The spectrum of surface magnetic polaritons in a ferromagnetic plate

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The spectrum of the electromagnetic vibrations of a ferromagnetic plate magnetized parallel to its surfaces is discussed. The dispersion law is obtained for wave propagation along the plate and normal to both the magnetic field and magnetization directions. In addition to ordinary wave solutions, an unusual mode analogous to the Damon–Eshbach wave is predicted. For low frequencies and sufficiently large wave vectors the velocity of the new waves is shown to be much slower than the speed of light.

Low frequency vibrations in magnetic materials continue to be a subject of much interest, largely because their spectra have proven amenable to study by laser radiation scattering techniques.[1]

In particular, considerable attention has been given to the Damon–Eshbach waves,[2] which are surface magnetostatic modes with amplitudes concentrated near the surface of the magnetic. The dispersion relations of the Damon–Eshbach waves are derived by analyzing the magnetostatic equations subject to appropriate boundary conditions (the continuity of the tangential components of the magnetic field and of the normal component of the magnetic induction).

Introducing the retardation effect (i.e., recognizing that the velocity of light c is finite) naturally modifies the dispersion law of the waves being studied as well as establishing the domain of applicability of magnetostatic equations and enabling the relation between the wave frequency ω and the wave vector $\mathbf{k}$ to be obtained for those values of parameters where the magnetostatic equations are no longer valid. Note that the Damon–Eshbach wave is slow: for $kd \gg 1$, $2d$ being the plate thickness, the wave velocity is $v = d\omega/dk \ll c$. Apart from the slow waves, the introduction of retardation into analysis enables waves to be obtained whose velocities are strongly dependent on the frequency or, for a fixed frequency, on the magnetic field H (the propagation direction being along the plate). In some of these waves $v \sim c$ holds, which result seems to offer promise for practical applications.

We consider the simplest geometry possible, one in which a ferromagnetic plate of thickness $2d$ is magnetized parallel to its surfaces and a wave of wavelength $2\pi/k$ and frequency ω travels parallel to the surfaces and perpendicular to both the magnetic field $\mathbf{H}$ and the magnetization $\mathbf{M}$ ($\mathbf{M} \| \mathbf{H}$ by assumption). (The case of a normally magnetized plate has been treated by Savchenko *et al.*[3]).

The variable fields of the problem are concentrated inside and near the plate and decay exponentially away from it with a logarithmic decrement

$$\gamma_0 = \sqrt{k^2 - \omega^2/c^2}\,. \tag{1}$$

Accordingly, our interest is in waves for which $\omega \leqslant kc$.

Inside the plate, let us denote by q the component of the wave vector normal to the plane of the plate. From Maxwell's equations,

$$k^2 + q^2 = \frac{\omega^2}{c^2}\,\varepsilon\mu_{eff}(\omega)\,, \tag{2}$$

where ε is the dielectric permittivity and $\mu_{\rm eff}(\omega)$ is the effective susceptibility of the plate material; from the Landau–Lifshitz equation,[4]

$$\mu_{eff}(\omega) = \frac{(\omega_0 + \omega_M)^2 - \omega^2}{\omega_0(\omega_0 + \omega_M) - \omega^2}\,. \tag{3}$$

The characteristic frequencies are

$$\omega_0 = gH_{eff}\,, \qquad \omega_M = 4\pi g M\,, \tag{4}$$

where g is the magnetomechanical ratio; $H_{\rm eff}$ is the effective magnetic field (incorporating the anisotropy field[4]); and M is the specific magnetization. Although generally of the same order of magnitude, ω_0 and ω_M may differ appreciably from one another.

The field patterns in the plate are determined by the sign of q^2. For $q^2 > 0$ (Case A) the field variables in the plate are superpositions of the trigonometric functions[1)] $\cos qz$ and $\sin qz$. For $q^2 = -\gamma^2$ ($\gamma^2 > 0$, Case B) they are superpositions of $\cosh qz$ and $\sinh \gamma z$.

The boundary conditions of the problem (the continuity of the tangential components of the electric and magnetic fields in the wave) enable the following dispersion relations to be formulated:

Case A

$$\begin{aligned} &2[\omega_0(\omega_0 + \omega_M) - \omega^2]\,\gamma_0 q\,\mathrm{ctg}(2qd) \\ &= \frac{\omega^2}{c^2}[(\omega_0 + \omega_M)^2 + \varepsilon(\omega_0 + \omega_M)\omega_0 - (1+\varepsilon)\omega^2] \\ &\quad - k^2[\omega_0^2 + (\omega_0 + \omega_M)^2 - 2\omega^2]\,. \end{aligned} \tag{5}$$

Case B

$$\begin{aligned} &2[\omega^2 - \omega_0(\omega_0 + \omega_M)]\,\gamma_0\gamma\,\mathrm{cth}(2\gamma d) \\ &= \frac{\omega^2}{c^2}[(1+\varepsilon)\omega^2 - (\omega_0 + \omega_M)^2 - \varepsilon\omega_0(\omega_0 + \omega_M)] \\ &\quad + k^2[\omega_0^2 + (\omega_0 + \omega_M)^2 - 2\omega^2]\,. \end{aligned} \tag{6}$$

From Eqs. (1), (2), (4), and (5), the wave frequency/wave vector dependence may be determined.

We shall first consider Case A. The multivalued nature of the cotangent gives rise to an infinite number of spectral branches, and it is easy to show that in Case A the branches $\omega = \omega_n(k)$ are all located outside the frequency range $(\sqrt{\omega_0(\omega_0 + \omega_M)},\ \omega_0 + \omega_M/2)$ and have their starting

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points on the straight line $\omega = ck$. As $k \to 0$, the frequency of the lowest branch $\omega = \omega_1(k)$ vanishes

$$\omega \approx ck\left(1 - \frac{1}{2}\left(\varepsilon - \frac{\omega_0}{\omega_0 + \omega_M}\right)^2 k^2 d^2\right), \quad kd \ll 1. \quad (7)$$

As $k \to \infty$, the frequency of this branch—indeed of *all* the branches which lie below $\sqrt{\omega_0(\omega_0 + \omega_M)}$ (and are infinite in number)—tends to the limiting value $\omega_{\rm lim} = \sqrt{\omega_0(\omega_0 + \omega_M)}$ as

$$\omega^2 \approx \omega_{\rm lim}^2\left(1 - \varepsilon\frac{\omega_M}{\omega_0}\frac{\omega_{\rm lim}^2}{c^2(k^2 + \pi^2 n^2/4d^2)}\right). \quad (8)$$

For $\omega > \omega_0 + \omega_M/2$, the branches are asymptotic to the straight line $\omega = ck/\sqrt{\varepsilon}$:

$$\omega \approx \frac{ck}{\sqrt{\varepsilon}}\left(1 + \frac{\pi^2 n^2}{4d^2 k^2}\right), \quad n = 1, 2, 3, ...; kd \gg 1. \quad (9)$$

For $n > 1$, the starting points of all the branches $\omega = \omega_n^A(k)$ are found from the condition that γ vanish [cf. Eq. (1)], which implies that cotangent must go to infinity and hence $q_n = \pi(n-1)/2d$. Equations (2) and (3) then yield

$$\frac{\omega_n^2}{c^2}\left(\varepsilon\frac{(\omega_0 + \omega_M)^2 - \omega_n^2}{\omega_0(\omega_0 + \omega_M) - \omega_n^2} - 1\right) = q_n^2. \quad (10)$$

The biquadratic equations (10) have roots $\omega_n^{(-)}$ located below $\omega_{\rm lim} = \sqrt{\omega_0(\omega_0 + \omega_M)}$ and giving rise to branches which crowd together at this frequency [see Eq. (8)]. The roots $\omega_n^{(+)}$ above the frequency $\omega_0 + \frac{1}{2}\omega_M$ give rise to the branches asymptotically approaching the straight line $\omega = ck/\sqrt{\varepsilon}$ [see Eq. (9)]. For $\varepsilon - 1 \ll 1$ it follows from (10) that

$$\omega_n^{(+)} \approx c\left\{\frac{\left(\frac{\pi(n-1)}{2d}\right)^2 + \frac{\omega_M(\omega_0 + \omega_M)}{c^2}}{\varepsilon - 1}\right\}^{1/2},$$

$$\omega_n^{(-)} \approx \left\{\frac{\omega_0(\omega_0 + \omega_M)}{1 + \omega_M(\omega_0 + \omega_M)\left(\frac{2d}{\pi(n-1)c}\right)^2}\right\}^{1/2}. \quad (11)$$

Note that as $\varepsilon \to 1$, the upper branches of the spectrum all go to infinity (at $\varepsilon = 1$ they are absent altogether!). On the initial portions of the dispersion curves ($k \gtrsim k_n = \omega_n/c$; $\omega \gtrsim \omega_n$), the group velocities of the wave differ only slightly from the velocity of light c:

$$v_{\rm gr}^{(n)} \approx c\left[1 - B_n\frac{\omega - \omega_n}{\omega_n}\right];$$

$$B_n = \frac{\omega_n^2}{c^2}\frac{d^2}{2}\left(\frac{2d}{\pi(n-1)}\right)^4\left[\varepsilon - 1 + \frac{\omega_0\omega_M}{\omega_0(\omega_0 + \omega_M) - \omega_n^2}\right]^2$$

$$\times\left\{\left[\left(\frac{\pi(n-1)^2}{2d}\right)^2 + \frac{\omega_n^2}{c^2}\right]\right.$$

$$\left.\times\left(\frac{\omega_0(\omega_0 + \omega_M)}{\omega_0(\omega_0 + \omega_M) - \omega_n^2} - \frac{\omega_n^2}{(\omega_0 + \omega_M)^2 - \omega_n^2}\right) - \frac{\omega_n^2}{c^2}\right\}^2. \quad (12)$$

For $\varepsilon = 1$, using the second of equations (11) simplifies (12) to

$$v_{\rm gr}^{(n,-)} = c\left[1 - B_n^-\frac{\omega - \omega_n^-}{\omega_n^-}\right], \quad (12')$$

$$B_n = \frac{\pi^2(n-1)^2\omega_0\left[c^2\left(\frac{\pi(n-1)}{2d}\right)^2 + \omega_M(\omega_0 + \omega_M)\right]^3}{8\omega_M^2(\omega_0 + \omega_M)^5}.$$

For large wave vectors [$\omega \to \omega_{\rm lim}$, see Eq. (8)], the group velocities of the lower branches tend to zero,

$$v_n^{(-)} \approx c\tilde{B}_n\left(\frac{\omega_{\rm lim} - \omega}{\omega_{\rm lim}}\right)^{3/2}, \quad \tilde{B}_n = 2\sqrt{\frac{2\omega_0}{\varepsilon\omega_M}}, \quad (13)$$

whereas those of the upper branches are relatively less temperature-dependent: for $\omega \to \infty$ the group velocities behave like

$$v_n^{(+)} \to c/\sqrt{\varepsilon}. \quad (13')$$

Case B. In the frequency interval

$$\omega_{\rm lim} = \sqrt{\omega_0(\omega_0 + \omega_M)} < \omega < \omega_0 + \omega_M/2 \quad (14)$$

there exists one (unusual) vibration branch, $\omega = \omega_{\rm un}(k)$, which, for $kd \gg 1$ and $c \to \infty$, "converts" to the Damon–Eshbach wave[2]

$$\omega = \omega_{DE} \equiv \omega_0 + \omega_M/2. \quad (15)$$

The starting point of the unusual branch is on the lower boundary of the interval (14). For $\omega \gtrsim \omega_{\rm lim}$

$$\omega_{\rm un} - \omega_{\rm lim} \approx \frac{c^2\omega_M^3}{2\varepsilon\omega_0^{5/2}(\omega_0 + \omega_M)^{3/2}}(k - k_0)^2,$$

$$k - k_0 \ll k_0, \quad (16)$$

$$k_0 = \frac{\omega_0}{\omega_M}\frac{(\omega_0 + \omega_M)^2}{c^2}, \quad V_{\rm un} = c\left\{\frac{2\omega_M^3(\omega_{\rm un} - \omega_{\rm lim})}{\varepsilon\omega_0\omega_{\rm lim}^3}\right\}^{1/2},$$

and for $\omega \lesssim \omega_{\rm DE}$

$$\frac{\omega_{\rm un} - \omega_{DE}}{\omega_{DE}} \approx -\frac{(1+\varepsilon)\omega_{DE}\omega_M}{8c^2k^2},$$

$$v_{\rm un} = c\left\{\frac{2\omega_{DE}}{(1+\varepsilon)\omega_M}\right\}^{1/2}\left(\frac{\omega_{DE} - \omega_{\rm oc}}{\omega_{DE}}\right)^{3/2}. \quad (17)$$

On the boundaries of the frequency interval (that is, for $\omega \to \omega_{\rm lim}$ or $\omega \to \omega_{\rm DE}$) the velocity of the unusual branch turns to zero, its maximum value in-between being close to the velocity of light, $(v_{\rm gr}^{\rm un})_{\rm max} \sim c$.

A further point to note is that at $\omega = \omega_{\rm lim}$ the penetration depth of the unusual wave is zero ($\gamma \to \infty$). As the frequency ω tends to ω_{DE}, the logarithmic decrement γ tends to infinity along with k. We have discussed elsewhere[5] the applicability of macroscopic electrodynamics equations in situations like this.

Now the existence of a crowding point for the lower branches of the spectrum raises the question of their resolution and hence requires an estimate for the term Im $\omega_n^{(-)}(k)$ responsible for dissipation processes in the system.

Let us modify the Landau–Lifshitz equation to incorporate a relaxation term containing the transverse (τ_1) and longitudinal (τ_2) relaxation times [cf. Ref. 4, § 6, Eq. (6.3.1)]. Writing $\omega = \omega' + i\omega''$ and neglecting terms quadratic in ω'', it is readily found that

$$\mu_{eff} \approx \frac{i\omega_M}{2}\sqrt{\frac{\omega_0+\omega_M}{\omega_0}}\,\frac{1}{\omega''+\omega_{DE}/\tau\omega_M},$$

$$\omega' = \omega_{\lim}, \qquad \tau^{-1} = \tau_1^{-1} + \tau_2^{-1}.$$

As $k\to\infty$, it follows from (2) that the effective susceptibility tends to infinity. Hence

$$\omega'' = -\frac{1}{\tau}\frac{\omega_{DE}}{\omega_M} \tag{18}$$

(Im $\omega < 0$ because of the assumed $e^{-i\omega t}$ frequency dependence). For $kd \gg 1$, the branches are distinct if $\omega_n^{(-)} - \omega_{n-1}^{(-)} > |\omega''|$. For $n \gg 1$, the distinguishability condition is

$$k^2 + \left(\frac{\pi n}{2d}\right)^2 < \frac{\pi}{2d}\sqrt{\varepsilon}\,\frac{\omega_M(\omega_0(\omega_0+\omega_M)^3)^{1/4}}{c\sqrt{\omega_{DE}}}\sqrt{\tau n}. \tag{19}$$

The above inequality is not the only limitation on the k and n values. The neglect of the inhomogeneous exchange interaction in the original formulas also implies that the wave vectors k and q should be relatively small,

$$ak \ll (\hbar\omega_M/I)^{1/2}, \qquad aq \ll (\hbar\omega_M/I)^{1/2}, \tag{20}$$

where the exchange integral I is of the order of T_c, the ferromagnetic's Curie temperature. The last inequality suggests that the plate be sufficiently thick and that the mode number n not too large,

$$d/n \gg a(I/\hbar\omega_M)^{1/2}. \tag{21}$$

Reference 5, which we quoted earlier, analyses the effects of the inhomogeneous exchange interaction (i.e., of spin waves, or magnons) on the dispersion of low-frequency electromagnetic vibrations in magnetics.

The unusual wave we have discussed appears to be of particular interest. It is a simple matter to show that if the

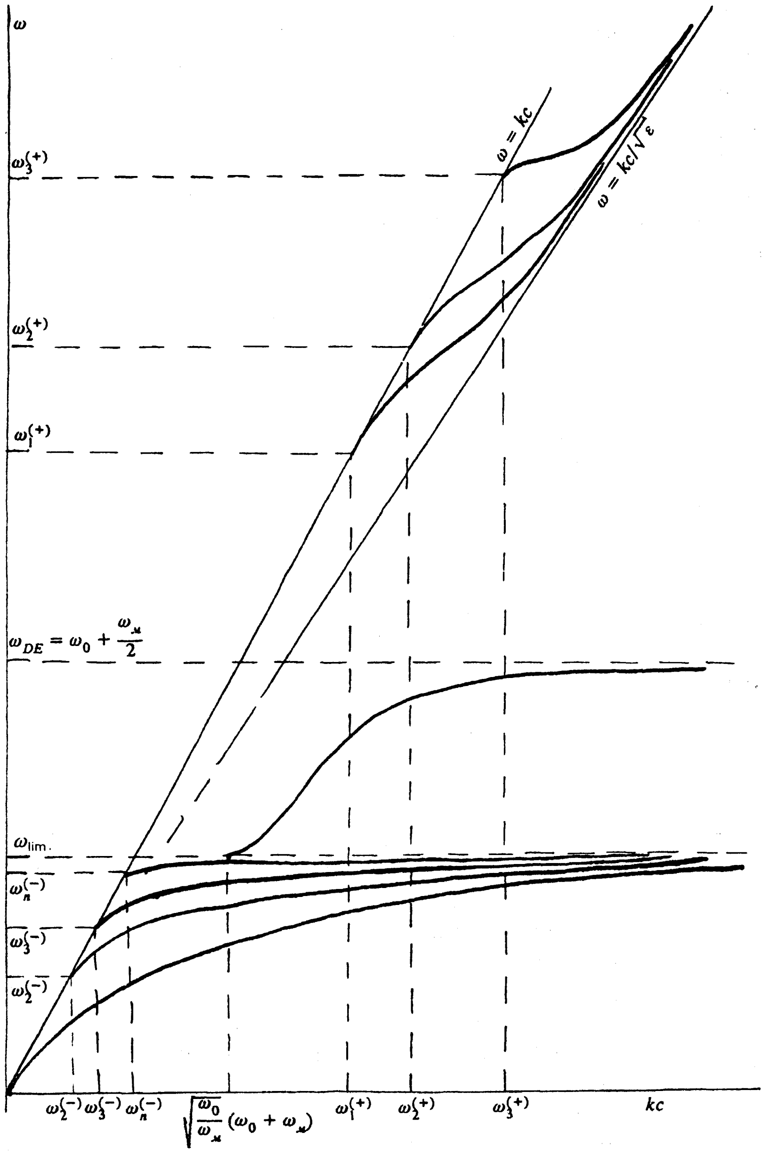

FIG. 1. Spectrum of electromagnetic vibrations of a ferromagnetic plate magnetized parallel to its surfaces.

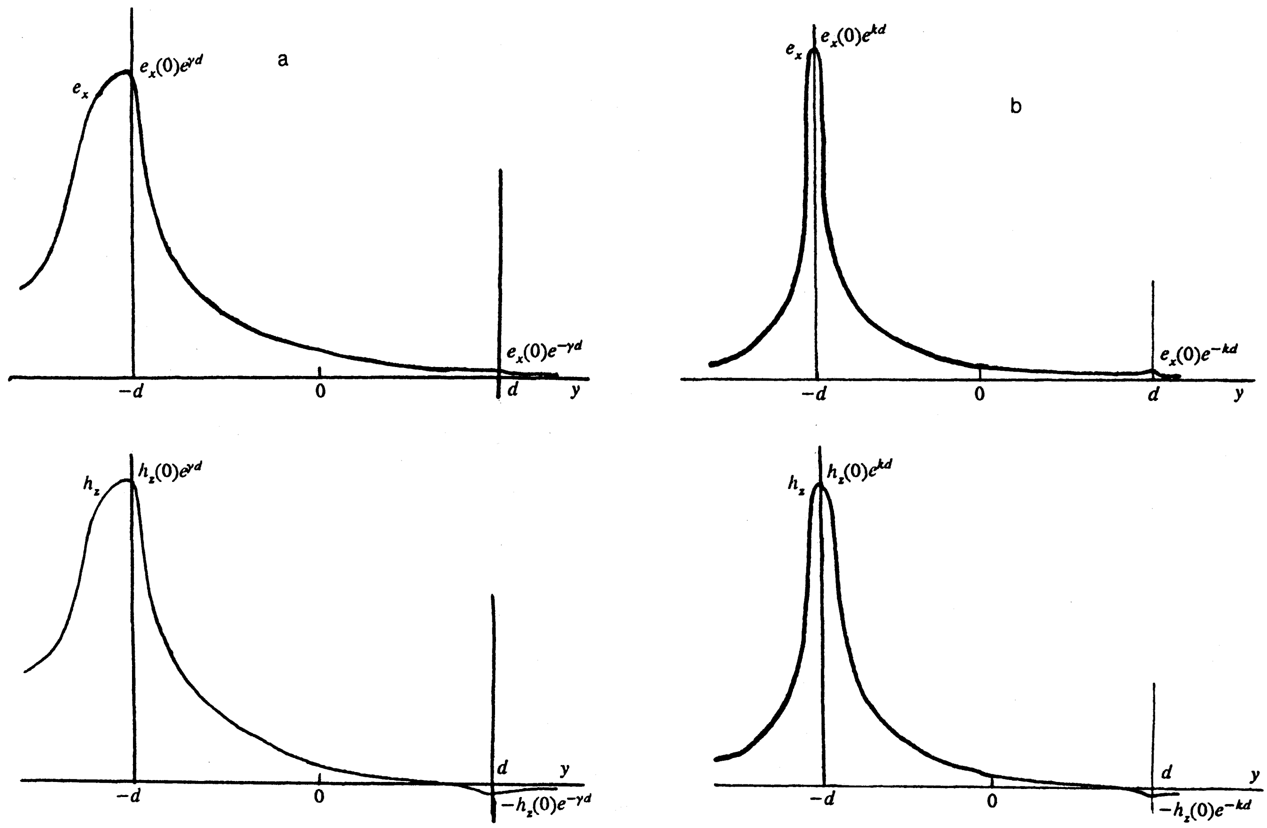

FIG. 2. a) Electric- and magnetic-field patterns in the wave for $\omega \gtrsim \omega_{\rm lim}$ ($\gamma \to \infty$). b) Electric- and magnetic-field patterns in the wave for $\omega \lesssim \omega_{DE}$, $kd \gg 1$ ($\gamma_0 \approx \gamma \approx k \to \infty$).

plate is thick, the electromagnetic field concentrates on one side of it and exponentially decays towards the other, which side is which depending on the particular wave propagation direction (cf. Ref. 5). Figures 1 and 2 illustrate the spectrum of vibrations and the electric- and magnetic-field patterns in the wave.

[1)]The problem has no $z \rightleftarrows -z$ symmetry (z is the coordinate along the plate, the origin of the coordinate system is at the midsurface of the plate. The plate is a layer $-d < z < d$). The reason is the existence of the vector $\mathbf{k} \times \mathbf{M}$ directed along the z axis.

[1]A. S. Borovik-Romanov and S. K. Sinha (eds.), *Spin Waves and Magnetic Excitations*, North-Holland, Amsterdam (1964).
[2]R. W. Damon and J. R. Eshbach, Phys. Rev. **118**, 1208 (1960).
[3]M. A. Panchenko, P. P. Povarov, and M. A. Savchenko, Fiz. Tverd. Tela (St. Petersburg) **20**, 1558 (1978) [Sov. Phys. Solid State **20**, 900 (1978)].
[4]A. I. Akhiezer, V. G. Bar'yakhtar, and S. V. Peletminskiĭ, *Spin Waves*, Wiley, 1968.
[5]M. I. Kaganov and T. I. Shalaeva, Zh. Eksp. Teor. Fiz. **96**, 2185 (1989) [Sov. Phys. JETP **69**, 1237 (1989)].

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