Four-wave interactions of intense radiation in vacuum

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The relatively low-threshold four-wave interactions of electromagnetic radiation in an electron–positron vacuum are analyzed. The efficiency with which three synchronized pump waves with frequencies ω_1, ω_2, and ω_3 produce the combination mode with frequency $\omega_1+\omega_2-\omega_3$ is shown. The nonlinear phase shifts of the interacting radiation pulses are determined. Transverse effects associated with the interaction of radiation beams (decay instability, mutual channeling, and mutual focusing) are considered. It is shown that vacuum polarization effects can be observed in laser experiments, including phase conjugation.

1. INTRODUCTION. INITIAL EQUATIONS

Progress in obtaining laser radiation with high luminance, and in particular, ultrashort (femtosecond) laser pulses with peak radiation of up to 10^{23} W/cm^2 (Ref. 1) is the motivation for studying quantum-electrodynamic phenomena in strong light fields. Among these phenomena the polarization of the vacuum has a fundamental significance. Although the critical value of the laser intensity at which electron–positron pairs are efficiently produced ($I_{cr}=2.5\cdot 10^{29}$ W/cm^2) will not be achieved anytime soon, the use of lasers in this field is promising for the following reasons. If we are interested in the changes in the properties of the laser radiation itself due to vacuum polarization, then they can be detected significantly below the critical intensity level because of changes accumulated along the path of the beam (propagation effects) and the high accuracy with which weak changes in the radiation properties can be measured. In addition, accumulated nonlinear distortions of the laser radiation can significantly enhance its local intensity, since, e.g., when radiation undergoes self-focusing in ordinary (focusing) media its peak intensity increases many times.[2,3]

The analysis carried out by Aleksandrov *et al.*[4] showed that vacuum polarization effects can realistically be observed in specially designed laser experiments, although this demands the attainment of exceptional accuracy in the optical measurements when powerful ultrashort laser pulses are used. The question of possible competing nonlinear effects such as, e.g., the decay instability, has remained open. In the present work we analyze and compare the achievability of a number of other four-wave nonlinear optical phenomena in vacuum, in which the synchronization conditions necessary for the effect to accumulate spatially can be satisfied.

We start with Maxwell's equation, including the Heisenberg–Euler radiation corrections caused by vacuum polarization. In the notation of Berestetskiĭ *et al.*[5] they have the same form as the equations for the electrodynamics of continuous media:

$$\text{div}\,\mathbf{B}=0,\quad \text{rot}\,\mathbf{E}=-\partial\mathbf{B}/\partial t,\quad \text{rot}\,\mathbf{H}=\partial\mathbf{D}/\partial t,\quad \text{div}\,\mathbf{D}=0, \tag{1.1}$$

$$\mathbf{D}=\mathbf{E}+4\pi\mathbf{P},\quad \mathbf{H}=\mathbf{B}-4\pi\mathbf{M}. \tag{1.2}$$

We use the relativistic system of units, in which we set Planck's constant $\hbar=1$ and the velocity of light $c=1$. The specific properties of the vacuum are expressed in the form of "constituent equations." For relatively low-frequency electric and magnetic fields **E** and **B** the vectors **P** and **M** of the electric and magnetic polarization of the vacuum are expressed in terms of the radiative correction to the Lagrangian density (see, e.g., Ref. 5; a more complete expression is given in Refs. 6 and 7). Since we will be interested in the effects with the lowest thresholds, it suffices to retain the terms which have the lowest powers of the fields. Then

$$\mathbf{P}=(e^4/180\pi^2m^4)(-4F\mathbf{E}+7G\mathbf{B}),\quad \mathbf{M}=(e^4/180\pi^2m^4)(4F\mathbf{E}+7G\mathbf{B}). \tag{1.3}$$

In (1.3) we have written $F=(\mathbf{B}^2-\mathbf{E}^2)/2$, $G=\mathbf{E}\,\mathbf{B}$, and m and e are the electron mass and charge, respectively (in our units $e^2=1/137$). The vacuum is transparent (the probability of producing actual electron–positron pairs is negligibly small). Note that Eqs. (2.1) and (2.2) have the semiclassical form[8] which is usual for nonlinear optics (the form of the constituent equations is determined from quantum theory, after which the electromagnetic field is described classically). The limits of applicability of this system according to Refs. 5 and 6 assume the form

$$E\ll E_{cr},\quad \omega\ll m(E/E_{cr}). \tag{1.4}$$

Here $E_{cr}=m^2/e$ is the critical value of the electric field strength corresponding to the value I_{cr}, the intensity at which electron–positron pairs are produced.

In contrast with the usual problems of nonlinear optics,[8] the vacuum is characterized by both the electric and the magnetic nonlinear polarizations simultaneously. For a plane electromagnetic wave the contributions of the electromagnetic components mutually annihilate (the invariants satisfy $F=G=0$) and there are no radiative corrections. The corrections appear when external fields are

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included; e.g., in a static magnetic field magnetic anisotropy of the vacuum[5] and other effects which are important for astrophysics occur.[9–12] Under laboratory conditions higher field strengths are produced in ultrashort laser pulses. Consequently, here we will consider only manifestations of vacuum polarization in the field of several crossed laser beams.

2. FOUR-WAVE INTERACTIONS OF PLANE WAVES

2.1. Reduced equations

We represent the field in the form of a set of plane waves:

$$\mathbf{E}=(1/2)\sum_f \mathbf{E}_f \exp[i(\mathbf{k}_f\mathbf{r}-\omega_f t)]+\text{c.c.},$$
$$\mathbf{B}=(1/2)\sum_f \mathbf{B}_f \exp[i(\mathbf{k}_f\mathbf{r}-\omega_f t)]+\text{c.c.} \tag{2.1}$$

If we neglect vacuum polarization ($\mathbf{P}=\mathbf{M}=0$) the individual plane waves do not interact with one another. Then their complex amplitudes $\mathbf{E}_f$ and $\mathbf{B}_f$ are constant. For them, and also for the wave vectors $\mathbf{k}_f$ and frequencies ω_f, it follows from the Maxwell equations that

$$\mathbf{k}_f^2=\omega_f^2,\quad \mathbf{B}_f=[(\mathbf{k}_f/\omega_f)\mathbf{E}_f]. \tag{2.2}$$

Substituting (2.1) in (1.2) yields the electric and magnetic polarizations of the vacuum in the form

$$\mathbf{P}=(1/2)\sum_v \mathbf{P}_v \exp[i(\mathbf{k}_v\mathbf{r}-\omega_v t)]+\text{c.c.},$$
$$\mathbf{M}=(1/2)\sum_v \mathbf{m}_v \exp[i(\mathbf{k}_v\mathbf{r}-\omega_v t)]+\text{c.c.} \tag{2.3}$$

If the fields are represented by a given set of wave vectors and frequencies, then for the polarization a larger set of these quantities would be required. Substitution of the polarizations $\mathbf{P}$ and $\mathbf{M}$ in Eq. (1.1) shows that components with new (combination) wave vectors and frequencies arise systematically. However, they are produced effectively only when the synchronization conditions hold, namely

$$\delta_v^2 \ll \omega_v^2,\quad \delta_v=\omega_v-k_v. \tag{2.4}$$

Moreover, when the region where the different beams overlap is small, the amplitude of the combination mode is also small (see below). In the representation of the field and the vacuum polarization we can therefore restrict ourselves to a finite (and small) set of plane waves.

In treating the interaction of waves in vacuum we regard the amplitude $\mathbf{E}_j$, $\mathbf{B}_j$, $\mathbf{P}_j$, and $\mathbf{M}_j$ as slowly varying (on the scale of the radiation wavelength λ_j) functions of the coordinate z (for simplicity we choose the z axis in the direction of the wave vector $\mathbf{k}_j$ and assume that all waves are monochromatic). Then, taking into account (2.4), the reduced equations for the amplitude components assume the form

$$dE_{jx}/dz=i\delta_j E_{jx}+2\pi i\omega_j(P_{jx}+M_{jy}),$$
$$dE_{jy}/dz=i\delta_j E_{jy}+2\pi i\omega_j(P_{jy}-M_{jx}),$$
$$dB_{jx}/dz=i\delta_j B_{jx}-2\pi i\omega_j(P_{jy}-M_{jx}), \tag{2.5}$$
$$dB_{jy}/dz=i\delta_j B_{jy}-2\pi i\omega_j(P_{jx}+M_{jy}).$$

Note that from (2.5) we can deduce the conservation of the quantities

$$(E_{jx}-B_{jy})\exp(i\delta_j z)=\text{const},$$
$$(E_{jy}+B_{jx})\exp(i\delta_j z)=\text{const}. \tag{2.6}$$

The longitudinal components of the field are quantities of higher order. For nonmonochromatic (pulsed) radiation we must substitute $d/dz\to\partial/\partial z+\partial/\partial t$ in the left-hand sides of Eqs. (2.5).

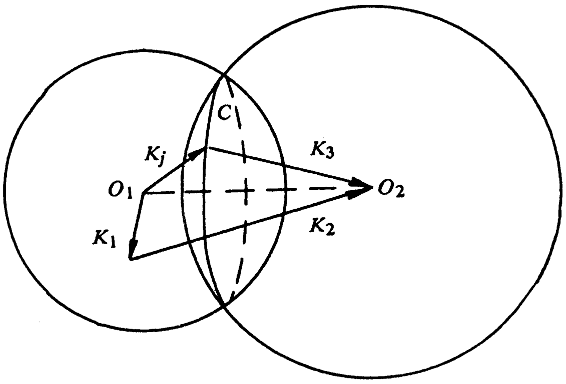

FIG. 1. Diagram of the wave vectors satisfying the synchronization condition.

2.2. Generation of combination modes

Since in this approximation the polarization (1.3) is of third order in the field (in the terminology of nonlinear optics the vacuum is a cubic medium), it suffices to follow the generation of a combination mode by three initial pump waves. We write the subscripts of these waves in Eq. (2.1) as $j=1$, 2, and 3. Note that for a wave with the sum frequency $\omega_j=\omega_1+\omega_2+\omega_3$ the synchronization condition is satisfied only when all pump waves are coparallel, $\mathbf{k}_f=\omega_f\mathbf{e}_z$, where $\mathbf{e}_z$ is the unit vector in the z direction. But the total field of these waves is a plane wave, for which the nonlinearity of the vacuum disappears. Hence it is effectively impossible to generate combination modes with the sum frequency in vacuum (see also Ref. 4).

Consider the generation of a combination mode of the form

$$\mathbf{k}_j=\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3,\quad \omega_j=\omega_1+\omega_2-\omega_3. \tag{2.7}$$

We will show that the synchronization conditions hold for this scheme. For example, assume that the frequencies of all the pump waves and the directions of the first two waves (i.e., $\omega_{1,2,3}$ and $\mathbf{k}_{1,2}$) are given. The definition of $\mathbf{k}_3$ (and $\mathbf{k}_j$) satisfying the synchronization condition $\delta_j=0$ is illustrated by the scheme of Fig. 1. Here we write the vector $\mathbf{O}_1\mathbf{O}_2=\mathbf{k}_1+\mathbf{k}_2$, where the spheres with center at O_1 and O_2 have radii of $\omega_1+\omega_2-\omega_3$, and ω_3, respectively. The

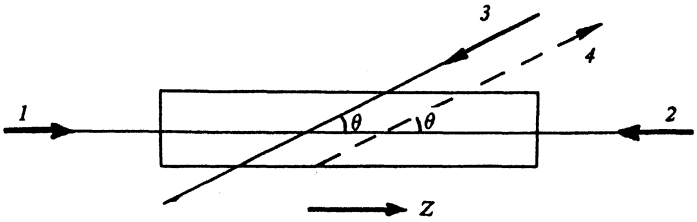

FIG. 2. Diagram of wavefront conjugation for degenerate four-wave interaction.

spheres intersect on the circle C. The wave vectors $\mathbf{k}_j$ and $\mathbf{k}_3$ act as generatrices for the cones with base C and vertices O_1 and O_2.

In calculating the components of the vacuum polarization that enter in (2.5) we can assume that relations (2.2) are satisfied. Thus, for an arbitrary direction of propagation and polarization states of the waves 1, 2, and 4 we find

$$P_{jx}+M_{jy}=(e^4/180\pi^2m^4)\{[\mathbf{E}_1\mathbf{E}_2-\mathbf{B}_1\mathbf{B}_2](E_{3x}-B_{3y})^* +[\mathbf{E}_1\mathbf{E}_3^*-\mathbf{B}_1\mathbf{B}_3^*](E_{2x}-B_{2y})+[\mathbf{E}_2\mathbf{E}_3^* -\mathbf{B}_2\mathbf{B}_3^*](E_{1x}-B_{1y})+(7/4) \times[[\mathbf{E}_1\mathbf{B}_2+\mathbf{E}_2\mathbf{B}_1](E_{3y}+B_{3x})^*+[\mathbf{E}_1\mathbf{B}_3^* +\mathbf{E}_3^*\mathbf{B}_1](E_{2y}+B_{2x})+[\mathbf{E}_2\mathbf{B}_3^*+\mathbf{E}_3^*\mathbf{B}_2] \times(E_{1y}+B_{1x})]\}, \tag{2.8}$$

$$P_{jy}-M_{jx}=(e^4/180\pi^2m^4)\{[\mathbf{E}_1\mathbf{E}_2-\mathbf{B}_1\mathbf{B}_2](E_{3y}+B_{3x})^* +[\mathbf{E}_1\mathbf{E}_3^*-\mathbf{B}_1\mathbf{B}_3^*](E_{2y}+B_{2x})+[\mathbf{E}_2\mathbf{E}_3^* -\mathbf{B}_2\mathbf{B}_3^*](E_{1y}+B_{1x})-(7/4) \times[[\mathbf{E}_1\mathbf{B}_2+\mathbf{E}_2\mathbf{B}_1](E_{3x}-B_{3y})^*+[\mathbf{E}_1\mathbf{B}_3^* +\mathbf{E}_3^*\mathbf{B}_1](E_{2x}-B_{2y})+[\mathbf{E}_2\mathbf{B}_3^*+\mathbf{E}_3^*\mathbf{B}_2] \times(E_{1x}-B_{1y})]\}.$$

The formal solution of (2.5) takes the form

$$E_{jx}(z)=E_{jx}(0)\exp(i\delta_jz)+2\pi i\omega_j\int_0^z(P_{jx}+M_{jy}) \times\exp[i\delta_j(z-z')]dz'. \tag{2.9}$$

Assuming that the field is given[8] amounts to ignoring the longitudinal variation of the pump wave amplitudes in evaluating the integral in (2.9). We take the quantity $P_{jx}+M_{jy}$ outside the integral, replacing the pump wave amplitudes [see (2.8)] appearing there by their original values (at the entrance to the beam intersection region):

$$E_{jx}(z)=E_{jx}(0)\exp(i\delta_jz)+2\pi(\omega_j/\delta_j)(P_{jx}+M_{jy}) \times[1-\exp(i\delta_jz)]. \tag{2.10}$$

The deviation from the exact synchronization condition is negligible if

$$|\delta_j|z\ll1. \tag{2.11}$$

Then taking this condition to hold and assuming that the original amplitude (at $z=0$) of the combination mode satisfies $E_{jx}(0)=0$ we find

$$E_{jx}(z)=2\pi i\omega_jz(P_{jx}+M_{jy}). \tag{2.12}$$

The expressions for the other field components have a similar form.

2.3. Phase conjugation

An important special case of the generation of combination waves studied above is the scheme for wavefront conjugation in degenerate four-wave interactions.[13,14] Here all waves have the same frequencies ($\omega_j=\omega_1=\omega_2=\omega_3=\omega$), and the wave vectors satisfy the relation (Fig. 2)

$$\mathbf{k}_2=-\mathbf{k}_1,\quad \mathbf{k}_j=-\mathbf{k}_3. \tag{2.13}$$

For simplicity we choose the polarization of all waves to be such that their magnetic field is directed perpendicular to $\mathbf{k}_1$ and $\mathbf{k}_3$ ($\mathbf{B}_j=B_j\mathbf{e}_y$, where $\mathbf{e}_y$ is the unit vector in the y direction, perpendicular to the plane of Fig. 2). As before the z axis is directed parallel to the vector $\mathbf{k}_j$. Then, using relations (2.2) to calculate the component of the polarization (2.8), we find

$$dE_j/dz=i(e^4\omega/45\pi m^4)(3+\cos^2\theta)E_1E_2E_3^*. \tag{2.14}$$

If we assume that the field (2.12) is known for $z=l$ we have

$$E_j(l)=i(e^4/45\pi m^4)(\omega l)(3+\cos^2\theta)E_1E_2E_3^*. \tag{2.15}$$

The amplitude E_j of the new (combination) mode with wave vector $\mathbf{k}_j=-\mathbf{k}_3$ is proportional to the length l of the wave interaction region (the effect builds up over a long trajectory), and also to the complex conjugate amplitude of wave 3. If the result is generalized by replacing plane waves with beams having a smooth transverse variation of the amplitude this implies that the radiation of the combination mode has a reversed (relative to pump wave 3) wave front, i.e., that phase conjugation occurs.

2.4. Birefringence

Another case in which the synchronization conditions are satisfied exactly corresponds to the interaction of two (rather than three) waves:

$$\mathbf{k}_2=\mathbf{k}_3,\quad \omega_2=\omega_3,\quad \mathbf{E}_2=\mathbf{E}_3=\mathbf{E}_0,\quad \mathbf{B}_2=\mathbf{B}_3=\mathbf{B}_0. \tag{2.16}$$

Choosing the z axis parallel to the direction $\mathbf{k}_1$, we can write Eqs. (2.5) in the form

$$d\mathbf{E}_{1\perp}/dz=i\omega_1\delta n\mathbf{E}_{1\perp},\quad d\mathbf{B}_{1\perp}/dz=i\omega_1\delta n\mathbf{B}_{1\perp}. \tag{2.17}$$

This corresponds to optical anisotropy of the vacuum and the presence of two intrinsic polarization states of wave 1 with different indices of refraction (birefringence of the vacuum in the field of a strong electromagnetic wave[4]):

$$\mathbf{E}_{1\perp}=\mathbf{E}_{1\perp}(0)\exp(i\omega_1\delta nz),\quad \mathbf{B}_{1\perp}=\mathbf{B}_{1\perp}(0)\exp(i\omega_1\delta nz), \tag{2.18}$$

$$\delta n=n-1=(k_1/\omega_1)-1=n_2|\mathbf{E}_0|^2. \tag{2.19}$$

Then from (2.8) and (2.16) it follows that

$$\delta n=(e^4/360\pi m^4)\{11[|E_{0x}-B_{0y}|^2+|E_{0y}-B_{0x}|^2] \pm 3|(E_{0x}-B_{0y})^2+(E_{0y}-B_{0x})^2|\}. \tag{2.20}$$

The two signs in (2.20) correspond to two different (linear and mutually orthogonal) intrinsic polarization states of wave 1. When wave 0 is linearly polarized we find for the polarization states $\rho=1$, 2 of wave 1

$$n_2=q_p(e^4/45\pi m^4)\sin^4(\theta/2). \tag{2.21}$$

Here θ is the angle between the propagation directions of the two waves (between their wave vectors) and we have $q_p=7$ or 4. The nonlinearity coefficient n_2 is largest for oppositely directed waves ($\theta=\pi$; this case was treated previously in Ref. 4). For small values of θ it satisfies $n_2\sim\theta^4$. The description of the change in an arbitrary polarization state of a test wave 1 reduces to the above relations after the field is expanded in a basis of the intrinsic polarization states.

To generalize to the case of pulsed radiation, instead of (2.17) we use the transport equations in the following form (for simplicity we consider the case of oppositely directed waves and two polarization states in which $\mathbf{E}=E\mathbf{e}_x$, $\mathbf{B}=B\mathbf{e}_y$) holds:

$$\partial E_1/\partial z+\partial E_1/\partial t=in_2\omega_1|E_2|^2E_1,\quad -\partial E_2/\partial z+\partial E_2/\partial t=in_2\omega_2|E_1|^2E_2. \tag{2.22}$$

We identify the real amplitudes and phases of the waves: $E_j=A_j\exp(i\Phi_j)$, $I_j=A_j^2$. For these it follows from (2.22) that

$$\partial I_1/\partial z+\partial I_1/\partial t=0,\quad -\partial I_2/\partial z+\partial I_2/\partial t=0, \tag{2.23}$$

$$\partial\Phi_1/\partial z+\partial\Phi_1/\partial t=n_2\omega_1I_2,\quad -\partial\Phi_2/\partial z+\partial\Phi_2/\partial t=n_2\omega_2I_1. \tag{2.24}$$

The solution (2.23) corresponds to transport of the pulse intensity profiles without any distortion:

$$I_1=I_1(u),\quad I_2=I_2(v),\quad u=z-t,\quad v=z+t. \tag{2.25}$$

The wave phases undergo nonlinear shifts:

$$\Phi_1(u,v)=\Phi_1(u,v_0)+(n_2\omega_1/2)\int_{v_0}^{v}I_2(v)dv,\quad \Phi_2(u,v)=\Phi_2(u_0,v)-(n_2\omega_2/2)\int_{u_0}^{u}I_1(u)du. \tag{2.26}$$

3. TRANSVERSE EFFECTS

Thus far we have disregarded the transverse variation of the amplitudes of the interacting waves, assuming them to be planar. In "ordinary" nonlinear media it is important to treat the transverse structure, mainly for the following reasons. First, the propagation of a plane wave in a nonlinear medium may be accompanied by the decay instability (small-scale self-focusing), which leads to filamentation: the beam radiation decays into separate filaments.[15] Second, the transverse variation of the beam intensity can give rise to effective nonlinear lenses in the medium (large-scale self-focusing).[2,3] The sharp rise in the radiation intensity at a nonlinear focus leads to a variety of nonlinear phenomena, all the way to breakdown of the medium.

In vacuum these effects have a somewhat different character. Thus, the decay instability is absent for a plane wave, but can occur for several intersecting waves (transverse instabilities of oppositely directed waves in "ordinary" nonlinear media were studied in Ref. 16 and in many subsequent publications). The large-scale self-focusing of a single beam in vacuum is also impossible. We can, however, speak of mutual focusing, e.g., of two beams (see also Ref. 2 and the references cited there); for pulses each beam induces a propagating distributed lens for the other beam.

For simplicity we will consider the interaction in vacuum of two oppositely directed beams in a quasisteady regime (long radiation pulses). Neglecting the weak variations in the radiation polarization (see Ref. 4), we can describe the transverse effects by coupled scalar quasioptical equations for the slowly varying amplitudes of the oppositely directed waves:[3]

$$2i\omega\partial E_1/\partial z+\Delta_\perp E_1+\omega^2n_2|E_2|^2E_1=0,\quad -2i\omega\partial E_2/\partial z+\Delta_\perp E_2+\omega^2n_2|E_1|^2E_2=0. \tag{3.1}$$

Here $\Delta_\perp=\partial^2/\partial x^2+\partial^2/\partial y^2$ is the transverse Laplacian, ω is the wave frequency, and n_2 is the nonlinearity coefficient [see Eqs. (2.19) and (2.21)]. Neglecting the diffraction ($\Delta_\perp E_{1,2}=0$) we can convert Eqs. (3.1) into the transport Eqs. (2.5) used in the previous sections.

3.1. Decay instability

We begin by considering the propagation of two oppositely directed waves whose amplitudes in some initial cross section $z=0$ are equal to E_{10} and E_{20}. Weak perturbations of the transverse structure are described by the relative variations $\delta E_{1,2}(x,y,z)$, $|\delta E_{1,2}|^2\ll1$ of the amplitudes:

$$E_1=E_{10}(1+\delta E_1)\exp(iM_1z/2\omega),\quad E_2=E_{20}(1+\delta E_2)\exp(-iM_2z/2\omega), \tag{3.2}$$

where $M_1=\omega^2n_2|E_{20}|^2$, $M_2=\omega^2n_2|E_{10}|^2$. We linearize (3.1) with respect to $\delta E_{1,2}$:

$$\begin{cases} 2i\omega\partial\delta E_1/\partial z+\Delta_\perp\delta E_1+M_1(\delta E_2+\delta E_2^*)=0\\ -2i\omega\partial\delta E_2/\partial z+\Delta_\perp\delta E_2+M_2(\delta E_1+\delta E_1^*)=0\end{cases} \tag{3.3}$$

The perturbations are easily expanded in a Fourier integral in the transverse coordinates $\mathbf{r}_\perp=(x,y)$. For the component of the perturbations with spatial frequency $\mathbf{q}$ we have

$$\begin{cases} \delta E_{1q} = a_1 \exp(i\mathbf{q}\mathbf{r}_\perp + \gamma z) + b_1^* \exp(-i\mathbf{q}\mathbf{r}_\perp + \gamma^* z) \\ \delta E_{2q} = a_2 \exp(i\mathbf{q}\mathbf{r}_\perp + \gamma z) + b_2^* \exp(-i\mathbf{q}\mathbf{r}_\perp + \gamma^* z) \end{cases} \quad (3.4)$$

Substituting (3.4) in (3.3) we find a homogeneous system of linear algebraic equations for the coefficients $a_{1,2}$ and $b_{1,2}$. From the condition that the determinant of this system vanish it follows that

$$4\omega^2\gamma^2 = -q^2(q^2 \pm q_m^2), \quad q_m^2 = 2(M_1M_2)^{1/2}. \quad (3.5)$$

For moderately large spatial frequencies $q^2 > 2q_m^2$ the values of γ are pure imaginary, so that the perturbations do not grow exponentially. But for $q^2 < 2q_m^2$ weak perturbations grow exponentially due to parametric amplification and the energy pumped into them from the main (underperturbed) waves. The growth rate γ is largest at $q^2 = q_m^2$, where $\gamma_{\max} = q_m^2/2\omega$. Thus, when oppositely directed plane waves propagate in vacuum the decay instability develops, causing them to undergo filamentation (small-scale mutual focusing). This circumstance must be taken into account when we describe real radiation in the plane-wave approximation.

3.2. Mutual channeling and large-scale self-focusing

Analysis of the beam interaction using Eqs. (3.1) is considerably more difficult. We will treat the symmetric case in which the oppositely directed beams 1 and 2 differ only in their direction of propagation. An estimate for the focusing conditions of one beam under the action of the other takes the form

$$\delta n = n_2 |E|^2 > \theta_d^2. \quad (3.6)$$

Here $\theta_d = \lambda/d$ is the diffraction angular divergence of the radiation beams and d is their transverse dimension. From (3.6) it follows that there is a critical power

$$P_{\rm cr} = |E|^2 d^2 \approx \lambda^2/n_2. \quad (3.7)$$

The critical power can also be closely estimated from (3.5) by equating the typical spatial frequency $q = 1/d$ to the spatial frequency q_m of the fastest growing perturbation.

When the power satisfies $P > P_{\rm cr}$ the nonlinear focusing dominates the diffractive spreading and the beam undergoes compression. For $P = P_{\rm cr}$ these two processes cancel out one another, which gives rise to mutual channeling of the oppositely directed beams. In this regime the transverse intensity profile of each beam remains constant as a function of z:

$$E_1 = R(\mathbf{r}_\perp)\exp(iMz/2\omega),$$
$$E_2 = R(\mathbf{r}_\perp)\exp(-iMz/2\omega). \quad (3.8)$$

Hence (3.1) reduces to a single equation

$$\Delta_\perp R + (\omega^2 n_2 |R|^2 - M)R = 0. \quad (3.9)$$

Equation (3.9), together with the requirement that the field vanish sufficiently rapidly in the transverse direction, determines the spectrum of the eigenvalues M and the corresponding eigenfunctions $R(\mathbf{r}_\perp)$. We need not solve it, since it is the same (except for notation) as the equation describing self-channeling of a single beam in a medium

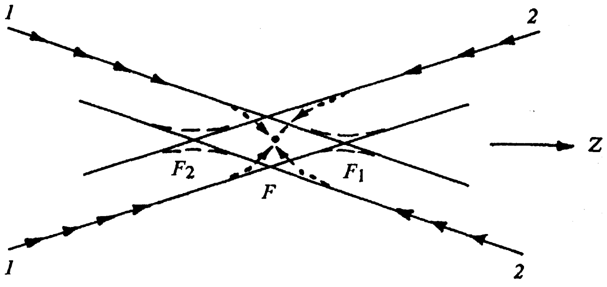

FIG. 3. Ray paths for the interaction between oppositely directed waves.

with a Kerr nonlinearity.[17] The lower eigenvalue corresponds to a smooth (not oscillating in the radial direction) axisymmetric intensity profile; the corresponding critical power differs from the estimate (3.7) only by a factor of order unity.

When the power is in excess of the critical value we should expect marked mutual focusing of the beams. Consider, e.g., the propagation of two oppositely directed beams with the linear foci F_1 and F_2 located as shown in Fig. 3 (at low intensities). As the power increases the nonlinear foci of these two beams approach one another, and for some value (depending on the original defocusing F_1F_2) of the power these foci merge (broken trace in Fig. 3). The radiation intensity in a general nonlinear focus can be considerably larger than the maximum intensity at the focus of single beams. This would tend to enhance nonlinear effects in vacuum. In the general case in self-focusing there is a mechanism limiting the power at the nonlinear foci, related to divergence (the decrease in the intersection region) of the interacting beams due to their focusing. For a given beam power level the conditions are optimized by appropriate choice of the defocusing F_1F_2 (in the case of oppositely directed radiation pulses the effect increases for an appropriate adjustment of the defocusing as a function of time). Hysteresis effects[18] are also possible for quasi-steady oppositely directed beams.

4. NUMERICAL CALCULATIONS AND DISCUSSION OF THE RESULTS

In this section we compare the possibilities for experimental observation of the nonlinear optical phenomena discussed above under laboratory conditions.

The efficiency of transverse nonlinear effects is determined by the ratio of the laser radiation power to the critical value (3.7), which can be estimated as $P_{\rm cr} \sim 10^3 I_{\rm cr}\lambda^2$. If the wavelength is $\lambda = 1$ μm the power is $P_{\rm cr} \sim 2.5 \cdot 10^{24}$ W, which is many orders of magnitude greater than that achieved thus far. Consequently, there is no point in expecting any pronounced mutual focusing of laser beams. It is even less likely that the decay instability (filamentation) of laser radiation will be observed; it would require a power $P \gg P_{\rm cr}$. The situation may change as a result of progress in constructing wide-aperture short-wavelength lasers (as the angular divergence λ/d decreases), or due to high-current detection of the nonlinear shift in the focus of one of the beams under the action of the other in the subthreshold regime.

Let us now consider the generation of combination modes. It is possible to obtain fairly complete overlapping of the focal regions of the laser beams in a scheme which is close to collinear. For this purpose in (2.15) we set $\theta^2 \ll 1$, $l = c\tau$ (here c is the velocity of light and τ is the pulse length) and $I_1 \approx I_2 \simeq I_3$, and transform from intensities to energies in the pulse according to $W = IS\tau$ (here S is the area of the transverse laser beam cross section in the focal region). Then for the pulse energy produced by the nonlinear interaction of three pump waves 1–3 we find the combination mode from (2.15)

$$W_j \sim 10^{-5} c^2 W_1^3 / (I_{cr} S \lambda)^2. \tag{4.1}$$

It is noteworthy that the laser pulse lengths do not appear in (4.1) so that when τ decreases not only does the intensity I grow but the overlap length l of the pulses also decreases. Consequently, going to ultrashort pulses is not necessary here.

We set $\lambda = 1\ \mu m$ and $S = 10^{-4}\ cm^2$. The laser pulse can be detected fairly accurately when it has an energy ~ 100 photons. To obtain this energy $W_j = 10^2 \hbar\omega_j = 2 \cdot 10^{-17}$ according to (4.1) we need a pump beam of energy $W_1 \sim 2.7 \cdot 10^3$ J, i.e., the total energy of the pump pulses is $W \sim 3W_1 \sim 8$ kJ. Note that laser pulses have already been obtained at Livermore Laboratory (USA) with an energy greater than 100 kJ, and a facility with energies 1–2 MJ is now being designed.[19]

The most serious difficulty in the experiment is securing the necessary signal-to-noise ratio. Scattering of pump beam radiation in the signal channel (in the direction $\mathbf{k}_j$) may be reduced by choice of the polarizations. The increase in the frequency mismatch is limited by the synchronization. The property of wave conjugation of the detected radiation with respect to the pump beam 3, described above, affords additional possibilities. According to the estimate of Ref. 4, the pressure of the residual gas can be chosen to be $\sim 10^{-11}$ torr.

In another type of experiment the birefringence of the vacuum is used in a scheme with oppositely directed laser pulses.[4] The ratios (2.26) yield the following estimates for the nonlinear phase shifts (which can be converted into a change in the polarization of the radiation):

$$\delta\Phi \sim (1/137)(I/I_{cr})(l/\lambda) \sim (1/137) cW/(I_{cr} S \lambda). \tag{4.2}$$

For $\lambda = 1\ \mu m$, $S = 10^{-4}\ cm^2$, and $W = 10^4$ J we find $\delta\Phi \sim 10^{-10}$ rad. These values can be measured using laser polarimeters.[4] In order to choose between this type of experiment and the other it is necessary to take into account the properties of high-energy laser facilities already in existence.

Thus, this treatment shows that in vacuum there exists a broad range of relatively low-threshold nonlinear optical phenomena. Manifestations of vacuum polarization can be observed in laser facilities now in existence, where both wave front combination approaches and extremely sensitive laser polarimeters appear to be promising.

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