$, we find that $\tilde{A}$ is related in the following way to the coefficient of k_y^2 in (2.13):

$$A\sim\omega_\parallel^2 (V_A^{\text{ex}})^2\tilde{A}. \quad (3.49)$$

Under condition (3.48), in the zeroth approximation in ε_1, the solutions of Eq. (2.13) have the form of (2.14) and

$$k_y^d=\omega_\parallel /v_y, \quad (3.50)$$

$$k_y^- = \pm\sqrt{\frac{iA}{V_s^2 v_m\omega_\parallel}}, \quad (3.51)$$

$$k_y^*=\frac{1}{A}[\omega_\parallel v_yF\pm\sqrt{\omega_\parallel^2 v_y^2F^2-A(k_x^2A-\omega_\parallel^4)}], \quad (3.52)$$

where $F=V_1^2 k_x^2-2\omega_\parallel^2$.

It follows from inequality (2.2) that wave vectors (2.14), (3.50), and (3.51) satisfy the condition $k_y \gg 1/a$. These perturbations satisfy the WKB approximation in the sheet. Dispersion relation (2.13) is valid for them, since in the limit $1/k_y a \to 0$ the terms in Eqs. (3.4)–(3.9) containing derivatives of the unperturbed quantities are negligibly small. Expressions (2.14), (3.50), and (3.51) give us four solutions of system (3.4)–(3.9). The perturbations in (3.52), on the other hand, do not satisfy the WKB approximation, since the condition $k_y \ll 1/a$ holds in their case. In this case we cannot ignore the derivatives of the unperturbed quantities in (3.4)–(3.9), so Eq. (2.13) does not hold. Such perturbations are described by (3.43)–(3.46).

4. SOLUTION OF THE LINEARIZED EQUATIONS AT THE SHEET BOUNDARY

To find boundary conditions we need to determine the magnitude of a perturbation at the boundary of the sheet, i.e., at $Q=Q^{\rm ex}$. In this case we have $a \ll y \ll 1/k_y^{\rm ex}$. If $Q=Q^{\rm ex}$, solution (3.43)–(3.46) does not apply, since the coefficients in (3.41) are much smaller than unity and we cannot ignore small parameters in the derivation of this equation.

We find solutions of Eqs. (3.26)–(3.31) near the sheet boundary, in the region

$$\widetilde{Q} \sim 1. \tag{4.1}$$

Since we have $p^{\rm in} \gg p^{\rm ex}, \omega_\parallel^{\rm in} \gg \omega_\parallel^{\rm ex}$, there exists a value $\widetilde{y}$ for which we have $\widetilde{p} \gg 1$, $\widetilde{\omega}_\parallel \gg 1$, although for $\widetilde{y} \gg 1$ we always have $\widetilde{Q}'/\widetilde{Q} \ll 1$.

We substitute (3.27) into (3.31), and then (3.31) and (3.28) into (3.30), as in the derivation of (3.41). However, we retain terms proportional to ε_0:

$$i\widetilde{\omega}_\parallel \left(1-\frac{\widetilde{p}}{\widetilde{\rho}\widetilde{\omega}_\parallel^2}\right)\delta\widetilde{p} = \widetilde{\omega}_\parallel\, \varepsilon_3 \left(\widetilde{p}\delta\widetilde{v}_y' + \frac{1}{\gamma}\widetilde{p}'\delta\widetilde{v}_y\right) - \frac{\widetilde{p}}{\widetilde{\rho}\widetilde{\omega}_\parallel}\widetilde{B}_x'\delta\widetilde{B}_y + \varepsilon_0\widetilde{v}_y\left(\frac{\widetilde{p}}{\widetilde{\omega}_\parallel}\delta\widetilde{v}_x' + \delta\widetilde{p}'\right). \tag{4.2}$$

Here we have used (3.38) and the inequalities $\varepsilon_0 \ll \varepsilon_2, \varepsilon_3$, which follow from (2.2).

Since the derivatives $\delta\widetilde{v}_x'$ and $\delta\widetilde{p}'$ appear in (4.2) with a small parameter, they can be found in a first approximation from Eqs. (3.34) and (3.35), which do not contain small parameters. We differentiate (3.34) and use (3.39) and (3.40). Setting $\widetilde{Q}' \ll 1$, and using (3.42), we then find an equation for the function $\delta\widetilde{v}_y$:

$$i\varepsilon_0 \widetilde{B}_x^2 \widetilde{v}_y \left(1+\frac{\widetilde{p}}{\widetilde{\rho}\widetilde{\omega}_\parallel^2}\right)\delta\widetilde{v}_y'' + \widetilde{\omega}_\parallel\, \widetilde{A}\delta\widetilde{v}_y' = iC_0\widetilde{\omega}_\parallel \left(1-\frac{\widetilde{p}}{\widetilde{\rho}\widetilde{\omega}_\parallel^2}\right) \tag{4.3}$$

[c.f. (3.41)].

If $1-\widetilde{p}/\widetilde{\rho}\widetilde{\omega}_\parallel^2 \gg \varepsilon_0$, then $\widetilde{A} \gg \varepsilon_0$ [see (3.47)], and (3.41) holds. We assume $1-\widetilde{p}/\widetilde{\rho}\widetilde{\omega}_\parallel^2 \lesssim \varepsilon_0$; then we have $\widetilde{A} \lesssim \varepsilon_0$, and all terms in Eq. (4.3) are important. In this case, in a first approximation, it is sufficient to substitute $\delta\widetilde{p}$ from (3.40), rather than (3.29), into (4.2), so ε_1 does not appear in Eq. (4.3).

At the layer boundary ($|\widetilde{Q}|=1$) we have

$$1=\widetilde{p}/\widetilde{\rho}\widetilde{\omega}_\parallel^2=0, \quad \widetilde{A}=0$$

and Eq. (4.3) becomes $\delta\widetilde{v}_y''=0$. After an integration, this equation reduces to

$$\delta\widetilde{v}_y = C_*\widetilde{y} + C. \tag{4.4}$$

Expression (4.4) along with (3.43)–(3.46) determines three solutions of system (3.26)–(3.31). The other three equations, with $|\widetilde{Q}|=1$, satisfy the WKB approximation with wave vectors (2.5), (2.14), and (2.15).

Let us go back to the neighborhood of the point $\widetilde{y}_0$, at which we have $\widetilde{A}=0$. It follows from Eq. (3.10) and condition (1.7) that the point $\widetilde{y}_0$ can, in general, be in either the region $\widetilde{y} \lesssim 1$ or the region $\widetilde{y} \gg 1$. If

$$\widetilde{y}_0 \lesssim 1, \tag{4.5}$$

then with $\widetilde{A}=0$ terms containing $\widetilde{v}_y'$ appear in the equation for $\delta\widetilde{v}_y$ in the first approximation. Since we have $\widetilde{v}_y' \sim 1$, these terms are comparable to the terms proportional to $\partial v_x/\partial x$, which we ignored in deriving system (3.4)–(3.9). In order to find $\delta\widetilde{v}_y$ near the point $\widetilde{y}_0$ in this case we thus need to solve a partial differential equation.

We assume

$$\widetilde{y}_0 \gg 1. \tag{4.6}$$

We then have $\widetilde{v}_y' \ll 1$, and at $\widetilde{y} = \widetilde{y}_0$ the function $\delta\widetilde{v}_y$ is described in the first approximation by an ordinary differential equation. In particular, in region (4.1) this is Eq. (4.3). This equation does not have a singularity at $\widetilde{A}=0$, and the solution of (3.26)–(3.31) near the point $\widetilde{y}_0$ is given by (4.4), (3.43)–3.46), (2.5), (2.14) and (2.15).

Finally, we wish to establish the correspondence between the perturbations inside and outside the sheet. We assume that (4.6) holds, and we assume that (3.48) holds for $\widetilde{y} \lesssim 1$.

Solving system (3.26)–(3.31) in the region $1 \ll (\widetilde{\rho}, \widetilde{\omega}_\parallel^2) \ll 1/\varepsilon_3$, we can show that the following correspondence prevails: Perturbations which are determined by the wave vectors k_y^d from (2.5) and k_y^0 from (2.14) outside the sheet become (3.50) and (2.14) inside the sheet. In other words, they are the same roots of Eq. (2.13) for different values of $\widetilde{y}$. Wave (2.15) becomes one of perturbations (3.51) (with the plus sign or the minus sign, depending on the sign of v_y). Accordingly, the superposition of (3.43)–(3.46) and the second of perturbations (3.51) corresponds to the set (2.16) and (2.17).

Furthermore, we can choose a frequency $\omega_\parallel$ from the interval (2.2) such that the solution proportional to C_0 is present in the sheet for any value of $\widetilde{y}$. In this case the solution which is proportional to C_* in region (4.1) becomes a perturbation with wave vector (3.51) and $\widetilde{y} \lesssim 1$.

The waves with $k_y^{\rm ex} \ll 1/a$ outside the sheet thus lead to perturbations inside the sheet for which the condition $k_y^{\rm in} \gg 1/a$ holds.

5. BOUNDARY CONDITIONS AND EVOLUTIONARITY CRITERION

To derive the conditions under which the current sheet is evolutionary, we derive boundary conditions on the current sheet as a discontinuity surface.

If the perturbation amplitudes (2.14), (3.50), and (3.51) with $k_y \gg 1/a$ inside the sheet are nonzero, then there are no boundary conditions on the surface of the sheet corresponding to the those which hold at $1D$ discontinuities. If these conditions do hold, then they tell us, in particular [see (3.17)], that δv_y remains constant as the sheet is crossed. In addition, the magnitude of the perturbations in (2.14), (3.50), and (3.51) changes substantially over a distance a, and (3.17) generally does not hold. Below we consider only those perturbations for which the mode amplitudes in (2.14), (3.50), and (3.51) are zero. This requirement is satisfied by a solution of Eqs. (3.4)–(3.9) in which the constant C_0 is nonzero, while the other constants are zero (see the discussion at the end of Sec. 4).

To find the boundary conditions satisfied by the perturbation proportional to C_0, we note that Eqs. (3.23) and (3.24), with (4.4) taken into account, lead to the boundary values in (3.16) and (3.17) for δv_y and δB_y. From (3.17) we find

$$\{\delta v_y\}=0. \tag{5.1}$$

Relation (3.16) does not lead to an additional boundary condition, since it is equivalent to (3.17). Expression (3.46) determines a second boundary condition:

$$\left\{\delta p+\frac{B_x\delta B_x}{4\pi}\right\}=0. \tag{5.2}$$

Finally, (3.45) implies

$$\delta B_x=0 \tag{5.3}$$

on both sides of the discontinuity, since $\delta\widetilde{v}_y'=0$ and $\widetilde{B}_x'=0$.

The reason for the appearance of condition (5.3) is that we are dealing not with an aribitrary perturbation but one for which only the constant C_0 is nonzero. If there are other perturbations inside the sheet, condition (5.3) generally does not hold. Since δB_x is not zero in magnetosonic waves, condition (5.3) along with (5.1) and (5.2) constitutes four boundary conditions relating the wave amplitudes outside the sheet. Equations (3.32) and (3.34) do not give us additional boundary conditions, since they hold for perturbations in magnetosonic waves.

We now write Eqs. (5.1)–(5.3) explicitly; i.e., we express all quantities in terms of density perturbations. As we mentioned toward the end of Sec. 4, the set of perturbations in (2.16)–(2.17) outside the current sheet corresponds to a superposition of (3.43)–(3.46) and (3.51) inside the sheet. Accordingly, if only the constant C_0 is nonzero inside the sheet, then outside the sheet we have waves (2.16) and (2.17), and the amplitudes in (2.5), (2.14), and (2.15) are zero. Using the relationship between the perturbations of the MHD properties in magnetosonic waves under approximation (2.2), we find from (5.1)–(5.3), respectively,

$$\sum_{i=1}^{3}\frac{k_{y+}^{(i)}}{(k^{(i)})^2}(\delta\rho_+^{(i)}+\delta\rho_-^{(i)})=0, \tag{5.4}$$

$$\sum_{i=1}^{3}\frac{1}{(k^{(i)})^2}(\delta\rho_+^{1}-\delta\rho_-^{(i)})=0, \tag{5.5}$$

$$\sum_{i=1}^{3}\left(\frac{k_y^{(i)}}{k^{(i)}}\right)^2\delta\rho_\pm^{(i)}=0. \tag{5.6}$$

The "+" and the "−" here specify quantities outside the sheet, on the $y=+\infty$ and $y=-\infty$ sides; the "i" specifies the three waves in (2.16) and (2.17). Here we have made use of the condition $k_{y+}^i=-k_{y-}^i$, which follows from the symmetry of the flow.

We wish to find solutions of these equations for the cases in which the medium is flowing in and out. In other words, we wish to find the amplitudes of the outgoing waves in terms of the amplitudes of the incoming waves.

If medium flows into the sheet, there are two outgoing waves (one on each side). Since there are four equations, the system (5.4)–(5.6) has solutions only if the amplitudes of the incident waves satisfy certain relations. For arbitrary amplitudes, Eqs. (5.4)–(5.6) do not have solutions. This means in particular that condition (5.3) cannot hold for such perturbations. Since Eq. (5.3) always holds if C_0 is the only nonzero arbitrary constant, the violation of this equality leads to nonzero values of the other constants—i.e., perturbation amplitudes with $k_y^{\text{in}}\gg 1/a$—inside the sheet. Consequently, there are no boundary conditions at the surface of the current sheet, i.e., the sheet is not a discontinuity, and we cannot conclude that it is not evolutionary.

Let us assume that medium is flowing out of the sheet. In this case there are four outgoing waves (two on each side). We specify these waves by the indices $i=1$, 2. The amplitudes $\delta\rho_\pm^{1,2}$ are then expressed in terms of the amplitudes $\delta\rho_\pm^3$ of the incident waves, as follows:

$$\delta\rho_\pm^{(1)}=-\frac{1}{2}\left(\frac{k^{(1)}}{k^{(3)}}\right)^2\frac{k_y^{(2)}-k_y^{(3)}}{k_y^{(2)}-k_y^{(1)}}\left[\frac{k_y^{(3)}}{k_y^{(1)}}(\delta\rho_+^{(3)}+\delta\rho_-^{(3)})\pm\frac{k_y^{(2)}+k_y^{(3)}}{k_y^{(2)}+k_y^{(1)}}(\delta\rho_+^{(3)}-\delta\rho_-^{(3)})\right], \tag{5.7}$$

$$\delta\rho_\pm^{(2)}=-\left(\frac{k^{(2)}}{k_y^{(2)}}\right)^2\left[\left(\frac{k_y^{(3)}}{k^{(3)}}\right)^2\delta\rho_\pm^{(3)}+\left(\frac{k_y^{(1)}}{k^{(1)}}\right)^2\delta\rho_\pm^{(1)}\right].$$

All the quantities k_y in (5.7) are taken on one side of the discontinuity. It follows from (5.7) that if we have $k_y^1=k_y^2$ and $k_y^2\neq k_y^3$, then $\delta\rho_\pm^1$ becomes infinite; i.e., the reflection coefficients and refractive indices are not bounded.

Under what conditions are the wave vectors of the two outgoing waves the same? It was shown back in Sec. 2 that if

$$|v_y^{\text{ex}}|<\frac{3\sqrt{3}}{16}\frac{V_s^3}{V_A^2}, \tag{5.9}$$

then there is always a resonant angle θ_0^* at which the expression in the radical in (2.17) is zero, and the two roots of (2.17) are the same. This angle is found from Eq. (2.18). At $\theta_0=\theta_0^*$, both waves in (2.17) are outgoing waves, since only one incoming wave exists in the case of an outflow of medium. In this case, this wave is given by (2.16), with $k_y^2 \neq k_y^3$.

If (5.9) does not hold, the expression in the radial in (2.17) is negative, and the waves are surface waves for any θ_0 (Sec. 2). In this case all the wave vectors are different, and we have $k_y^i \neq k_y^j$ for $i \neq j$. The reflection coefficients and the refractive indices are therefore bounded.

CONCLUSION

In summary, if a medium is flowing into the current sheet, or if inequality (5.9) does not hold, we cannot draw the conclusion that the sheet is not evolutionary. Let us assume that relation (4.6) holds in the case of an outflow of medium and that the outflow velocity is smaller than the projection of the group velocity of the slow magnetosonic wave onto the normal to the sheet [see (5.9)]. Then there exists a perturbation such that (first) boundary conditions hold at the surface of the sheet and (second) the amplitudes of the outgoing waves which are determined from these conditions are arbitrarily large in comparison with that of the incoming wave in the limit as $\varepsilon_i \to 0$, i.e., in the case of a sufficiently high conductivity. In this case the perturbation is not described by linear equations. Accordingly, a reconnecting current sheet is not evolutionary as a discontinuity, since there is no initial change in the flow. This change might be a splitting of the sheet into $1D$ evolutionary discontinuities, as has been observed in numerical simulations.[2,3]

[1] S. I. Syrovatskiĭ, Zh. Eksp. Teor. Fiz. **60**, 1727 (1971) [Sov. Phys. JETP **33**, 933 (1971)].

[2] K. V. Brushlinskiĭ, A. M. Zaborov, and S. I. Syrovatatskiĭ, Fiz. Plazmy **6**, 297 (1980) [Sov. J. Plasma Phys. **6**, 165 (1989)].

[3] D. Biskamp, Phys. Fluids **29**, 1520 (1986).

[4] S. I. Syrovatskiĭ, *Proceedings of the Lebedev Physics Institute*, Vol. 8, 13, Nauka, Moscow.

[5] A. I. Akhiezer, G. Ya. Lyubarskiĭ, and R. V. Polovin, Zh. Eksp. Teor. Fiz. **35**, 731 (1958) [Sov. Phys. JETP **8**, 507 (1959)].

[6] S. I. Syrovatskiĭ, Zh. Eksp. Teor. Fiz. **35**, 1466 (1958) [Sov. Phys. JETP **8**, 1024 (1959)].

[7] S. I. Syrovatskiĭ, in *Neutral Current Sheets in Plasmas (Vol. 74, Lebedev Institute Proceedings)*, Consultants Bureau, New York, 1976.

[8] S. I. Syrovatskiĭ, Pis'ma Astron. Zh. **2**, 35 (1976) [Sov. Astron. Lett. **2**, 13 (1976)].

[9] V. M. Kontorovich, Zh. Eksp. Teor. Fiz. **35**, 1216 (1958) [Sov. Phys. JETP **8**, 851 (1959).

[10] D. I. Blokhintsev, Dokl. Akad. Nauk SSSR **47**, 22 (1945).

[11] Z. B. Roĭkhvarger and S. I. Syrovatskiĭ, Zh. Eksp. Teor. Fiz. **66**, 1338 (1974) [Sov. Phys. JETP **39**, 654 (1974)].

Translated by D. Parsons