Absence of stabilization for a particle in a short-range potential

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We present an exact solution of the problem of a particle in a zero-range potential perturbed by a circularly polarized high-frequency ultrastrong electromagnetic field. We show that no stabilization effect is present in this problem, that is, the probability of particle ionization continuously grows with the strength of the field in the electromagnetic wave. In the case of an ultrastrong field there is only a slight decrease in the growth of the ionization probability per unit time (the ionization rate) in comparison with the case of a weak field. For an ultrastrong field of strength F we find that the ionization rate w is proportional to $F^{2/3}$. The results and conclusions are generalized to the case of ground states of atoms in high-frequency ultrastrong fields.

In connection with the interaction between ultrastrong electromagnetic fields and atoms there has been energetic discussion recently of what is known as the stabilization effect (see, e.g., the review in Ref. 1). The essence of the effect is that at a fixed frequency of the electromagnetic radiation the ionization probability first increases with the amplitude of the field strength of the electromagnetic wave and then, in the range of ultrastrong fields, begins to decrease and tends to zero as the field strength tends to infinity. Such a dependence occurs not at just any frequency of the radiation but at a value considerably exceeding the ionization potential of the given system. The higher the radiation frequency, the more dramatically the stabilization effect manifests itself.

The reliability of such results is doubtful because usually the solution of this problem requires a number of various additional assumptions.[2] The ionization of the ground and excited states of the hydrogen atom by a high-frequency field of arbitrary intensity outside the scope of the standard perturbation theory has been investigated most thoroughly.

Two types of stabilization are distinguished. The first has to do with the atomic ionization probability per unit time (the ionization rate), that is, with an analysis of how the imaginary part of the energy of the particular bound state of the atom depends on the field strength in the high-frequency limit. At the same time, of course, the dependence of the real part of the energy of the atomic state on the field strength is studied, namely, the dynamic Stark shift in the high-frequency ultrastrong field with respect to the edge of the continuous spectrum. (The average vibrational energy of a free electron in the field of the electromagnetic wave is added both to the energy of the bound atomic state and to the energy of the edge of the continuous spectrum, thus cancelling out in the difference of these two energies.)

The second type of stabilization concerns to the total ionization probability over the duration of the laser pulse in the regime of strong ionization saturation. Basically, the ionization rate is the primary characteristic. But from the practical viewpoint, for the sake of comparing with experimental data it is important to know the ion yield over the entire period during which the laser pulse acts.

As for the hydrogen atom, a stabilization effect for the ground state and for the excited and highly excited states is observed in both the ionization rate and the total ionization probability over the duration of the laser pulse. This has been established by various approximate methods and by direct numerical solution of the appropriate time-dependent Schrödinger equation.[3] But it leaves open the possibility that stabilization is a consequence of boundary conditions, that is, that an atom is placed in a closed volume of finite dimensions, in which case an electron sink is clearly absent. Another viewpoint meriting attention is that for any atomic system with the field strength tending to infinity and a fixed radiation frequency we must obtain a limit coinciding with that of a constant ultrastrong electric field acting on the system. In the latter case there can be no stabilization.

An example of the results of such calculations is shown in Fig. 1 taken from Ref. 4, which gives the dependence of the ionization rate for the hydrogen atom on the intensity of radiation of frequency $\omega = 0.65$ a.u., a value higher than the ionization potential $\omega = 0.50$ a.u. We see that at low radiation intensities the rate grows with intensity, reaches a maximum at $I_a = 1.1 \times 10^{16}$ W cm^{-2} (the atomic intensity for the given radiation frequency), and finally decreases, tending to zero. The curve in Fig. 1 was obtained in Ref. 4 by expanding in Floquet states, i.e., the Floquet Hamiltonian was diagonalized numerically, and the basic approximation consists of truncating the diagonalized matrix after a finite number of elements. We believe that the stronger the field,

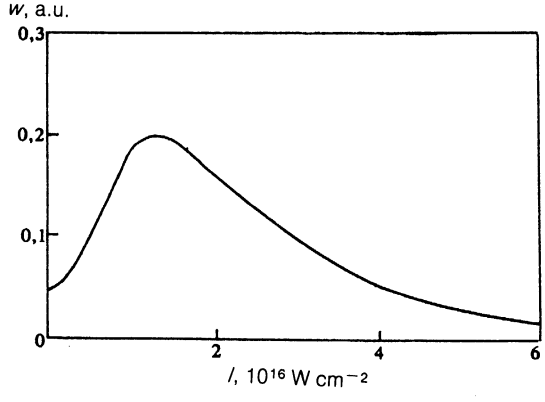

FIG. 1. The ionization rate for the ground state of the hydrogen atom vs the intensity of linearly polarized radiation of frequency $\omega = 0.65$ a.u. according to calculations done in Ref. 4.

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the greater the number of Floquet states that must be taken into account in calculations. This leads to doubts about the reliability of the conclusion arrived at in Ref. 4 regarding the occurrence of stabilization.

In Ref. 5 the same problem was solved by the Kramers–Henneberger method. The method, which consists in going over to a system of coordinates that oscillates together with the free electron placed in the field of the electromagnetic wave, is exact. However, actual calculations use the high-frequency expansion, and even in this approximation results are achieved only numerically. Figure 2 shows the dependence of the ionization rate for the ground state of the hydrogen atom on the intensity of radiation of frequency $\omega = 5$ a.u., which exceeds the atom's ionization potential. The pattern is similar to that of Fig. 1, but $I_a = 200$ a.u. $= 7.2\times 10^{18}$ W cm^{-2}. Thus, the higher the radiation frequency, the greater the atomic intensity. Again, we cannot exclude the possibility that as the intensity grows the number of terms in the high-frequency expansion increases, which lowers the reliability of the results.

Note that the diagrams depicted in Figs. 1 and 2 belong to the case of linearly polarized radiation. With circularly polarized radiation the stabilization effect manifests itself even more strongly.

Reiss[6] employed the so-called strong-field approximation, which consists in taking the wave function of the final state in the form of a Volkov solution. This means that in the final state the effect of the atomic potential on the emitted electron is ignored. The statement is justified by the postulate that the stronger the electromagnetic field the weaker the role of the atomic potential in the final-state wave function. Figure 3 shows the dependence of the lifetime of the hydrogen-atom ground state, which is the inverse of the ionization rate, on the intensity of linearly polarized radiation of frequency $\omega = 1$ a.u. We see that the stabilization effect is present, with $I_a = 1.8\times 10^{17}$ W cm^{-2}. This agrees with the above statement about the frequency dependence of the atomic intensity. However, the extent to which the strong-field approximation is applicable cannot be considered well established.

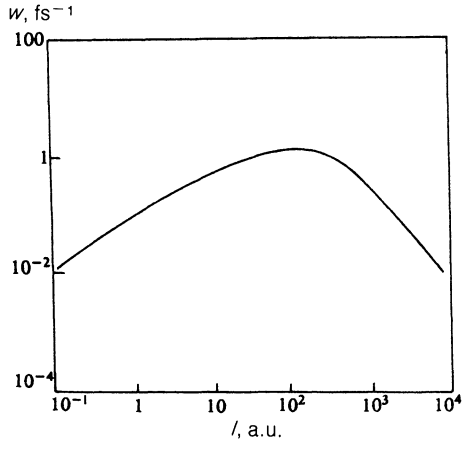

FIG. 2. The ionization rate for the ground state of the hydrogen atom vs the intensity of linearly polarized radiation of frequency $\omega = 4$ a.u. according to calculations done in Ref. 5.

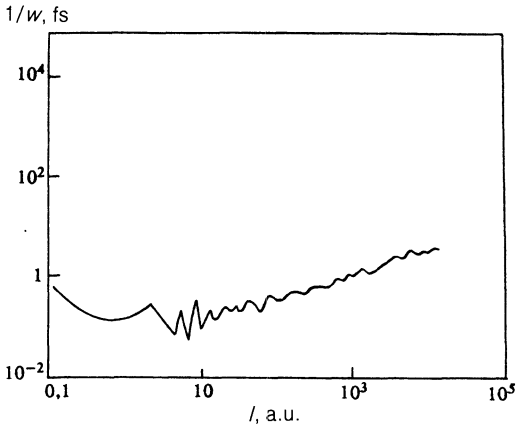

FIG. 3. The lifetime of the ground state of the hydrogen atom vs the intensity of linearly polarized radiation of frequency $\omega = 1$ a.u. according to calculations done in Ref. 6.

Thus, the fact that the ground state of the hydrogen atom in a high-frequency ultrastrong electromagnetic field is stabilized cannot be considered theoretically proven. In contrast to this, there is convincing theoretical reasoning concerning the mechanism of stabilization of highly excited states, reasoning based on the idea of Raman scattering of radiation through states in the continuous spectrum and return to the highly excited states. This reasoning was developed by Fedorov[7] and also in the work of other researchers. There are simple analytical estimates for the ionization rate and the values of atomic intensity as a function of the frequency and energy of a highly excited (Rydberg) state of the atom.

The strong-field approximation discussed above has been used by Reiss[8] to calculate the probability of ionizing a particle from a zero-range potential by a low-frequency circularly polarized field (the radiation wavelength was 10.6 μm). No stabilization was observed. However, a possible explanation could be too low a frequency, corresponding to a practically quasistationary electric field. Besides, at ultrahigh values of the field strength F the probability of ionization, per unit time, from a zero-range potential in the strong-field approximation is, according to Ref. 9, proportional to $F^{1/3}$. As the reader will see, this dependence contradicts the one established as a result of the exact solution with the correct asymptotic behavior $E\sim F^{2/3}$ (Fig. 4), which raises doubts about the applicability of the above approximation for arbitrary field strengths.

Finally, as the data from the literature imply, the nature of the dependence of the ionization rate is greatly influenced by whether the polarization of the radiation is linear or circular.

In this paper we have attempted analytically to solve the question of the possibility of stabilization for a particle in a zero-range potential perturbed by a circularly polarized monochromatic field of an arbitrary strength and frequency.

As is known, for the case of circular polarization there is an exact transcendental equation to determine the complex-valued energy,[10,11] that is, to determine, among other things, the ionization rate as twice the imaginary part of this energy. The equation was obtained in Refs. 10 and 11 in the following form:

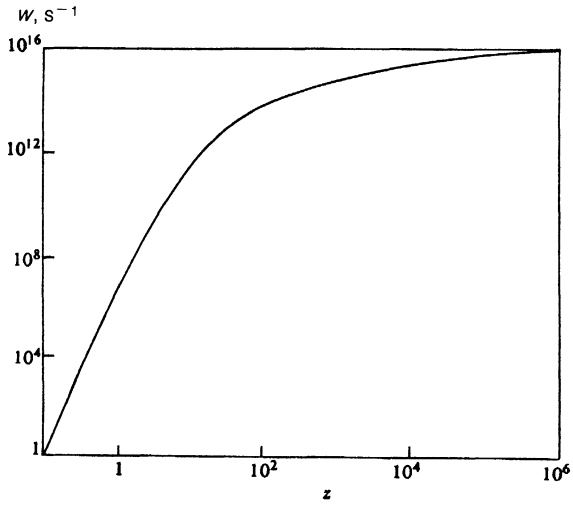

FIG. 4. The ionization rate for a particle in a zero-range potential vs the intensity of a circularly polarized wave (the Reiss intensity parameter z is laid off on the horizontal axis) with wavelength $\lambda = 10.6\,\mu$m, according to calculations done in Ref. 8.

$$(-2E)^{1/2} = (-2E_i)^{1/2} - (2\pi i)^{-1/2}\int_0^\infty dt\, t^{-3/2}\exp(iEt)$$
$$\times\{1 - \exp[-(iF^2t/2\omega^2)(1 - (4/\omega^2t^2)\sin^2(\omega t/2))]\}. \quad (1)$$

Here E is the complex-valued energy of the unique quasistationary level in the zero-range potential perturbed by the circularly polarized electromagnetic field, E_i the level's unperturbed energy (for more details see Ref. 12), F the amplitude of the field strength of the circularly polarized electromagnetic wave, and ω the wave frequency. Here and in what follows we employ a system of units in which $e = h = m_e = 1$.

Analyzing Eq. (1), we see that in contrast to the low-frequency limit $\omega \ll E_i$, where the nature of ionization (multiphoton or tunneling) is determined by the adiabaticity parameter (the Keldysh parameter)

$$\gamma = \omega(-2E_i)^{1/2}/F \quad (2)$$

(see, e.g., Ref. 13), the ionization process in the high-frequency limit $\omega \gg E_i$, to which we restrict our discussion, is determined by the Reiss intensity parameter[14]

$$z = F^2/\omega^3. \quad (3)$$

We start by studying the limit in which this parameter is small, $z \ll 1$. For weak fields we evaluate the integral in Eq. (1) by the saddle-point method and arrive at the following expression for the ionization rate $w = -2\,\mathrm{Im}\,E$:

$$w = (4/3)(-E_i)^{1/2}\omega^{-5/2}F^2. \quad (4)$$

Of course, the same result can be achieved in a simpler way if we start with Fermi's golden rule, that is, remain within the first order of perturbation theory in the field amplitude, and use the condition $\omega \gg E_i$ for the high-frequency limit.

When the Reiss intensity parameter is low, $z \ll 1$, we obtain not only the imaginary part of E but also the real part, which, as expected, obeys the following relation:

$$\mathrm{Re}\,E = E_i + F^2/2\omega^2. \quad (5)$$

The high-frequency Stark shift in this case is equal to the vibrational energy of a free electron in the field of the circularly polarized electromagnetic wave. Note that Eq. (5) is valid both for small Stark shifts $F^2/2\omega^2 < |E_i|$ and for large Stark shifts $F^2/2\omega^2 < |E_i|$. Although in the latter case the dynamic Stark shift proves to be large, it is practically unobservable since not only is the energy of the unique bound state shifted upward but so is the edge of the continuous spectrum (by the same quantity of the electron vibrational energy). The observable quantity is a correction term of the order of F^2/ω^4.

Now let us consider the opposite limit of very large values of the Reiss intensity parameter, $z \gg 1$. This means, first, that we can ignore the term $\sqrt{-2E_i}$ in Eq. (1), with the result that in the limit of a high-frequency ultrastrong field both the real and imaginary parts of energy E are independent of E_i. After this simplification Eq. (1) acquires a self-similar form, which means we can isolate the dependence of E on the field strength F explicitly:

$$E = AF^{2/3}. \quad (6)$$

Substituting this into Eq. (1), we arrive at the following transcendental equation for the complex-valued constant A:

$$(-2A)^{1/2} = -(2\pi i)^{-1/2}\int_0^\infty du\, u^{-3/2}\exp(iAu)$$
$$\times[1 - \exp(-iu^3/24)]. \quad (7)$$

Since ω does not appear in Eq. (7), the equation coincides with the one obtained in the limit of a constant ultrastrong electric field F perturbing a particle that is in a zero-range potential.

Rotating the integration contour in Eq. (7) in the complex u plane through an angle of $-\pi/6$ and introducing a real-valued integration variable x via the relation $u = \exp(-i\pi/6)x$, we find that the phase of the constant A can be established explicitly,

$$A = a\exp(-i\pi/3), \quad (8)$$

where for the real-valued constant a we obtain the real equation

$$(2a)^{1/2} = (2\pi)^{-1/2}\int_0^\infty dx\, x^{-3/2}\exp(ax)[1 - \exp(-x^3/24)]. \quad (9)$$

A numerical solution yields

$$a = 0{,}350. \quad (10)$$

Of course, the dependence given by Eq. (6) is not universal but is determined by the type of potential. For instance, in the case of the ground state of the hydrogren atom a similar dependence of the energy E of this state on the ultrastrong electric field F was obtained in Ref. 15:

$$E = 2^{-5/3}\exp(-i\pi/3)(F\ln F)^{2/3}. \quad (11)$$

There is reason to believe that in a varying field $F \to \infty$ as well

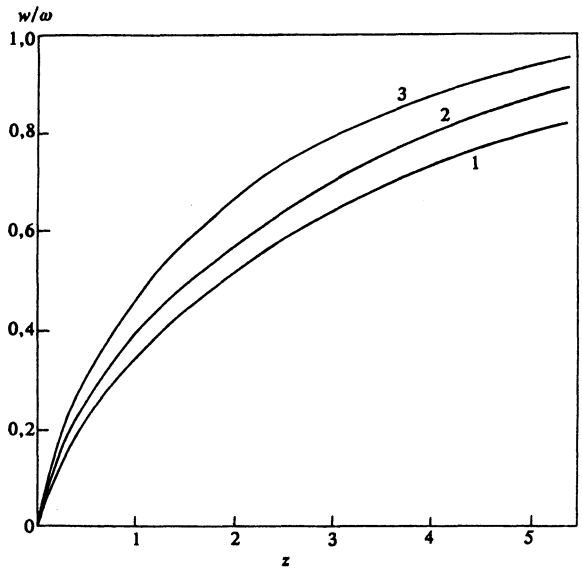

FIG. 5. The ionization rate for a particle in a zero-range potential (in units of the radiation frequency ω) and circularly polarized radiation vs the Reiss intensity parameter z at different values of the binding energy E_i of the unperturbed level. The curves were obtained by solving Eq. (1) numerically: curve *1* corresponds to $E_i/\omega = 0.5$, curve *2* to $E_i/\omega = 1$, and curve *3* to $E_i/\omega = 2$.

the energy of the ground state of the hydrogren atom is determined by (11) rather than tending to zero (see Figs. 1–3), that is, no stabilization is present.

Figure 5 shows how the ionization rate $w = -2\,\mathrm{Im}\,E$ depends on the Reiss intensity paramater $z = F^2/\omega^3$ in the high-frequency limit $\omega \gg E_i$, calculated as a solution to Eq. (1) for three values of E_i. The reader can see that this dependence lacks a peak, that is, the ionization rate in an ultrastrong electromagnetic field continues to grow with field the strength, although somewhat less rapdily.

Thus, we can conclude that no stabilization effect is observed in the ionization rate for a particle in a zero-range potential perturbed by a circularly polarized high-frequency ultrastrong electromagnetic field (to say nothing of the case of a low-frequency field). There is every reason to believe that this remains true for the ground states of atoms (say, of the hydrogen atom). Thus, highly excited (Rydberg) states of an atom, which are absent in a zero-range potential, play the main role in the stabilization process.

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