Resonant interaction of electromagnetic beams with surface waves at an interface between linear and nonlinear media: excitation and scattering

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We consider effects resulting from the interaction of volume and surface waves at the interface between linear and nonlinear media. We investigate the resonant interaction of weakly nonlinear p-polarized surface waves due to the diffraction of intersecting coherent light beams by the self-consistent grating produced when those beams interfere in the nonlinear medium. Expressions are derived for the amplitudes of surface waves leaving the interaction region. We study various beam interaction regimes in the nonlinear medium, and determine optimal surface-mode excitation conditions both inside and outside the beam intersection region. We propose a theory to account for scattering of the incident bulk waves at the surfaces, and show that for a strong enough incident field, the incident beam pumps energy into the scattered beam and the surface wave and causes instability. Back-scattering occurs at certain definite angles of incidence. The characteristics of the scattered field are calculated.

INTRODUCTION

A great deal of effort has been devoted of late to studies of how electromagnetic radiation penetrates and is reflected by nonlinear media with a refractive index that depends on the square of the electric field amplitude (see, e.g., Refs. 1–4). Of particular interest are the linear and nonlinear excitation of surface waves by electromagnetic beams.[5–10] This type of problem has gained prominence through its relationship to channeling of strong electromagnetic waves,[11,12] the creation of new nonlinear materials,[13,14] optical machining of planar microelectronic surface structures,[15,16] and the burgeoning field of polariton spectroscopy of condensed media.[17,18]

We consider two related effects in the present paper: the resonant excitation of weakly nonlinear p-polarized surface waves by intersecting electromagnetic beams, and the scattering of radiation by a surface wave with the same frequency. The physical basis is the same for both phenomena—incident waves are scattered by variations in the refractive index of the nonlinear medium that are produced by the interference of coherent wave fields. Relatively recent studies of the kinetics of fast processes in solids, liquids, and gases[19–23] have shown that if the electromagnetic beams overlap in a nonlinear medium, a self-consistent inhomogeneity is produced in the latter—a light-induced diffraction grating, which can scatter both the original radiation and a probe wave. One would expect that surface wave excitation ought to be possible[1)] if an appropriate grating is produced near the interface between two media, just as in the case of diffraction gratings that are artificially deposited on a surface.[24] Likewise, the diffraction grating produced by the interference of surface and bulk waves leads to scattering of the latter.[2)] The development of resonant processes involving surface waves entails a significant rise in the local electromagnetic fields near the interface,[28] with a consequent nonlinear change in the phase relationships among the interacting waves. This can in turn become the dominant field-constraining mechanism in the interaction region, and can appreciably affect the excitation efficiency of the surface waveguide and the characteristics of the scattered radiation. An appropriate theory therefore requires a self-consistent approach, which we now describe.

1. STATEMENT OF THE PROBLEM. BASIC EQUATIONS

Let the half-space $z<0$ be a vacuum, with dielectric constant $\varepsilon_0=1$, and let the half-space $z>0$ be occupied by a nonlinear medium with dielectric constant ε_1. Setting aside the specific mechanism producing the nonlinearity, we assume in the theory below that the dielectric constant of the nonlinear medium is proportional to the square of the amplitude of the electric field,

$$\varepsilon_1=\varepsilon_{10}+\alpha_0|\mathbf{E}|^2, \tag{1}$$

where α_0 is a nonlinear coefficient of the medium that can be either positive (focusing medium) or negative (defocusing medium), and ε_{10} is the unperturbed value of the dielectric constant. In considering the excitation of surface electromagnetic waves (SEW), we will assume that two coherent p-polarized beams (B_y, E_x, E_z; $\mathbf{B}$ is the magnetic field, $\mathbf{E}$ the electric field) are incident from the vacuum side and overlap somewhere on the interface. The magnetic field distribution at $z=0$ due to the beams can be written in the form[3)]

$$B_{y1}=F_1(x\cos\theta_1/a_{\perp 1})\exp(-i\omega t+ik_0\gamma_1 x)+\text{c.c.}, \tag{2a}$$

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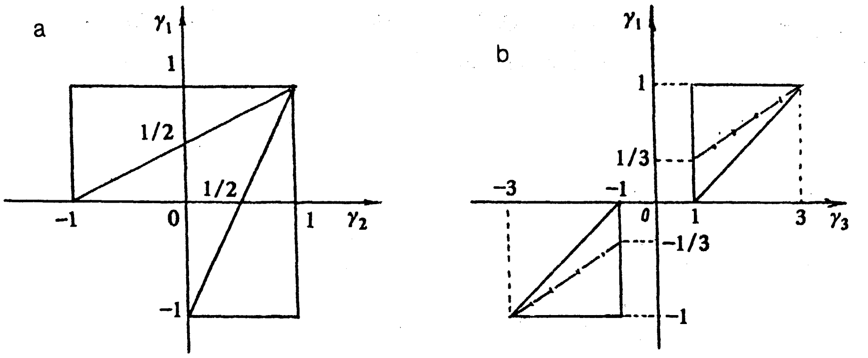

FIG. 1. (a) Domain for excitation of surface electromagnetic waves by volume waves; (b) domain for scattering of volume waves by surface electromagnetic waves. Dash-dot lines show locus of backscattering.

$$B_{y2}=F_2(x\cos\theta_2/a_{\perp 2})\exp(-i\omega t+ik_0\gamma_2 x)+\text{c.c.},\tag{2b}$$

where ω is the frequency of the electromagnetic field, $k_0=\omega/c$ is the wave number in vacuum, c is the speed of light, $F_{1,2}(\zeta)$ specifies the transverse structure of the incident fields, $a_{\perp 1,2}$ is the characteristic transverse beamwidth,[4)] $\theta_{1,2}$ is the angle of incidence at the interface, and $\gamma_{1,2}=\sin\theta_{1,2}$. If we are concerned with the scattering of incident radiation by an SEW, we will need the expression corresponding to (2b) for the SEW magnetic field ($z=0$),

$$B_{y3}=F_3(x)\exp(-i\omega t+ik_0\gamma_3 x)+\text{c.c.},\tag{3}$$

in which $\gamma_3>1$, since the phase velocity of the SEW is less than the speed of light in vacuum, and $F_3(x)$ is the self-consistent field distribution of the SEW. With $F_3=\text{const}$, we will treat the electromagnetic field described by (2) and (3) as a zeroth approximation, since it is the solution to the linear problem.[5)] We can thus neglect self-interaction effects in the pump field, such as self-focusing and bistability[29,30] (for a treatment of SEW excitation in the presence of bistability by an electromagnetic beam in thin plasma films, see Ref. 8). For this to be the case, it is at least necessary that perturbations of the dielectric constant in the nonlinear medium be relatively small ($\delta\varepsilon^{nl}=\alpha_0|\mathbf{E}|^2\ll|\varepsilon|$), which we will assume below. Interference between the pump waves then produces a grating in the nonlinear dielectric constant with corresponding wave numbers ($\gamma_1-\gamma_2,\gamma_2-\gamma_1$) (if the second pump wave is a surface wave γ_2 must be replaced by γ_3), giving rise to a scattered field with wave numbers ($\gamma_1,2\gamma_1-\gamma_2,\gamma_2,2\gamma_2-\gamma_1$). The scattered field can have a phase velocity greater than the speed of light, and can be radiated into the vacuum (if the corresponding wave number is less than unity), or it can be a retarded wave, producing an evanescent near field at the interface. In the SEW excitation problem, we will be interested in the situation in which a slow spatial harmonic is in resonance with the surface wave; in the SEW scattering problem, we will study the structure of the radiated field. For clarity, we show in Fig. 1 the regions in which SEW excitation (Fig. 1a) and bulk scattering (Fig. 1b) can take place, and we have included in the latter the line on which backscattering is possible.

Adopting the foregoing assumptions and approximations, we will use perturbation theory (in the nonlinearity parameter α_0, which is assumed to be small) to construct the theory; this approach is well-developed for resonantly interacting waves, in particular (see, e.g., Ref. 31). We point out that by virtue of the resonant nature of the interaction, first-order perturbation theory suffices when surface electromagnetic waves are excited by incident beams, while SEW scattering requires a second-order analysis.

Our starting point is Maxwell's equations for an inhomogeneous, nonlinear medium:

$$\text{rot}\,\mathbf{E}=-\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},\quad \text{rot}\,\mathbf{B}=\frac{1}{c}\frac{\partial}{\partial t}\varepsilon(|\mathbf{E}|^2,z)\mathbf{E}.\tag{4}$$

The electromagnetic field components (E_x,E_z,B_y) in (4) correspond to p-polarized waves.

We can now proceed with the solution of the problems at hand.

2. EXCITATION OF SURFACE ELECTROMAGNETIC WAVES BY INTERSECTING ELECTROMAGNETIC BEAMS

We assume that the beams are wide enough that their interaction with the surface looks locally like the interaction of a plane wave of appropriate amplitude. Since the grating period in the nonlinear dielectric constant is dictated by interference between the incident beams, this approach will be justified if the characteristic width (L_F) of the beam intersection region is much greater than the period of the interference pattern. We thus require that

$$k_0L_F\gg(\gamma_1-\gamma_2)^{-1}.$$

We represent the electric and magnetic fields in the nonlinear medium in the form

$$B_y=\frac{1}{\alpha_0^{1/2}}\sum_{p=1}^{3}b_p(t,x)\exp(-k_0\varkappa_p z+i\gamma_p k_0 x-i\omega t)+\mathscr{B}_y(t,x,z),\tag{5a}$$

$$E_x=\frac{i}{\alpha_0^{1/2}\varepsilon_{10}}\sum_{p=1}^{3}\varkappa_p b_p(t,x)\exp(-k_0\varkappa_p z+i\gamma_p k_0 x-i\omega t)+\mathscr{E}_x(t,x,z),\tag{5b}$$

$$E_z=-\frac{1}{\alpha_0^{1/2}\varepsilon_{10}}\sum_{p=1}^{3}\gamma_p b_p(t,x)\exp(-k_0\varkappa_p z+i\gamma_p k_0 x$$
$$-i\omega t)+\mathscr{E}_z(t,x,z), \tag{5c}$$

where the $b_p(t,x)$ are dimensionless, slowly varying magnetic field amplitudes of the spatial harmonics [which depend only on x in the case of stationary incident beams $b_{1,2}$; see Eqs. (2a,b)], and $\varkappa_p=(\gamma_p^2-\varepsilon_{10})^{1/2}$. Terms in the sums in (5) with $p=3$ correspond to a resonantly excited internal surface wave synchronized with the incident fields in both space and time. To be definite, let

$$\gamma_3=2\gamma_1-\gamma_2. \tag{6}$$

In turn, γ_3 satisfies a linear dispersion relation for SEW at the interface,

$$\gamma_3^2=\frac{\mathrm{Re}\,\varepsilon_{10}(\omega)}{\mathrm{Re}\,\varepsilon_{10}(\omega)+1}. \tag{7}$$

The existence criterion for weakly damped SEW is $\mathrm{Re}\,\varepsilon_{10}<-1$. The corrections $\mathscr{B}_y$, $\mathscr{E}_{x,z}$, and the surface-wave amplitude b_3 in (5) can be found via first-order perturbation theory in the nonlinearity parameter (as the latter relates to the fields in the zeroth approximation).

Substituting the expressions in (5) into the basic equations (4) and equating terms with the same γ and the equal powers of the perturbation parameter, thereby satisfying the continuity conditions on E_x and B_y at the interface, and likewise making use of a well-known method based on Fredholm's alternative theorem[31] to eliminate corrections in (5) that grow in space and time, a series of lengthy calculations yields an equation describing SEW generation in the field of two intersecting electromagnetic beams:

$$\frac{1}{v_g}\frac{\partial b_3}{\partial t}+\frac{\partial b_3}{\partial x}=iF_0(t,x)e^{-i\Omega t+iqx}+ip_0^{nl}(t,x)b_3$$
$$-\delta b_3+iA_0\beta_{333}|b_3|^2b_3, \tag{8}$$

where Ω and q are linear phase mismatches from resonance; the adopted normalization of time and space coordinates is $\bar{t}=\omega t$, $\bar{x}=(\omega/c)x$ (overbars have been omitted);

$$F_0=A_0\beta_{112}b_1^2b_2^*,\quad p_0^{nl}=A_0(\beta_{113}|b_1|^2+\beta_{223}|b_2|^2),$$

$$\delta=1/L_s\approx\mathrm{Im}\,\varepsilon_{10}/2[\mathrm{Re}\,\varepsilon_{10}(\mathrm{Re}\,\varepsilon_{10}+1)^3]^{1/2},$$

$$\mathrm{Im}\,\varepsilon_{10}\ll\mathrm{Re}\,\varepsilon_{10},$$

δ is a phenomenological damping constant introduced into Eq. (8) (the reciprocal of the surface wave's mean free path), and is due to weak absorption in the nonlinear medium; $v_g=\omega^{-1}\,\partial\omega/\partial\gamma$ is the SEW group velocity, normalized to the speed of light; and

$$A_0=\varkappa_3/\gamma_3\varepsilon_{10}^3(1-\varepsilon_{10}^2);\quad \beta_{333}=(\gamma_3^2+\varkappa_3^2)^2/4\varkappa_3;$$

$$\beta_{112}=(\gamma_1\gamma_2+\varkappa_1\varkappa_2)(\gamma_1\gamma_3+\varkappa_1\varkappa_3)/(2\varkappa_1+\varkappa_2+\varkappa_3);$$

$$\beta_{113}=[2(\gamma_1\gamma_3+\varkappa_1\varkappa_3)^2+(\gamma_1\varkappa_3-\gamma_3\varkappa_1)^2]/2(\varkappa_1+\varkappa_3);$$

$$\beta_{223}=[2(\gamma_2\gamma_3+\varkappa_2\varkappa_3)^2+(\gamma_2\varkappa_3-\gamma_3\varkappa_2)^2]/2(\varkappa_2+\varkappa_3).$$

The parameters in corresponding coefficients are related by (6) and (7), and are assumed to be real. The space-time distribution of the incident field at the interface is spelled out by the functions $b_1(t,x)$ and $b_2(t,x)$, which are assumed to be given. By redefining the SEW amplitude to be $b_3=b_3'\exp(-i\Omega t+iqx)$, we can eliminate the exponential factor in (8); the linear and nonlinear phase mismatches are then represented in the same way, and Eq. (8) takes the form (omitting the prime on b_3)

$$\frac{1}{v_g}\frac{\partial b_3}{\partial t}+\frac{\partial b_3}{\partial x}=+iF_0(t,x)+i(p_0^l+p_0^{nl})b_3-\delta b_3$$
$$+iA_0\beta_{333}|b_3|^2b_3, \tag{9}$$

where $p_0^l=(\Omega/v_g-q)$. For Eq. (9) to hold, perturbation theory requires that the linear and nonlinear phase mismatches (p_0^{nl} and p_0^l) and the linear SEW damping factor all be small. In dimensionless form,

$$p_0^l,\quad p_0^{nl},\quad \delta,\quad A_0\beta_{333}|b_3|^2\ll 1.$$

We point out that these coefficients bear no particular size relationship to one another.

Now consider the stationary ($\partial/\partial t=0$) excitation of surface electromagnetic waves by intersecting beams, and above all, by a spatially uniform pump, $b_{1,2}=\mathrm{const}$. We can find the appropriate spatially uniform solution for the surface wave amplitude from a cubic equation in $|b_3|^2$,

$$|b_3|^2=\frac{|F_0|^2}{\delta^2+(p_\Sigma+A_0\beta_{333}|b_3|^2)^2}, \tag{10}$$

where $p_\Sigma=p_0^l+p_0^{nl}$. Clearly, when $p_0^l>-(\sqrt{3}\,\delta+p_0^{nl})$, Eq. (10) has a single real root. When the inequality has the opposite sense, there is a range of amplitudes for which Eq. (10) has three real roots, giving rise to optical bistability in SEW generation (it can easily be shown that two roots are stable and the third is not). With a uniform pump, however, there are also oscillatory regimes produced by beats associated with the loss of synchronism between the SEW and the incident radiation; the latter results from both linear and nonlinear phase mismatches.

For simplicity, we first consider the lossless case ($\delta=0$). Let the SEW field be $b_3=\mathscr{A}\exp i\Phi$. The amplitude and phase are then

$$\frac{d\mathscr{A}}{dx}=F_0\sin\Phi,$$

$$\mathscr{A}\frac{d\Phi}{dx}=F_0\cos\Phi+(p_\Sigma+A_0\beta_{333}\mathscr{A}^2)\mathscr{A},$$

where the functions $b_{1,2}$ can be assumed to be real with no loss of generality. Figure 2 shows phase-plane portraits in the variables $\mathscr{A}$ and Φ for various values of the linear mismatch p_0^l. The solutions plotted in Fig. 2 depict the periodic variation of SEW amplitude along the surface of the nonlinear medium; the period depends on the pump-wave amplitudes. This sort of surface-wave field behavior suggests that there is a distinct nonlinear scale length (corresponding to the period), which in turn gives rise to op-

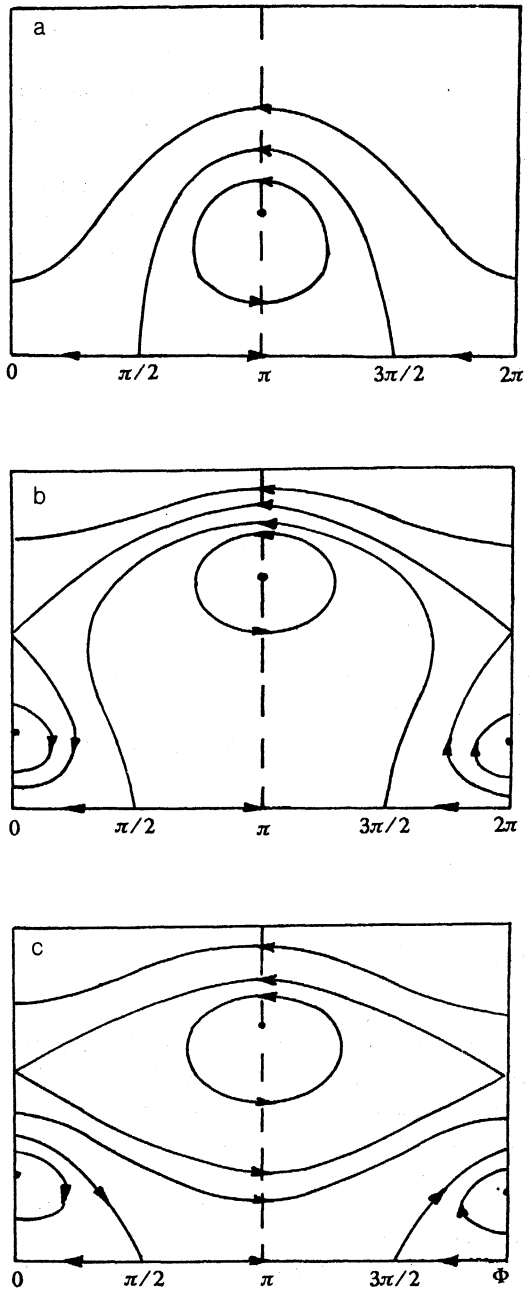

FIG. 2. Amplitude vs phase for a surface electromagnetic wave excited by two uniform incident plane waves. a) $p_\Sigma > p_{c1}$; b) $p_{c2} < p_\Sigma < p_{c1}$; c) $p_\Sigma < p_{c2}$, where $p_{c1} = -[3^{3/2}|A_0F_0|\beta_{333}/2]^{2/3}$, $p_{c2} = 2^{1/3}p_{c1}$.

timum scale sizes for the beam intersection and focusing regions that produce SEW of maximum amplitude.

To determine the corresponding scale length and calculate the amplitude of the SEW that is excited, we linearize Eq. (9) by neglecting the term proportional to $\sim|b_3|^2$, which takes SEW self-interaction effects into account:

$$\frac{db_3}{dx} = iF_0(x) + ip_\Sigma b_3 - \delta b_3. \qquad (11)$$

Equation (11) admits of an analytic solution; noting that $b_3 \to 0$ when $x \to -\infty$, we can write the solution in the form

$$b_3(x) = -i\exp\left[-\delta x - i\int_{-\infty}^{x} p_\Sigma(\xi)d\xi\right] \times \int_{-\infty}^{x} F_0(\xi)\exp\left[\delta\xi + i\int_{-\infty}^{\xi} p_\Sigma(\zeta)d\zeta\right]d\xi. \qquad (12)$$

For purposes of estimation, we start by assuming that the fields in the beams have a rectangular transverse distribution, i.e.,

$$F_0(x) = \begin{cases} F_m, & 0 < x < L_F, \\ 0, & x < 0; \quad x > L_F. \end{cases}$$

From (12), then, the expression for the squared SEW amplitude in the beam intersection region becomes

$$|b_3|^2 = \frac{|F_m|^2}{p_\Sigma^2 + \delta^2}\,[1 + e^{-2\delta x} - 2e^{-\delta x}\cos(p_\Sigma x)]. \qquad (13)$$

The amplitude of the SEW exiting the beam intersection region can be found from (13), letting $x = L_F$.

We now analyze Eq. (13) and determine the optimal conditions for SEW excitation both inside and outside the interaction region. Different wave-generation conditions prevail, depending on the ratio between p_Σ and δ. From Eq. (13), it can be seen that the most favorable regime corresponds to $p_\Sigma = 0$, i.e., cancellation of the linear and nonlinear phase mismatches. The solution (13) can then be rewritten:

$$|b_3|^2 = \frac{|F_m|^2}{\delta^2}\,(1 - e^{-\delta x})^2. \qquad (14)$$

Equation (14) describes an SEW that attains its maximum amplitude at the boundary $x = L_F$ of the beam intersection region. The optimum size of that region is then a scale length significantly greater than the SEW mean free path, $L_F \gg L_s$. This condition simultaneously ensures that the surface waveguide is excited at maximum efficiency.

Consider now the situation in which $|p_\Sigma| \gg \delta$, which can only come about if the linear mismatch is large enough, $(|p_0^l| \gg \delta)$, or the pump fields are strong enough, $(p_0^{nl} \gg \delta)$. The SEW attains its maximum amplitude, in general, inside the intersection region at a point $x_m \approx \pi/p_\Sigma$ (if $x_m < L_F$, of course); that amplitude is

$$|b_3|^2_{\max} \approx 4|F_m|^2/p_\Sigma^2.$$

An SEW leaving the interaction region will have its maximum amplitude when $x_m = L_F$, which then sets the corresponding optimum scale length. Note that the SEW amplitude varies quasiperiodically, with period $\Delta x \approx 2\pi/p_\Sigma$, depending on the width of the intersection region. The behavior of the emerging SEW as a function of δL_F is illustrated in Fig. 3.

Analysis of the solution of Eq. (12) for beams with a smooth transverse intensity distribution is much more complicated than for beams with a rectangular profile. But if the function $F_0(x)$ is smooth on a scale $1/\delta$, it can be

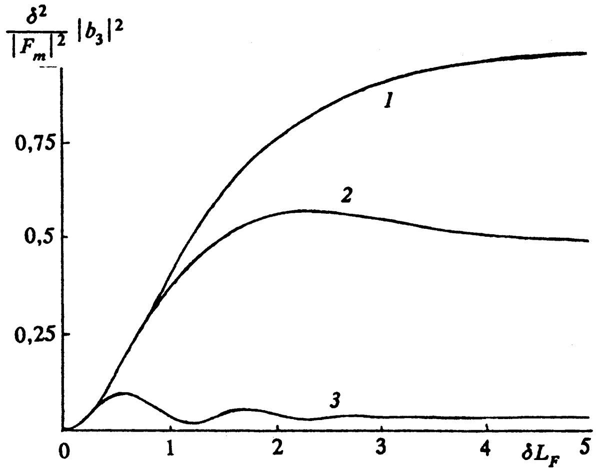

FIG. 3. Squared amplitude of a surface electromagnetic wave emerging from beam intersection region, as a function of δL_F: *1*—$p_\Sigma/\delta=0$; *2*—$p_\Sigma/\delta=1$; *3*—$p_\Sigma/\delta=5$. The squared amplitude has been normalized to its maximum possible value, $|b_3|^2_{\max}=|F_m|^2/\delta^2$.

taken outside the integral in (12) and assigned its value at the upper limit; an approximate expression for the SEW amplitude is then

$$b_3(x)\approx -iF_0(x)\int_{-\infty}^{x}\exp\left[\delta(\xi-x)+i\int_x^{\xi}p_\Sigma(\zeta)d\zeta\right]d\xi. \quad (15)$$

One feature of the solution (15) is that the SEW amplitude vanishes at the boundary of the beam intersection region ($b_3\to 0$ as $F_0\to 0$). Meanwhile, an SEW is excited within the interaction region, and it reaches a maximum amplitude $b_{3\,\max}\sim F_{0\,\max}/\delta$ when the linear and nonlinear phase mismatches cancel,

$$p_0^l L_F\approx -\int_{-\infty}^{\infty}p_0^{nl}(\zeta)d\zeta.$$

To ensure the most efficient possible generation of the outgoing SEW, it is necessary in this case that $F_0(x)$ have two distinct scale lengths: a width scale $L_F\gg 1/\delta$ and an outer scale $L_b\ll 1/\delta$, which can be achieved in practice by using apertures to define the beams. For high-power pump fields for which the phase mismatches do not cancel, L_F should be close to the appropriate optimal scale length calculated above for incident beams with a rectangular transverse intensity profile.

Recall that the solutions obtained so far hold when $A_0\beta_{333}|b_3|^2\ll \max\{p_\Sigma,\delta\}$. If this is not in fact the case, it will not be possible to obtain a stationary solution of Eq. (9), necessitating the use of numerical methods. As an example, we have plotted in Fig. 4 the calculated SEW amplitude distribution for three different linear phase offsets, one of which (curve *3*) corresponds approximately to cancellation of the nonlinear phase.

Similar considerations apply to the excitation of surface electromagnetic waves by electromagnetic pulses in the spatially homogeneous problem [$\partial/\partial x=0$ in (9)] when they are long enough that the "source" F_0 in Eq. (9) can be assumed to depend solely on time, i.e., $F_0=F_0(t)$. The above conclusions will then hold if we make the substitution $x\to v_g t$ in the appropriate solutions.

The results derived above still hold for more complicated interface structures, which for instance might contain a thin transition layer.[6] The only modifications are in the quantitative relationships and are due to changes in the

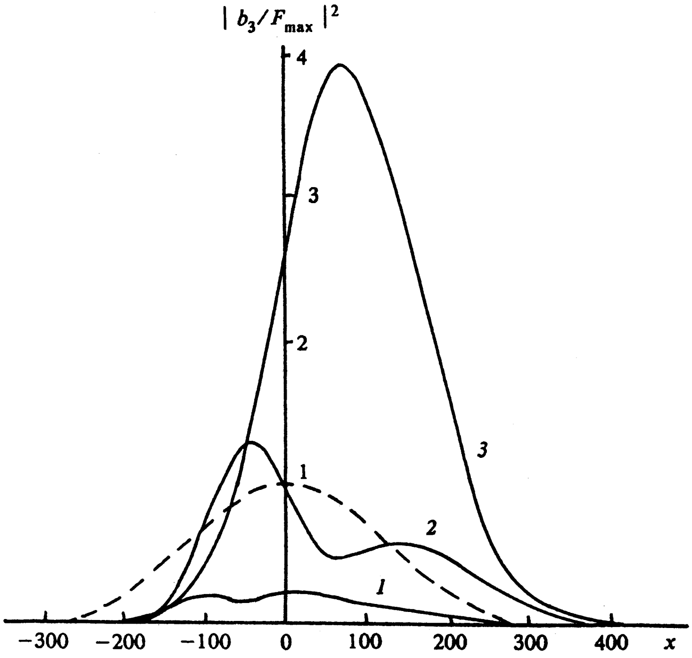

FIG. 4. Spatial profile of the amplitude of a surface electromagnetic wave generated by intersecting beams: *1*) $1-p_0^l=0$; *2*) $2-p_0^l=-0.03$; *3*) $3-p_0^l=-0.06$. The dashed curve shows the function $|F_0(x)|^2$ (normalized to $|F_0|^2_{\max}$). $L_F\approx 600$, $\delta=0.01$ ($\delta\cdot L_F\approx 6$), $b_{1\max}=b_{2\max}=0.1$, $\gamma_1=0.8$, $\gamma_2=-0.4$ ($\gamma_3=2$, $\varepsilon_{10}\approx -1.33$).

constant coefficients in Eq. (9). In Appendix 1, we present expressions for the appropriate coefficients in several "complex" interface examples. Note that the existence of an interface transition layer will occasionally lead to a nonmonotonic SEW dispersion relation, with two types of surface waves—forward and backward (having oppositely directed phase and group velocities)—at each frequency.[32] Under those circumstances, double the number of SEW can be excited, depending on how the beams' angles of incidence vary.

3. BEAM SCATTERING BY SURFACE ELECTROMAGNETIC WAVES

When we consider the scattering of electromagnetic volume waves incident upon the surface of a nonlinear medium, the surface electromagnetic waves are basically governed, as before, by Maxwell's equations (4). Let the electromagnetic field be given in the form (5), where the pump waves consist of an incident volume wave ($p=1$) and an internal surface wave ($p=3$). When these interact they produce electromagnetic fields with wave numbers $\gamma_4=2\gamma_3-\gamma_1$ and $\gamma_2=2\gamma_1-\gamma_3$, corresponding to an external evanescent wave (γ_4) and a radiated (for $|\gamma_2|<1$) scattered field. It is the structure of the latter in which we are interested.

First-order perturbation theory in the nonlinearity enables us to derive expressions for the scattered and near fields, and likewise for the corrections to the specularly reflected field and the propagation constant of the SEW. As before, omitting details of the lengthy derivation, we can immediately write out the expressions for the dimensionless magnetic field amplitudes [see Eq. (5)] at the surface,

$$b_2=A_2b_1^2b_3^*, \tag{16}$$

$$b_4=A_4b_3^2b_1^*. \tag{17}$$

The nonlinear correction to the magnetic field amplitude for the specularly reflected wave takes the form

$$b_1'=A_{11}|b_1|^2b_1+A_{13}\cdot|b_3|^2b_1,$$

where

$$A_2=\frac{(\varkappa_1\varkappa_3+\gamma_1\gamma_3)(\gamma_1\gamma_2+\varkappa_1\varkappa_2)}{\varepsilon_{10}^4(2\varkappa_1+\varkappa_3+\varkappa_2)(i\chi_2+\varkappa_2/\varepsilon_{10})},$$

$$A_4=\frac{(\varkappa_1\varkappa_3+\gamma_1\gamma_3)(\gamma_3\gamma_4+\varkappa_3\varkappa_4)}{\varepsilon_{10}^4(2\varkappa_3+\varkappa_1+\varkappa_4)(\chi_4+\varkappa_4/\varepsilon_{10})},$$

$$A_{11}=\frac{(\varkappa_1^2+\gamma_1^2)^2}{4\varepsilon_{10}^4\varkappa_1(-i\chi_1+\varkappa_1/\varepsilon_{10})},$$

$$A_{13}=\frac{(\varkappa_3^2+\gamma_3^2)(\varkappa_1^2+\gamma_1^2)+(\varkappa_1\varkappa_3+\gamma_1\gamma_3)^2}{2\varepsilon_{10}^4(\varkappa_1+\varkappa_3)(-i\chi_1+\varkappa_1/\varepsilon_{10})},$$

$$\chi_{1,2}=(1-\gamma_{1,2}^2)^{1/2},\quad \chi_{3,4}=(\gamma_{3,4}^2-1)^{1/2}.$$

The variation of the SEW amplitude, allowing for energy exchange between the interacting waves, is given by second-order perturbation theory:

$$\frac{1}{v_g}\frac{\partial|b_3|}{\partial t}+\frac{\partial|b_3|}{\partial x}=A_3|b_1|^4|b_3|-\delta|b_3|, \tag{18}$$

where

$$A_3=\frac{\chi_2(\gamma_1\gamma_2+\varkappa_1\varkappa_2)^2(\gamma_1\gamma_3+\varkappa_1\varkappa_3)^2}{\varepsilon_{10}^6\gamma_3(1/\chi_3+1/\varepsilon_{10}\varkappa_3)(\varkappa_2^2+\varepsilon_{10}^2\chi_2^2)(2\varkappa_1+\varkappa_2+\varkappa_3)^2}.$$

When $A_3|b_1|^4>\delta$, Eq. (18) describes convective SEW instability, which transfers energy out of the volume pump wave. It is clear from (16) that the scattered field is algebraically related to the fields of the pump waves, and it grows in concert with the SEW.

Now consider stationary scattering ($\partial/\partial t=0$) of the beam. We can write the solution of Eq. (18) in the form

$$|b_3(x)|=b_{30}\exp\left(-\delta x+\int_0^x A_3\left|b_1(\xi)\right|^4 d\xi\right), \tag{19}$$

where we set $x=0$ at the point at which the amplitude (b_{30}) of the SEW entering the interaction region has been specified. The SEW field traversing the interaction region will be amplified if

$$\int_0^d A_3|b_1|^4dx>\delta\cdot d,$$

where $d=a_{\perp 1}/(1-\gamma_1^2)^{1/2}$ is the size of the illuminated spot at the interface. Making use of (16) and (19), we obtain the structure of the scattered field at the surface of the nonlinear medium,

$$|b_2(x)|=|A_2||b_1|^2b_{30}\times\exp\left(-\delta x+\int_0^x A_3|b_1(\xi)|^4d\xi\right). \tag{20}$$

To obtain an estimate, assume as before that the field in the interface region illuminated by the incident beam has a rectangular profile,

$$b_1=\begin{cases}b_{1m}, & |x|\leqslant d,\\ 0, & |x|>d.\end{cases}$$

In the region occupied by the beam, the solution of Eq. (18) will be

$$|b_3|=b_{30}\exp[\Gamma(x+d)], \tag{21}$$

where $\Gamma=A_3|b_{1m}|^4-\delta$ is the SEW spatial growth rate. From (20), we obtain the transverse structure of the scattered field,

$$|b_2(x)|=\begin{cases}|A_2|b_{1m}^2b_{30}\exp[\Gamma(x+d)], & |x|\leqslant d\\ 0, & |x|>d\end{cases}. \tag{22}$$

Thus, if $\Gamma d\ll 1$, Eq. (22) will describe a transverse magnetic field distribution in the scattered beam that closely resembles the distribution in the incident wave. Conversely, when $\Gamma d\gg 1$, the structure of the scattered radiation differs significantly from that of the pump. The characteristic transverse scale length in the scattered beam is $1/\Gamma\ll d$, i.e., the scattered beam is narrower (with a broader spatial spectrum) than the incident beam (in the spatially uniform ($\partial/\partial x=0$) temporal problem this means that the scattered pulse is shorter than the incident one).

Physically, this results from the nonlinear interaction region being smaller than the illuminated spot width. Furthermore, its center of gravity, as given by

$$\Delta=\int_{-\infty}^{\infty} x|b_2|^2dx\Big/\int_{-\infty}^{\infty}|b_2|^2dx,$$

is displaced (we assume that $\int_{-\infty}^{\infty}x|b_1|^2dx=0$). For an assumed rectangular incident profile, the displacement Δ is

$$\Delta=d[\mathrm{cth}(2\Gamma d)-1/2\Gamma d].$$

There is no displacement at the threshold of instability ($\Gamma=0$), and when $\Gamma d\gg 1$, the quantity $\Delta\to d$ amounts to a generalization of the nonlinear Goos–Hänchen effect to scattering (see, e.g., Ref. 9). The angle between the outgoing scattering beam and the surface normal is given by

$$\sin\theta_2\equiv\gamma_2=2\gamma_1-\gamma_3. \tag{23}$$

When $\gamma_2=-\gamma_1$ (or $\gamma_1=\gamma_3/3$), [as follows from (23)], we have backscattering. Bearing in mind that $|\gamma_3|>1$ and $|\gamma_2|<1$, we obtain the range of angles of incidence over which backscattering from SEW is possible:

$$\theta_1>\theta_c=\arcsin(1/3)\approx 19.47°.$$

To the accuracy of the constant coefficients in Eqs. (16)–(18), these results are unchanged for an interface consisting of a thin transition layer. The expressions for the coefficients appropriate to a thin nonlinear film deposited on a metallic surface are given in Appendix 2.

CONCLUSION

We have described and studied the excitation of surface electromagnetic waves (SEW) by electromagnetic beams, and the scattering of such beams by SEW at the interface between linear and nonlinear media. Our results demonstrate that a significant transformation of field structure is possible when radiation interacts with such an interface, which may well prove useful for diagnostics of nonlinear media and surfaces. The resonant nature of SEW excitation by means of two interfering incident beams makes it possible to transform volume waves into surface waves much more efficiently than is the case for linear SEW excitation by grazing-incidence electromagnetic beams (see Ref. 6). The excitation of SEW therefore does not require complicated optical coupling elements like frustrated total internal reflection prisms, slit transducers, diffraction gratings, and so forth. We predict several novel effects (such as pulse shortening and scattered-beam compression) that could be usefully verified in a real experiment. The present theory enables one to take account of nonlinear relaxation and the excitation of low-frequency wave motion (sound, for example) in a nonlinear medium; these processes must typically be factored into any study of the interaction between short electromagnetic pulses and a nonlinear medium.

APPENDIX 1

We give two examples of expressions for the constant coefficients in Eq. (9) for the case of two linear media separated by a thin nonlinear transition layer of thickness h, ($k_0h\ll 1$, $k_0(|\varepsilon_s|)^{1/2}h\ll 1$, ε_s is the dielectric constant of the transition layer, $\varepsilon_s=\varepsilon_{s0}+\alpha_s|\mathbf{E}|^2$).

a) For a nonresonant transition layer that weakly affects SEW dispersion, the SEW dispersion relation is

$$\gamma_3^2=\frac{\varepsilon_{10}}{\varepsilon_{10}+1}+k_0h\frac{(\varepsilon_{s0}-1)(\varepsilon_{s0}-\varepsilon_{10})2\varepsilon_{10}^2}{\varepsilon_{s0}(1-\varepsilon_{10})(-1-\varepsilon_{10})^{5/2}}.$$

In fact, the transition layer in this case serves to produce a diffraction grating. The coefficients are

$$A_0=k_0h\cdot\frac{\varepsilon_{10}\varkappa_3\chi_3}{\gamma_3(\varepsilon_{10}\varkappa_3+\chi_3)},$$

$$\beta_{112}=\frac{\chi_3}{\varepsilon_{10}}\left(\frac{\gamma_1^2\gamma_2\varkappa_1}{\varepsilon_{s0}^2}+\frac{\varkappa_1^2\varkappa_2}{\varepsilon_{10}^2}\right)-\frac{\gamma_3}{\varepsilon_{s0}^2}\times\left(\frac{\gamma_1^2\gamma_2}{\varepsilon_{s0}^2}+\frac{\varkappa_1\varkappa_2\gamma_1}{\varepsilon_{10}}\right),$$

$$\beta_{113}=\frac{\chi_3}{\varepsilon_{10}}\left(\frac{\gamma_1\gamma_3\varkappa_1}{\varepsilon_{s0}^2}+\frac{\gamma_1^2\varkappa_3}{\varepsilon_{s0}^2}+\frac{2\varkappa_1^2\varkappa_3}{\varepsilon_{10}^2}\right)-\frac{\gamma_3}{\varepsilon_{s0}^2}\times\left(\frac{\varkappa_1\varkappa_3\gamma_1}{\varepsilon_{10}^2}+\frac{2\gamma_1^2\gamma_3}{\varepsilon_{s0}^2}+\frac{\varkappa_1^2\gamma_3}{\varepsilon_{10}^2}\right),$$

$$\beta_{223}=\frac{\chi_3}{\varepsilon_{10}}\left(\frac{\gamma_2\gamma_3\varkappa_2}{\varepsilon_{s0}^2}+\frac{\gamma_2^2\varkappa_3}{\varepsilon_{10}^2}+\frac{2\varkappa_2^2\varkappa_3}{\varepsilon_{10}^2}\right)-\frac{\gamma_3}{\varepsilon_{s0}^2}\times\left(\frac{\varkappa_2\varkappa_3\gamma_2}{\varepsilon_{10}^2}+\frac{2\gamma_2^2\gamma_3}{\varepsilon_{s0}^2}+\frac{\varkappa_2^2\gamma_3}{\varepsilon_{10}^2}\right),$$

$$\beta_{333}=\left(\frac{\chi_3\varkappa_3}{\varepsilon_{10}}-\frac{\gamma_3^2}{\varepsilon_{s0}^2}\right)\left(\frac{\gamma_3^2}{\varepsilon_{s0}^2}+\frac{\varkappa_3^2}{\varepsilon_{10}^2}\right).$$

These dimensionless coefficients effect a renormalization of the electromagnetic field [as in Eq. (5)] by the characteristic nonlinear field strength, i.e., by $(\alpha_s)^{-1/2}$. This situation is typical of experiments on nonlinear interactions that involve surface electromagnetic waves.[18] A nonlinear thin-film medium is most commonly deposited on a metallic surface, in which the mean free path of infrared SEW is quite large.

b) For a resonant nonlinear thin film on a perfect conductor ($\sigma\to\infty$), $|\varepsilon_s|\ll 1$, and $\varepsilon_s<0$. The SEW dispersion properties are largely determined by the film. The corresponding SEW dispersion relation is

$$\frac{k_0h}{\varepsilon_{s0}}+\chi_3/\gamma_3^2=0.$$

We point out that there is an opportunity here for significant enhancement of nonlinear effects (compared with the simple interface considered earlier in this paper) due to resonant growth in the film of the electric field component normal to the surface.[33,34]

The coefficients are

$$A_0=-\gamma_3(\gamma_3^2-1)/\varepsilon_{s0}(\gamma_3^2-2);\quad \beta_{112}=1;$$

$$\beta_{113}=2;\quad \beta_{223}=2;\quad \beta_{333}=1.$$

These coefficients appear in an equation of the form (9) that describes SEW generation, and which contains the normal components of the electric field in the corresponding wave within the thin film instead of the magnetic fields $b_{1,2,3}$ at the interface. Note here that excitation of both forward and backward SEW[30] are possible at two corresponding angles of incidence if the wave number of the backward mode $\gamma_3^{(\mathrm{back})} < 3$; otherwise, there will be no backward wave.

APPENDIX 2

For the case corresponding to part 2 of Appendix 1, the coefficients in Eqs. (16)–(18) take the form

$$A_2 = -i\gamma_2 k_0 h/(\varepsilon_{s0}\chi_2 + i\gamma_2^2 k_0 h);$$

$$A_3 = -\gamma_2^2 \chi_3^{3/2}/\varepsilon_{s0}^2 \gamma_3 \chi_2 (\gamma_3^2 - 2)(1 + \chi_3^2\gamma_2^2/\chi_2^2\gamma_3^2);$$

$$A_4 = -\gamma_4 k_0 h/(\varepsilon_{s0}\chi_4 - \gamma_4^2 k_0 h);$$

$$A_{11} = -i\gamma_1 k_0 h/(\varepsilon_{s0}\chi_2 + i\gamma_1^2 k_0 h);$$

$$A_{13} = 2A_{11}.$$

Here we point out the possibility, in principle, of resonant scattering when the "slow" field with wave number γ_4 is in resonance with the backward SEW. The resonant field should then have a scattered wave of much higher intensity.

1) Control of the spatial spectrum of scattered radiation induced in thin plasma films by a diffraction grating is treated in Ref. 25.
2) Inverse problems—in a certain sense—related to nonlinear radiation from thin films of plasma-like media by surface waves have been solved in Refs. 26 and 27.
3) The media are assumed to be nonmagnetic, with $\mathbf{B} = \mathbf{H}$.
4) The beams are assumed to be much broader than a wavelength.
5) The structure of the electromagnetic field in z is obtained from the corresponding linear solution. The electric field components can then be expressed in terms of the magnetic field components: $E_{zi} = -(\gamma_i/\varepsilon) B_{yi}$, $E_{\xi} = -(i/k_0\varepsilon) dB_{yi}/dz$; $i = 1, 2, 3$.
6) Calculations show that the nature of the equations depends solely on the type of nonlinearity, and not on the structure of the interface.

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