

# Dipole emission of surface electromagnetic waves in superlattices

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The problem of emission of surface electromagnetic waves (SEWs) by a dipole located near the boundary between a uniform medium and a semi-infinite superlattice is solved. The conditions under which SEWs exist and the dependence of the intensity of the emitted SEW on the frequency, the superlattice parameters, and the position of the dipole is found. It is shown that interference suppression of SEW radiation is possible.

In the last few years progress in the experimental techniques of molecular-beam epitaxy has led to rapid growth of investigations of superlattices (SLs), whose optical and electronic properties already have wide practical applications.

Cossel<sup>1</sup> predicted, on the basis of the similarity between the wave equation and the Schroedinger equation, that media with spatially periodic dielectric permittivity should exhibit a number of optical phenomena which are analogous to the properties of electrons in crystals. Interest in this idea has increased rapidly in the last few years because possibilities for observing these effects experimentally have appeared. The most striking effect is the appearance of band structure for optical waves in isotropic media with three-dimensional periodicity.<sup>2–4</sup>

Another, less obvious, effect is the existence of surface electromagnetic waves (SEWs) analogous to Tamm surface states.<sup>5</sup> These SEWs become localized at the boundary between periodic and uniform media due to Bragg diffraction in the periodic medium and the total internal reflection in the uniform medium.

These SEWs have been observed experimentally in the superlattice GaAs/Al<sub>0.2</sub>Ga<sub>0.8</sub>As with excitation from the end of the sample.<sup>6</sup> These SEWs can also be generated in other optical processes: luminescence and Rayleigh and Raman scattering. Theoretical study of these processes leads to the problem of radiation from a point dipole in a superlattice. The solution of this problem is the subject of this paper.

## 1. CONDITIONS FOR EXISTENCE OF SEWs IN A SUPERLATTICE

Consider a semi-infinite superlattice consisting of alternating transparent isotropic slabs with thicknesses  $d_1$  and  $d_2$  and dielectric permittivities  $\epsilon_1$  and  $\epsilon_2$ . Let the thickness  $\tilde{d}$  of the slab closest to the boundary be different from the other slabs, i.e., let this slab be "defective." Then the index of refraction has the following spatial dependence:

$$\epsilon(z) = \begin{cases} \epsilon_0, & z < 0 \\ \epsilon_1, & 0 < z < \tilde{d} \\ \epsilon_2, & l_m - d_2 < z < l_m \\ \epsilon_1, & l_m < z < l_m + d_2 \end{cases} \quad (1)$$

Here  $l_m = \tilde{d} - d_1 + mD$ , where  $D = d_1 + d_2$  is the period of the superlattice and  $m = 1, 2, \dots$  is the period number.

The solution of Maxwell's equations for SEWs can be found in the form

$$E(\mathbf{R}) = \exp[i(\mathbf{q}\mathbf{R} - \omega t)]\mathcal{E}(q, z), \quad (2)$$

$$\mathcal{E}(q, z) = \begin{cases} b_0 \exp(\mu z), & z < 0 \\ \bar{a} \exp(ik_1 z) + \bar{b} \exp(-ik_1 z), & 0 < z < \tilde{d} \\ \{a_j \exp[ik_1(z - l_m)] + b_j \exp[-ik_1(z - l_m)]\} \exp[iKD(m - 1)], & z > \tilde{d} \end{cases} \quad (3)$$

Here  $\mu^2 = q^2 - \lambda_0^2$ ,  $k_j^2 = \lambda_j^2 - q^2$ ,  $\lambda_j^2 = \epsilon_j \omega^2 / c^2$ , and  $K$  is the Bloch wave vector, satisfying the equation

$$\cos(KD) = \cos(k_1 d_1) \cos(k_2 d_2) - \frac{1}{2} (u + u^{-1}) \sin(k_1 d_1) \sin(k_2 d_2), \quad (4)$$

where  $u_E = k_1/k_2$  for TE waves with nonzero components ( $\mathcal{E}_\eta, \mathcal{H}_\xi, \mathcal{H}_z$ ) and  $u_M = \epsilon_1 k_2 / \epsilon_2 k_1$  for TM waves ( $\mathcal{H}_\eta, \mathcal{E}_\xi, \mathcal{E}_z$ ).

The  $\xi\eta z$  coordinate system is tied to  $\mathbf{q}$  ( $\mathbf{q} \parallel 0\xi$ ).

The condition for SEWs to be localized near the boundary of the superlattice [ $\text{Im}(K) > 0$ ] is satisfied in the Bragg reflection zones, where  $KD = mn + i\kappa$ , where  $m$  is the order of Bragg reflection. It is convenient to represent the localization parameters  $\kappa_{E,M}$  for TE and TM waves in these forbidden bands in the form

$$\text{sh}(\kappa_{E,M}) = \frac{1}{2} (u_{E,M} - u_{E,M}^{-1}) \sin(k_2 d_2) \sin \beta_{E,M}, \quad (5)$$

where as  $q$  increases in the zone the phase  $\beta_{E,M}$  changes from 0 to  $\pi$  or from  $\pi$  to  $2\pi$ , so that  $\kappa > 0$ . It is obvious that we have  $\kappa = 0$  for a)  $\beta = 0$ , at the zone boundary; b)  $k_2 d_2 = n\pi$ , interference bleaching; and c)  $u = 1$ , which is possible only for TM waves and corresponds to Brewster's angle. In the last two cases the wave should be observed to "collapse" inside the forbidden band.

The condition for the existence of SEWs is found from the equations of continuity at the boundaries of the defective slab ( $z = 0, \tilde{d}$ ). For TE waves these equations

$$\begin{aligned} a_0 &= \bar{a} + \bar{b}, \\ \mu a_0 &= ik_1(\bar{a} - \bar{b}), \\ \bar{b} &= r_E \bar{a} \exp(2ik_1 \tilde{d}) \end{aligned} \quad (6)$$

have a solution for

$$\Delta_E = \tilde{r}_E \exp(-2i\alpha_E) - 1 = 0, \quad (7)$$

where  $a_E = \tan^{-1}(\mu/k_1)$  and  $\tilde{r}_E = r_E \exp(2ik_1 \tilde{d})$ . Inside the forbidden band the reflection coefficient at the boundary ( $z = \tilde{d}$ ) is  $r_E = \exp[i(\beta - k_1 d_1)]$ , and for this reason Eq. (7) reduces to the condition

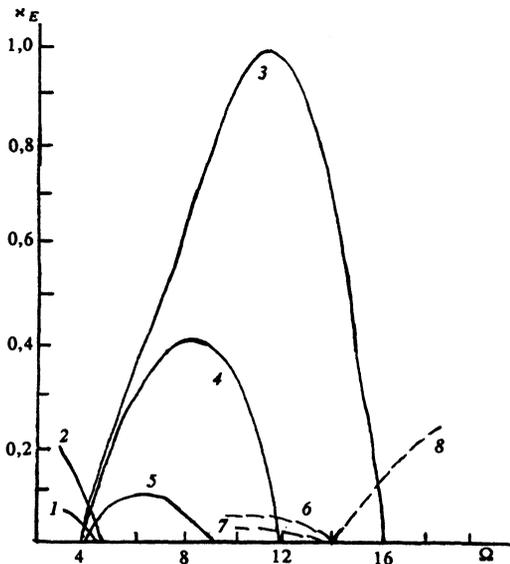


FIG. 1. TE-SEW localization coefficient as a function of the dimensionless frequency  $\Omega = [\omega(d_1 + d_2)/c][(\epsilon_1 + \epsilon_2)/2]^{1/2}$ . Superlattice parameters:  $\epsilon_1 = 3.0$ ,  $\epsilon_2 = 2.5$ ,  $d_2/d_1 = 1.5$ . The curves correspond to different values of the ratio  $\tilde{d}/d_1$ : 0.8 (1), 1.0 (2), 1.4 (3), 1.6 (4), 1.8 (5), 1.2 (6), 1.4 (7), and 1.6 (8). The solid and dashed lines correspond to first- and second-order Bragg reflection, respectively.

$$\beta_E + k_1(2\tilde{d} - d_1) - 2\alpha_E = (2n - m)\pi, \quad (8)$$

which for fixed thicknesses of the slabs and frequency  $\omega$  determines in each admissible forbidden band one possible value of  $q_E$ , and the corresponding phase  $\beta_E$  must satisfy the condition  $\kappa_E > 0$  [Eq. (5)].

Figure 1 displays the frequency dependences of the TE-SEW localization coefficient  $\kappa_E$  for the characteristic parameters of the superlattice and different thicknesses of the defective slab. As expected, SEWs do not exist for all  $\tilde{d}$ , since the value of  $\kappa_E$  found from Eqs. (5) and (8) must be positive. For each order of Bragg reflection there exists a lower limiting frequency  $\omega_c$ , determined by the condition of total internal reflection in the uniform medium. Surface electromagnetic waves of first (solid curves) and second (dashed curves) orders exist in the frequency interval shown in the figure. The frequency dispersion of  $\kappa_E$  is due mainly to the change in phase of  $\beta_E$  (5), which determines the position of the SEW frequency in the forbidden band and depends on the dispersion of  $a$  and  $k_1$ . Since  $\alpha$  increases with  $\omega$  and the sign of the change in  $\kappa_E$  is determined by the sign of  $\beta_E$ , the dispersion of  $\kappa$  can be monotonic (curves 1, 2, 6-8) or non-monotonic (curves 3-5) (Fig. 1).

For TM waves the equations of continuity at the boundaries of the defective slab

$$\begin{aligned} \epsilon_0 b_0 &= \epsilon_1(\tilde{a} + \tilde{b}), \\ \mu b_0 &= ik_1(\tilde{a} - \tilde{b}), \\ \tilde{b} &= r_M \tilde{a} \exp(2ik_1 \tilde{d}) \end{aligned} \quad (6a)$$

have an analogous solution [Eqs. (7) and (8)] with

$$\alpha_M = \text{arctg}(\epsilon_1 \mu / \epsilon_0 k_1).$$

Figure 2 displays the frequency dependence of the TM-SEW localization coefficient  $\kappa_M$ . The overall behavior of the curves is similar to that of the TE curves (Fig. 1), but  $\kappa_M$  is

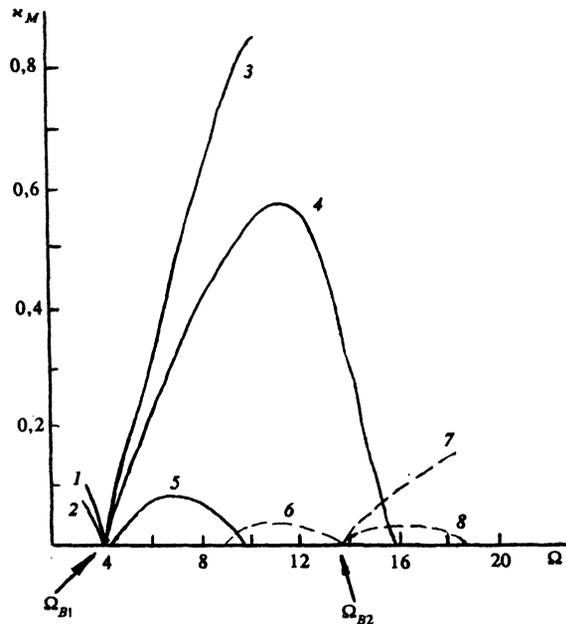


FIG. 2. TM-SEW localization coefficient versus the dimensionless frequency  $\Omega$ . Superlattice parameters:  $\epsilon_1 = 3.0$ ,  $\epsilon_2 = 2.5$ ,  $d_2/d_1 = 1.5$ . The curves correspond to different values of the ratio  $\tilde{d}/d_1$ : 1.8 (1), 2.0 (2), 0.8 (3), 1.0 (4), 1.2 (5), 1.4 (6), 1.8 (7), and 2.0 (8). The solid and dashed lines correspond to first- and second-order Bragg reflection, respectively.

somewhat smaller than  $\kappa_E$  and the TM curves have pronounced features at the frequencies  $\omega_{B1}$  and  $\omega_{B2}$  corresponding to the Brewster angles for Bragg reflection of first and second orders.

## 2. DIPOLE RADIATION IN THE SUPERLATTICE

Consider a dipole  $\mathbf{p}_0$  located at the point  $\mathbf{R}_0 = (0, 0, z_0)$  of the defective slab and emitting radiation at the frequency  $\omega$ . In order to solve the wave equation we employ the Fourier transform

$$E = \frac{1}{(2\pi)^2} \int dq \mathcal{E}(q, z) \quad (9)$$

and we analyze the contributions of the TE and TM harmonics separately. For TE waves the solution in the defective slab has the form

$$\begin{aligned} \mathcal{E}_\eta(q, z) &= \tilde{a} \exp(ik_1 z) + \tilde{b} \exp(-ik_1 z) \\ &+ \theta(z_0 - z) P_\eta \sin[k_1(z - z_0)], \end{aligned} \quad (10)$$

where

$$P_\eta = \frac{i\omega^2}{k_1 c^2} p_\eta,$$

and  $\theta(x)$  is the Heaviside step function.

It follows from Eqs. (10) that  $\mathcal{E}_\eta$  can be represented in the form

$$\mathcal{E}_\eta = \frac{4\pi\omega^2}{c^2} \frac{W_E(q, z)}{\Delta_E(q)} p_\eta.$$

For this reason, returning to the coordinate representation, it is convenient to transform to a cylindrical coordinate system  $(\rho, \phi, z)$

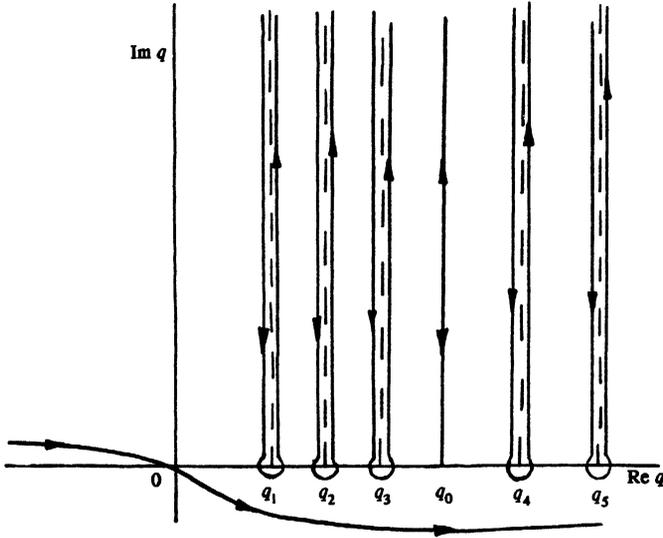


FIG. 3. Contour of integration  $L + L_1$ . Branch points:  $q_1 = \lambda_0$ ,  $q_2 = \lambda_1$ ,  $q_5 = \lambda_2$ , and  $q_3$  and  $q_4$  are the limits of the first allowed zone;  $q_0$ —pole corresponding to a surface wave.

$$E_{\rho,\phi} = \frac{2\pi\omega^2}{c^2} p_{\rho,\phi} \int_L q \frac{W_E(q, z)}{\Delta'_E(q)} [J_0(q\rho) \pm J_2(q\rho)] dq, \quad (11)$$

where the contour  $L$  is shown in Fig. 3. In order to perform the integration we employ the following equality for cylindrical functions

$$J_n(x) = \frac{1}{2} [H_n^1(x) - e^{i\pi n} H_n^1(-x)],$$

switch to the integration contour  $L + L_1$ , and close the contour (Fig. 3), taking into account the singularity of the integrand and the radiation condition (no sources at infinity<sup>7</sup>). It follows from Eqs. (3), (4), and (11) that the branch points are  $\lambda_j$  and  $q_k^\pm$ , corresponding to the edges of the  $k$ th forbidden band, and when the condition of existence of SEWs (8) is satisfied the corresponding value of  $q_E$  is a pole. It can be shown that the SEW field is determined by the residue of the pole  $q = q_E$ , and the integrals along the edges of the cuts determine the radiation intensity of internal waves, which will not be calculated here.

### 3. TE-SEW RADIATION INTENSITY

The field of the emitted SEW is described by the residue of the pole  $q = q_E$ , which, using Eq. (11), gives the expression

$$E_{\rho,\phi} = \frac{\pi\omega^2}{c^2} p_{\rho,\phi} \frac{W_E(q_E, z) q_E [H_0^1(q_E \rho) \pm H_2^1(q_E \rho)]}{\Delta'_E}, \quad (12)$$

where

$$\Delta'_E = \frac{d\Delta_E}{dq} = \frac{d\beta_E}{dq} - \frac{q(2\tilde{d} - d_1)}{k_1} - 2 \frac{d\alpha}{dq},$$

$$W_E(z, q_E) = \begin{cases} we^{\mu z}, & z < 0 \\ w \frac{k_1 - i\mu}{2k_1} [\exp(ik_1 z) + \exp(-ik_1 z)], & 0 < z < \tilde{d} \end{cases},$$

$$w = \exp(ik_1 z_0) + \tilde{r}_E \exp(-ik_1 z_0).$$

It follows from the last formulas that  $E_\rho \sim \rho^{-3/2}$  as  $\rho \rightarrow \infty$  and does not contribute to the SEW intensity. For this

reason, the Poynting vector has a single nonzero component  $S_\rho$  in the far zone; to calculate it we must find

$$H_z = \frac{\pi\omega}{c} p_\phi \frac{W_E(q_E, z) q_E^2 H_1^1(q_E \rho)}{\Delta'_E}. \quad (13)$$

Using the asymptotic expression for the Hankel functions

$$H_n^1(x) \sim \sqrt{\frac{2}{\pi x}} \exp[i(x - \frac{\pi}{4} - \pi n)],$$

the TE-SEW intensity can be written in the form

$$\frac{dI}{d\phi} = I_0 \sin^2 \phi \sin^2 \theta (S_0 + S_{SL} + \tilde{S}), \quad (14)$$

where

$$I_0 = \frac{8\omega^4}{c^3} p_0^2,$$

$S_0$ ,  $\tilde{S}$ , and  $S_{SL}$  are, respectively, the radiation flux densities in the uniform medium, the defective slab, and the superlattice and are normalized to the integrated intensity of dipole radiation in the uniform medium with  $\varepsilon = \varepsilon_1$ :

$$S_0 = \frac{q_E C}{(k_1^2 + \mu^2) \Delta_E'^2},$$

$$\tilde{S} = \frac{2q\tilde{d}}{k_1^2 \Delta_E'^2} \left[ 1 + \frac{\sin(k_1 d) \cos(k_1 d - \alpha)}{k_1 \tilde{d}} \right] C,$$

$$S_{SL} = \frac{2}{k_1^2 \Delta_E'^2} \left\{ q_E d_2 |r|^2 \left[ 1 + \frac{\sin(k_2 d_2)}{k_2 d_2} \right] + q_E d_1 e^{-2\alpha} \right. \\ \left. \times \left[ 1 + \frac{\sin(k_1 d_1) \cos(k_1 d_1 - \alpha)}{k_1 d_1} \right] \right\} \frac{C}{1 - e^{-2\alpha}}, \quad (15)$$

where

$$r = \cos \frac{\alpha}{2} - i \frac{k_1}{k_2} \sin \frac{\alpha}{2}, \quad C = \cos^2(k_1 z_0 - \alpha).$$

#### 4. PROPERTIES OF TM WAVES

For TM harmonics the solution of the inhomogeneous wave equation has the form

$$\begin{aligned} \mathcal{E}_z(q, z) = & \bar{a} \exp(ik_1 z) + \bar{b} \exp(-ik_1 z) \\ & + \theta(z_0 - z) \{ \mathcal{P}_z \sin[k_1(z - z_0)] \\ & + \mathcal{P}_\xi \cos[k_1(z - z_0)] \}, \end{aligned} \quad (9a)$$

where  $\mathcal{P}_z = (2\omega^2 i/c^2) p_z$  and  $\mathcal{P}_\xi = (2q\omega^2/\lambda_1 c^2) p_\xi$ .

As in the case of TE waves, the  $E$  and  $H$  fields can be expressed in terms of integrals along the contour  $L_1 + L_2$  (Fig. 3). Since  $\mathcal{E}_z$  can be represented in the form

$$\mathcal{E}_z = \frac{4\pi\omega^2}{c^2} \frac{W_\xi p_\xi + W_z p_z}{\Delta_M}, \quad (16)$$

the TM-SEW fields are described by the formulas

$$\begin{aligned} \mathcal{E}_z = & \frac{2\pi i\omega^2}{c^2 \lambda_1^2 k_1 \Delta'_E} e^{i\alpha} q_M W_M [\exp(ik_1 z) + r_M \exp(-ik_1 z)] \\ & \times [k^2 \cos(k_1 z_0 - \alpha) p_z H_0^1(q_M \rho) \\ & - q_M k_1 \sin(k_1 z_0 - \alpha) p_x H_1^1(q_M \rho)], \\ \mathcal{H}_y = & \frac{2\pi i\omega^2}{c^2 \lambda_1^2 k_1 \Delta'_E} e^{i\alpha} q_M W_M [\exp(ik_1 z) + \bar{r}_M \exp(-ik_1 z)] \\ & \times \{ ik_1 \cos(k_1 z_0 - \alpha) p_z H_1^1(q_M \rho) \\ & + \frac{i q_M k_1}{2} \sin(k_1 z_0 - \alpha) p_x [H_0^1(q_M \rho) - H_2^1(q_M \rho)] \}, \end{aligned} \quad (17)$$

$W_M$

$$= \begin{cases} -2ik_1 \varepsilon_1 e^{\mu z}, & z < 0 \\ -(i\varepsilon_0 k_1 - \varepsilon_1 \mu) [\exp(ik_1 z) + \bar{r}_M \exp(-ik_1 z)], & 0 < z < \tilde{d}. \end{cases}$$

Next, using the asymptotic expansions of the Hankel functions, the TM-SEW intensity is likewise given by the formula (14), where

$$\begin{aligned} S_0 = & B_0 \frac{q_M}{\mu} t, \\ \bar{S} = & 2B_0 q_M \tilde{d} \left[ 1 + \frac{\sin(k_1 \tilde{d}) \cos(k_1 \tilde{d} - \alpha_M)}{k_1 \tilde{d}} \right] t, \\ S_{SL} = & 2B_1 t \left\{ q_M d_2 |r|^2 \left[ 1 + \frac{\sin(k_2 d_2) \cos(k_2 d_2)}{k_2 d_2} \right] + q_M d_1 e^{-2\alpha} \right. \\ & \left. \times \left[ 1 + \frac{\sin(k_1 d_1) \cos(k_1 d_1 - \alpha_M)}{k_1 d_1} \right] \right\}, \\ B_0 = & \cos^2 \theta \cos^2(k_1 z_0 - \alpha) + \frac{q_0^2 k_1^2}{\lambda_1^4} \sin^2 \theta \cos^2 \phi \\ & \times \sin^2 \left( k_1 z_0 - \frac{\alpha}{2} \right) (k_1 \Delta'_E)^2, \\ B_1 = & \frac{B_0}{1 - e^{-2\alpha}}, \quad t = \frac{\mu^2 \varepsilon_1^2 + k_1^2 \varepsilon_0^2}{k_1^2 \varepsilon_1^2}, \\ r = & \cos \frac{\alpha}{2} - i \frac{\varepsilon_1 k_1}{\varepsilon_2 k_2} \sin \frac{\alpha}{2}. \end{aligned} \quad (18)$$

#### 5. COMPUTATIONAL RESULTS AND DISCUSSION

First, we note that the contributions of TE and TM harmonics to the SEW intensity can be studied independently, since the polarizations of these harmonics are mutually orthogonal [Eqs. (12) and (17)].

Figure 4 displays the frequency dependence of the normalized integrated intensity of TE-SEW radiation when the dipole lies at the center of the defective slab and is oriented parallel to the slabs. The parameters of the superlattice and the thicknesses of the defective slab correspond to Fig. 1. The SEW intensity as a function of the dipole position is described by the interference factor  $\cos^2(k_1 z_0 - \alpha)$ , which also depends on the frequency because of the frequency dispersion of  $k_1$  and  $\alpha$ .

At low frequencies, close to  $\omega_c$ , the main SEW radiation flux propagates in the uniform medium and is described by the formula

$$\frac{dI}{d\phi} \approx I_0 \frac{\cos^2(k_1 z_0)}{\lambda_0} \mu \sin^2 \phi. \quad (19)$$

As the frequency increases, the flux  $S_0$  becomes small compared with  $S_{SL}$ , since in this case  $\mu \ll \kappa$ . Then the factors  $\Delta'_E$  and  $\cos^2(k_1 z_0 - \alpha)$  make the main contribution to the dispersion of the intensity.

As a rule, the difference between the dielectric permittivities of the constituent semiconductors of the superlattice is small,  $|\varepsilon_1 - \varepsilon_2| \ll \varepsilon_1 + \varepsilon_2$ , and in this case the main radiation flux propagates in the superlattice and

$$\Delta'_E \approx -\frac{1}{\sin \beta_E} \frac{d(\cos \beta_E)}{dq}.$$

Since in this case, due to the narrowness of the forbidden band, the scan rate over the band ( $\sim d\beta/dq$ ) will be quite high. This makes it possible to find the approximate dependence of the SEW intensity on the localization coefficient  $\kappa$  [see Eq. (5)], since the cofactor  $d(\cos \beta)/dq$  is virtually independent of the position of the SEW in the forbidden

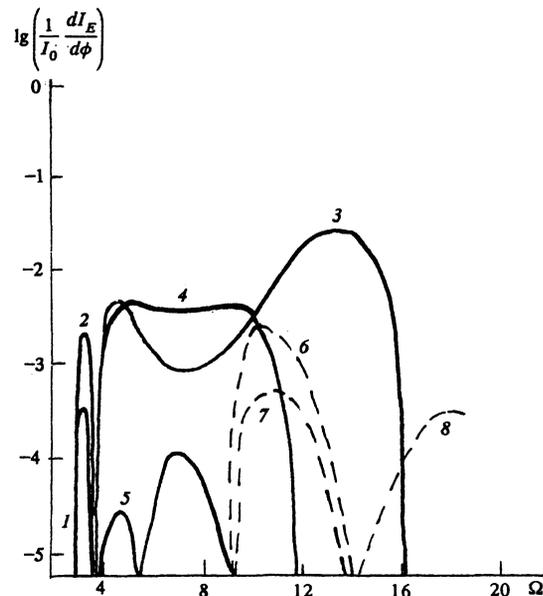


FIG. 4. Intensity of TE waves versus the frequency  $\Omega$ . All parameters are the same as in Fig. 1.

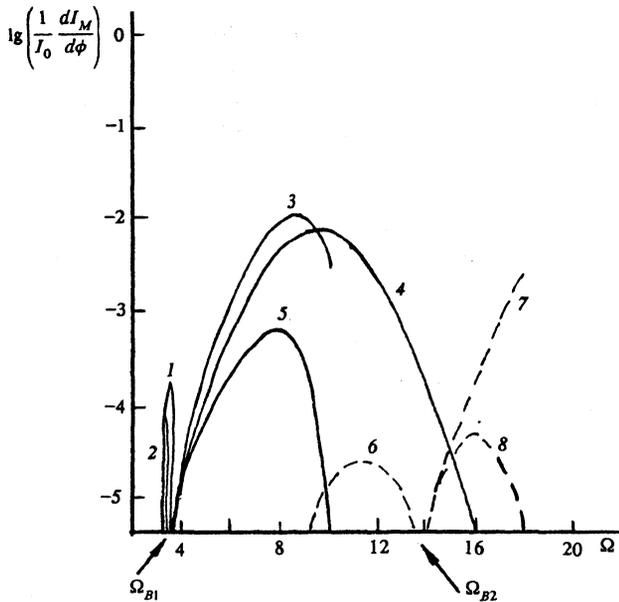


FIG. 5. Intensity of the anisotropic component of a TM wave as a function of the frequency  $\Omega$ . All parameters correspond to Fig. 2.

band:

$$I \sim e^{\kappa} \text{sh } \kappa. \quad (20)$$

Thus, far from  $\omega_c$  the frequency dependence of the intensity, as a rule, repeats the frequency dependence of the localization coefficient  $\kappa$  (see curves 1, 2, 4, and 6–8). The reason for the dip at  $\Omega \approx 5.6$  in curve 5 is that the interference cofactor vanishes (interference suppression of SEW radiation).

The expression for the intensity of TM waves is more complicated due to the presence of an isotropic contribution from the perpendicular component ( $p_z$ ) and the anisotropic contribution from the parallel ( $p_x$ ) component of the dipole. Since the contributions to the field for these components are shifted by  $\pi/2$  [Eq. (17)], the cross term in the formula for the intensity is absent, and the interference factors add constructively (18).

Figures 5 and 6 display the frequency dependence of the intensity of the isotropic and anisotropic TM–SEW components. The intensity of the anisotropic component decreases with increasing frequency, since waves undergoing Bragg diffraction are emitted almost parallel to the axis of the dipole.

The remaining behavior of the curves is similar to that for TE waves (Fig. 4), but the intensities are somewhat lower than for TE waves and have pronounced features at  $\Omega_{B1}$  and  $\Omega_{B2}$ , corresponding to Brewster's angle for Bragg reflection of first and second orders.

## 6. CONCLUSIONS

In this paper we have derived expressions for the electromagnetic radiation field of a point dipole located in the defective slab of a superlattice near the boundary with a uniform medium [Eqs. (11) and (16)].

The obtained expression made is possible to calculate the radiation intensity of TE and TM surface waves [Eqs. (15) and (18)] existing when the conditions (8) are satisfied.

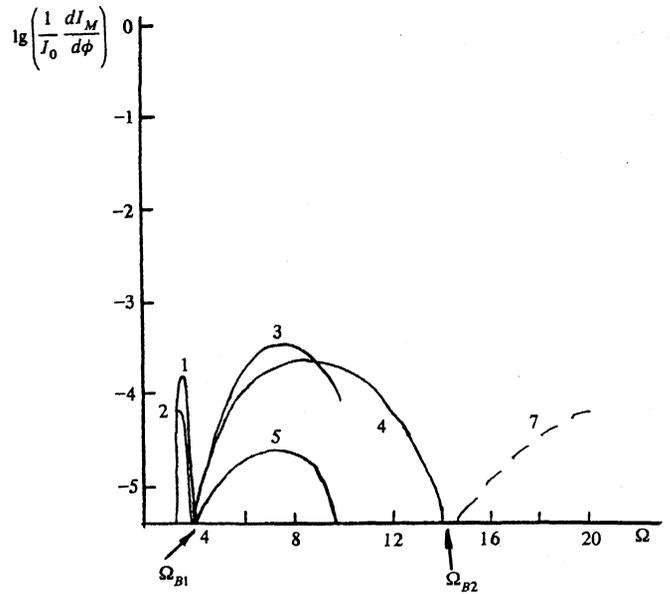


FIG. 6. Intensity of the isotropic component of a TM wave as a function of the frequency  $\Omega$ . All parameters correspond to Fig. 2.

The dependence of the SEW intensity on the position of the dipole is determined by the interference factor [see Eqs. (15) and (18)], which can give rise to complete suppression of TE–SEW radiation. For TM–SEW complete suppression is possible only in a direction perpendicular to the dipole, or in cases when the dipole is perpendicular or parallel to the slabs of the superlattice.

The frequency dependence of the SEW intensity has a complicated resonance behavior, which is determined by both interference in the slabs of the superlattice and scanning of SEW over the forbidden band. For each order of Bragg reflection there exists a lower cutoff frequency  $\omega_c$ , determined by the condition of total internal reflection in the uniform medium. At low frequencies, close to  $\omega_c$ , the main SEW flux propagates in the uniform medium and is described by Eq. (19). At high frequencies the main flux is localized in the superlattice and increases with increasing  $\kappa$  [Eq. (20)].

The formulas obtained can be used to find the intensity of luminescence and Rayleigh and Raman scattering by adding together the fields of separate emitters, taking into account the corresponding averaging over the coordinates of the emitting and scattering centers. The calculation of these relations is a problem for future investigations.

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