Kinetics of the interaction of strong radiation with a plasma

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We obtain simplified expressions for the photon-electron collision operators for interaction processes of weakly relativistic electrons with strong radiation under conditions when the relative change in the photon frequency during their elastic scattering is small. The collision operator in the kinetic equation for the electrons has then the form of a Fokker–Planck equation. The analogous collision operator in the kinetic equation for the photons generalizes the well known Kompaneets expression to the case of arbitrary optical thickness and arbitrary motion of the electron gas in an inhomogeneous medium.

The interaction between plasma and radiation plays an important role in many astrophysical objects. The intensity of the radiation then often happens to be so large that it approaches the Eddington limit when the radiation pressure becomes so significant that it compensates gravitation and thanks to that restrains the accretion flux of matter. Such conditions are realized, for instance, in x-ray pulsars and γ-bursters. A consistent description of the interaction between a plasma flow, accreting onto the object and the strong radiation appearing as a result of this accretion is therefore an important physical problem.

The problem discussed here has been studied before by a number of authors using a hydrodynamic approach.[1–3] However, under conditions when the radiation makes an important contribution to the total balance of forces one needs more detailed kinetic considerations. Indeed, estimates show that under such conditions the effective electric field E caused by the action of the radiation on an electron can be comparable with, or even exceed, the critical run-away field[4] E_c and this usually leads to an appreciable deformation of the electron distribution function. In a one-component plasma there occurs in the hydrodynamic limit a practically complete balance of forces between gravitation, electric fields, and radiation, but taking into account the energy-dependence of the interaction cross-sections shows that the balance relation for the forces is only satisfied on average and is violated for separate groups of particles. Such a violation of the balance may lead to an appreciable deformation of the electron distribution function. Finally, only the use of a consistent kinetic approach makes it possible to obtain a complete set of macroscopic equations correctly describing the interaction processes between radiation and plasma in the hydrodynamic limit.

We see thus that one needs a kinetic approach for a detailed study of the physical phenomena under the conditions which are of interest to us. The aim of the present paper is a consistent derivation of the kinetic equations describing the dynamics of the electrons and a strong photon flux taking their interaction fully into account.[1)]

In Sec. 1 of the present paper we derive, in covariant form, the kinetic equation for weakly relativistic electrons which takes into account their collisions both with other electrons and ions and with photons. We carry out possible simplifications of the kinetic equation for the conditions of interest to us.

Section 2 is devoted to the derivation of the photon-electron collision integral. Such a collision integral was found by Kompaneets[5] for the case of strong angular scattering of the photons. In the present paper we consider a broader class of conditions realized in actual astrophysical processes when the angular scattering of the photons may be arbitrary. As a result we obtain a new, more general expression for the photon-electron collision operator which has an integro-differential nature and which generalizes the Kompaneets expression to the case of arbitrary scattering of the photons.

The kinetic equation obtained will be used in future papers for a consistent description of physical processes in the vicinity of accretion centers in the presence of strong radiation.

1. KINETIC EQUATION FOR ELECTRONS TAKING ELECTRON-PHOTON COLLISIONS INTO ACCOUNT

The interaction of a plasma with radiation is in the present paper considered in a rather general form. However, bearing in mind well defined astrophysical applications we use a few simplifications taking into account actual circumstances. First of all it follows from the many observations of the objects which are of interest to us (x-ray pulsars, quasars, γ-bursters, and so on) that the main radiative flux consists of x-ray and γ-quanta of a relatively high energy while their effective temperature $T_{\rm ph}$ is of the order of the average temperature T_e of the electrtons interacting with them which is of the order of a few or tens of keV. One sees therefore easily that one may assume that for most photons and electrons the condition

$$k \ll p \ll m \tag{1}$$

is satisfied, where $p = |\mathbf{p}|$ is the electron momentum, m is their mass, and $k = |\mathbf{k}|$ is the photon momentum. Here and in most of the formulas which follow we use an abbreviated system of notation in which Dirac's constant $\hbar$ and the velocity of light c are equal to unity; in those units the frequency of the radiation and the photon energy are the same as its momentum k. Condition (1) means that, on the one hand, the photons are "light" particles in relation to the "heavy" electrons and, on the other hand, the electrons themselves are, in the main, weakly relativistic: $\gamma - 1 \ll 1$. Here $\gamma = (1 - v^2)^{-1/2}$, $v = |\mathbf{v}|$, $\approx p/m$ is the electron velocity. In that case the electron momentum changes little when it col-

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lides with a photon. The enables us to give a kinetic description of the electron-photon collisions using a Fokker–Planck equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{R}} + e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = \left(\frac{\delta f}{\delta t}\right)_{coll},$$

$$\left(\frac{\delta f}{\delta t}\right)_{coll} = \left(\frac{\delta f}{\delta t}\right)_{e,ph} + \left(\frac{\delta f}{\delta t}\right)_{e,e} + \left(\frac{\delta f}{\delta t}\right)_{e,i},$$

in which the electron-photon collision operator $(\delta f/\delta t)_{e,\mathrm{ph}}$, like the electron-electron and electron-ion collision operators, $(\delta f/\delta t)_{e,e}$ and $(\delta f/\delta t)_{e,i}$, can be written as divergences of a flux in momentum space.

We obtain the appropriate expression for $(\delta f/\delta t)_{e,\mathrm{ph}}$ under condition (1). We shall start from the Boltzmann electron-photon collision integral[2)]

$$\left(\frac{\delta f}{\delta t}\right)_{e,ph} = \int \{f(\mathbf{p}')F(\mathbf{k}')[1 + n(\mathbf{k})]w(\hat{\mathbf{p}}, \hat{\mathbf{k}}; \hat{\mathbf{p}}', \hat{\mathbf{k}}')$$

$$- f(\mathbf{p})F(\mathbf{k})[1 + n(\mathbf{k}')]w(\hat{\mathbf{p}}', \hat{\mathbf{k}}'; \hat{\mathbf{p}}, \hat{\mathbf{k}})\}d^3p'd^3kd^3k'. \qquad (2)$$

Here $\hat{\mathbf{p}}$ and $\hat{\mathbf{k}}$ are the electron and photon four-momenta, respectively, and $n(\mathbf{k})$ and $F(\mathbf{k})$ are, respectively the quantum occupation number of the photon states and their distribution function, which are (in a complete notation system) connected by the simple relation

$$F(\mathbf{k}) = \frac{2}{(2\pi\hbar)^3}n(\mathbf{k})$$

(the factor 2 in the enumerator takes into account two possible directions of the photon polarization; we assume that the radiation is on average unpolarized). The additional terms in (2) leading to a quadratically nonlinear n- or F-dependence are caused by the Bose quantum statistics for photons and are connected with their induced scattering. They are important only for a rather dense photon gas, close to the radiation of an absolutely black body. The primes indicate, as usual, the momenta of all the colliding particles after the scattering. The quantity $w(\hat{\mathbf{p}}',\hat{\mathbf{k}}';\hat{\mathbf{p}},\hat{\mathbf{k}}) \equiv w(\hat{\mathbf{p}},\hat{\mathbf{k}};\hat{\mathbf{p}}',\hat{\mathbf{k}}')$ is the differential probability for a transition of photons from the range d^3k into the range d^3k' and electrons from the range d^3p into the range d^3p'; it includes a δ-function taking into account the energy and momentum conservation laws (see Sec. 2 below). We have taken into account in (2) that the relative velocity of the colliding particles is constant and equal to the velocity of light c.

We first of all consider the part $(\delta f/\delta t)^{\mathrm{lin}}_{e,\mathrm{ph}}$ of the Boltzmann collision integral (2) which is linear in the photon distribution function. Introducing the notation

$$\mathcal{F}(\mathbf{p};\mathbf{q}) = \int w(\hat{\mathbf{p}}', \hat{\mathbf{k}}'; \hat{\mathbf{p}}, \hat{\mathbf{k}})F(\mathbf{k})d^3kd^3k',$$

$$\mathbf{q} = \mathbf{p}' - \mathbf{p} \equiv \mathbf{k} - \mathbf{k}',$$

we find

$$\left(\frac{\delta f}{\delta t}\right)^{lin}_{e,ph} = \int [f(\mathbf{p}+\mathbf{q})\mathcal{F}(\mathbf{p}+\mathbf{q}; -\mathbf{q}) - f(\mathbf{p})\mathcal{F}(\mathbf{p};\mathbf{q})]d^3q. \quad (3)$$

Taking into account the momentum and energy conservation laws during the collision we find that the absolute magnitude of the momentum transfer $q = |\mathbf{q}|$ from the photons to the electrons,

$$q \le 2k$$

is under the conditions (1) small as compared to p. This means that the quantity $\mathcal{F}(\mathbf{p},\mathbf{q})$, as function of its second argument is entirely concentrated in a narrow $\mathbf{q}$ range. At the same time this quantity is a smooth function of its second argument. Making in the first term of the integrand in (3) the substitution $\mathbf{q} \to -\mathbf{q}$ and expanding this expression up to second-order terms in $\mathbf{q}$ we get

$$\left(\frac{\delta f}{\delta t}\right)^{lin}_{e,ph} \approx \frac{\partial}{\partial p_\alpha}\int \{(-q_\alpha)f(p)\mathcal{F}(\mathbf{p};\mathbf{q}) + \frac{q_\alpha q_\beta}{2}\frac{\partial}{\partial p_\beta}[f(\mathbf{p})\mathcal{F}(\mathbf{p};\mathbf{q})]\}d^3q,$$

where the Greek indices denote the spatial components of the corresponding vectors and where it is understood that we sum over repeated indices. Carrying out a similar procedure with the part of (2) which is nonlinear in n and afterwards again changing from an integration over d^3q to an integration over d^3p' and further by standard methods changing to the differential scattering cross-section[6] we get finally

$$\left(\frac{\delta f}{\delta t}\right)_{e,ph} = \frac{\partial J_\alpha}{\partial p_\alpha}, \qquad (4)$$

$$J_\alpha = \int F(\mathbf{k})\{[1 + n(\mathbf{k})]A_\alpha f(\mathbf{p}) + \frac{\partial}{\partial p_\beta}[B_{\alpha\beta}f(\mathbf{p})]\}d^3k, \qquad (5)$$

$$A_\alpha = \int q_\alpha d\sigma, \qquad B_{\alpha\beta} = \frac{1}{2}\int q_\alpha q_\beta d\sigma.$$

Here we have $d\sigma = (d\sigma/d\Omega)d\Omega$ where $d\sigma/d\Omega$ is the differential cross-section for the scattering of photons into an element of solid angle $d\Omega$ (integration over d^3k is assumed, in particular, to include integration over all Ω); the distribution function F depends on the magnitude of k and on the direction of the momentum of the incident photon, $n = \mathbf{k}/k$. The integration in (5) is over all momenta of the incident photons and it does not involve the electron distribution function $f(p)$.

It is convenient to obtain expressions for A_α and $B_{\alpha\beta}$ in the laboratory frame of reference by forming the contravariant four-vector U^i and four-tensor W^{ij}, with spatial parts which are, respectively, equal to $k\mathscr{E}A_\alpha$ and $k\mathscr{E}B_{\alpha\beta}$ where $\mathscr{E} = mc^2\gamma$ is the electron energy. The factor $k\mathscr{E}$ is chosen starting from the relativistic invariance of the quantity[7] $k\mathscr{E}d\sigma$ and of the four-vector q^i, the spatial part of which is the same as q_α. One must after that express the quantities U^i and W^{ij} in an invariant manner in terms of the four-vector momenta of the electron, p^i, and of the incident photon, k^i. This can be most conveniently done in the frame of reference in which the electron is at rest before the collision. In that frame of reference the photon frequency remains unchanged in zeroth approximation in the small parameter k/m of (1) and the differential scattering cross-section is by virture of the smallness of the same parameter described by the classical Thomson expression.[7] The main relativistic corrections to the Thomson cross-section arise when we change to the laboratory frame of reference in our case (1) due to the Doppler shift in the frequency.

To first order in the expansion in k/m we must use the relativistic Klein–Nishina formula

$$\frac{d\sigma}{d\Omega}=\frac{r_0^2}{2}\frac{\tilde{k}'^2}{\tilde{k}^2}\left(\frac{\tilde{k}'}{\tilde{k}}+\frac{\tilde{k}}{\tilde{k}'}-\sin^2\tilde{\vartheta}\right)$$

$$\approx\frac{r_0^2}{2}(1+\cos^2\tilde{\vartheta})[1-2\frac{\tilde{k}}{m}(1-\cos\tilde{\vartheta})],$$

where $\tilde{k}$ and $\tilde{k}'$ are the photon momenta before and after the scattering, while $\tilde{\vartheta}$ is the corresponding scattering angle in the rest frame of reference of the electron before the scattering and $r_0=e^2/mc^2$ is the classical electron radius. Using in that frame of reference the formula for the transformation of the photon frequency in the scattering,[8]

$$\frac{\tilde{k}'}{\tilde{k}}=\frac{1}{1+(\tilde{k}/m)(1-\cos\tilde{\vartheta})}\approx 1-\frac{\tilde{k}}{m}(1-\cos\tilde{\vartheta}),$$

we get the following expressions for the quantities U^i and W^{ij}

$$U^i\approx\sigma_{Th}mk^2\left[\frac{k^i}{k}\left(1-\frac{16k}{5m}\right)-\frac{p^i}{m}\left(1-\frac{21k}{5m}\right)\right],\qquad(6)$$

$$W^{ij}\approx\frac{\sigma_{Th}}{20}mk^3\left[-3\left(1-4\frac{k}{m}\right)G^{ij}+\left(14-88\frac{k}{m}\right)\frac{p^ip^j}{m^2}\right.$$

$$\left.+\left(11-48\frac{k}{m}\right)\frac{k^ik^j}{k^2}-\frac{(11-62k/m)}{mk}(p^ik^j+k^ip^j)\right],\qquad(7)$$

where G^{ij} is the metric tensor of the four-dimensional flat space ($G^{00}=1$, $G^{11}=G^{22}=G^{33}=-1$, $G^{ij}=0$ when $i\neq j$), and $\sigma_{Th}=(8\pi/3)r_0^2$ is the total Thomson cross-section. Changing to the laboratory frame of reference, using the formula for the frequency transformation,

$$\tilde{k}=k\gamma(1-\mathbf{vn}),$$

we get after separating the spatial components ($i,j=1,2,3$)

$$A_\alpha=\sigma_{Th}k(1-\mathbf{vn})\left[(1-\mathbf{vn})\left(1-\frac{21k}{5m}\right)\gamma^2v_\alpha\right.$$

$$\left.-n_\alpha\left(1-\frac{16k}{5m}\right)\right],\qquad(8)$$

$$B_{\alpha\beta}=\frac{\sigma_{Th}k^2}{20}(1-\mathbf{vn})\left[3\left(1-4\frac{k}{m}\right)(1-\mathbf{vn})^2\gamma^2\delta_{\alpha\beta}\right.$$

$$+\left(14-88\frac{k}{m}\right)(1-\mathbf{vn})^2\gamma^2v_\alpha v_\beta+\left(11-48\frac{k}{m}\right)n_\alpha n_\beta$$

$$\left.-\left(11-62\frac{k}{m}\right)(1-\mathbf{vn})\gamma(v_\alpha n_\beta+v_\beta n_\alpha)\right],$$

where $\delta_{\alpha\beta}$ is the Kronecker δ-symbol ($\delta_{\alpha\beta}=1$ when $\alpha=\beta$, $\delta_{\alpha\beta}=0$ when $\alpha\neq\beta$).

Equations (4), (5), and (8) determine the form of the required electron-photon collision integral under the conditions (1).

We further consider the case one often encounters when in the usual space, characterized by the vector **R**, there is a single preferred direction. In that case the distribution function in momentum space will depend only on the absolute magnitude of the electron velocity v and on $\mu=\cos\theta$, where θ is the angle between the velocity vector and the preferred direction. The electron-photon collision integral can then be simplified. Using the fact that it has the form (4) of a divergence we get:

$$\left(\frac{\delta f}{\delta t}\right)_{e,ph}=\frac{\partial}{\partial v}I_v^{(1)}+\frac{\partial}{\partial\mu}I_\mu^{(1)},\qquad(9)$$

where the quantities $I_v^{(1)}$ and $I_\mu^{(1)}$ describe the electron-photon collisions; their actual form depends on the photon distribution function F. Under the conditions of interest to us there is often strong angular scattering of the photons, i.e., the condition

$$l_{ph}=1/N_e\sigma_{Th}\ll L_{ph}\qquad(10)$$

is satisfied where l_{ph} is the photon mean free path, N_e is the electron density, and L_{ph} is a characteristic scale on which the photon distribution function changes. The photon distribution function is then nearly isotropic and, hence, can be written in the form:

$$F(\mathbf{k},\mathbf{n},\mathbf{R})=F_0(\mathbf{k},\mathbf{R})-l_{ph}(\mathbf{n}\nabla)F_0(\mathbf{k},\mathbf{R})=F_0(1+2\alpha\cos\vartheta),$$

$$F_0(\mathbf{k},\mathbf{R})\equiv\frac{2}{(2\pi\hbar)^3}n_0(\mathbf{k},\mathbf{R}),\qquad(11)$$

where ϑ is the angle between the photon momentum and the gradient of the distribution function, while α is a small parameter in (10):

$$\alpha=-\frac{l_{ph}}{2}\frac{\partial}{\partial R}(\ln F_0).\qquad(12)$$

One must substitute expression (11) into (5), using (8), and integrate over the photon momenta,

$$d^3k=k^2dkd\Omega.$$

Restricting ourselves to the main corrections in the parameters $v\approx p/m$ of (1) and α of (12) and to the zeroth approximation in the parameter k/m of (1) we get from (5) the following expressions for $I_v^{(1)}$ and $I_\mu^{(1)}$:

$$I_v^{(1)}=\nu_{ph}v^3\left(f+\frac{v_{ph}^2\partial f}{v\partial v}-\alpha\mu\left(v+\frac{1}{2v}\right)f\right),\qquad(13)$$

$$I_\mu^{(1)}=\nu_{ph}(1-\mu^2)(v_{ph}^2\frac{\partial f}{\partial\mu}-\frac{\alpha}{2}vf),$$

$$\nu_{ph}=\frac{4}{3}\frac{U\sigma_{Th}}{m},\qquad v_{ph}=\left(\frac{T_{ph}}{m}\right)^{1/2},$$

$$U=4\pi\int k^3F_0dk,\qquad T_{ph}=\frac{1}{4}\frac{\int k^4(n_0+n_0^2)dk}{\int k^3n_0dk}.$$

Here ν_{ph} is the effective electron-photon collision frequency, U the photon energy density, and T_{ph} the effective photon temperature (it is the same as the true photon temperature if their distribution function is a Planck one).

For a complete kinetic description of the electrons we must add to the right-hand side of the kinetic equation (4) the Landau electron-electron and electron-ion collision integrals.[6] Taking into account that significant deformations arise only in the tail of the distribution function we restrict ourselves here to a consideration of only the superthermal electrons:

$$v_{T_e} < v, \tag{14}$$

where v_{T_e} is the mean random electron velocity. This enables us to use the linearized collision operator (4). The linearization takes into account that by virture of (14) the number of superthermal electrons is small so that only their collisions with the bulk of the thermal electrons and ions is important. In that case the terms in the formula which is the analog of (9) take in the weakly relativistic limit the form:[4]

$$\left(\frac{\delta f}{\delta t}\right)_{e,e} + \left(\frac{\delta f}{\delta t}\right)_{e,i} = \frac{\partial}{\partial v} I_v^{(2)} + \frac{\partial}{\partial \mu} I_\mu^{(2)}, \tag{15}$$

$$I_v^{(2)} = \nu_{T_e} v_{T_e}^3 \left(f + \frac{v_{T_e}^3}{v}\frac{\partial f}{\partial v}\right),$$

$$I_\mu^{(2)} = \frac{1+Z}{2}\nu_{T_e}\frac{v_{T_e}^3}{v}(1-\mu^2)\frac{\partial f}{\partial \mu},$$

$$\nu_{T_e} = \frac{4\pi e^4 N_e}{m^2 v_{T_e}^3}\ln\Lambda, \qquad v_{T_e} = \left(\frac{T_e}{m}\right)^{1/2}$$

Here Z is the charge number of the ions and ln Λ is the Coulomb logarithm.

Expressions (9), (13), and (15) completely describe the electron collision integral under the conditions (1) which are of interest to us. We note that the terms in (13) which are independent of α describe the relaxation of the electron distribution function caused by their collisions with the isotropic part of the photon distribution function. These terms are similar to the Coulomb collision operator (15). The difference consists in the energy dependence of the corresponding coefficients. The frequency of the Coulomb collisions decreases with increasing velcotiy as v^{-3} whereas the electron-photon collision frequency is independent of the velocity in the nonrelativistic approximation.

2. PHOTON-ELECTRON SCATTERING INTEGRAL

The kinetic equation for the photons describes the change in their occupation numbers $n(\mathbf{k},\mathbf{R},t)$ due to the usual transfer in space and scattering. The determining factor is then the scattering of the photons by the electrons. The general expression for the Boltzmann collision operator $(\delta n/\delta t)_{\text{ph},e}$ which describes the scattering of photons by electrons has a form similar to (2):

$$\left(\frac{\delta n}{\delta t}\right)_{ph,e} = \int \{f(\mathbf{p}')n(\mathbf{k}')[1+n(\mathbf{k})] - f(\mathbf{p})n(\mathbf{k})[1+n(\mathbf{k}')]\} \times w(\hat{\mathbf{p}}', \hat{\mathbf{k}}'; \hat{\mathbf{p}}, \hat{\mathbf{k}}) d^3p d^3p' d^3k'. \tag{16}$$

The probability $w(\hat{\mathbf{p}}',\hat{\mathbf{k}}';\hat{\mathbf{p}},\hat{\mathbf{k}})$ can be written in relativistically invariant form as follows (see, e.g., Ref. 8):

$$w(\hat{\mathbf{p}}', \hat{\mathbf{k}}'; \hat{\mathbf{p}}, \hat{\mathbf{k}}) = \frac{e^4}{2} R \frac{\delta(\hat{\mathbf{p}} + \hat{\mathbf{k}} - \hat{\mathbf{p}}' - \hat{\mathbf{k}}')}{\mathcal{E}\mathcal{E}' k k'}, \tag{17}$$

$$R = \left(1 + \frac{m^2}{A} - \frac{m^2}{B}\right)^2 + \frac{A}{B} + \frac{B}{A} - 1.$$

Here e is the electron charge, $\mathcal{E}$ the total electron energy, and A and B are kinematic invariants differing from the standard ones[8] only by the normalization and constant corrections:

$$A = \hat{\mathbf{k}}\hat{\mathbf{p}} = k(\mathcal{E} - p\cos\xi) \equiv \hat{\mathbf{k}}'\hat{\mathbf{p}}', \tag{18}$$
$$B = \hat{\mathbf{k}}'\hat{\mathbf{p}} = k'(\mathcal{E} - p\cos\xi') \equiv \hat{\mathbf{k}}\hat{\mathbf{p}}',$$

where ξ is the angle between the vectors $\mathbf{p}$ and $\mathbf{k}$ and ξ', correspondingly, between the vectors $\mathbf{p}$ and $\mathbf{k}'$. The delta-function in (17) describes the energy and momentum conservation laws during the collision

$$\delta(\hat{\mathbf{p}} + \hat{\mathbf{k}} - \hat{\mathbf{p}}' - \hat{\mathbf{k}}') = \delta(\mathcal{E} + k - \mathcal{E}' - k')\delta(\mathbf{p} + \mathbf{k} - \mathbf{p}' - \mathbf{k}').$$

Writing the conservation laws in the form

$$\hat{\mathbf{p}}' = \hat{\mathbf{p}} + \hat{\mathbf{k}} - \hat{\mathbf{k}}'$$

and multiplying both sides of that equation scalarly with themselves we get the relation:

$$A = B + C. \tag{19}$$

Here C is yet another kinematic invariant

$$C = \hat{\mathbf{k}}\hat{\mathbf{k}}' = kk'(1 - \cos\chi) = \hat{\mathbf{p}}\hat{\mathbf{p}}' - m^2,$$

where χ is the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$.

We consider first the "departure" term, i.e., the second term in the curly brackets in (16). We integrate it over the momenta p' of the scattered electrons. After this the energy of the scattered electrons becomes a function of the quantities p, k, and k':

$$\mathcal{E}'^2 = \mathcal{E}^2 + k^2 + k'^2 + 2pk\cos\xi - 2pk'\cos\xi' - 2kk'\cos\chi. \tag{20}$$

Using the fact that $d^3k' = k'^2 dk' d\Omega'$ (where Ω' is the solid angle in the momentum space of the scattered photons) we integrate the expression obtained also over k', removing the remaining δ-function. We then obtain

$$\int (...)\delta(\mathcal{E}' + k' - \mathcal{E} - k)dl' = (...)|_{k'=k'(\mathbf{p},\mathbf{k},\Omega')}\frac{1}{J},$$

where $J = (\partial\mathcal{E}'/\partial k' + 1)$ is the corresponding Jacobian of the transformation for the argument of the function $\delta(\mathcal{E} + k - \mathcal{E}'(k') - k')$. From the energy conservation law and Eq. (20) we have

$$\mathcal{E}'J = \mathcal{E} - p\cos\xi' + k(1 - \cos\chi).$$

From (19) we have also the following relations:

$$k'[\mathcal{E} - p\cos\xi' + k(1-\cos\chi)] = k(\mathcal{E} - p\cos\xi), \tag{21}$$

$$R \equiv \left[1 - \frac{m^2(1-\cos\chi)}{(\mathcal{E} - p\cos\xi)(\mathcal{E} - p\cos\xi')}\right]^2 + \frac{(\mathcal{E} - \cos\xi')}{\mathcal{E} - p\cos\xi' + k(1-\cos\chi)} + \frac{\mathcal{E} - p\cos\xi' + k(1-\cos\chi)}{(\mathcal{E} - \cos\xi')} - 1,$$

$$\frac{k'^2}{kk'\mathcal{E}\mathcal{E}'J} = \frac{k'^2}{k^2(\mathcal{E} - p\cos\xi)}. \tag{22}$$

We now consider the arrival term, i.e., the first term in the curly brackets of (16). After changing the notation p to p' in (16), there remains in (16) everywhere only the function $f(p)$, rather than $f(p')$. However, in that case there occurs at the same time also a change in the expression for the probability (17):

$$w(\mathbf{p}', \mathbf{k}'; \mathbf{p}, \mathbf{k}) \to w(\mathbf{p}, \mathbf{k}'; \mathbf{p}', \mathbf{k}),$$

and, in particular, the form of the conservation laws and of the functional dependence (21) of $\mathbf{k}'$ on $\mathbf{p}$, $\mathbf{k}$, and Ω' also changes. To avoid confusion we introduce here for the momentum of the scattered photon another notation, $\tilde{k}$, instead of k'. We note also that the kinematic invariants change also: A changes to $B(\tilde{k}) \equiv \tilde{B}$ and B changes to $A(\tilde{k}) \equiv \tilde{A}$, while the invariant C remains unchanged. Instead of Eqs. (19) and (21) we now have:

$$\tilde{B} = \tilde{A} + \tilde{C},$$
$$\tilde{k}(\mathscr{E} - p\cos\xi') = k(\mathscr{E} - p\cos\xi) + k\tilde{k}(1 - \cos\chi). \quad (23)$$

After integration of the δ-function, similar to what was done before, the quantity J changes to $\tilde{J} = \partial\mathscr{E}'/\partial\tilde{k} - 1$ and, moreover,

$$\frac{k'^2}{kk'\mathscr{E}\mathscr{E}'J} \to \frac{\tilde{k}^2}{k^2(\mathscr{E} - p\cos\xi)}.$$

The probability kernel R of (17) and (22) changes into an expression $\tilde{R}$ differing from (22) only in the sign in front of k. As a result the collision operator takes the form:

$$\left(\frac{\delta n}{\delta t}\right)_{ph,e} = \frac{e^4}{2k^2}\int f(\mathbf{p})d^3p d\Omega' \times \frac{\tilde{R}\tilde{k}^2 n(\tilde{\mathbf{k}})[1 + n(\mathbf{k})] - Rk'^2 n(\mathbf{k})[1 + n(\mathbf{k})]}{\mathscr{E}(\mathscr{E} - p\cos\xi)}. \quad (24)$$

So far we have not made any approximations in all the transformations so that Eq. (24) is applicable for any energy. We now use the fact that under the conditions (1) of interest to us one can expand in the small parameters p/m and k/m and we shall consider terms of order p/m to be of first order and those of order k/m and $(p/m)^2$ to be of second order. It is clear from (22) that the quantities R and $\tilde{R} = R(-k)$ are up to second-order terms independent of k and are thus the same. We then get instead of (24) the following equation:

$$\left(\frac{\delta n}{\delta t}\right)_{ph,e} = \frac{e^4}{2k^2}\int \frac{f(\mathbf{p})(Z_1 + Z_2)R}{\mathscr{E}} d^3p d\Omega', \quad (25)$$

$$Z_1 = \frac{\tilde{k}^2 n(\tilde{k}, \Omega') - k'^2 n(k, \Omega)}{(\mathscr{E} - p\cos\xi)},$$

$$Z_2 = \frac{n(k, \Omega)[\tilde{k}^2 n(\tilde{k}, \Omega') - k'^2 n(k', \Omega')]}{(\mathscr{E} - p\cos\xi)}. \quad (26)$$

We first consider the expression for Z_1. We use (23) to express the quantity k as function of $\tilde{k}$, Ω', and Ω, introducing for that function the notation $\mathfrak{R}$:

$$k = \mathfrak{R}(\tilde{k}, \Omega, \Omega') = \frac{(\mathscr{E} - p\cos\xi')\tilde{k}}{\mathscr{E} - p\cos\xi + \tilde{k}(1 - \cos\chi)}. \quad (27)$$

It is clear from (21) that k' can be expressed in terms of k similarly to (27),

$$k' = \mathfrak{R}(k, \Omega', \Omega). \quad (28)$$

Moreover, we easily get from (23) the relation

$$\frac{\partial\tilde{k}(k, \Omega, \Omega')}{\partial k} = \frac{\tilde{k}^2}{k^2}\frac{\mathscr{E} - p\cos\xi'}{\mathscr{E} - p\cos\xi}. \quad (29)$$

Using (27)–(29) we can rewrite Z_1 in the form:

$$Z_1 = \frac{1}{\mathscr{E} - p\cos\xi'}[\mathfrak{R}^2(\tilde{k}, \Omega, \Omega')n(\tilde{k}, \Omega')\left(\frac{\partial\tilde{k}}{\partial k} - 1\right) + \mathfrak{R}^2(\tilde{k}, \Omega, \Omega')n(\tilde{k}, \Omega') - \mathfrak{R}^2(k, \Omega, \Omega')n(k, \Omega')] + \frac{\mathfrak{R}^2(k, \Omega, \Omega')}{\mathscr{E} - p\cos\xi'}n(k, \Omega') - \frac{\mathfrak{R}^2(k, \Omega', \Omega)}{\mathscr{E} - p\cos\xi}n(k, \Omega). \quad (30)$$

It is now convenient to carry out an expansion in $\delta\tilde{k}$ in the square brackets in Eq. (30),

$$\delta\tilde{k}(k, \Omega, \Omega') = \tilde{k} - k \approx k\left[\frac{p}{m}(\cos\xi' - \cos\xi)(1 + \frac{p}{m}\cos\xi') + \frac{k}{m}(1 - \cos\chi)\right], \quad (31)$$

$$\mathfrak{R}^2(\tilde{k}, \Omega, \Omega')n(\tilde{k}, \Omega') \approx \mathfrak{R}^2(k, \Omega, \Omega')n(k, \Omega') + \delta\tilde{k}\frac{\partial}{\partial k}[\mathfrak{R}^2(k, \Omega, \Omega')n(k, \Omega')] + \frac{\delta\tilde{k}^2}{2}\frac{\partial^2}{\partial k^2}[\mathfrak{R}^2(k, \Omega, \Omega')n(k, \Omega')]. \quad (32)$$

As a result we have for Z_1:

$$Z_1 \approx \frac{1}{\mathscr{E} - p\cos\xi'}\frac{\partial}{\partial k}\{\delta\tilde{k}(k, \Omega, \Omega')\mathfrak{R}^2(k, \Omega, \Omega')n' + \frac{\delta\tilde{k}^2(k, \Omega, \Omega')}{2}\frac{\partial}{\partial k}[\mathfrak{R}^2(k, \Omega, \Omega')n']\} + \frac{\mathfrak{R}^2(k, \Omega, \Omega')n'}{\mathscr{E} - p\cos\xi'} - \frac{\mathfrak{R}^2(k, \Omega', \Omega)n}{\mathscr{E} - p\cos\xi}, \quad (33)$$

where we have introduced the notation:

$$n = n(k, \Omega), \qquad n' = n(k, \Omega').$$

We now turn to the quantity Z_2 and carry out a similar expansion with the required accuracy; as a result we get:

$$Z_2 \approx \frac{2k^2}{m^3}(1 - \cos\chi)n\frac{\partial}{\partial k}(k^2 n') \equiv \frac{(1 - \cos\chi)}{m^3}\left[\frac{\partial}{\partial k}(k^4 nn') + k^4\left(n\frac{\partial n'}{\partial k} - n'\frac{\partial n}{\partial k}\right)\right]. \quad (34)$$

Similar to (31), the expansion of the quantities occurring in (25), (26), and (33) in the small parameters (p/m) and (k/m) has the form:

$$R \approx 1 + \cos^2\chi - 2\frac{p}{m}\cos\chi(1-\cos\chi)(\cos\xi + \cos\xi')$$

$$+ \frac{p^2}{m^2}(1-\cos\chi)[(1-3\cos\chi)(\cos^2\xi + \cos^2\xi') + 2\cos\xi\cos\xi'(1-2\cos\chi) + 2\cos\chi], \qquad (35)$$

$$\delta\tilde{k}(k,\Omega,\Omega') \approx k\left[\frac{p}{m}(\cos\xi' - \cos\xi) + \frac{p^2}{m^2}\cos\xi'(\cos\xi' - \cos\xi) + \frac{k}{m}(1-\cos\chi)\right],$$

$$\Re^2(k,\Omega,\Omega') \approx k^2[1 + \frac{2p}{m}(\cos\xi - \cos\xi') + \frac{p^2}{m^2}(3\cos^2\xi - 4\cos\xi\cos\xi' + \cos^2\xi') - \frac{2k}{m}(1-\cos\chi)],$$

$$\delta\tilde{k}\Re^2 + \frac{\delta\tilde{k}^2}{2}\frac{\partial}{\partial k}\Re^2 \approx \frac{k^3}{m}\{p(\cos\xi' - \cos\xi)(1 + \frac{p}{m}\cos\xi') + k(1-\cos\chi)\},$$

$$\frac{1}{\mathcal{E}(\mathcal{E} - p\cos\xi)} \approx \frac{1}{m^2}[1 + \frac{p}{m}\cos\xi + \frac{p^2}{m^2}(\cos^2\xi - 1)].$$

Substituting all these expansions into (25) we get the final expression for the collision integral:

$$\left(\frac{\delta n}{\delta t}\right)_{ph,e} = 2\pi r_e^2\int f(\mathbf{p})d^3p\left\langle\frac{1}{k^2}\frac{\partial}{\partial k}\left\{k^3\frac{p}{m}(\cos\xi' - \cos\xi)\mathcal{R}(\Omega,\Omega')n' + \frac{k^4}{m}(1+\cos^2\chi)\left[(1-\cos\chi)n'(1+n) + \frac{p^2}{2m}(\cos\xi' - \cos\xi)^2\frac{\partial n'}{\partial k}\right]\right\} + \tilde{\mathcal{M}}(\Omega,\Omega')n' - \tilde{\mathcal{M}}(\Omega',\Omega)n + \frac{k}{m}(1-\cos\chi)(1+\cos^2\chi)\left(n\frac{\partial n'}{\partial k} - n'\frac{\partial n}{\partial k}\right)\right\rangle_{\Omega'}, \qquad (36)$$

$$\mathcal{R}(\Omega,\Omega') = 1 + \cos^2\chi + \frac{p}{m}(\cos\xi + \cos\xi') \times (1 - 2\cos\chi + 3\cos^2\chi), \qquad (37)$$

$$\tilde{\mathcal{M}}(\Omega,\Omega') = 1 + \cos^2\chi + \frac{p}{m}[2\cos\xi(1-\cos\chi + 2\cos^2\chi) + \cos\xi'(\cos^2\chi - 2\cos\chi - 1)] + \frac{p^2}{m^2}[2\cos^2\xi(2 - 4\cos\chi + 5\cos^2\chi) + \cos^2\xi'(1 - 2\cos\chi + \cos^2\chi) + 4\cos\xi\cos\xi'\cos\chi(\cos\chi - 2) - (1 - 2\cos\chi + 3\cos^2\chi)] - \frac{2k}{m}(1-\cos\chi)(1+\cos^2\chi), \qquad (38)$$

$$\tilde{\mathcal{M}}(\Omega',\Omega) \equiv \tilde{\mathcal{M}}(\Omega,\Omega') + 3\frac{p}{m}(\cos\xi' - \cos\xi)\mathcal{R}(\Omega,\Omega'),$$

where the pointed brackets indicate averaging over angles,

$$\langle\ldots\rangle_{\Omega'} = \frac{1}{4\pi}\int(\ldots)d\Omega'.$$

The integration in Eq. (36) over the electron momentum p acts only on the coefficients depending on p, ξ, and ξ' and the averaging over Ω' on the occupation number n' and coefficients depending on ξ' and χ. Both these operations can be carried out under the derivative sign. Equation (36) thus has the divergence Fokker–Planck part of the flux in mommentum space connected with the weak change of the photon frequency in the scattering by the "heavy" electrons [see (1)] and the integral part in the angular variables caused by the arbitrary angular scattering of the photons. The latter changes sign when we interchange the arguments Ω and Ω'. Hence it is clear that Eq. (36) automatically satisfies the conservation law for the number of photons.

Equations (36)–(38) are valid for any photon and electron distributions over the solid angle Ω. However, often there arise situations when there is a single preferred direction. This enables us considerably to simplify the collision operator (36).

We consider a spherical system of coordinates in momentum space with the axis along the preferred direction. We then have:

$$\cos\xi = \cos\theta\cos\vartheta + \sin\theta\sin\vartheta\cos(\varphi - \psi),$$
$$\cos\xi' = \cos\theta\cos\vartheta' + \sin\theta\sin\vartheta'\cos(\varphi - \psi'),$$
$$\cos\chi = \cos\vartheta\cos\vartheta' + \sin\vartheta\sin\vartheta'\cos(\psi - \psi'),$$

where θ, ϑ, and φ, ψ are the polar and axial angles of the electron and photon distributions, respectively. Since the electron distribution function and the photon occupation numbers are independent of the axial angles φ, ψ, and ψ' we can average over them in (36)–(38). Introducing the notation

$$\mu = \cos\theta, \quad \lambda = \cos\vartheta, \quad \lambda' = \cos\vartheta',$$

to simplify the formulas we get

$$\left(\frac{\delta n}{\delta t}\right)_{ph,e} \approx \sigma_{Th}\int f(p,\mu)\left\langle\frac{1}{k^2}\frac{\partial}{\partial k}\times\left\{k^3\left[\mathcal{A}n' + \mathcal{B}n'(1+n) + \mathcal{P}\frac{\partial n}{\partial k}\right]\right\} + \mathcal{M}(\lambda,\lambda')n' - \mathcal{M}(\lambda',\lambda)n + \mathcal{B}\left(n\frac{\partial n'}{\partial k} - n'\frac{\partial n}{\partial k}\right)\right\rangle_{\lambda'}d^3p, \qquad (39)$$

where

$$\langle\ldots\rangle_{\lambda'} = \frac{1}{2}\int_{-1}^{1}(\ldots)d\lambda'.$$

The coefficients $\mathcal{A}$, $\mathcal{B}$, $\mathcal{P}$, and $\mathcal{M}$ are given by the relations

$$\mathcal{A} = \frac{p}{m}(\lambda' - \lambda)\mu G + \frac{p^2}{m^2}(\lambda'^2 - \lambda)\frac{3\mu^2 - 1}{2} \times [3G - 2(1 + \lambda\lambda')],$$

$$\mathcal{B} = \frac{k}{m}\{G + \frac{3}{8}\lambda\lambda'[3(\lambda^2 + \lambda'^2) - 5(1 + \lambda^2\lambda'^2)]\},$$

$$\mathcal{P} = \frac{p^2k}{3m^2}\{G[1 + \frac{1}{4}(3\mu^2 - 1)(3\lambda^2 + 3\lambda'^2 - 2)] + \frac{3}{8}\lambda\lambda'[3(\lambda^2 + \lambda'^2) - 5(1 + \lambda^2\lambda'^2) - 2(3\mu^2 - 1)(1 + \lambda^2\lambda'^2)]\}, \quad (40)$$

$$\mathcal{M}(\lambda, \lambda') = G + \frac{p}{m}\mu\left\{\frac{3}{8}(3\lambda^2 - 1)(3\lambda'^3 - 5\lambda') + \frac{3}{2}\lambda[\lambda^2(3\lambda'^2 - 1) + 2(1 - \lambda'^2)] - \lambda'\right\}$$

$$+ \frac{p^2}{m^2}\left\{\frac{1}{4}(3\mu^2 - 1)\left[\frac{3}{2}(3\lambda'^2 - 1)(5\lambda^4 - 1) + \frac{1}{4}(3\lambda^2 - 1) \times(3\lambda'^4 - 30\lambda'^2 + 19) + \lambda\lambda'(5 - 3\lambda'^2 - 12\lambda^2 + 4\lambda^2\lambda'^2)\right] - \frac{1}{4}(3\lambda^2 - 1)(3\lambda'^2 - 1) + \frac{1}{2}\lambda\lambda'[1 + 5\lambda^2\lambda'^2 - 3(\lambda^2 + \lambda'^2)]\right\} - 2\mathcal{B},$$

$$G = 1 + \frac{1}{8}(3\lambda^2 - 1)(3\lambda'^2 - 1).$$

Moreover there are the relations:

$$\mathcal{M}(\lambda', \lambda) \equiv \mathcal{M}(\lambda, \lambda') + 3\mathcal{A},$$

$$\langle\mathcal{M}(\lambda', \lambda)n\rangle_{\lambda'} = \left(1 - \frac{p}{m}\mu\lambda - 2\frac{k}{m}\right)n.$$

For our further discussion we write the angular dependence of the photon distribution as a series of Legendre polynomials:

$$n(\lambda, k) = \sum_j P_j(\lambda)n_j(k), \qquad n_j(k) = (2j + 1)\langle n(\lambda, k)P_j(\lambda)\rangle_\lambda. \quad (41)$$

The collision integrals then also decompose into a chain of integrals corresponding to different n. We give the complete expansion in the Appendix, and here we restrict ourselves to the first two terms of the chain:

$$\left(\frac{\delta n_0}{\delta t}\right)_{ph,e} \approx \frac{\sigma_{Th}N}{k^2}\frac{\partial}{\partial k}\left\{k^3\left[\frac{k}{m}\left(n_0 + T_s\frac{\partial n_0}{\partial k} + n_0^2 - \frac{2n_1^2}{15} + \frac{n_2^2}{50} - \frac{3n_3^2}{490}\right) + \bar{V}\frac{n_1}{3} + \frac{\bar{q}}{25}[8n_2 + \frac{k}{7}\frac{\partial}{\partial k}(7n_0 + 2n_2 + 2n_4)]\right]\right\}, \quad (42)$$

$$\left(\frac{\delta n_1}{\delta t}\right)_{ph,e} \approx -\left\{n_0 + \bar{V}\left[k\frac{\partial}{\partial k}\left(n_0 + \frac{n_2}{25}\right) + \frac{6n_2}{25}\right]\right\} + \Lambda_1, \quad (43)$$

$$m\Lambda_1 = 2n_1\frac{\partial}{\partial k}(k^2n_0) - \frac{4}{25}(5n_0 + 2n_2)\frac{\partial}{\partial k}(k^2n_1) + \frac{1}{175}(14n_1 + 9n_3)\frac{\partial}{\partial k}(k^2n_2) - \frac{3}{35}\left(\frac{9}{35}n_2 + \frac{4}{21}n_4\right)\frac{\partial}{\partial k}(k^2n_3),$$

where

$$N = \int fd^3p, \qquad \bar{V} = \frac{1}{N}\int\frac{p}{m}\mu fd^3p,$$

$$T_s = \frac{1}{N}\int\frac{p^2}{3m}fd^3p, \qquad \bar{q} = \frac{1}{N}\int\frac{p^2}{m^2}P_2(\mu)fd^3p,$$

and $P_2(\mu)$ is the second Legendre polynomial of the argument μ.

It is clear from (42) and (43) that the expansion in the Legendre polynomials is rather efficient in the case when we have $l_{\rm ph} \ll L_{\rm ph}$ for the parameters in (10). If we restrict ourselves solely to the zeroth term of the expansion (42) we get

$$\left(\frac{\delta n_0}{\delta t}\right)_{ph,e} \approx \frac{\sigma_{Th}N}{k^2}\frac{\partial}{\partial k}\left[\frac{k^4}{m}\left(n_0 + n_0^2 + T_s\frac{\partial n_0}{\partial k}\right)\right],$$

i.e., the well known Kompaneets equation.[5] The expansion up to the first polynomial corresponds to Eq. (11).

We have thus in this paper obtained a complete set of kinetic equations for the electrons and photons describing the interaction betwen plasma and radiation under the conditions (1). We note that the collision integrals for collisions of the electrons and the photons with each other, (4), (5), (8), and (36)–(39), possess all necessary properties; in particular, they satisfy the energy and momentum conservation laws.

APPENDIX

The equation for the photon-electron collision operator as a chain of equations for the expansion coefficients $n(\lambda,k)$ in the series (40) in Legendre polynomials has the form:

$$\left(\frac{\delta n_i}{\delta t}\right)_{ph,e} \approx \frac{\sigma_{Th}N}{k^2}\frac{\partial}{\partial k}\left[k^3\left(A_i\bar{V} + B_i\bar{q} + \frac{k}{m}C_i + \frac{T_sk}{m}\frac{\partial D_i}{\partial k} + \bar{q}k\frac{\partial E_i}{\partial k}\right)\right] + \alpha_i + \bar{V}\beta_i + \frac{T_s\gamma_i}{m} + \bar{q}\delta_i + \frac{k\rho_i}{m} + \Lambda_i,$$

$$A_i = \frac{n_1}{3}\delta_{i0} - \left(n_0 + \frac{n_2}{25}\right)\delta_{i1} + \left(\frac{n_1}{3} + \frac{3n_3}{14}\right)\frac{\delta_{i2}}{2} - \frac{3n_2}{50}\delta_{i3},$$

$$B_i = \frac{2}{5}\left[\frac{4n_2}{5}\delta_{i0} - \frac{3n_3}{7}\delta_{i1} + \left(\frac{2n_4}{7} - 4n_0\right)\delta_{i2} + n_1\delta_{i3} - \frac{18}{35}n_2\delta_{i4}\right],$$

$$C_i = n_0\delta_{i0} - \frac{2}{5}n_1\delta_{i1} + \frac{n_2}{10}\delta_{i2} - \frac{3n_3}{70}\delta_{i3},$$

$$D_i = n_0\delta_{i0} - \frac{4}{5}n_1\delta_{i1} + \frac{n_2}{10}\delta_{i2} - \frac{3n_3}{35}\delta_{i3},$$

$$E_i = \frac{1}{15}\left[\frac{7n_0 + 2n_2 + 2n_4}{35}\delta_{i0} - (43n_1 + \frac{36}{7}n_3)\frac{\delta_{i1}}{10} + \frac{2}{49}(7n_0 + 100n_2 + 2n_4)\delta_{i2} - \frac{6}{5}(n_1 + \frac{2}{7}n_3)\delta_{i3}\right],$$

$$\alpha_i = \delta_{i0}n_0 + \frac{n_2}{10}\delta_{i2} - n_i,$$

$$\beta_i = -\frac{n_1}{3}\delta_{i0} + \left(2n_0 - \frac{n_2}{25}\right)\delta_{i1} + \left(\frac{3n_3}{7} - \frac{8n_1}{3}\right)\frac{\delta_{i2}}{10} + \frac{6n_2}{25}\delta_{i3} + \frac{in_{i-1}}{2i-1} + \frac{(i+1)n_{i+1}}{2i+3},$$

$$\gamma_i = \frac{1}{5}\left(-2n_1\delta_{i1} - 3n_2\delta_{i2} + \frac{6n_3}{7}\delta_{i3}\right),$$

$$\delta_i = \left[-\frac{1}{5}\left(\frac{4n_1}{3} + \frac{3n_3}{7}\right)\delta_{i1} + 2\left(6n_0 - \frac{n_2}{7} + \frac{n_4}{27}\right)\delta_{i2} - \frac{8}{5}\left(2n_1 - \frac{n_3}{7}\right)\delta_{i3} + \frac{12n_2}{7}\delta_{i4}\right],$$

$$\rho_i = -\left(2\delta_{i0} - \frac{4}{5}n_1\delta_{i1} + \frac{n_2}{5}\delta_{i2} - \frac{3n_3}{35}\delta_{i3}\right) + 2n_i,$$

$$\Lambda_i = 2n_i\frac{\partial}{\partial k}\left(\frac{k^2}{m}n_0\right) - \frac{4}{5}\left(\frac{in_{i-1}}{2i-1} + \frac{(i+1)n_{i+1}}{2i+3}\right)\frac{\partial}{\partial k}\left(\frac{k^2}{m}n_1\right) + \frac{1}{10}\left[\frac{3(i-1)i}{(2i-3)(2i-1)}n_{i-2} + \frac{2i(i+1)}{(2i-1)(2i+3)}n_i + \frac{3(i+1)(i+2)}{(2i+3)(2i+5)}n_{i+2}\right]$$
$$\times\frac{\partial}{\partial k}\left(\frac{k^2}{m}n_2\right) - \frac{3}{70}\left[\frac{5(i-2)(i-1)i}{(2i-5)(2i-3)(2i-1)}n_{i-3} + \frac{3i(i-1)(i+1)}{(2i-3)(2i-1)(2i+3)}n_{i-1} + \frac{3i(i+1)(i+2)}{(2i-1)(2i+3)(2i+5)}n_i + \frac{5(i+1)(i+2)(i+3)}{(2i+3)(2i+5)(2i+7)}n_{i+3}\right]\frac{\partial}{\partial k}\left(\frac{k^2}{m}n_3\right).$$

[1)]Note that in this work only two-particle encounters involving elastic scattering of the interacting particles are considered. Collisions in which a larger number of particles participate and the various inelastic processes, including those in which particles are created or annihilated (bremsstrahlung and inverse bremsstrahlung, creation and annihilation of electron–positron pairs, etc.), can be treated independently by including additional terms in the appropriate kinetic equations.

[2)]Here we use classical statistics for the electrons, assuming that the electron gas is nondegenerate. Hence its density N_e and temperature T_e must satisfy the condition $T_e \gg \hbar^2 N_e^{2/3}/m$.

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