Suppression of weak localization in backscattering of particles undergoing inelastic collision in a randomly inhomogeneous medium

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(Submitted 9 November 1992)
Zh. Eksp. Teor. Fiz. **103**, 1800–1828 (May 1993)

A theory of backscattering of particles inelastically colliding in a randomly inhomogeneous medium is formulated under the assumption the elastic and inelastic scatterings are sequential. Universal expressions obtained for the doubly differential backscattering spectra are valid for inelastic collisions of any physical nature. Manifestations of weak localization in the inelastic-scattering channel are considered. Coherent effects in the inelastic channel are shown to be manifested in the case $ql \ll 1$ (q is the wave vector lost by the particle in an elastic collision, l is the particle mean free path in a disordered medium.

1. INTRODUCTION

The weak localization of light and of particles scattered in a disordered medium is known to be due to interference effects that enhance strongly the backscattering near the "exactly backward" direction.[1–7] Interest in this phenomenon has greatly increased of late, as attested by the large number of theoretical papers devoted to weak localization of waves of various types in randomly inhomogeneous media.[8–22] These papers report investigations of coherent effects in an elastic scattering channel; the weak localization was due there to multiple elastic scatterings of the scattered particle with randomly located elementary scatterers, which generally speaking cannot be regarded as being exactly in succession. In other words, under conditions of weak localization the next elastic collisions begins before the preceding one ends. For particles moving in a medium in which the scattering processes are symmetric with respect to time reversal, this corresponds to interference of waves passing through identical inhomogeneities in the forward and backward direction.[20–22]

In addition to the study of the weak localization proper, it is no less of interest to gain an idea of the physical processes that destroy coherent effects in disordered media.

It was shown in Ref. 20 that the coherent effect will be suppressed if the system is subject to interactions that destroy the symmetry of the scattering processes with respect to time reversal. Such interactions are random motions of the inhomogeneities and gyrotropy of the medium,[20,23] spin-spin and spin-orbit interactions with the scattering centers,[24,25] and the presence of an external magnetic field.[26,27]

Furthermore, as will be shown below, the coherent effects that lead to weak localization of scalar waves can be destroyed also in T-invariant systems. In the present paper we investigate the onset of classically weak localization of particles undergoing inelastic collisions in a disordered medium. Considering weak localization in an inelastic channel, we assume that the elastic and inelastic collisions of a scattered particle succeed one another, therefore the new weak-localization type predicted in a recent paper[28] will be disregarded. This approach is valid because the estimates in that paper demonstrate the existence of inelastic processes that can be regarded as fully consistent with elastic collisions of the scattered particle. One such process is collision of a charged particle with a "Cherenkov" bulk plasmon, when the frequency of all the particle collisions is much lower than the plasma frequency.

We begin with the Schrödinger equation of the wave function of a particle in a disordered medium, and consider on its basis the mutual-incoherence function of a particle wave function in an inelastic scattering channel.

2. AVERAGE WAVE FIELD OF A PARTICLE IN AN INELASTIC SCATTERING CHANNEL

The wave function $\Psi_n(\mathbf{r})$ of a particle scattered in a disordered medium with a random potential, subjected to a single inelastic collision, and located in the scattering n-channel, satisfies the Schrödinger equation

$$\Delta\psi_n(\mathbf{r})+2m\hbar^{-2}[E_n-U(\mathbf{r})]\psi_n(\mathbf{r})$$
$$=2m\hbar^{-2}T(\mathbf{r},i\to n)\psi_i(\mathbf{r}), \qquad (1)$$

in which m and E_n are the mass and energy of the particle, $T(\mathbf{r},i\to n)$ is a matrix element of the operator of the inelastic interaction between the particle and the medium, calculated from the wave functions of the aggregate of the medium particles in ground (i) and excited (n) states, while $\psi_i(\mathbf{r})$ is the wave function of a particle having in the elastic channel an energy E_i satisfying the equation

$$\Delta\psi_i(\mathbf{r})+2m\hbar^{-2}[E_i-U(\mathbf{r})]\psi_i(\mathbf{r})=0. \qquad (2)$$

We define also the Green's function $G_n(\mathbf{r},\mathbf{r}')$ of Eq. (1):

$$\Delta G_n(\mathbf{r},\mathbf{r}')+2m\hbar^{-2}[E_n-U(\mathbf{r})]G_n(\mathbf{r},\mathbf{r}')$$
$$=2m\hbar^{-2}\delta(\mathbf{r}-\mathbf{r}'). \qquad (3)$$

When solving (1)–(3) it must be borne in mind that the scattering medium occupies the half-space $z\geqslant 0$, and take into account the boundary conditions at $z=-\infty$ as well as the continuity of the corresponding solutions and of their normal derivatives at $z=0$.

The solution of Eqs. (2) and (3) can be represented by infinite series suitable for finding subsequently the average wave field of a particle in an inelastic scattering channel. Before we do this, we define the integral operator

$$U_\alpha^k=\delta U G_{k0}\delta U \ldots G_{k0}\delta U, \qquad (4)$$

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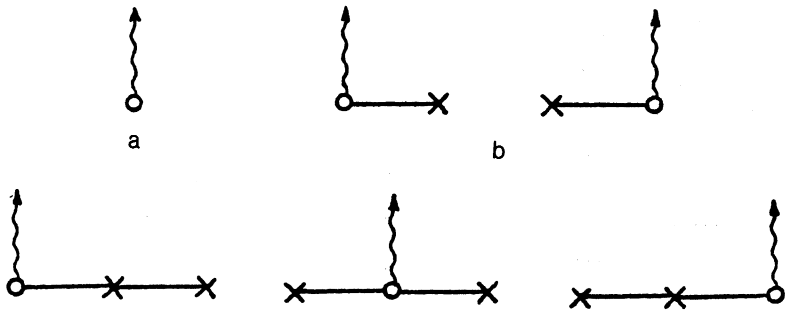

FIG. 1. Graphic representation of the coherent wave field of a particle in an inelastic scattering channel. Diagrams of zeroth (a), first (b), and second (c) orders in the incoherent elastic interaction operator δU.

in which the symbol δU corresponds to the fluctuating part of the random potential $U(\mathbf{r})$:

$$\delta U(\mathbf{r})=U(\mathbf{r})-U_{av}(\mathbf{r}),\quad U_{av}(\mathbf{r})=\langle U(\mathbf{r})\rangle,$$

G_{k0} is an integral operator corresponding to a Green's function of a particle having an energy E_k in a mean field $U_{av}(\mathbf{r})$. The subscript α indicates that the operator δU is encountered in (4) α times. The action of the operator U^k_α on certain functions is defined by the relation

$$U^k_\alpha f(\mathbf{r})=\int d\mathbf{r}' U^k_\alpha(\mathbf{r},\mathbf{r}')f(\mathbf{r}').$$

The solution of Eqs. (2) and (3) can now be represented by the infinite series:

$$G_n=G_{n0}+G_{n0}\sum_{\alpha=1}^{\infty} U^n_\alpha G_{n0}, \tag{5}$$

$$\psi_i=\psi_{i0}+G_{i0}\sum_{\beta=1}^{\infty} U^i_\beta\psi_{i0}. \tag{6}$$

In Eq. (6) ψ_{i0} denotes a wave function of a particle of energy E_i in an average field.

Bearing expansions (5) and (6) in mind, we can represent the solution of Eq. (1) in the symbolic form:

$$\psi_n(\mathbf{r})=G_{n0}\left[\hat{I}+\sum_{\alpha=1}^{\infty} U^n_\alpha G_{n0}\right]T(i\to n)\times\left[\hat{I}+G_{i0}\sum_{\beta=1}^{\infty} U^i_\beta\right]\psi_{i0}(\mathbf{r}). \tag{7}$$

The expansion (7) for a coherent wave field of a particle in an inelastic scattering channel (of first order in the operator of inelastic interaction of a particle with a medium) takes into account multiple elastic incoherent scattering of a particle both before and after the inelastic collision of the scattered particle. The wave function ψ_n accords with all possible elastic-collision combinations, in each of which the particle momentum changes noticeably, and with a collision that alters the internal state of the disordered medium.

The expansion (7) (without the "factors" G_{n0} and ψ_{i0}) can be set in correspondence with an infinite set of diagrams shown in Fig. 1, where a thin solid line corresponds to the operator G_{k0} $(k=i,n)$, a cross to the operator δU of elastic incoherent scattering, and a light circle with an outgoing wavy line to the operator $T(i\to n)$ that describes an inelastic collision of a particle with the medium whereby the particle and the disordered medium go over from the ground i-state to an excited n-state.

A diagram formulation of the expansion (7) permits a clearer description of the scattering processes and a simple averaging of the coherent wave field ψ_n over the random disposition of the scattering centers.

We assume next that the random field $\delta U(\mathbf{r})$ is Gaussian. We assume furthermore, when we refer to excitation of any one of the internal states of a medium, that this medium is fully homogeneous. This means that the matrix element $T(i\to n)$ in Eq. (7) is not a random quantity.

Bearing this in mind we find that a contribution to the average wave field is made only by diagrams with even numbers of crosses, joined pairwise in all possible manners[29,30] (see Fig. 2). Not all the diagrams in this figure make contributions of like order to the average wave field.

Consider diagrams containing two crosses (see Fig. 2a). Two diagrams of second order in the elastic interaction—the second and third in the figure—differ, topologically speaking, from the fourth diagram in which the dashed line encloses a vertex corresponding to an inelastic collision. Let us compare the contributions of these diagrams to the average wave field. To this end we change to the impulse approximation (see Fig. 3), in which a thin solid line to the left of the inelastic-interaction vertex corresponds to

$$G_{i0}(\mathbf{p})=2m\hbar^{-2}(k^2-p^2+i\nu)^{-1},$$

and a similar line on the right of the inelastic-scattering vertex

$$G_{n0}(\mathbf{p}-\mathbf{q})=2m\hbar^{-2}[k^2-2m\omega_\mathbf{q}\hbar^{-1}-(\mathbf{p}-\mathbf{q})^2+i\nu]^{-1},$$

corresponds to two crosses joined by a dashed line, $n|U_0(\mathbf{Q})|^2$, and the vertex of the inelastic interaction cor

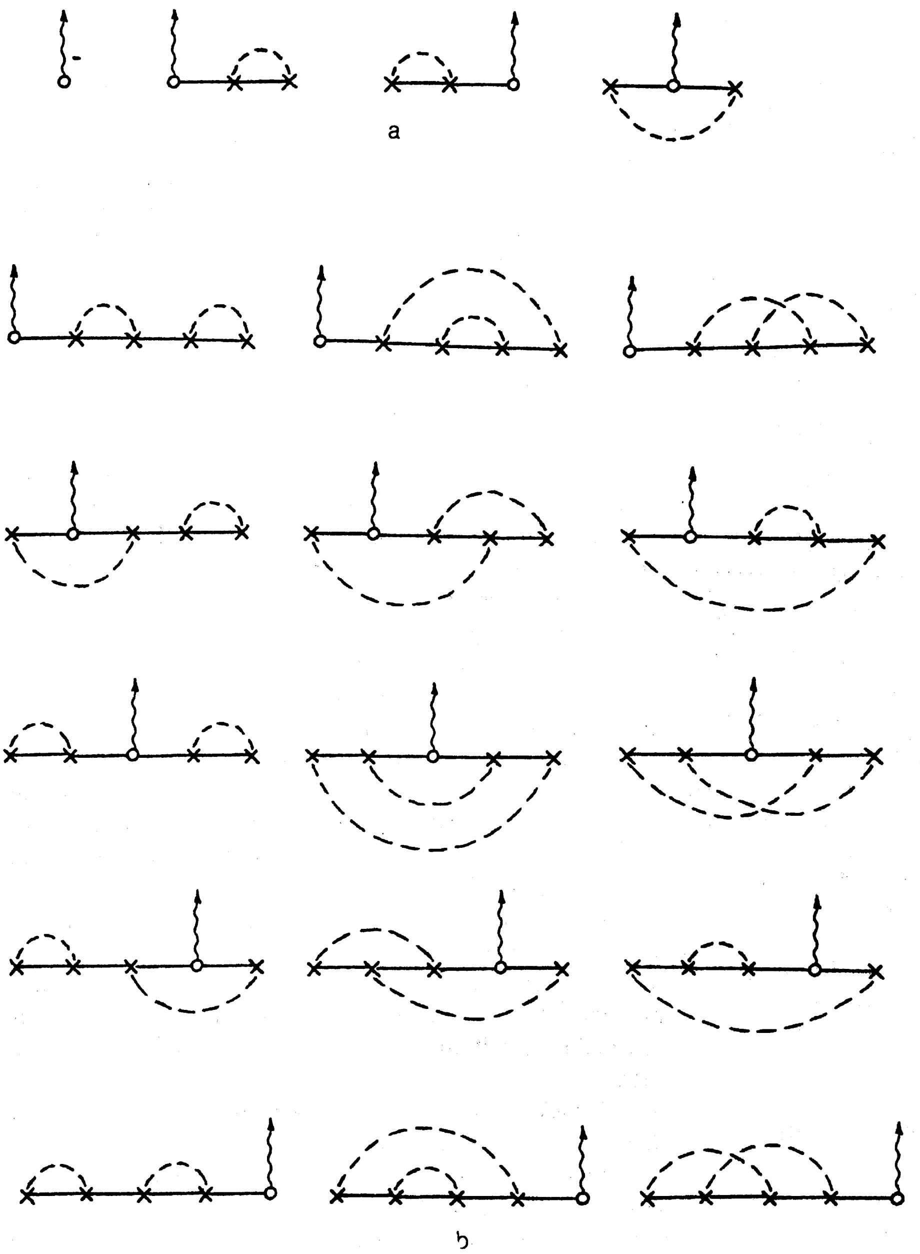

FIG. 2. Graphic representation of the average wave field of a particle in an inelastic scattering channel. Diagrams of zeroth, second (a) and fourth (b) orders in the incoherent elastic interaction operator δU.

responds to the Fourier transform $T(\mathbf{q})$ of the matrix element $T(\mathbf{r},i\rightarrow n)$. In all the equations $k^2=2m\hbar^{-2}E_i$, where n is the number of scattering centers per unit volume, $\mathbf{q}$ and $\mathbf{Q}$ are the wave vectors lost by the particle in inelastic and elastic collisions, respectively, $\hbar\omega_{\mathbf{q}}$ is the energy lost, $U_0(\mathbf{Q})$ is the Fourier transform of the fluctuation part of the potential of a single scatterer. The quantity ν describes the particle wave field damping due to its motion in the average field U_{av}.

The contribution of the diagram shown in Fig. 3a is determined by the equation

$$A_{nc}(\mathbf{p},\mathbf{p}-\mathbf{q})=n\int d\mathbf{Q}\,|U_0(\mathbf{Q})|^2G_{i0}(\mathbf{p}-\mathbf{Q})G_{i0}(\mathbf{p})T(\mathbf{q}). \qquad (8)$$

Owing to the presence of $G_{i0}(\mathbf{p})$ in (8), we shall be interested in the value of A_{nc} near $p\approx k$. The presence of the

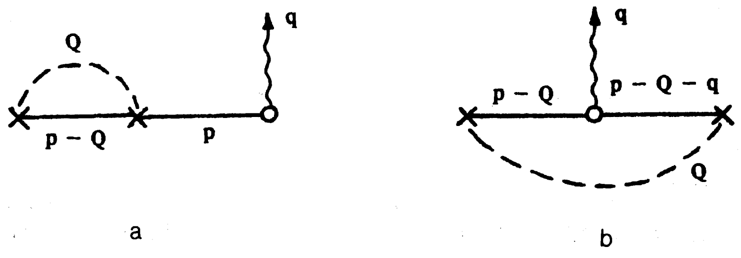

FIG. 3. Diagrams of second order in the incoherent elastic interaction operator δU in the momentum representation.

function $G_{i0}(\mathbf{p}-\mathbf{Q})$ in the integral causes the main contribution to the integral to be made by the values of Q in a small region $\delta p'$ close to $p'=|\mathbf{p}-\mathbf{Q}|\approx k$. Changing to integration over $\mathbf{p}'=\mathbf{p}-\mathbf{Q}$ we find thus that contributions to the integral are made on by the values of $\mathbf{p}'$ from a thin spherical layer $(k,k+\delta p')$. Using the estimates

$$\delta p'\approx l^{-1}, \quad \text{and} \quad G_{i0}(\mathbf{p}')\approx\frac{2m}{\hbar^2}\frac{l}{k}$$

(Ref. 31), where l is the mean free path of a particle in a medium with an average potential U_{av}, we obtain

$$A_{nc}(\mathbf{p},\mathbf{p}-\mathbf{q})\approx16\pi^2 nl\sigma_{el}T(\mathbf{q}). \tag{9}$$

In (9),

$$\sigma_{el}=(m/2\pi\hbar^2)^2\int d\Omega_{\mathbf{p}'}|U_0(\mathbf{p}-\mathbf{p}')|^2$$

is the cross section for elastic scattering by a unit center in the Born approximation.

The contribution to the average field of the diagram shown in Fig. 3b is determined by the integral

$$A_{sc}(\mathbf{p},\mathbf{p}-\mathbf{q})=n\int d\mathbf{p}'|U_0(\mathbf{p}-\mathbf{p}')|^2 G_{i0}(\mathbf{p}') \times G_{n0}(\mathbf{p}'-\mathbf{q})T(\mathbf{q}) \tag{10}$$

(we have changed directly from integration over $\mathbf{Q}$ to integration over $\mathbf{p}'$). The simultaneous requirements $p'\approx k$ and $|\mathbf{p}'-\mathbf{q}|\approx(k^2-2m\omega_{\mathbf{q}}\hbar^{-1})^{-1/2}$ (owing to the presence of two Green's functions) limits the region of those solid angles $\delta\Omega_{\mathbf{p}'}$ in the momentum $\mathbf{p}'$ space which contribute substantially to A_{sc}. Let us estimate this region. Variation of the denominators G_{i0} and G_{n0} yields, respectively,

$$|\delta G_{i0}^{-1}(\mathbf{p}')|=\hbar^2 p'\delta p'/m\approx\hbar^2 k/ml, \tag{11a}$$

$$|\delta G_{n0}^{-1}(\mathbf{p}'-\mathbf{q})|=\hbar^2|p'\delta p'-\delta p' q\cos\gamma-p'q\delta \times\cos\gamma|/m\approx\hbar^2 k/ml, \tag{11b}$$

where γ is the angle between the vectors $\mathbf{p}'$ and $\mathbf{q}$. We find from (11a) and (11b) that $\sin\delta\gamma\sin\gamma=\cos\gamma/kl$, or

$$\delta\Omega_{\mathbf{p}'}=2\pi\delta\sin\gamma\leqslant2\pi/kl.$$

We can now easily obtain the integral (10):

$$A_{sc}(\mathbf{p},\mathbf{p}-\mathbf{q})\leqslant8\pi^2 n\sigma_{el}T(\mathbf{q})/k. \tag{12}$$

It follows thus from the estimates (9) and (12) that the relative contribution of the second-order diagram in which the dashed line surrounds the vertex that corresponds to inelastic collision (see Fig. 3b) is a fraction

$$A_{sc}/A_{nc}\leqslant1/2kl$$

of the contribution of a second-order diagram without such a topological singularity (see Fig. 3a). Under the weak localization conditions we have $kl\gg1$, so that the contribution made to the mean field from diagrams of the type shown in Fig. 3b can be neglected.

Similarly, not all diagrams of fourth order in the elastic interaction (see Fig. 2b) make contributions of the same order to the mean field. It follows from the analysis above, the contribution of the fourth diagram of the figure in question is small and of order $1/2kl$ compared with the first diagram, etc. The result means in fact that if a diagram contains crosses both to the left and to the right of the inelastic-interaction vertex, the averaging on the left and on the right must be carried out independently, i.e.,

$$\sum_{\alpha=1}^{\infty}\sum_{\beta=1}^{\infty}\langle U_\alpha^n G_{n0}T(i\to n)G_{i0}U_\beta^i\rangle = \sum_{\alpha=1}^{\infty}\langle U_\alpha^n\rangle G_{n0}T(i\to n)G_{i0}\sum_{\beta=1}^{\infty}\langle U_\beta^i\rangle. \tag{13}$$

It was furthermore shown in [31] that the contribution of diagrams with intersecting dashed lines is exactly of the same order of smallness (i.e., the contribution of the third diagram of Fig. 2b is much lower than that of the first, etc.). Taking this into account, we should retain only the first, second, seventh,thirteenth and fourteenth diagrams with four crosses. Exactly the same would be done with higher-order diagrams.

It follows from the foregoing that a contribution to the inelastic-scattering average wave field is made by diagrams with even numbers of crosses (disregarding the first diagram of Fig. 2a), having neither non-intersecting dashed lines nor dashed lines that surround the inelastic-collision vertex.

Taking the identity (13) and Eqs. (5) and (6) into account when (7) is averaged, we get

$$\langle\psi_n\rangle=\langle G_n\rangle T(i\to n)\langle\psi_i\rangle. \tag{14}$$

Here $\langle\psi_i\rangle$ is the average wave number in the elastic scattering channel, and $\langle G_n\rangle$ is the average Green's function of a particle in an inelastic scattering channel; both averages satisfy the equations[29]

$$\Delta\langle\psi_i\rangle+2m\hbar^{-2}[E_i-U_{av}-M_i]\langle\psi_i\rangle=0, \tag{15}$$

$$\Delta\langle G_n\rangle+2m\hbar^{-2}[E_n-U_{av}-M_n]\langle G_n\rangle =2m\hbar^{-2}\delta(\mathbf{r}-\mathbf{r}'), \tag{16}$$

in which M_k is the mass operator connected with U_k is a mass operator connected with U_α^k by the relation

$$M_k=\sum_{\alpha=1}^{\infty}\langle U_\alpha^k\rangle\left[\hat{I}+G_{k0}\sum_{\beta=1}^{\infty}\langle U_\beta^k\rangle\right]^{-1} \tag{17}$$

and determined by the sum of the strongly connective single-row diagrams.[29,30]

3. WAVE-FIELD MUTUAL COHERENCE FUNCTION IN AN INELASTIC SCATTERING CHANNEL

We define the mutual-coherence function (the density matrix) for a wave field of particles in the nth inelastic-scattering channel:

$$\rho_{nn}(\mathbf{r}_1,\mathbf{r}_2)=\rho_{12}^{nn}=\langle\psi_n^*(\mathbf{r}_2)\psi_n(\mathbf{r}_1)\rangle, \tag{18}$$

and the mutual-coherence function for a wave field of a particleinthesamescatteringchannel,butwithoutincoherentscattering:

$$\rho^0_{nn}(\mathbf{r}_1,\mathbf{r}_2)=\rho_{12}^{nn0}=\langle\psi_n^*(\mathbf{r}_2)\rangle\langle\psi_n(\mathbf{r}_1)\rangle, \qquad (19)$$

where the function ψ_n is defined by the expansion (7).

Bearing in mind the definitions (18) and (19) and the expansion (7), we calculate the difference between (18) and (19):

$$\begin{aligned}
\rho_{12}^{nn}-\rho_{12}^{nn0}=&\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}\psi_{i0}\right)_1\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}\psi_{i0}\right)_2^*\right\rangle-\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}\psi_{i0}\right)_1\right\rangle\\
&\times\left\langle\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}\psi_{i0}\right)_2^*\right\rangle+\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}\psi_{i0}\right)_1\left(G_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_2^*\right\rangle\\
&-\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}\psi_{i0}\right)_1\right\rangle\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_2^*\right\rangle\\
&+\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}U_\alpha^i\psi_{i0}\right)_1\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}\psi_{i0}\right)_2^*\right\rangle-\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}U_\alpha^i\psi_{i0}\right)_1\right\rangle\\
&\times\left\langle\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}\psi_{i0}\right)_2^*\right\rangle+\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}U_\alpha^i\psi_{i0}\right)_1\left(G_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_2^*\right\rangle\\
&-\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}U_\alpha^i\psi_{i0}\right)_1\right\rangle\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_2^*\right\rangle\\
&+\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}\psi_{i0}\right)_1\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}G_{i0}\sum_{\gamma=1}^{\infty}U_\gamma^i\psi_{i0}\right)_2^*\right\rangle-\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}\psi_{i0}\right)_1\right\rangle\\
&\times\left\langle\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}G_{i0}\sum_{\gamma=1}^{\infty}U_\gamma^i\psi_{i0}\right)_2^*\right\rangle\\
&+\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}U_\alpha^i\psi_{i0}\right)_1\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}G_{i0}\sum_{\gamma=1}^{\infty}U_\gamma^i\psi_{i0}\right)_2^*\right\rangle-\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\alpha=1}^{\infty}U_\alpha^i\psi_{i0}\right)_1\right\rangle\\
&\times\left\langle\left(G_{n0}\sum_{\beta=1}^{\infty}U_\beta^nG_{n0}T_{in}G_{i0}\sum_{\gamma=1}^{\infty}U_\gamma^i\psi_{i0}\right)_2^*\right\rangle\\
&+\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_1\left(G_{n0}\sum_{\gamma=1}^{\infty}U_\gamma^nG_{n0}T_{in}\psi_{i0}\right)_2^*\right\rangle\\
&-\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_1\right\rangle\left\langle\left(G_{n0}\sum_{\gamma=1}^{\infty}U_\gamma^nG_{n0}T_{in}\psi_{i0}\right)_2^*\right\rangle\\
&+\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_1\left(G_{n0}T_{in}G_{i0}\sum_{\gamma=1}^{\infty}U_\gamma^i\psi_{i0}\right)_2^*\right\rangle\\
&-\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_1\right\rangle\left\langle\left(G_{n0}T_{in}G_{i0}\sum_{\gamma=1}^{\infty}U_\gamma^i\psi_{i0}\right)_2^*\right\rangle\\
&+\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_1\left(G_{n0}\sum_{\gamma=1}^{\infty}U_\gamma^nG_{n0}T_{in}G_{i0}\sum_{\delta=1}^{\infty}U_\delta^i\psi_{i0}\right)_2^*\right\rangle\\
&-\left\langle\left(G_{n0}\sum_{\alpha=1}^{\infty}U_\alpha^nG_{n0}T_{in}G_{i0}\sum_{\beta=1}^{\infty}U_\beta^i\psi_{i0}\right)_1\right\rangle\left\langle\left(G_{n0}\sum_{\gamma=1}^{\infty}U_\gamma^nG_{n0}T_{in}G_{i0}\sum_{\delta=1}^{\infty}U_\delta^i\psi_{i0}\right)_2^*\right\rangle. \qquad (20)
\end{aligned}$$

We simplify Eq. (20) by averaging under the assumption that the elastic and inelastic collisions follow one another in succession. By the same token, we discard effects connected with the new type of weak localization.[28]

Equation (20) contains nine paired terms, each comprising the difference between the averaged product of two brackets, labeled 1 and 2, and the product of the corresponding mean values. In virtue of this, if (20) is graphically represented by two-row diagrams, some of them will correspond to non-connective ones (in the "elastic" sense), i.e., such that the upper and lower rows are not interconnected by even one dashed line, whereas such diagrams are present in a combination consisting of a wavy line and an inelastic-interaction. We shall henceforth call such diagrams "unconnected," for short.

Diagrams corresponding to the expansion (20) (without external G_{n0} and ψ_{i0} lines), up to fourth order inclusive (relative to incoherent scattering) are shown in Fig. 4. We have left out here diagrams of two types. The first contain intersections of a wavy line with a dashed line joining two crosses, one of which is located in the upper row and the second in the lower. This type of diagram describes, as we have already mentioned, weak localization of particles of a new type in the inelastic channel.

The second type consists of two-row diagrams in which at least one dashed line surrounds an inelastic-interaction vertex. Diagrams of this type, as follows from our analysis in Sec. 2, make under conditions of a negligibly weak interaction a small contribution to the mutual-coherence function. Note that in this approximation the second and third paired terms in (20) make no contribution at all to the mutual-coherence function.

Summing the diagrams of this type and taking relations (6) and (19) into account, we obtain the following expression for the mutual-coherence function of the wave field of a particle in inelastic scattering channeling n:

$$\begin{aligned}
\rho_{nn}(\mathbf{r},\mathbf{r}')-\rho^0_{nn}(\mathbf{r},\mathbf{r}')
&=\int d\mathbf{r}_1 d\mathbf{r}_1' d\mathbf{r}_1'' d\mathbf{r}_2 d\mathbf{r}_2'' \langle G_n(\mathbf{r},\mathbf{r}_1)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_2)\rangle\Gamma_{nn}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\langle G_n(\mathbf{r}_1',\mathbf{r}_1'')\rangle\langle G_n^*(\mathbf{r}_2',\mathbf{r}_2'')\rangle T_{in}(\mathbf{r}_1'')T_{ni}(\mathbf{r}_2'')\rho^0_{ii}(\mathbf{r}_1'',\mathbf{r}_2'')\\
&+\int d\mathbf{r}_1 d\mathbf{r}_1' d\mathbf{r}_1'' d\mathbf{r}_2 d\mathbf{r}_2' d\mathbf{r}_2'' \langle G_n(\mathbf{r},\mathbf{r}_1'')\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_2'')\rangle T_{in}(\mathbf{r}_1'')T_{ni}(\mathbf{r}_2'')\langle G_i(\mathbf{r}_1'',\mathbf{r}_1)\rangle\langle G_i^*(\mathbf{r}_2'',\mathbf{r}_2)\rangle\Gamma_{ii}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\rho^0_{ii}(\mathbf{r}_1',\mathbf{r}_2')\\
&+\int d\mathbf{r}_1 d\mathbf{r}_1' d\mathbf{r}_1'' d\mathbf{r}_2 d\mathbf{r}_2' d\mathbf{r}_2'' d\mathbf{R}_1 d\mathbf{R}_1' d\mathbf{R}_2 d\mathbf{R}_2' \langle G_n(\mathbf{r},\mathbf{r}_1)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_2)\rangle\Gamma_{nn}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\langle G_n(\mathbf{r}_1',\mathbf{r}_1'')\rangle\\
&\times\langle G_n^*(\mathbf{r}_2',\mathbf{r}_2'')\rangle T_{in}(\mathbf{r}_1'')T_{ni}(\mathbf{r}_2'')\langle G_i(\mathbf{r}_1'',\mathbf{R}_1)\rangle\langle G_i^*(\mathbf{r}_2'',\mathbf{R}_2)\rangle\Gamma_{ii}(\mathbf{R}_1,\mathbf{R}_1';\mathbf{R}_2,\mathbf{R}_2')\rho^0_{ii}(\mathbf{R}_1',\mathbf{R}_2'). \qquad (21)
\end{aligned}$$

The function Γ_{kk} describes the evolution of a coherent wave field in multiple scattering of a particle of energy E_k in a substance, and is determined by the sum of all the connected diagrams without external $\langle G\rangle$ lines, but with internal propagation $\langle G\rangle$ lines; any two neighboring vertices of one row cannot be interconnected by a dashed line. The function ρ^0_{ii} is the function of mutual coherence of waves in the elastic channel, which underwent no incoherent scattering in the substance in the elastic channel.

Equation (21) for the mutual-coherence function ρ_{nn} can be represented in graphic form (see Fig. 5. The thick solid line corresponds to the averaged Green's function, the shaded rectangle to the matrix Γ, and the partial circle to the density matrix of a particle not experiencing incoherent elastic scattering in the elastic channel.

The first diagram corresponds to inelastic collision of a particle not experiencing incoherent elastic scattering in the substance [the term ρ^0_{nn} in (21)].

The second diagram of this figure describes a particle which it experiences first an inelastic collision, and as it continues to move it is subject to multiple elastic incoherent scattering. The third diagram corresponds to a process with a reversed sequence of events—first multiple incoherent elastic scattering, and then a single inelastic collision in the interval between the repeated incoherent elastic scattering.

Thus, in contrast to multiple particle scattering in an elastic channel (see, e.g., Refs. 8 and 19), the mutual-coherence function in an inelastic channel contains also terms quadratic in the operator Γ.

Note that our assumption that the field δU is Gaussian is of no fundamental principal significance and served only to decrease the number of displayed diagrams.

We shall be interested henceforth not in the mutual-coherence function for the wave field of a particle in the inelastic-scattering in the n-channel, but in one for a situation in which the final situation of the medium is not recorded:

$$\rho_{\text{inel}}=\sum_n \rho_{nn}.$$

We must thus sum (21) over all possible excited states of the medium.

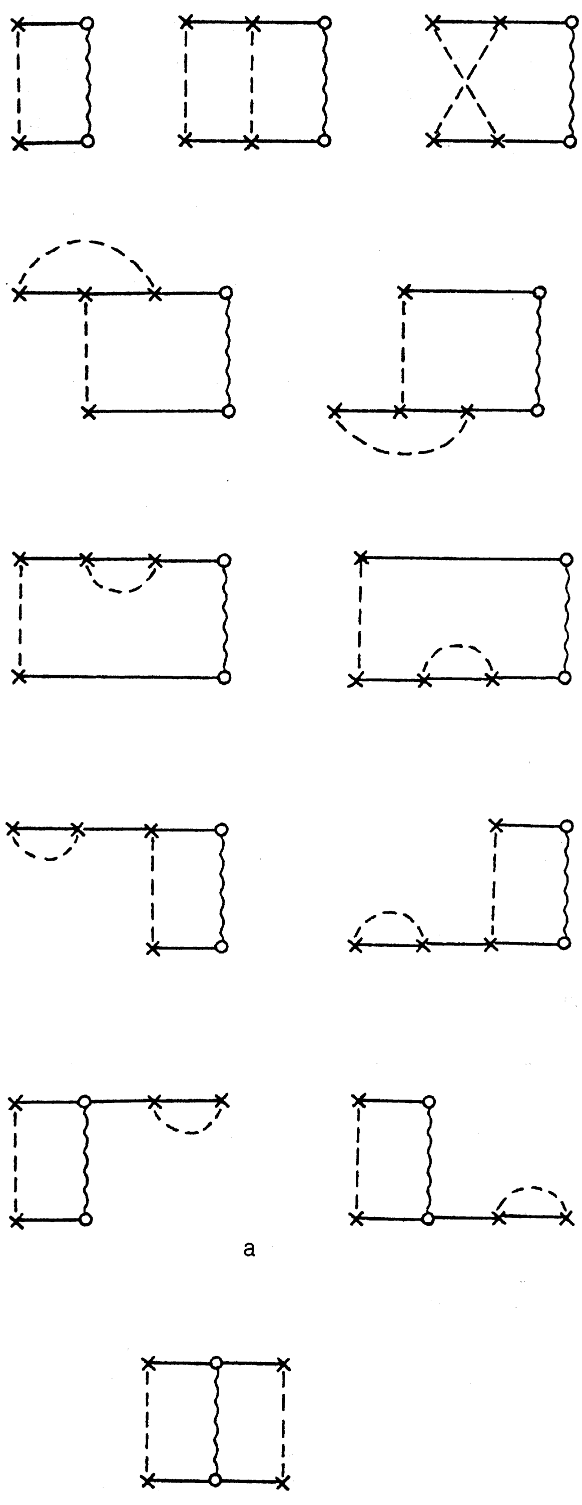

FIG. 4. Graphic representation of the mutual-coherence function of the wave field in the inelastic scattering channel of second and fourth order in the operator δU when (a) the incoherent elastic scattering is preceded by an inelastic collision; (b) the inelastic collision occurs between elastic incoherent scattering acts. Scattering with the sequence (a) reversed corresponds to diagrams obtained from (a) by reflection about the axis passing through the inelastic-collision line.

4. DENSITY MATRIX OF A PARTICLE NOT UNDERGOING INCOHERENT ELASTIC SCATTERING IN THE *s*-SCATTERING APPROXIMATION

The Hamiltonian of the interaction of a particle with elementary scatterers in a randomly inhomogeneous medium takes in the s-scattering approximation the form

$$U(\mathbf{r}) = \sum_a U_0(\mathbf{r}-\mathbf{r}_a).$$

$$U_0(\mathbf{r}-\mathbf{r}_a) = -2\pi\hbar^2 m^{-1} f\delta(\mathbf{r}-\mathbf{r}_a). \tag{22}$$

Strictly speaking, $U(\mathbf{r})$ is a pseudopotential in which f is the exact amplitude for particle scattering by a single scatterer (Ref. 32, §151), and $\mathbf{r}_a$ is the radius vector of the elementary scatterer.

The averaged value of the potential is

$$U_{av} = -2\pi\hbar^2 m^{-1} nf, \tag{23a}$$

and is fluctuating part is

$$\delta U(\mathbf{r}) = -2\pi\hbar^2 m^{-1} f\left[\sum_a \delta(\mathbf{r}-\mathbf{r}_a) - n\right]. \tag{23b}$$

The particle's average wave field with energy E_n is determined by an equation such as (15). We shall use an approximation in which the average wave field is determined by the particle motion in the averaged potential U_{av}. In this approach the average Green's function is $G_k = G_0(E_k - U_{av})$, where $G_0(E)$ is the Green's function of the particle in free space; this simplifies the summation of the diagrams for the density matrix ρ_{nn}.

Neglecting coherence effects on the interface between the vacuum and the medium, the solution (15) can be written in the form:

$$\langle\psi(\mathbf{k}_n,\mathbf{r})\rangle = \exp(i\mathbf{k}_n\mathbf{r})\exp(-z/2l_n|\mu|), \tag{24}$$

where $\mu = (\mathbf{k}_n)_z/k_n, \mathbf{k}_n$ is the wave vector of the particle, $l_n = (n\sigma^n_{\text{tot}})^{-1}$ is the mean free path of a particle of energy E_n, and $\sigma^n_{\text{tot}} = 4\pi k_n^{-1}$ Im f is the corresponding total cross section for scattering by a single center (it is assumed that the scattering amplitude is independent of the particle energy E_n). We have left out the index i that relates this wave function to a particle in an elastic channel.

Let us calculate the density matrix of a particle not subjected to incoherent scattering:

$$\rho^0_{\text{inel}}(\mathbf{r},\mathbf{r}') = \sum_n \rho^0_{nn}(\mathbf{r},\mathbf{r}')$$

$$= \sum_n \langle\psi^*_n(\mathbf{k}_n,\mathbf{r}')\rangle\langle\psi_n(\mathbf{k}_n,\mathbf{r})\rangle. \tag{25}$$

Bearing (14) in mind, we obtain

$$\rho^0_{\text{inel}}(\mathbf{r},\mathbf{r}') = \sum_n \int\int d\mathbf{r}_1 d\mathbf{r}_2 \langle G^*_n(\mathbf{r}',\mathbf{r}_2)\rangle \times\langle G_n(\mathbf{r},\mathbf{r}_1)\rangle T(\mathbf{r}_1, i\to n)T(\mathbf{r}_2, n\to i) \times\langle\psi(\mathbf{k},\mathbf{r}_1)\rangle\langle\psi^*(\mathbf{k},\mathbf{r}_2)\rangle. \tag{26}$$

We use the Lehmann expansion[33] for the function $D(\mathbf{r},\mathbf{r}',\omega)$, which is equal to the difference between the

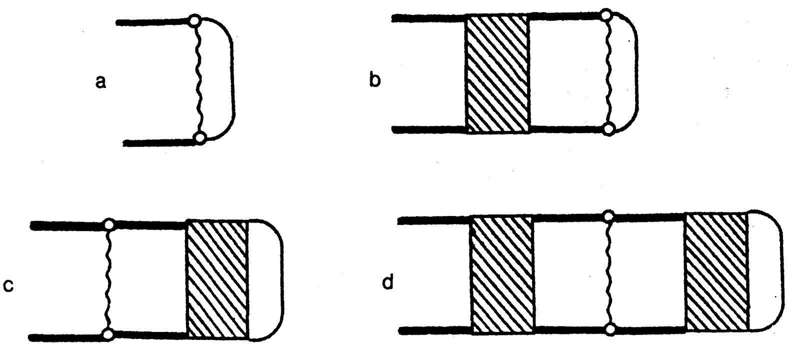

FIG. 5. Graphic representation of the mutual-coherence function of a wave field in an inelastic scattering channel.

retarded $[D_R(\mathbf{r},\mathbf{r}',\omega)]$ and advanced $[D_A(\mathbf{r},\mathbf{r}',\omega)]$ Green's functions of the electric field in the substance (here $\hbar\omega$ is the energy of the internal excitation of the medium. With the aid of this expansion we obtain for the arbitrary function $F(E_n=E-\varepsilon_n)$ the following identity which will be frequently used below:

$$\sum_n F(E-\varepsilon_n)T(\mathbf{r}_1,i\to n)T(\mathbf{r}_2,n\to i)$$
$$=\sum_n \int d\omega\delta(\omega-\varepsilon_n/\hbar)F(E-\varepsilon_n)T(\mathbf{r}_1,i\to n)$$
$$\times T(\mathbf{r}_2,n\to i)=(2\pi i)^{-1}\int d\omega\theta(\omega)F$$
$$\times(E-\hbar\omega)D(\mathbf{r}_1,\mathbf{r}_2,\omega). \quad (27)$$

In view of the identity (27), we obtain from (26)

$$\rho^0_{\text{inel}}(\mathbf{r},\mathbf{r}')=(2\pi i)^{-1}\int d\omega\theta(\omega)$$
$$\times\int\int d\mathbf{r}_1 d\mathbf{r}_2\langle G^*_{\omega\mathbf{k}_\omega}(\mathbf{r}',\mathbf{r}_2)\rangle$$
$$\times\langle G_{\omega\mathbf{k}_\omega}(\mathbf{r},\mathbf{r}_1)\rangle D(\mathbf{r}_1,\mathbf{r}_2,\omega)\langle\psi(\mathbf{k},\mathbf{r}_1)\rangle$$
$$\times\langle\psi^*(\mathbf{k},\mathbf{r}_2)\rangle. \quad (28)$$

We have used in (28) $\langle G_{\omega\mathbf{k}_\omega}\rangle = \langle G_n\rangle|_{E_n=E-\hbar\omega}$ to denote the Green's function of a particle having an energy $E-\hbar\omega$ and a wave vector $\mathbf{k}_\omega$, where $k^2_\omega=2m\hbar^{-2}(E-\hbar\omega)$. The integration over ω in (28) has the meaning of integration over the energy lost by the particle. This allows us to consider the spectral density matrix $\rho^0_{\text{inel}}(\omega,\mathbf{r},\mathbf{r}')$:

$$\rho^0_{\text{inel}}(\mathbf{r},\mathbf{r}')=\int d\omega\theta(\omega)\rho^0_{\text{inel}}(\omega,\mathbf{r},\mathbf{r}').$$

We represent the Green's function $\langle G_{\omega\mathbf{k}_\omega}\rangle$ as a Fourier integral

$$\langle G_{\omega\mathbf{k}_\omega}(\mathbf{r},\mathbf{r}')\rangle=(2\pi)^{-2}\int d\mathbf{q}G_{\omega\mathbf{k}_\omega}(\mathbf{q},z,z')\exp(i\mathbf{q}(\mathbf{R}-\mathbf{R}')), \quad (29)$$

in which $\mathbf{q}=(q_x,q_y)$ is a vector parallel to the interface between the vacuum and the medium, while $\mathbf{R}$ and $\mathbf{R}'$ are the parallel components of the vectors $\mathbf{r}$ and $\mathbf{r}'$. Similar representations are valid for the wave function $\langle\psi(\mathbf{k},\mathbf{r})\rangle$ and the function $D(\mathbf{r},\mathbf{r}',\omega)$:

$$D(\mathbf{r},\mathbf{r}',\omega)=(2\pi)^{-2}\int d\mathbf{q}D(\mathbf{q},\omega,z,z')\exp(i\mathbf{q}(\mathbf{R}-\mathbf{R}'), \quad (30)$$

$$\langle\psi(\mathbf{k},\mathbf{r})\rangle=\exp(i\mathbf{k}_\parallel\mathbf{R})\psi_{\mathbf{k}}(z). \quad (31)$$

Here $\mathbf{k}_\parallel$ is the component of the wave vector $\mathbf{k}$ and is parallel to the interface.

Substituting (29)–(31) in (28) and changing to the spectral density matrix we obtain for

$$\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z')=\int d(\mathbf{R}-\mathbf{R}')\rho^0_{\text{inel}}(\omega,\mathbf{r},\mathbf{r}')$$
$$\times\exp(-i\mathbf{q}(\mathbf{R}-\mathbf{R}')) \quad (32)$$

the expression

$$\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z')=-\frac{i}{2\pi}\int\int dz_1dz_2G^*_{\omega\mathbf{k}_\omega}(\mathbf{q},z',z_2)$$
$$\times G_{\omega\mathbf{k}_\omega}(\mathbf{q},z,z_1)D(\mathbf{q}-\mathbf{k}_\parallel,\omega,z_1,z_2)$$
$$\times\psi^*_{\mathbf{k}}(z_2)\psi_{\mathbf{k}}(z_1). \quad (33)$$

Let us show that the quantity $\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z')$ can be expressed in terms of the cross section for a single inelastic particle collision in the substance, calculated neglecting coherence effects. We use to this end an integral representation for the Green's function:[34,35]

$$\langle G_{\omega\mathbf{k}_\omega}(\mathbf{r},\mathbf{r}')\rangle=\frac{1}{(2\pi)^3}\int d\mathbf{Q}\frac{\langle\psi_{\omega\mathbf{Q}}(\mathbf{r})\rangle\langle\psi^*_{\omega\mathbf{Q}}(\mathbf{r}')\rangle}{E_{\mathbf{k}_\omega}-E_{\mathbf{Q}}+i\delta},$$

in which $E_{\mathbf{Q}}=\hbar^2\mathbf{Q}^2/2m$, while the function $\langle\psi_{\omega\mathbf{Q}}(\mathbf{r})\rangle$ is defined by relation (31) with $\mathbf{k}=\mathbf{Q}$ and $Q=k_\omega$.

The last relation may be put in the form

$$G_{\omega\mathbf{k}_\omega}(\mathbf{q},z,z')=\frac{1}{2\pi}\int d\mathbf{Q}\frac{\psi_{\omega\mathbf{Q}}(z)\psi^*_{\omega\mathbf{Q}}(z')}{E_{\mathbf{k}_\omega}-E_{\mathbf{Q}}+i\delta}\delta(\mathbf{q}-\mathbf{Q}_\parallel). \quad (34)$$

We now transform (33) with allowance for (34). Note, first of all, that when wave processes are considered the dimension of the near zone is of the order of $R\approx\lambda$, where λ is the de Broglie wavelength of the scattered particles. At the same time, the average distance between individual scattering centers is of the order of $d \simeq n^{-1/3}$. If the density of the elementary scatterers in a randomly inhomogeneous medium is low, i.e., $n\lambda^3 \ll 1$, we have $d \gg R$. The latter circumstance allows us to take $G_{\omega k_\omega}(\mathbf{q},z,z_1)$ in (33) to mean its asymptotic value as $z \to \infty$. A similar statement holds also for $G^*_{\omega k_\omega}(\mathbf{q},z',z_2)$ (with $z' \to \infty$).

Bearing this in mind, we can transform (34) in the above approximation into

$$G_{\omega k_\omega}(\mathbf{q},z,z') = -im\hbar^{-2}(k_\omega^2 - q^2)^{-1/2}\psi_{\omega \mathbf{k_q}(\omega)} \times (z)\psi^*_{\omega \mathbf{k_q}(\omega)}(z'), \quad (35)$$

with the vector $\mathbf{k_q}(\omega) = (\mathbf{q},(k_\omega^2 - q^2)^{1/2})$. Substituting (35) in (33) we get:

$$\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z') = -\frac{im^2}{2\pi\hbar^4}(k_\omega^2 - q^2)^{-1}\psi_{\omega \mathbf{k_q}(\omega)}(z)\psi^*_{\omega \mathbf{k_q}(\omega)}(z') \times \int\int dz_1 dz_2 \psi_{\omega \mathbf{k_q}(\omega)}(z_2)\psi^*_{\omega \mathbf{k_q}(\omega)}(z_1) \times D(\mathbf{q}-\mathbf{k}_\parallel, \omega, z_1, z_2)\psi^*_{\mathbf{k}}(z_2)\psi_{\mathbf{k}}(z_1). \quad (36)$$

We separate in (36) the doubly differential inelastic-collision cross section. To this end we calculate the amplitude of an inelastic collision of a particle located in the scattering n-channel:

$$f_n(\mathbf{k}\to\mathbf{Q}) = -\frac{m}{2\pi\hbar^2}\int d\mathbf{r}\langle\psi^*(\mathbf{Q}=\mathbf{k}_n,\mathbf{r})\rangle \times T(\mathbf{r},i\to n)\langle\psi(\mathbf{k},\mathbf{r})\rangle. \quad (37)$$

Defining the inelastic-scattering cross section differentiated with respect to the angle as[1)]

$$(d\sigma_{\text{inel}}/d\Omega)_{\mathbf{k}\to\mathbf{Q}} = \sum_n |f_n(\mathbf{k}\to\mathbf{Q})|^2,$$

where $\mathbf{k}$ and $\mathbf{Q}$ are the wave vectors of the particle in the initial and final states, summing over all the excited states with use of the identity (27), we obtain for the doubly differential inelastic-collision integral the equation

$$(d^2\sigma_{\text{inel}}/d\Omega d\omega)|_{\mathbf{k}\to\mathbf{Q}} = -\frac{iS}{(2\pi)^3}\left(\frac{m}{\hbar^2}\right)^2\int\int dz_1 dz_2 \psi_{\omega\mathbf{Q}}(z_2)\psi^*_{\omega\mathbf{Q}}(z_1) \times D(\mathbf{Q}_\parallel - \mathbf{k}_\parallel, \omega, z_1, z_2)\psi^*_{\mathbf{k}}(z_2)\psi_{\mathbf{k}}(z_1). \quad (38)$$

Here S is the area of the interface. Comparing (36) with (38) we get

$$\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z') = \frac{(2\pi)^2}{S}(k_\omega^2 - q^2)^{-1}\psi_{\omega \mathbf{k_q}(\omega)}(z)\psi^*_{\omega \mathbf{k_q}(\omega)}(z') \times (d^2\sigma_{\text{inel}}/d\Omega d\omega)_{\mathbf{k}\to\mathbf{k_q}(\omega)}. \quad (39)$$

Here $z,z' > 0$. Note that the function $\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z)$ is the distribution function, in the substance, of particles which have not been incoherently scattered, with respect to the energy loss $\hbar\omega$ and the longitudinal momentum transfer $\hbar\mathbf{q}$, and which move in the medium at a distance z from the surface.

The distribution of these particles with respect to energy loss at a distance z from the surface in the medium is given by the function

$$\rho^0_{\text{inel}}(\omega,z) = \frac{1}{(2\pi)^2}\int d\mathbf{q}\rho^0_{\text{inel}}(\mathbf{q},\omega,z,z) = \frac{1}{S}\int\frac{d\mathbf{q}}{k_\omega^2 - q^2}|\psi_{\omega \mathbf{k_q}(\omega)} \times (z)|^2(d^2\sigma_{\text{inel}}/d\Omega d\omega)_{\mathbf{k}\to\mathbf{k_q}(\omega)}. \quad (40)$$

5. GENERAL EQUATION FOR ANGLE SPECTRUM OF BACKSCATTERED PARTICLES

To determine the angle spectrum of particles that are backscattered in a medium and have lost an energy $\hbar\omega$, it suffices to know the distribution of the emitted particles over the components of the wave vector $\mathbf{q}$ parallel to the surface, for $z=-0$.

$$J_{\text{inel}}(\mu_0 \to |\mu|,\omega) = \frac{k_\omega^2\mu^2}{(2\pi)^2 S}\rho_{\text{inel}}(\mathbf{q},z=-0,\mathbf{q},z = -0,\omega)|_{\mathbf{q}=(\mathbf{k}'_\omega)_\parallel}. \quad (41)$$

Here $q = k_\omega(1-\mu^2)^{1/2}$, μ_0 is the cosine of the angle between the inward normal to the vacuum-medium interface and the direction of particle incidence on the surface, and μ is the cosine of the angle of particle escape from the medium ($\mu \leqslant 0$) for backward scattering), and $(\mathbf{k}'_\omega)_\parallel$ is the wave-vector component parallel to the surface of the particle in the final state.

Let us calculate the contribution made to the backscattering angle spectrum from each of the four diagrams of Fig. 5.

The diagram 5a describes the contribution made to the backscattering spectrum by particles that underwent no incoherent elastic scattering in the substance. Clearly, the contribution of this term will be significant only if the inelastic collision in the bulk of the substance is accompanied by a large change of the momentum of the particle moving in the medium, so that the particle motion changes direction following an inelastic collision.

To use Eq. (41) in this case, we must know the distribution (39) in the region $z<0$. To this end we must repeat the reasoning of the preceding section, using in lieu of (35) the equation

$$G_{\omega k_\omega}(\mathbf{q},z=-0,z'>0) = -im\hbar^{-2}(k_\omega^2 - q^2)^{-1/2}\psi^*_{\omega \mathbf{k}^-_{\mathbf{q}}(\omega)}(z'), \quad (42)$$

which, in contrast to (5), is exact; $\mathbf{k}^-_{\mathbf{q}}(\omega) = [\mathbf{q}, -(k_\omega^2 - q^2)^{1/2}]$. We ultimately obtain

$$\rho^0_{\rm inel}(\mathbf{q},\omega,z=-0,z'=-0)$$

$$=\frac{(2\pi)^2}{S(k^2_\omega-q^2)}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mathbf{k}\to\mathbf{k}^-_{\mathbf{q}}(\omega)}.$$

With (41) taken into account, the contribution of this diagram to the spectrum is determined by the doubly differential cross section for inelastic collision of the particle in the medium, calculated neglecting incoherent elastic scattering:

$$J_i(\mu_0\to|\mu|,\omega)=\frac{1}{S}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_0\to\mu}. \tag{43}$$

The contribution of the three remaining diagrams can be expressed in terms of the doubly differential cross section for an inelastic collision and in terms of the elastic-scattering angle spectrum. To demonstrate this, we obtain the connection between the angle spectrum $J_{\rm el}(\mu_0\to|\mu|)$ for backscattering in an elastic channel, with a matrix $\Gamma_{kk}(\mathbf{r}_1,\mathbf{r}'_1;\mathbf{r}_2,\mathbf{r}'_2)$, describing the multiple scattering of a particle with energy E_k in a randomly inhomogeneous medium.

The density matrix of incoherently scattered particles of energy E_k in an elastic channel is determined by the integral[19]

$$\rho^{\rm inc}_{\rm el}(\mathbf{r},\mathbf{r}')=\int\int d\mathbf{r}_1 d\mathbf{r}'_1\int\int d\mathbf{r}_2 d\mathbf{r}'_2\langle G_k(\mathbf{r},\mathbf{r}_1)\rangle$$

$$\times\langle G^*_k(\mathbf{r},\mathbf{r}_2)\rangle\Gamma_{kk}(\mathbf{r}_1,\mathbf{r}'_1;\mathbf{r}_2,\mathbf{r}'_2)\rho^0_{kk}(\mathbf{r}'_1,\mathbf{r}'_2). \tag{44}$$

The backscattering spectrum in an elastic channel is determined by an equation such as (41), with k replacing k_ω and a density matrix $\rho^{\rm inc}_{\rm el}$ instead of $\rho_{\rm inel}$. From (44) and from analogs of (31), (34), and (42) one obtains

$$J_{\rm el}(\mu_0\to|\mu|)=\left(\frac{m}{2\pi\hbar^2}\right)^2\frac{1}{S}\Gamma_{kk}$$

$$\times(\mathbf{k}'_\parallel,K^*,\mathbf{k}_\parallel,K_0,\mathbf{k}'_\parallel,$$

$$K,\mathbf{k}_\parallel,K^*_0), \tag{45}$$

where

$$K=-k|\mu|+i(2|\mu|l)^{-1},\quad K_0=k\mu_0+i(2\mu_0 l)^{-1}, \tag{46}$$

while $\mathbf{k}'_\parallel$ is the longitudinal component of the particle's wave vector in the final state. The matrix Γ_{kk} in (45) is defined in the momentum representation as

$$\Gamma_{kk}(\mathbf{q}_1,K_1,\mathbf{q}'_1,K'_1,\mathbf{q}_2,K_2,\mathbf{q}'_2,K'_2)$$

$$=\int d\mathbf{r}_1 d\mathbf{r}'_1\int d\mathbf{r}_2 d\mathbf{r}'_2\Gamma_{kk}(\mathbf{r}_1,\mathbf{r}'_1;\mathbf{r}_2,\mathbf{r}'_2)$$

$$\times\exp(-i\mathbf{q}_1\mathbf{R}_1-iK_1z_1)\exp(i\mathbf{q}'_1\mathbf{R}'_1+iK'_1z'_1)$$

$$\times\exp(i\mathbf{q}_2\mathbf{R}_2+iK_2z_2)\exp(-i\mathbf{q}'_2\mathbf{R}'_2-iK'_2z'_2). \tag{47}$$

Here $\mathbf{R}$ and z are, respectively, the components or the radius vector $\mathbf{r}$ parallel and perpendicular to the surface.

Consider now the contribution made by the remaining diagrams to the angle spectrum by the inelastically scattered particles.

The diagram of Fig. 5b describes the contribution made to the angle spectrum by backscattering particles undergoing first inelastic collision and then multiple elastic incoherent scattering. The corresponding density matrix for a particle in the inelastic-scattering n-channel is given by

$$\rho^{(1)}_{nn}(\mathbf{r},\mathbf{r}')=\int d\mathbf{r}_1 d\mathbf{r}'_1 d\mathbf{r}''_1 d\mathbf{r}_2 d\mathbf{r}'_2 d\mathbf{r}''_2\langle G_n(\mathbf{r},\mathbf{r}_1)\rangle$$

$$\times\langle G^*_n(\mathbf{r}',\mathbf{r}_2)\rangle\Gamma_{nn}(\mathbf{r}_1,\mathbf{r}'_1;\mathbf{r}_2,\mathbf{r}'_2)\langle G_n(\mathbf{r}_1,\mathbf{r}''_1)\rangle$$

$$\times\langle G^*_n(\mathbf{r}_2,\mathbf{r}''_2)\rangle T_{in}(\mathbf{r}''_1)T_{ni}(\mathbf{r}''_2)\rho^0_{ii}(\mathbf{r}''_1,\mathbf{r}''_2). \tag{48}$$

After summing over n (the procedure is fully analogous to that considered in Sec. 4), we change over to a spectral density matrix and use Eqs. (35), (42), and (47). The result is

$$\rho^{(1)}_{\rm inel}(\mathbf{q},z=-0;\mathbf{q},z=-0;\omega)=\left(\frac{m}{\hbar^2}\right)^2\frac{1}{S(k^2_\omega-q^2)}\int$$

$$\times\int\frac{d\mathbf{q}'}{(k^2_\omega-q'^2)}\Gamma_\omega(\mathbf{q},K^*_\mathbf{q},\mathbf{q}',K_{\mathbf{q}'},\mathbf{q},K_\mathbf{q},\mathbf{q}',K^*_{\mathbf{q}'})$$

$$\times\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mathbf{k}\to\mathbf{k}_{\mathbf{q}'}(\omega)}, \tag{49}$$

where

$$K_\mathbf{q}=-k_\mathbf{q}(\omega)+i(2k_\mathbf{q}(\omega)l_\omega/k_\omega)^{-1},$$

$$K_{\mathbf{q}'}=k_{\mathbf{q}'}(\omega)+i(2k_{\mathbf{q}'}(\omega)l_\omega/k_\omega)^{-1}. \tag{50}$$

The subscript ω in these equations labels quantities corresponding to a particle energy $E_\omega=E-\hbar\omega$.

We take into account expression (45) for the angular spectrum of backscattering in an elastic channel and change from integration over $\mathbf{q}'$ to integration over μ', where

$$\mu'=(1-q'^2/k^2_\omega)^{1/2}.$$

The corresponding part of the spectrum is then determined by the formula (accurate to $\hbar\omega/E\ll 1$):

$$J_{i-e}(\mu_0\to|\mu|,\omega)$$

$$=\frac{2\pi}{S}\int_0^1\frac{d\mu'}{\mu'}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_0\to\mu'}J_{\rm el}(\mu'\to|\mu|). \tag{51}$$

Similar treatment of the contribution of the diagram of Fig. 5c and corresponding to a reverse sequence of the events in Fig. 5b leads, as is easily verified by applying the arguments above, to the following contribution to the density matrix

$$\rho_{nn}^{(2)}(\mathbf{r},\mathbf{r}')=\int d\mathbf{r}_1 d\mathbf{r}_1' d\mathbf{r}_1'' d\mathbf{r}_2 d\mathbf{r}_2' d\mathbf{r}_2'' \langle G_n(\mathbf{r},\mathbf{r}_1'')\rangle \times\langle G_n^*(\mathbf{r}',\mathbf{r}_2'')\rangle T_{in}(\mathbf{r}_1'' T_{ni}\mathbf{r}_2'')\langle G_n(\mathbf{r}_1'',\mathbf{r}_1)\rangle \times\langle G_n^*(\mathbf{r}_2'',\mathbf{r}_2)\rangle \Gamma_{ii}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\rho_{ii}^0(\mathbf{r}_1',\mathbf{r}_2'), \quad (52)$$

and to the spectrum (we omit the detailed calculations)

$$J_{e-i}(\mu_0\rightarrow|\mu|,\omega)=\frac{2\pi}{S}\int_0^1 \frac{d\mu'}{\mu'} J_{\rm el}(\mu_0\rightarrow\mu') \times\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu'\rightarrow|\mu|}. \quad (53)$$

Consider, finally, the contribution of the last diagram, Fig. 5d, to the backscattering angle spectrum. The diagram considered describes the contribution made to the spectrum by particles that underwent multiple elastic incoherent scattering both before and after an inelastic collision in a disordered medium.

The contribution of such particles to the density matrix is [see (21) and (52)]

$$\rho_{nn}^{(3)}(\mathbf{r},\mathbf{r}')=\int d\mathbf{r}_1 d\mathbf{r}_1' d\mathbf{r}_2 d\mathbf{r}_2' \langle G_n(\mathbf{r},\mathbf{r}_1)\rangle\langle G_n^*(\mathbf{r}',\mathbf{r}_2)\rangle \times\Gamma_{nn}(\mathbf{r}_1,\mathbf{r}_1';\mathbf{r}_2,\mathbf{r}_2')\rho_{nn}^{(2)}(\mathbf{r}_1',\mathbf{r}_2'). \quad (54)$$

We proceed now to a the spectral density matrix, using Eqs. (35), (42) and (47) and recognizing that in the far-field approximation we have

$$\rho_{\rm inel}^{(2)}(\mathbf{q},z;\mathbf{q},z';\omega) =\left(\frac{m}{\hbar^2}\right)^2 \frac{1}{S(k_\omega^2-q^2)}\psi_{\omega\mathbf{k_q}(\omega)}(z)\psi^*_{\omega\mathbf{k_q}(\omega)}(z') \times\int\frac{d\mathbf{q}}{(k^2-q'^2)}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mathbf{k}_{\mathbf{q}'}\rightarrow\mathbf{k_q}(\omega)} \times\Gamma(\mathbf{q}',K^*_{\mathbf{q},},\mathbf{k}_\parallel\ ,K_0,\mathbf{q}',K_{\mathbf{q}'},\mathbf{k}_\parallel\ ,K_0^*). \quad (55)$$

We then obtain for the quantity $\rho_{\rm inel}^{(3)}(\mathbf{q},z=-0;\mathbf{q},z=-0;\omega)$ that determines the corresponding angular-spectrum component:

$$\rho_{\rm inel}^{(3)}(\mathbf{q},z=-0;\mathbf{q},z=-0;\omega) =\left(\frac{m}{\hbar^2}\right)^4 \frac{1}{S^2(k_\omega^2-q^2)(2\pi)^2}\int\frac{d\mathbf{q}}{(k_\omega^2-q_1^2)}\Gamma_\omega \times(\mathbf{q},K^*_{\mathbf{q}},\mathbf{q}_1,K_{\mathbf{q}_1},\mathbf{q},K_{\mathbf{q}},\mathbf{q}_1,K^*_{\mathbf{q}_1}) \times\int\frac{d\mathbf{q}_2}{(k^2-q_2^2)}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\psi}\right)_{\mathbf{k}_{\mathbf{q}_2}\rightarrow\mathbf{k}_{\mathbf{q}_1}(\omega)} \times\Gamma(\mathbf{q}_2,K^*_{\mathbf{q}_2},\mathbf{k}_\parallel\ ,K_0,\mathbf{q}_2,K_{\mathbf{q}_2},\mathbf{k}_\parallel\ ,K_0^*). \quad (56)$$

K_0 and $K_{\mathbf{q}}$ in (56) are defined by Eqs. (46) and (50), respectively, and

$$K_{\mathbf{q}1}=k_{\mathbf{q}_1}(\omega)+i[2k_{\mathbf{q}_1}(\omega)l_\omega/k_\omega]^{-1},$$

$$K_{\mathbf{q}_2}=-k_{\mathbf{q}_2}+i(2k_{\mathbf{q}_2}l/k)^{-1}. \quad (57)$$

The corresponding part of the backscattering angle spectrum is determined with the aid of (41) and (45):

$$J_{e-i-e}(\mu_0\rightarrow|\mu|,\omega) =\frac{(2\pi)^2}{S}\int_0^1\frac{d\mu_1}{\mu_1}J_{\rm el}(\mu_0\rightarrow\mu_1) \times\int_0^1\frac{d\mu_2}{\mu_2}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_1\rightarrow\mu_2} J_{\rm el}(\mu_2\rightarrow|\mu|). \quad (58)$$

In the derivation of (58) we changed to integration over the variables μ_1 and μ_2 [a similar procedure was used earlier, see (51)], and neglected quantities of order $\hbar\omega/E$.

We have thus solved the problem of finding the backscattering angle spectrum of particles following a single inelastic collision and a multiple incoherent elastic scattering in a disordered medium. The angle spectrum of such a particle, which has lost an energy $\hbar\omega$ by inelastic collision, is

$$J_{\rm inel}(\mu_0\rightarrow|\mu|,\omega) =\frac{1}{S}\Bigg\{\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_0\rightarrow\mu} +2\pi\int_0^1\frac{d\mu_1}{\mu_1}\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_0\rightarrow\mu_1}J_{\rm el}(\mu_1\rightarrow|\mu|) +2\pi\int_0^1\frac{d\mu_1}{\mu_1}J_{\rm el}(\mu_0\rightarrow\mu_1)\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_1\rightarrow|\mu|} +(2\pi)^2\int_0^1\frac{d\mu_1}{\mu_1}J_{\rm el}(\mu_0\rightarrow\mu_1)\int_0^1\frac{d\mu_2}{\mu_2} \times\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_1\rightarrow\mu_2}J_{\rm el}(\mu_2\rightarrow|\mu|)\Bigg\}. \quad (59)$$

It is seen from (59) that this angle spectrum is expressed in terms of the doubly differential cross section for a particle subject to inelastic collisions the medium without undergoing incoherent elastic scattering, and in terms of the exact angle spectrum of the particle backscattering in an elastic channel. Note that the angle spectrum (59) satisfies the reciprocity theorem (symmetry with respect to interchange of μ_0 and $|\mu|$). At the same time the spectrum components J_{i-e} and I_{e-i}, resulting from scattering processes with opposite sequence of the inelastic collision and the multiple incoherent elastic scattering, do not satisfy separately the reciprocity theorem.

6. BACKSCATTERING ANGLE SPECTRUM IN THE CASE OF AN ISOTROPIC INELASTIC COLLISION IN THE BULK OF A SUBSTANCE

Consider the case when the cross section for inelastic collisions in the bulk of a substance is independent of the scattering angle (isotropic inelastic scattering in the bulk).

The doubly differential cross section in Eq. (59) should in principle take into account the influence of the surface on the inelastic-scattering process. This influence can be manifested in two ways—firstly, via the influence of the interface between the vacuum and the medium on the wave function of the particle in the substance (boundary conditions in the Schrödinger equation), and secondly, via the influence of the surface on the inelastic-scattering channel in the bulk.

The role of the second factor can, generally speaking, be extraordinarily great (see, e.g., Ref. 36), so that at glancing incidence of the particle on the surface the angle picture of the inelastic picture can be unusually greatly altered. It is therefore meaningless to consider, making the most general assumptions concerning the character and nature of the inelastic collision, the coherent effects due to the presence of an abrupt vacuum-medium boundary, similar to the effects of elastic incoherent scattering.[37] This is precisely why coherent effects that occur at glancing entry and exit angles, were disregarded above.

Nonetheless, the effect of the surface on the wave function at not too inclined incidence angles can be taken into account. In the absence of absorption, the cross section for isotropic inelastic collision corresponds to a D-function whose Fourier transform with respect to the spatial variables does not depend on the momentum loss (38) in the collision:

$$D(\mathbf{Q},\omega)=\frac{2i\pi^2\hbar^4}{m^2V}\,\sigma_{\rm col}(\omega), \tag{60}$$

where $\sigma_{\rm col}(\omega)$ is the cross section, integral over the angles and differential with respect to the energy loss $\hbar\omega$, of an isolated inelastic collision in the bulk of the substance in the absence of absorption, and V is the volume of the medium.

We find now the double differential inelastic-collision cross section with allowance for the influence of the surface on the wave field of the scattered particle (at not too grazing an incidence); we shall assume here that at such incidence angles the surface does not influence the collision process in the bulk, and that the processes that attenuate the wave field of the particle moving in the medium do not affect greatly the properties of the material subsystem which is associated with the inelastic-scattering channel singled out by us. Consequently, the doubly differential cross section can be calculated by using Eq. (60). From (38) we obtain (with accuracy $\hbar\omega/E\ll 1$):

$$\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_0\to\mu}=\frac{|\mu|\mu_0}{|\mu|+\mu_0}\,\frac{S}{\pi n\sigma_{\rm tot}v}\,W_{\rm col}(\omega). \tag{61}$$

We have assumed here that $\sigma_{\rm tot}$ depends little on the energy lost in the elastic collision, and have changed from an angle- integrated cross section for inelastic collision in the bulk to an angle-integrated probability $W_{\rm col}=v\sigma_{\rm col}/V$ of such a collision per unit time; v is the fast-particle velocity.

Bearing in mind (61) and the elastic-channel backscattering angle spectra, expressions for which were obtained in Ref. 19 by summing ladder and fan diagrams in the expansion for the matrix Γ [see (44)] and are of the form

$$J_{\rm el}(\mu_0\to|\mu|)=J_{\rm el}^l(\mu_0\to|\mu|)+J_{\rm el}^f(\mu_0\to|\mu|), \tag{62a}$$

$$J_{\rm el}^l(\mu_0\to|\mu|)=\frac{\omega_0}{4\pi}\,\frac{|\mu|\mu_0}{|\mu|+\mu_0}\,H(|\mu|,\omega_0)H(\mu_0,\omega_0), \tag{62b}$$

$$J_{\rm el}^f(\mu_0\to|\mu|)=\frac{\omega_0}{4\pi}\,\frac{|\mu|\mu_0}{|\mu|+\mu_0}\times\left[\left|H\left(\tilde{\mu},\omega_0\,\Big|\,\frac{|\mathbf{k}_\parallel+\mathbf{k}'_\parallel|}{n\sigma_{\rm tot}}\right)\right|^2-1\right], \tag{62c}$$

one can obtain an analytic expression for the angle spectra of inelastically scattered particles in a randomly inhomogeneous medium. Note that (62b) describes the contribution of ladder diagrams to the angle spectrum of elastically scattered particles, and describes weak localization in the elastic channel; ω_0 is the single-scattering albedo, $H(\mu,\omega_0|\tilde{v})$ is a generalization of the Chandrasekhar function,[19] $H(\mu,\omega_0)=H(\mu,\omega_0|\tilde{v}=0)$, and

$$\tilde{\mu}^{-1}=(\mu_0^{-1}+|\mu|^{-1})/2-ik(\mu_0-|\mu|)/n\sigma_{\rm tot}. \tag{62d}$$

The contribution made to $J_{\rm inel}$ by particles not subjected to incoherent elastic scattering is determined by the quantity (61):

$$J_i(\mu_0\to|\mu|,\omega)=\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\,\frac{\mu_0|\mu|}{\mu_0+|\mu|}. \tag{63}$$

We calculate now the contribution of the second term in (59), assuming that $\hbar\omega\ll E$ and taking into account for the time being only the contribution of the ladder diagrams to the elastic-scattering angle spectrum.

$$J_{i\text{-}e}(\mu_0\to|\mu|,\omega)=\frac{\omega_0}{2}\,\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\,\mu_0|\mu|H(|\mu|,\omega_0)I(\mu_0,|\mu|),$$

$$I(\mu_0,|\mu|)=\int_0^1\frac{\mu_1H(\mu_1,\omega_0)}{(\mu_1+\mu_0)(\mu_1+|\mu|)}\,d\mu_1.$$

The integral $I(\mu_0,|\mu|)$ can be calculated by using the integral equation for the Chandrasekhar function:[38]

$$I(\mu_0,|\mu|)=\frac{2}{\omega_0}\,\frac{1}{|\mu|-\mu_0}\times[H^{-1}(\mu_0,\omega_0)-H^{-1}(|\mu|,\omega_0)],$$

so that

$$J_{i\text{-}e}(\mu_0\to|\mu|,\omega)=\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\frac{\mu_0|\mu|}{|\mu|-\mu_0}\left[\frac{H(|\mu|,\omega_0)}{H(\mu_0,\omega_0)}-1\right]. \quad (64)$$

The third term of (59) can be written in the form:

$$J_{e\text{-}i}(\mu_0\to|\mu|,\omega)=\frac{\omega_0}{2}\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\mu_0|\mu|H(\mu_0,\omega_0)I(\mu_0,|\mu|),$$

or

$$J_{e\text{-}i}(\mu_0\to|\mu|,\omega)=\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\frac{\mu_0|\mu|}{|\mu|-\mu_0}\left[1-\frac{H(\mu_0,\omega_0)}{H(|\mu|,\omega_0)}\right]. \quad (65)$$

We point out that the spectra (64) and (65) taken separately do not satisfy the reciprocity theorem. At the same time, their sum is symmetric with respect to interchange of μ_0 and $|\mu|$.

Let us calculate the contribution of the last (fourth) term of (59):

$$J_{e\text{-}i\text{-}e}(\mu_0\to|\mu|,\omega)=\left(\frac{\omega_0}{2}\right)^2\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\mu_0|\mu|H(\mu_0,\omega_0)H|\mu|,\omega_0)\times\int_0^1 d\mu_1\int_0^1\times d\mu_2\frac{\mu_1\mu_2}{(\mu_0+\mu_1)(\mu_1+\mu_2)(\mu_2+|\mu|)}\times H(\mu_1,\omega_0)H(\mu_2,\omega_0).$$

We use the results of integrating J_{i-e}. Then

$$J_{e\text{-}i\text{-}e}(\mu_0\to|\mu|,\omega)=\left(\frac{\omega_0}{2}\right)^2\frac{W_{\rm col}(\psi)}{\pi n\sigma_{\rm tot}v}\mu_0|\mu|H\mu_0,\omega_0)H(|\mu|,\omega_0)\times\int_0^1 d\mu_1\frac{\mu_1}{(\mu_0+\mu_1)}H(\mu_1,\omega_0)I(|\mu|,\mu_1).$$

The remaining integral can be taken by using the identity (75) of Ref. 38. We obtain ultimately

$$J_{e-i-e}(\mu_0\to|\mu|,\omega)=\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\frac{\mu_0|\mu|}{|\mu|+\mu_0}\Big\{H(\mu_0,\omega_0)H(|\mu|,\omega_0)-1-\frac{\omega_0}{2}H(\mu_0,\omega_0)H(|\mu|,\omega_0)\times\left[|\mu|\ln\frac{1+|\mu|}{|\mu|}+\mu_0\ln\frac{1+\mu_0}{\mu_0}\right]\Big\}. \quad (66)$$

Thus, in the case of an isotropic inelastic collision the backscattering angle spectrum is determined by the formula

$$J_{\rm inel}(\mu_0\to|\mu|,\omega)=\frac{W_{\rm col}(\omega)}{\pi n\sigma_{\rm tot}v}\Big\{\frac{\mu_0|\mu|}{|\mu|+\mu_0}H(\mu_0,\omega_0)H(|\mu|,\omega_0)\times\left[1-\frac{\omega_0}{2}\left(|\mu|\ln\frac{1+|\mu|}{|\mu|}+\mu_0\ln\frac{1+\mu_0}{\mu_0}\right)\right]+\frac{\mu_0|\mu|}{|\mu|-\mu_0}\left[\frac{H(|\mu|,\omega_0)}{H(\mu_0,\omega_0)}-\frac{H(\mu_0,\omega_0)}{H(|\mu|,\omega_0)}\right]\Big\}. \quad (67)$$

Strictly speaking, in the derivation of (67) we have used only the "ladder" component of the angle spectrum of backscattering in an elastic channel. It is easy to show that the contribution to $J_{\rm inel}$ of the "fan" component of the spectrum in the inelastic channel, a component due to weak localization, has an order of smallness $(kl)^{-1}\ll 1$, so that at the indicated accuracy Eq. (67) is an analytic solution of the problem of backscattering in an inelastic channel.

7. ANGLE SPECTRUM OF BACKSCATTERING IN THE CASE OF A SMALL-ANGLE INELASTIC COLLISION IN THE BULK OF THE SUBSTANCE

To determine the backscattering angle spectrum in this case, we must also obtain some expression for the doubly differential cross section for small-angle inelastic collision of a particle that has not undergone an incoherent elastic scattering. To do this, we examine the physical meaning of the analogous equation (61) in the case of isotropic inelastic collision.

The factor $|\mu|\mu_0/(|\mu|+\mu_0)$ is due to the presence of a surface separating the vacuum from the substance. The remaining multiplicative quantity is none other than the cross section for inelastic collision in an unbounded absorbing medium. In fact, in a medium without absorption $\sigma_{\rm inel}$ is proportional to the volume V occupied by the medium. When the absorption is turned, damping of the wave field causes the particle to penetrate into the medium to a depth $d_{\rm eff}\approx l$, where l is the mean free path. In a medium with damping the corresponding scattering cross section can therefore be estimated at $\sigma_{\rm inel}Sd_{\rm eff}/V$.

This estimate corresponds to the results of Ref. 39 and permits the use of Eq. (61) in which the inelastic-collision probability per unit time must be replaced by the doubly differential inelastic collision probability $W_{\rm col}(\omega,\mu_0\to\mu)$ multiplied by 4π:

$$\left(\frac{d^2\sigma_{\rm inel}}{d\Omega d\omega}\right)_{\mu_0\to\mu}=\frac{\mu_0|\mu|}{|\mu|+\mu_0}\frac{4S}{n\sigma_{\rm tot}v}W_{\rm col}(\omega,\mu_0\to\mu). \quad (68)$$

The differential probability of inelastic collision is different from zero in this case in a narrow range of scattering angles, $|\theta-\theta_0|\leqslant\Delta\theta_S$, where $\Delta\theta_S$ is the characteristic region of scattering angles; $\mu=\cos\theta$, and $\mu_0=\cos\theta_0$. The integral probability of inelastic collision of a particle in the bulk of the substance per unit time is

$$W_{\rm col}^{\rm int}(\omega)=2\pi\int_{-1}^{+1}d\mu W_{\rm col}(\omega,\mu_0\to\mu). \quad (69)$$

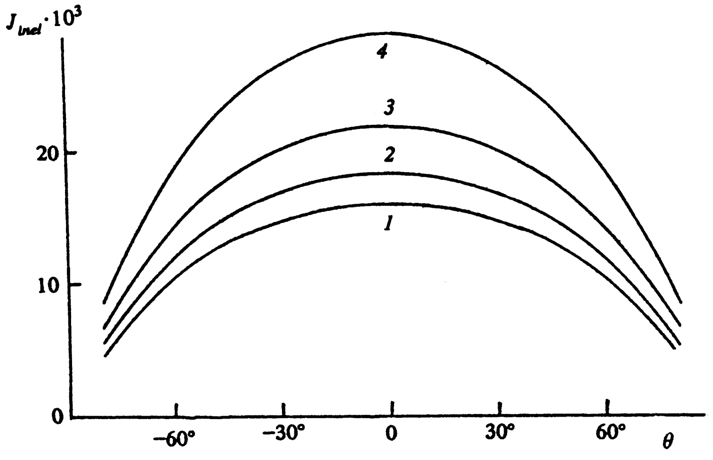

FIG. 6. Angle spectrum of backscattering in an inelastic channel. Case of isotropic inelastic collision: *1*—$\omega_0=0$, *2*—$\omega_0=0.25$, *3*—$\omega_0=0.50$, *4*—$\omega_0=0.75$; $W_{\rm col}(\omega)/nv\sigma_{\rm tot}=0.1$; $\mu_0=1$.

7.1. The case $\Delta\theta_l \ll \Delta\theta_S \ll 2\pi$

Let us carry out the integration in the case of small-angle inelastic scattering, in which the characteristic scattering angle exceeds the angle range $\Delta\theta_l \approx (kl)^{-1}$ in which weak localization appears.

The first term of (59), due to inelastically scattered particles not subjected to incoherent elastic scattering, does not contribute to the backscattering angle spectrum, since $W_{\rm col}(\omega,\mu_0\to\mu<0)\simeq 0$.

Integration of the second term of (59) with allowance only for the ladder contribution to the elastic-scattering angle spectrum can be readily seen to yield

$$J_{i\text{-}e}(\mu_0\to|\mu|,\omega)=\frac{4\pi}{n\sigma_{\rm tot}v}J^l_{\rm el}(\mu_0\to|\mu|)\times\int_0^{+1}d\mu_1 W_{\rm col}(\omega,\mu_0\to\mu_1)=\frac{2}{n\sigma_{\rm tot}v}J^l_{\rm el}(\mu_0\to|\mu|)W^{\rm int}_{\rm col}(\omega).$$

In the integration over μ_1 we took it into account that in the region of nonzero $W_{\rm col}(\omega,\mu_0\to\mu_1)$, the angle spectrum in the elastic channel, calculated neglecting the weak localization effect, is a slowly varying quantity and can therefore be moved outside the integral sign. Furthermore, the integration region in (69) can be narrowed down to $\mu\in[0,1]$ for small-angle inelastic scattering.

Integration of the remaining terms in (59) yields:

$$J_{\rm inel}(\mu_0\to|\mu|,\omega)=\frac{4W^{\rm int}_{\rm col}(\omega)}{n\sigma_{\rm tot}v}\Big[J^l_{\rm el}(\mu_0\to|\mu|)+2\pi\times\int_0^1 d\mu_1 J^l_{\rm el}(\mu_0\to\mu_1)J^l_{\rm el}(\mu_1\to|\mu|)\Big]. \quad (70)$$

Elementary estimates show that the contribution of fan diagrams does not exceed in this case the value $\max(\Delta\theta_l/\Delta\theta_S,\Delta\theta_l)\ll 1$.

7.2. The case $\Delta\theta_S \ll \Delta\theta_l$

In this limiting case the contribution of the ladder diagrams to the angle spectrum of the backscattering in the inelastic channel is determined by Eq. (70). The contribution of the fan diagrams, however, is substantially altered.

In fact, when integrating the second term of (59) it is now necessary to recognize that in the μ region with nonzero differential probability $W_{\rm col}(\omega,\mu_0\to\mu)$ the angle spectrum $J^f_{\rm el}$ due the weak localization, is by virtue of the condition $\Delta\theta_S \ll \Delta\theta_l$ a slowly varying function and should be moved outside the integral sign, so that the corresponding correction is

$$\delta J_{i\text{-}e}(\mu_0\to|\mu|,\omega)=\frac{2}{n\sigma_{\rm tot}v}J^f_{\rm el}(\mu_0\to|\mu|)W^{\rm int}_{\rm col}(\omega).$$

A similar contribution is made to the spectrum by a correction stemming from the third term of (59). The contribution of the fourth term does not exceed the fraction $(kl)^{-1}$ of the contribution of the second and third terms.

Thus, the correction to the spectrum (70) under the condition $\Delta\theta_S \ll \Delta\theta_l$ is given by

$$\delta J_{\rm inel}(\mu_0\to|\mu|,\omega)=\frac{4W^{\rm int}_{\rm col}(\omega)}{n\sigma_{\rm tot}v}J^f_{\rm el}(\mu_0\to|\mu|). \quad (71)$$

8. DISCUSSION OF RESULTS

The backscattering angle spectrum in the inelastic channel, for an isotropic inelastic collision in the substance, is shown in Fig. 6 [Eq. (67)]. The calculations show that the contribution $J_{e\text{-}i\text{-}e}$ to the angle spectrum of scattering processes in which the inelastic collision occurs in an interval between elastic incoherent scattering events is of the same order as the contributions $J_{i\text{-}e}$ and $J_{e\text{-}i}$ of the scattering processes that start or end with an inelastic collision. A characteristic feature of the backscattering in this case is that it is preserved even when the cross section $\sigma_{\rm el}$ of elastic scattering by a single scattering center is zero. The reason, as can be readily seen, is that even in the absence of backscattering of the particle by scattering centers the back-

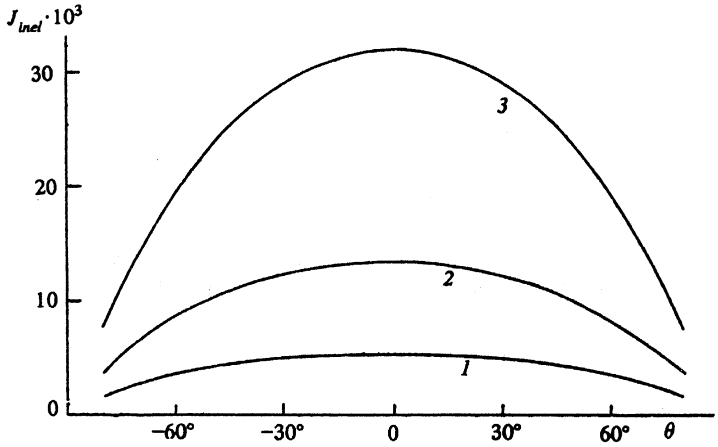

FIG. 7. Angle spectrum of backscattering in an inelastic channel. Case of small-angle inelastic collision, $\Delta\theta_S \gg \Delta\theta_l$. *1*—$\omega_0=0.25$, *2*—$\omega_0=0.50$, *3*—$\omega_0=0.75$; $W_{\rm col}^{\rm int}(\omega)/nv\sigma_{\rm tot}=0.1$; $\mu_0=1$.

scattering of the particles is preserved on account of scattering by the inelastic-interaction potential of the type considered.

The angle spectrum for the case of small-angle inelastic collision at $\Delta\theta_S \gg \Delta\theta_l$ is shown in Fig. 7 [Eq. (70)]. In this case the contribution J_{e-i-e} of scattering processes in which the inelastic collision takes place between elastic incoherent scattering events amounts to not more than 20% of the contribution of processes following the $i\rightarrow e$ and $e\rightarrow i$ schemes. In contrast to isotropic inelastic collision, backscattering vanishes completely at $\sigma_{\rm el}=0$.

Greatest interest attaches, of course, to small-angle inelastic scattering in which the characteristic scattering angle $\Delta\theta_S$ is much smaller than the angle region $\Delta\theta_l$ in which classical weak localization appears. Under these conditions the general picture of the angle spectrum remains the same as in the case of the small-angle inelastic collision at $\Delta\theta_S \gg \Delta\theta_l$, except for a narrow angle region close to the "strictly backward" scattering direction [Eqs. (70) and (71)]. The gain

$$\eta = 1 + \delta J_{\rm inel}(\mu_0 \rightarrow |\mu|,\omega)/J_{\rm inel}(\mu_0 \rightarrow |\mu|,\omega)$$

of backscattering is shown for this case in Fig. 8.

Let us examine the scattering picture of this case in greater detail. It is known that the drastic enhancement of the backscattering in the elastic channel is due to interference of waves that pass through identical inhomogeneities in the forward and backward directions.[5,20] The situation differs substantially for backscattering of particles subjected to inelastic collisions in a randomly inhomogeneous medium. An inelastic collision disrupts the coherence of the waves, which pass in the absence of this collision through one and the same inhomogeneity. Obviously, the coherence of the waves considered is restored in the case of extreme small-angle scattering as $\mathbf{q}\rightarrow 0$ ($\mathbf{q}$ is the wave vector lost in the inelastic collision).

A criterion of the violation of the coherence of the interfering waves is the relation between the angle $\Delta\theta_S$ of the inelastic scattering and the region of angles $\Delta\theta_l$ where weak localization occurs. The wave-vector transfer in small-angle inelastic scattering can be estimated at $q \approx k\Delta\theta_S$. Weak localization in the inelastic channel is preserved here if

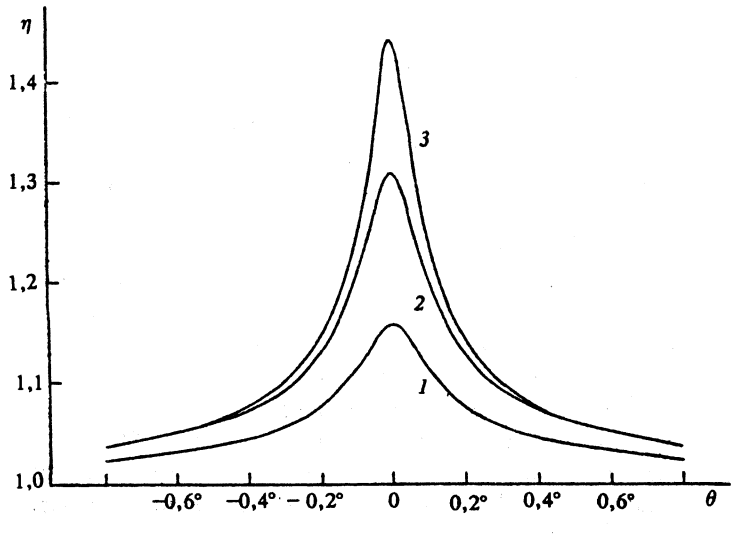

FIG. 8. Gain η of backscattering in an inelastic channel. Case of small-angle inelastic collision, $\Delta\theta_S \ll \Delta\theta_l$. *1*—$\omega_0=0.25$, *2*—$\omega_0=0.50$, *3*—$\omega_0=0.75$; $kl=800$; $\mu_0=1$.

$$q \approx k\Delta\theta_S \ll k\Delta\theta_l \approx l^{-1}. \quad (72)$$

Thus, the condition $ql \ll 1$ is the main criterion of the onset of classically weak localization in an inelastic scattering channel.

Strictly speaking, for inelastic scattering in a semi-infinite medium one must take inelastic polarization processes into account.[39] It can be shown that allowance for them amounts only to a redefinition of the single-scattering albedo, which should be taken to mean

$$\omega_0 = \sigma_{el}[\sigma_{tot} + N^{-1}\int d\omega \sigma_{col}(\omega)]^{-1},$$

where $\sigma_{col}(\omega)$ is the cross section, integrated over the angles and differentiated with respect to the lost energy, of the selected inelastic collision in the bulk of the substance [Eq. (60)], and N is the total number of scattering centers. The particle mean free path l of the particle in the substance should be redefined similarly.

In conclusion, we call attention once more that the analytic theory above was developed for a three-dimensional randomly inhomogeneous medium[2)] in an approximation in which elastic and inelastic collisions are regarded as fully consecutive. Generally speaking, such a picture may not be obtained.[28,40] This more complicated case is beyond the scope of the present paper.

[1)]Strictly speaking, the squared scattering amplitude should be preceded by a factor k_n/k (Ref. 32, §144), which will be neglected throughout under the assumption that $\hbar\omega \ll E$.

[2)]Generalization to the case of two-dimensional disordered media entails no difficulty.

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Translated by J. G. Adashko