Emission spectrum from the elastic scattering by ions of electrons, oscillating in a superradiant laser field: stochastic harmonic generation

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We show that the emission spectrum of electrons which oscillate in a superradiant laser and which collide elastically with ions has its first well-defined maximum at a frequency which is higher than the laser frequency by a factor which is the ratio of the amplitude of the oscillations to the electron mean free path. The harmonic which is generated with a very broad spectrum has an amplitude which is sufficient for recording and this makes it possible to use this effect as a direct method for diagnosing the medium and the laser field.

1. Recent work on the propagation of superradiant laser radiation in a plasma[1,2] demonstrates the appearance of a self-channelling regime with an increasing field amplitude in the nonlinear channel. Such a field (and smaller ones; the necessary intensity is $I > 5\times 10^{16}$ W/cm^2) leads to an appreciable electron oscillation amplitude which, in turn, guarantees the rapid further ionization and excitation of the ions.[3,4] Apart from the inelastic ionization and excitation processes considerably more frequent elastic "collisions" between electrons and ions then also occur in which the energy of the scattered electron remains unchanged. This distortion of the straight-line motion of an electron in a linearly polarized laser field causes it to radiate (in addition to the emission caused by its oscillations in the laser field). One may expect that the intensity of such radiation will be rather large, since the number of elastic collisions is rather large in a dense plasma—at least two per electron in a period of the laser radiation.[5] Note that one can call this medium a plasma only for simplicity, since the ions in it remain cold during the whole of the laser pulse ($\tau_i < 10^{-12}$ s) while it takes the electron component approximately the same (in a very dense plasma) or a longer time to acquire a Maxwell distribution. The amplitude and spectral characteristics of the emission by the elastically scattered electrons may give important information about the state of the medium and of the laser field. In particular, if the average number of collisions per oscillation period is M we may expect that the center of the spectrum of the radiation considered will correspond to a value $\omega M/2\pi$, where $\omega/2\pi$ is the frequency of the radiation of the laser pump. If the spectrum turns out to be rather narrow, the result of such a physical process can be interpreted as a stochastic generation of harmonics.

2. We consider an electron in a cold ionized medium into which external strong electromagnetic linearly polarized radiation is introduced. We assume that the stochastic component of the electron velocity is small and that their dominant motion will be oscillations in the external field (see Ref. 6 for a quantitative criterion). The electron will collide with ions while oscillating; we denote its acceleration in a single collision by W.

What will be recorded by an appropriate apparatus as a result of this elementary process? Let the apparatus recording the radiation be at a distance R_0 from the active region where the interaction of the superradiant field and the substance takes place. Using present-day laser sources this field may be produced in a very small volume of the substance (even the pulse length in 1 ps is 0.3 mm) so that we assume that R_0 is much larger than the linear dimensions of the active medium. Let the angle between the "ion-electron" dipole moment and the line of observing the radiation be ϑ in the elementary collision process; the field is then scattered into a solid angle dO with an intensity

$$dI = \left(\frac{\partial^2\mathcal{D}}{\partial t^2}\right)^2 \frac{1}{4\pi c^3 R_0^2} \sin^2\vartheta dO \tag{1}$$

at a distance R_0. Here we have $\partial^2\mathcal{D}/\partial t^2 = eW$, $\mathcal{D}$ is the dipole moment, c is the velocity of light, and e is the electron charge. Since the change in the dipole moment is in a direction at right angles to the direction of the oscillations, $\mathcal{D}$ is completely determined by the Coulomb interaction between the ion and the electron.

The average field in the point of observation is clearly equal to zero, so that one must measure the intensity (1) of the radiation and its spectrum. The temporal picture of the radiation intensity when a single electron undergoes collisions will then be a sequence of random pulses. What will be random?

First of all, the amplitude of each pulse is random. Indeed, in a collision in a screened Coulomb potential we have

$$\left(\frac{\partial^2\mathcal{D}}{\partial t^2}\right)_{max} = ev_0^2\left(1-\frac{2d}{r_e}\right)\left\{d\left[\left(1+\frac{D^2}{d^2}\left(1-\frac{2d}{r_e}\right)\right)^{1/2}-1\right]\right\}^{-1}, \tag{2}$$

where we have written $d = Ze^2/mv_0^2$, Z is the ion charge state, m is the electron mass, r_e is the screening radius (defined below), v_0 is the initial (random) electron velocity in the collision, and D is the impact parameter, which is also random. We note that for all intensities which are of interest to us we have $D \gg d$ (the oscillation velocity remains nonrelativistic) for the elastic collision processes, and we also have $D \gg r_e$ by virtue of the physical fact that there are many parti-

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cles in the screening "sphere." Expression (2) then reduces to a simpler one:

$$(\partial^2\mathcal{D}/\partial t^2)_{max} = ev_0^2/D. \tag{2a}$$

We determine the distribution function of the random quantity $(\partial^2\mathcal{D}/\partial t^2)_{max}$. The velocity v_0 has a distribution function which is the usual one for the given class of problems:[3]

$$f(v)dv = 2dv/\pi(v_m^2 - v^2)^{1/2}, \tag{3}$$

where we have $v_m = eE_{max}/m\omega$ and E_{max} is the amplitude of the laser field.

The impact parameter D has a uniform distribution. Its limits can be determined from the following considerations. From a classical point of view the more "exactly" an electron is incident upon an ion, the larger the energy it transfers to the ion and the higher the energy involved in the (inelastic) process it can trigger. For elastic processes the impact distance can thus not be less than some magnitude D_{min} which corresponds to the cross-section of the inelastic process (excitation of an ion) involving the least energy,

$$\sigma_{min} = \pi D_{min}^2.$$

The upper limit for the distribution function of the impact parameter D will be the radius for the screening of an ion by the electrons. One can deduce this radius r_e easily from the same considerations about the distribution of charges in a plasma as the Debye radius (n is the ion density):

$$r_e = [\langle mv_0^2/2\rangle/4\pi ne^2Z^2]^{1/2},$$

where the averaging is carried out with the distribution function $f(v)$ of (3). Taking into account the above physical restrictions, we can write the distribution function of the dimensionless random quantity (2a) in the form

$$f(x) = \frac{C}{x^2}\begin{cases}\arcsin x^{1/2} - \arcsin(\alpha x)^{1/2} + (\alpha x - \alpha^2x^2)^{1/2} + (x - x^2)^{1/2}, & x \le 1\\ \arcsin(1 - \alpha x)^{1/2} + (\alpha x - \alpha^2x^2)^{1/2}, & x \ge 1\end{cases}. \tag{4}$$

Here, as we noted, α is determined by the first inelastic process, $\alpha = (\sigma_{min}/\pi)^{1/2}/r_e$. Since we have $\alpha \ll 1$, it follows that $C \approx 2/\pi(1-\alpha)$. We have thus completely determined the random quantity (2a).

Second, the field at the point of observation is determined not only by the amplitude of the random pulse, but also by its temporal shape (for a two-particle Coulomb collision the length of the pulse is infinite). In dense gases the Coulomb potential is screened, so that it will be very reasonable to assume that the electron collides only with the nearest ions and after that undergoes the next collision, and so on. For such an approach we must choose a model shape of the pulse which should correspond well to the physics of the problem. The simplest and most natural approximation here would be a rectangular pulse. Indeed, there are many collisions per period, while an analysis shows that the velocity and the impact parameter cannot change strongly during a single collision and that the electron goes straight from one collision into another one. The length of the pulse is then a random quantity in exactly the same way as the magnitude of the time interval between pulses.

In the two-particle model of the collisions considered the length of the pulses and the time interval between them are thus independent random quantities which are identical in the sense of distribution functions. These quantities are determined by the relation $\tau = (n\sigma v)^{-1}$, where in principle we must take into account the v dependence of the cross-section σ for elastic scattering. The reciprocal quantity—the collision frequency (this is a physically exact definition for the quantity which is the reciprocal of the time interval) thus has a distribution which is close to (3). It turns out that when we calculate experimentally measurable quantities we need know only the first and the second moments of the quantities τ or $\tau^{-1} = \nu$. The mean collision frequency determines the amplitude of the spectrum and the spread in time of the intervals between them determines its width.

3. Rytov[7] solved the analogous problem of calculating the spectrum of self-oscillations with pulses of random shape and length, and also with random intervals between them. The spectrum of the signal (in our case the spectrum of the intensity of the emission by the electrons) can be written in the form:

$$g(\omega) = 2\pi\nu_{av}a^2\left\{K(\omega)\frac{(\Sigma^2 + a^2)}{a^2} + 2|H(\omega)|^2\,\mathrm{Re}\left[\frac{\Phi(\omega)}{1 - \Phi(\omega)}\right]\right\}, \tag{5}$$

with ν_{av} the average number of pulses per unit time which under our conditions is the same as the average number of collisions of a single electron:

$$\nu_{av} = \langle n\sigma v\rangle, \tag{6}$$

where the averaging is carried out with the function (3);

$$a^2 = 3v_m^4n^{2/3}/16\pi c^3R_0^2\alpha \tag{7}$$

is the mathematical expectation value of the pulse amplitude [this is a dimensional normalization of a dimensionless random quantity using the distribution function (4)], and Σ is the dispersion of the pulse amplitude. One finds easily that $a^2 \gg \Sigma^2$, which we shall use everywhere in what follows. The functions $K(\omega)$ and $H(\omega)$ are[7] the averages of the square of the Fourier transform of the pulse shape and of the Fourier transform of the pulse shape (we bear in mind that the pulse intensities are always positive). Under our assumptions their evaluation gives ($\nu_{av} = 2\nu_m/\pi$):

$$K(\omega) = \frac{1}{\pi \nu_m \omega^2} \int_0^\omega J_0 \left(\frac{\omega}{2\pi\nu_m}\right) d\omega,$$

$$|H(\omega)| = \frac{1}{\pi\nu_m\omega} \left\{ \left[\int_0^\omega J_0 \left(\frac{\omega}{2\pi\nu_m}\right) d\omega \right]^2 + \left[\int_0^\omega Y_0 \left(\frac{\omega}{2\pi\nu_m}\right) d\omega \right]^2 \right\}^{1/2}, \qquad (8)$$

where J_0 and Y_0 are zero-order Bessel and Neumann functions and $\Phi(\omega)$ is the characteristic function of the interval between the pulses.

It is easy to show that for the physically interesting gap in the spectrum $\omega \approx 2\pi\nu_m$ the first term in the curly brackets in (5) is much smaller than the second one, i.e.,

$$g(\omega) = 4\pi\nu_{av} a^2 |H(\omega)|^2 \operatorname{Re}\{\Phi(\omega)/[1 - \Phi(\omega)]\}. \qquad (5a)$$

In Ref. 7 it is shown that when the dispersion of the time intervals between the pulses is not too large the function $\Phi(\omega)$ can be written as an expansion in a Taylor series:

$$\Phi(\omega) = 1 - \omega^2 D(\tau)/2.$$

It is rather complicated to calculate the dispersion $D(\tau)$ directly, since one must construct the distribution function of a random quantity, the reciprocal of which has the distribution function (3). We proceed as follows: the dispersion of the quantity

$$\nu = n\sigma v = \tau^{-1} [f(\nu) = f(nv\sigma)]$$

is

$$D^2(\nu) = \int_0^\infty f(\nu)(\nu - \nu_{av})^2 d\nu = \nu_{av}^2 \int_0 \nu^2 f(\nu)(1/\nu - 1/\nu_{av})^2 d\nu.$$

For a sufficiently narrow distribution we can in this expression replace ν by ν_{av}, and according to the formula for the dispersion of a random quantity we then have

$$D^2(\tau) \approx D^2(\nu)/\nu_{av}^4, \quad D^2(\nu) = \nu_{av}^2(\pi^2/8 - 1).$$

Therefore it follows that

$$\Phi(\omega) \approx 1 - \omega^2 D^2(\tau)/2 = 1 - \omega^2(\pi^2/8 - 1)/2\nu_{av}^2.$$

Then we have

$$2\operatorname{Re}\left[\frac{\Phi(\omega)}{1 - \Phi(\omega)}\right] \approx \frac{[1 - \nu^2 D^2(\tau)/2][\cos(2\pi\nu/\nu_{av}) - 1 + \nu^2 D^2(\tau)]}{[\nu^2 D^2(\tau)/2]^2 - 2[1 - \nu^2 D^2(\tau)/2][\cos(2\pi\nu/\nu_{av}) - 1]}. \qquad (9)$$

The minimum of this function is approximately -1 and its maximum approximately $4/n'^2 D^2(\tau)\nu_{av}^2$ for $2\pi\nu/\nu_{av} = n' = 1,2,3,...$. In actual fact because of the effect of the function $K(\omega)$ the spectrum is equal to zero at the origin, its first maximum is expression (9), and (5a) has the value ≈ 17.2. Hence the spectrum is resolved. Its half-width at half maximum in the vicinity of the first maximum is equal to

$$\delta_1 = D^2(\tau)\nu_{av}^3/2 \approx 0.12\nu_{av}.$$

At the zero level the half-width has the magnitude $\delta_2 = D(\tau)\nu_{av}^2$ and the spectrum is to a large extent smeared out.

Thus, if $M \approx 8$, for

$$\nu_{av} = M\omega/2\pi, \qquad (10)$$

the seventh and tenth harmonics of the laser radiation will be observed in the wings of the spectrum and the emission considered here can then again be called the stochastic generation of harmonics of the laser pump. For larger M neighboring harmonics enter the spectrum, and it is therefore clearly no longer possible to call this emission, but for diagnostics the spectrum is useful (see figure)—the number M, ν_{av}, and, hence, the amplitude of the exciting laser radiation can be determined.

4. We give a few estimates. The explicit expression of the spectrum obtained at the point of observation is a combination of the spectra from all the electrons in the volume of the active medium. We bear in mind that the point of observation is situated in the far zone. We shall assume the laser pulse to have the shape of a cylinder in space (the space-time shape of the pulse does not give a contribution to the spectrum comparable to the ones considered). The total spectrum $G(\omega)$ is

$$G(\omega) = n_e V g(\omega). \qquad (11)$$

Here V is the volume occupied by the pulse and $n_e = Zn$ is the electron density. For $Z = 5$, a pulse cylinder with radius 10^{-3} cm and length 0.3 cm, $n = 2.67 \times 10^{19}$ cm^{-3}, $v_n \approx 0.1c$ (which for a KrF laser corresponds to an amplitude of 1.24×10^{10} W/cm), $\sigma_{min} \approx 10^{-16}$ cm^2, and $R_0 \sim 10$ cm, we have $G_{max} \approx 0.3$ g/s^2. To find the maximum intensity which corresponds to that quantity we make the simple estimate $G(\omega)\Delta\omega \sim I_{max}$, and it is reasonable in the light of what has been said before to estimate that $\Delta\omega \sim \omega$. We then have $I_{max} \approx 10^9$ W/cm^2, which is experimentally well observable.

The maximum in the intensity of the spectrum is determined by the relation $\nu_{av}/(\omega/2\pi) = l_{osc}/l$ where l_{osc} is the amplitude of the electron oscillations and l their mean free path with respect to elastic collisions. The cross-section for elastic collisions in a Yukawa potential with a screening length r_e is always of order r_e^2. In a plasma of moderate density with an ion density less than 10^{21} cm^{-3} the mean free path turns out to be less than the mean distance between ions, which is physically impossible. In such cases we must therefore take for the mean free path the quantity $n^{-1/3}$ and then we have $\nu_{av} = 2v_m n^{1/3}/\pi$ and $M \approx 4.72$, i.e., one should see a distinguishable maximum in the region of the fifth harmonic (see figure).

We make a few remarks about the angular spectrum of the scattered radiation. Since the original laser radiation is linearly polarized the "electron-ion" dipole shows maximum emission in the plane at right angles to the direction of motion of the electron. The directivity diagram of the emission will therefore have a toroidal shape $\sim \sin^2 \vartheta$, where ϑ is the angle between the polarization vector of the laser radi-

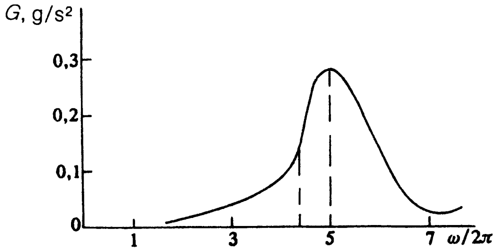

FIG. 1. Emission spectrum of electrons which are scattered elastically by ions in a strong field (see text for the physical parameters of the problem).

ation and the direction of observation. Further analysis is needed to show whether an exact angular spectrum will give information about the medium and about the propagation process of the laser radiation.

The frequency spectrum of the emission occurring when electrons which oscillate in a strong field collide elastically with ions is thus rather well resolved (i.e., it changes by approximately a factor 20 from its minimum to its maximum) and has a limited width, of the order of the frequency of the laser radiation. The maximum of the spectrum determines the average frequency of the collisions of an electron with the ions and the amplitude of the laser field in the medium. Apparently, the proposed method is the first direct means of measuring the amplitude of such a strong field.

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