Velocity selection of atoms in coherent scattering by traveling electromagnetic waves

E. A. Korsunskiĭ, D. V. Kosachev, B. G. Matisov and Yu. V. Rozhdestvenskiĭ

St. Petersburg State Technical University, Russia
(Submitted 10 July 1992)
Zh. Eksp. Teor. Fiz. **103**, 396–416 (February 1993)

We analyze theoretically the velocity selection of multilevel atoms in coherent scattering by traveling light waves. Selection of two-, three-, and four-level atoms is studied in detail. We show that in a certain quantum atomic state narrow velocity structures of atomic density ("atomic jets") may form, and find the conditions for their formation. We estimate the widths of such atomic jets and demonstrate that they can be much smaller than the atomic recoil velocity.

1. INTRODUCTION

Recent years have seen an upsurge of interest in the dynamics of coherent scattering of atomic beams by light waves.[1–4] This is largely due to the possibility of obtaining ultranarrow atomic-density structures with a width much smaller than the atomic recoil velocity $v_R = \hbar k/M$ (the typical values of recoil velocity for optical transitions in alkali-metal atoms range from $V_R \approx 1$ to 3 cm s^{-1}). Hence, in a number of cases such a coherent mechanism of forming narrow velocity distributions can be considered as an alternative for various mechanisms of laser cooling of metals, in which incoherent processes caused by spontaneous relaxation play a decisive role.[5,6]

Coherent scattering by a resonant electromagnetic field always results in the splitting of the atomic wave packet, which is due to the fractional velocity shift of the amplitudes of the probability of an atom existing in different quantum states by $\hbar k$, the resonant-photon momentum. The interference of such probability amplitudes creates an atomic wave packet with several atomic-density maxima.

Two physically different types of coherent atomic scattering can be distinguished: scattering by traveling waves[1–3] and scattering by standing waves.[4] A distinctive feature of scattering of atoms by a standing light wave is the presence of a spatial structure in the light wave, which leads to diffraction of the atomic wave packet with a characteristic size Δz ($\Delta z \approx \lambda \approx \hbar/m\Delta v_\perp$) on the spatial grating generated by the standing wave (the optical analog of the Kapitza–Dirac effect). As a result, narrow ($\approx v_R$) structures form in the velocity distribution, and the parameters of these structures are determined chiefly by the time of flight of the atoms through the interaction region and the initial spread of the transverse atomic velocities in the beam, $\Delta v_\perp$.

The scattering of atoms by traveling light waves leads to velocity structures of an entirely different type: narrow peaks in the velocity distribution of the population of different quantum states, $n_i(v,t)$ (what is known as "atomic jets"; see Ref. 2). Formation of such jets in the process of coherent interaction with traveling electromagnetic waves is due to the velocity selectivity of atom excitation caused by the Doppler effect. There is always a value of the atomic velocity at which the excitation is most effective. For one thing, only at such "resonant" values of the velocity can there be total population inversion in an atom (a π pulse). In other words, at fixed frequency detuning and exciting-field intensity there are certain moments in time when all atoms with a certain velocity value find themselves in one of the quantum states.

Thus, in real ensembles of quantum particle a π pulse is always selective in velocity, and the extent of this selectivity depends only on the intensities of the applied fields and the interaction time. For certain values of the parameters of the exciting waves, the widths δv of the velocity peaks forming can be much smaller than the recoil velocity, $\delta v \ll v_R$ (see Refs. 1–3). Hence, selection of atoms that are in one of the quantum states and have a narrow velocity distribution can be considered an extremely effective mechanism for the formation of atomic beams of a high degree of monochromaticity. For instance, in experiments in Raman scattering of a beam of sodium atoms by traveling light waves, Kasevich *et al.*[1] obtained the widths δv of the velocity distribution of the atoms selected and found that these were smaller by a factor of 100 than the recoil velocity, $\delta v \approx 10^{-2} v_R \approx 3\times 10^{-2}$ cm s^{-1}, which corresponds to an effective temperature $T_{\text{eff}} \approx 10^{-11}$ K.

In this paper we study the processes of atomic jet formation in velocity distributions for different schemes of coherent interaction of atoms with the electromagnetic field and specify the conditions in which the widths of these structures are smaller than the recoil velocity. We start by examining the formation of narrow atomic-density structures in the case of a two-level scheme for the interaction of an atom with a traveling electromagnetic wave. This is followed by a detailed study of the Raman scattering of a beam of three-level Λ atoms.[1–3] It appears that velocity selection of three-level atoms in Raman scattering is preferable, since it allows atomic jets to form on one of the lower (long-lived) states of a three-level atom. It should also be noted that there are many ways in which a three-level atom can interact with the field of traveling waves. In this connection we consider two cases important from the practical viewpoint: scattering by the field of two waves propagating in the same direction, and scattering by the field of waves of different frequencies traveling in opposite directions. We find that in both cases there are conditions that must be met for velocity jets to form in one of the lower states of a Λ atom. There are some differences here, of course, caused by the way the atom-field interaction depends on the direction of the wave vectors of the exciting waves.

We then examine the velocity selection of atoms for the case of double radio-optical resonance, which makes it possible to broaden the class of objects to which such a method of

 1063-7761/93/020210-09$10.00

forming narrow velocity distributions can be applied.

The possibility of carrying out this method of velocity selection for more complicated schemes of atom-field interaction is demonstrated by the example of a four-level double Λ-system, which is currently under intensive investigation.[7] In such a system the total phase of the applied fields has a crucial effect on the population dynamics. This also manifests itself in the formation of atomic jets in one of the lower states of a double Λ-system.

We conclude the paper by discussing the possibility of carrying out two- and three-dimensional velocity selection, which makes it possible to obtain a considerable number of atoms with velocities $v < v_R$ in magnetic storage rings.[8]

We also note that ensembles of atoms with energies considerably lower than the recoil energy $R = \hbar^2 k^2/2M$ (the effective temperature of the ensemble, $T_{\rm eff}$, is roughly 10^{-12} K) are, undoubtedly, of interest to various fields of atomic physics (e.g., atomic interferometry).

2. SCATTERING OF TWO-LEVEL ATOM BY A TRAVELING WAVE

We start with the simple case of scattering of a two-level atom by a traveling light wave. Ignoring spontaneous relaxation in the system and employing the results of Ref. 9, we write the population values of the atomic states $|m\rangle (m = 1,2)$ of a two-level atom as follows:

$$n_1\left(v + \frac{v_R}{2}, t\right) = n_0(v)\left[\frac{(\Omega - kv)^2}{\Delta^2} + \frac{4g^2}{\Delta^2}\cos^2\left(\frac{\Delta t}{2}\right)\right],$$

$$n_2\left(v - \frac{v_R}{2}, t\right) = n_0(v)\frac{4g^2}{\Delta^2}\sin^2\left(\frac{\Delta t}{2}\right),$$

$$\Delta^2 = 4g^2 + (\Omega - kv)^2, \tag{1}$$

where $\Omega = \omega - \omega_0$ is the detuning of the light wave of frequency ω from the atomic $|1\rangle$–$|2\rangle$ transition of frequency ω_0 (Fig. 1a), g is the Rabi frequency, $k = \omega/c$, v is the projection of the atomic velocity on the direction in which the light wave propagates, and $n_0(v)$ is the initial population in the system:

$$n_1(v, t = 0) = n_0(v), \quad n_2(v, t = 0) = 0.$$

Equations (1) show that the probability of discovering an atom in one of the states is a function of velocity v. Total population inversion in an atom ($n_1 = 0$ and $n_2 = n_0$) is possible only at the resonant velocity $v_{\rm res} = \Omega/k$ after a time interval

$$\tau_n = n\pi/g, \quad n = 1, 2, 3, \ldots, \tag{2}$$

which determines the length of the π pulse, has elapsed. For all other velocities the population probability for the second level, n_2, decrease in proportion to v^{-2}. Thus, on level $|2\rangle$ in the course of τ_n a velocity distribution may form (Fig. 1b,c) peaked at $v_0 = \Omega/k + v_R/2$ and with a width δv determined solely by the Rabi frequency:

$$\delta v \approx \frac{2g}{k} = v_R\frac{g}{\omega_R}, \quad \omega_R = \frac{\hbar k^2}{2M}, \tag{3}$$

which implies that for $g \ll \omega_R$ the width δv of such a structure is considerably smaller than the recoil velocity v_R. For example, for optical transitions in metallic atoms the recoil frequency ω_R is roughly 10^5 Hz, and at $g = 0.1\omega_R$ a narrow velocity distribution (an atomic jet) takes $\tau = g^{-1} = 10^{-4}$ s to form in state $|2\rangle$, with the width δv of this distribution being approximately $0.1 v_R \approx 0.1 - 0.3$ cm s^{-1}. Here, as we will shortly see, because of the fractional shift in the populations of levels $|1\rangle$ and $|2\rangle$ by the resonant-photon momentum $\hbar k$, the wave packet of the atom, $w(v) = n_1(v) + n_2(v)$, splits, and at time τ_n has the form depicted in Fig. 1c.

Since time intervals of roughly 10^{-4} s are needed for the formation of distributions with $\delta v \ll v_R$ by the velocity selection method, the lifetime of an atom in the upper state must be of the same order of magnitude. Otherwise such a velocity structure cannot form on the second level. Hence, the velocity selection method based on the scattering of a two-level atom by a traveling wave can be realized in practice in the IR

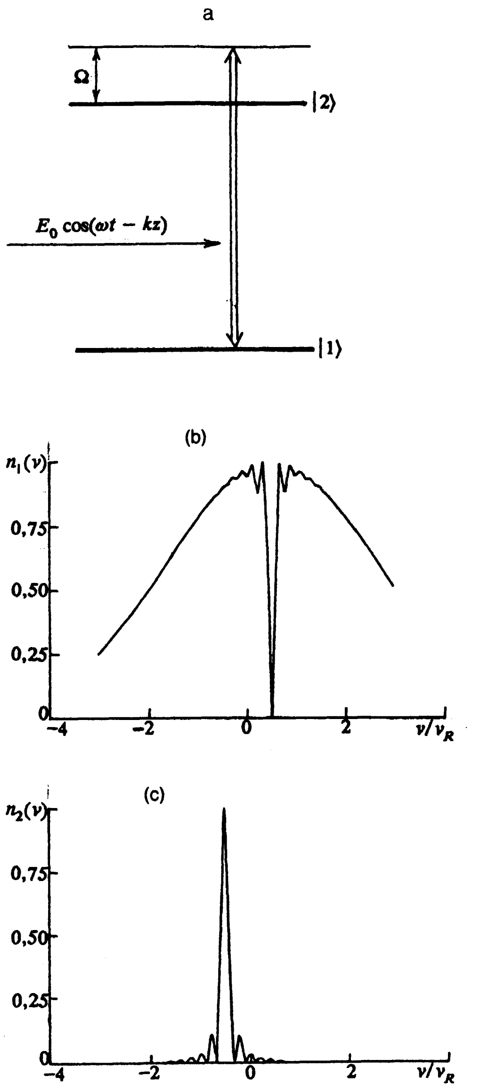

FIG. 1. The velocity selection patterns for the scattering of a two-level atom by a traveling light wave. (a) the transition diagram, (b) and (c) the velocity selection patterns.

region of the spectrum for which the excited state has a lifetime t^* of approximately 10^{-3} s and in optical intercombination transitions in Ca, Mg, and Zn atoms, where the lifetime of the upper excited state, t^*, is also roughly 10^{-3} s. Note that in such a time a thermal beam of atoms covers a distance of only roughly 10 cm. This estimate determines the size of the region where an atom interacts with the field during the velocity selection of an atomic beam.

3. VELOCITY SELECTION IN COHERENT SCATTERING OF THREE-LEVEL ATOMS

Let us now consider velocity selection in the coherent scattering of three-level Λ atoms (Fig. 2) by the field of two traveling waves propagating along the z axis,

$$\mathbf{E} = E_1\mathbf{e}_1\cos(\omega_1 t - k_1 z) + E_2\mathbf{e}_2\cos(\omega_2 t \mp k_2 z), \quad (4)$$

where the E_m are the amplitudes, ω_m are the frequencies, $k_m = \omega_m/c$ are the wave numbers and $\mathbf{e}_m$ are the unit polarization vectors of the applied fields ($m = 1,2$), the upper sign corresponds to the case of waves propagating in the same direction, and the lower sign to oppositely directed waves.

Here we are interested only in the coherent scattering of Λ atoms by field (4). Hence, the state of the system at every instant in time can be described by a wave function Ψ, which we write as an expression in the eigenfunctions $\chi_n(\xi)$ of the unperturbed Hamiltonian $\hat{H}_0$:

$$\Psi(z, \xi, t) = \sum_{n=1,2,3} \psi_n(z, t)\chi_n(\xi)\exp\left(-\frac{i}{\hbar}E_n t\right), \quad (5)$$

where z is the center-of-mass coordinate of an atom, ξ is the set of coordinates of internal movements, E_n are the energies of the unperturbed states of the Λ atom, and the χ_n functions are defined by the following relation:

$$\hat{H}_0\chi_n(\xi) = E_n\chi_n(\xi).$$

The dynamics of coherent scattering is determined by a Hamiltonian $\hat{H}_{\text{int}}$ of the form

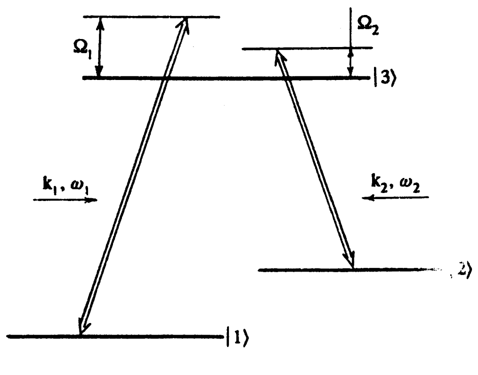

FIG. 2. An atom with a Λ configuration of levels in the field of two traveling electromagnetic waves. The wave with frequency ω_1 and wave vector $\mathbf{k}_1$ interacts with the atom in the $|1\rangle$–$|3\rangle$ transition, and the wave with ω_2 and $\mathbf{k}_2$ interacts with the atom in the $|2\rangle$–$|3\rangle$ transitions.

$$\hat{H}_{int} = -\frac{\hbar^2}{2M}\frac{\partial^2}{\partial z^2} + \hat{V}, \quad (6)$$

where $\hat{V}$ is the operator of the interaction of the atom with the field (4), and M is the atomic mass.

Substituting (5) into the time-dependent Schrödinger equation with the Hamiltonian (6) and introducing new functions a_1, a_2, and a_3 via the relations

$$\psi_1(z, t) = a_1(z, t)\exp[i\Omega_1 t - ik_1 z],$$
$$\psi_2(z, t) = a_2(z, t)\exp[i\Omega_2 t \mp ik_2 z], \quad (7)$$
$$\psi_3(z, t) = a_3(z, t),$$

in the resonance approximation we arrive at a system of equations describing the coherent motion of the Λ atom:

$$i\frac{\partial a_1}{\partial t} = -\frac{\hbar}{2M}\frac{\partial^2}{\partial z^2}a_1 + iv_{R1}\frac{\partial}{\partial z}a_1 + (\Omega_1 + \omega_{R1})a_1 - g_1 a_3,$$

$$i\frac{\partial a_2}{\partial t} = -\frac{\hbar}{2M}\frac{\partial^2}{\partial z^2}a_2 \pm iv_{R2}\frac{\partial}{\partial z}a_2 + (\Omega_2 + \omega_{R2})a_2 - g_2 a_3,$$

$$i\frac{\partial a_3}{\partial t} = -\frac{\hbar}{2M}\frac{\partial^2}{\partial z^2}a_3 - g_1 a_1 - g_2 a_2, \quad (8)$$

where $g_m = (2\hbar)^{-1}\langle\chi_m|\hat{V}|\chi_3\rangle$ is the Rabi frequency, $\Omega_m = \omega_m - (E_3 - E_m)/\hbar$ is the detuning from resonance for the wave with the frequency ω_m ($m = 1,2$), $v_{Rm} = \hbar k_m/M$ is the recoil velocity, and $\omega_{Rm} = \hbar k_m^2/2M$ is the recoil frequency ($m = 1,2$).

Now we go on to the momentum representation of wave functions:

$$a_n(z, t) = \int_{-\infty}^{+\infty} b_n(\varkappa, t)\exp\left[-\frac{i}{\hbar}\mathscr{E}_\varkappa + i\varkappa z\right]d\varkappa, \quad n = 1, 2, 3,$$

$$\mathscr{E}_\varkappa = \hbar^2\varkappa^2/2M.$$

Substituting into (8) yields the following system of equations for b_n:

$$i\dot{b}_1 = (\Omega_1 - y_1)b_1 - g_1 b_3,$$
$$i\dot{b}_2 = (\Omega_2 - y_2)b_2 - g_2 b_3, \quad (9)$$
$$i\dot{b}_3 = -g_1 b_1 - g_2 b_2,$$

where $y_1 = k_1 v - \omega_{R1}$, $y_2 = \pm k_2 v - \omega_{R2}$, and $v = \hbar\varkappa/M$ is the atom's velocity. In Eqs. (7)–(9) the upper sign corresponds, as usual, to the case of light waves propagating in the same direction and the lower sign to oppositely directed waves.

We write the general form of the solution to Eqs. (9) as

$$b_1(v,t) = \sum_{l=1,2,3} A_l(v)\exp(i\lambda_l t),$$

$$b_2(v,t) = \sum_{l=1,2,3} A_l(v)\frac{\lambda_l+\Omega_1-y_1}{\lambda_l+\Omega_2-y_2}\frac{g_2}{g_1}\exp(i\lambda_l t),$$

$$b_3(v,t) = \sum_{l=1,2,3} A_l(v)\frac{\lambda_l+\Omega_1-y_1}{g_1}\exp(i\lambda_l t), \qquad (10)$$

where the λ_l $(l=1,2,3)$ are the roots of the characteristic equation

$$\lambda^3 + (\Omega_1+\Omega_2-y_1-y_2)\lambda^2 + [(\Omega_1-y_1)(\Omega_2-y_2)+g_1^2+g_2^2]\lambda + g_1^2(\Omega_2-y_2)+g_2^2(\Omega_1-y_1) = 0. \qquad (11)$$

Note that the probability amplitudes $B_m(v,t)$ of the quantum states of the atom in momentum space, which are the Fourier images of the $\psi_m(z,t)$,

$$\psi_m(z,t) = \int_{-\infty}^{+\infty} B_m(\zeta,t)\exp\left[-\frac{i}{\hbar}\frac{\hbar^2\zeta^2}{2M}+i\zeta z\right]d\zeta,$$

are shifted with respect to the amplitudes $b_m(v,t)$ by the resonant-photon momenta $\hbar k_m$ $(m=1,2)$:

$$|B_1(v,t)|^2 = |b_1(v+\hbar k_1/M,t)|^2,$$
$$|B_2(v,t)|^2 = |b_2(v\pm\hbar k_2/M,t)|^2, \qquad (12)$$
$$|B_3(v,t)|^2 = |b_3(v,t)|^2,$$

which must be taken into account in establishing the evolution of the wave packet of an atom as a whole. Below, however, we are interested only in the probabilities of finding an atom in certain quantum states. Hence, in what follows we disregard the time-independent velocity shifts in the state amplitudes.

Let us now study the solution (10) separately for light waves propagating in the same $(\mathbf{k}_1\uparrow\uparrow\mathbf{k}_2)$ and opposite $(\mathbf{k}_1\uparrow\downarrow\mathbf{k}_2)$ directions, and for the case of double radio-optical resonance $(k_2\ll k_1)$.

a) Light waves propagating in the same direction ($\mathbf{k}_1\uparrow\uparrow\mathbf{k}_2$)

For waves propagating in the same direction the solution (10) assumes the form

$$|b_1(v,t)|^2 = |b_1(v,0)|^2\left\{1-\frac{2g_1^2g_2^2}{g_0^4}-\frac{4g_1^4}{g_0^2\Delta_0^2}\sin^2\left(\frac{\Delta_0 t}{2}\right) + \frac{2g_1^2g_2^2}{g_0^4}\left[\frac{\Delta_0+\Delta}{2\Delta_0}\cos\left(\frac{\Delta_0-\Delta}{2}t\right)+\frac{\Delta_0-\Delta}{2\Delta_0}\cos\left(\frac{\Delta_0+\Delta}{2}t\right)\right]\right\},$$

$$|b_2(v,t)|^2 = |b_1(v,0)|^2\frac{2g_1^2g_2^2}{g_0^4}\left\{1-\frac{2g_0^2}{\Delta_0^2}\sin^2\left(\frac{\Delta_0 t}{2}\right) - \left[\frac{\Delta_0+\Delta}{2\Delta_0}\cos\left(\frac{\Delta_0-\Delta}{2}t\right)+\frac{\Delta_0-\Delta}{2\Delta_0}\cos\left(\frac{\Delta_0+\Delta}{2}t\right)\right]\right\},$$

$$|b_3(v,t)|^2 = |b_1(v,0)|^2\frac{4g_1^2}{\Delta_0^2}\sin^2\left(\frac{\Delta_0 t}{2}\right), \qquad (13)$$

where it is assumed that $\Omega_m=\Omega$, $k_m=k$, $g_0^2=g_1^2+g_2^2$, $\Delta=\Omega+\omega_R-kv$, $\Delta_0^2=\Delta^2+4g_0^2$ and that the entire population at $t=0$ is concentrated in level $|1\rangle$:

$$|b_1(v,t=0)|^2 = |b_1(v,0)|^2,$$
$$|b_2(v,t=0)|^2 = |b_3(v,t=0)|^2 = 0.$$

This condition can be realized, for instance by the preliminary optical pumping $|1\rangle$ condition (as in experiment in Ref. 1). From (13) it follows directly that the probability of finding an atom in one of its quantum states strongly depends on the translational velocity of the atom. Hence velocities can be specified at which the probability of discovering the atom, say, in state $|2\rangle$ is close to unity while particles with other velocities are practically absent from this state. In other words, in the field of two light waves propagating in the same direction, velocity selection of a beam of Λ atoms can be carried out, as shown in Sec. 2 for the case of coherent scattering of a beam of two-level atoms. However, when velocity selection involves three-level atoms, narrow velocity structures may form [according to (13)] in one of the lower (long-lived) states of a Λ atom, which considerably simplifies the realization of the method in practice.[1,2] Characteristically, the excitation of a Λ atom via the stimulated Raman transitions $|1\rangle\leftrightarrow|3\rangle\leftrightarrow|2\rangle$ can be considered as that of an effectively two-level atom with states $|1\rangle$ and $|2\rangle$ (Ref. 10). The necessary condition for this in our case is the absence of particles from the upper level of the Λ atom, $|b_3(v,t)|^2=0$. According to (13), this is possible only at times τ specified by the condition

$$\Delta_0\tau/2 = \pi n, \quad n=\pm1,\pm2,\ldots \qquad (14)$$

Actually, this condition determines the length of the π pulse in the effectively two-level system with states $|1\rangle$ and $|2\rangle$ in the excitation of the system through the upper state $|3\rangle$.

Assuming that condition (14) is met, we use (13) to establish the probability of finding the atom in the lower states $|1\rangle$ and $|2\rangle$ at $g_1=g_2\equiv g$:

$$|b_1(v,\tau)|^2 = \frac{1}{2}|b_1(v,0)|^2\left[1+(-1)^n\cos\left(\frac{\Delta}{\Delta_0}\pi n\right)\right],$$

$$|b_2(v,\tau)|^2 = \frac{1}{2}|b_1(v,0)|^2\left[1-(-1)^n\cos\left(\frac{\Delta}{\Delta_0}\pi n\right)\right]. \qquad (15)$$

We have assumed, as usual, that initially $(t=0)$ all the atoms were in state $|1\rangle$, or

$$\int_{-\infty}^{+\infty}|b_1(v,0)|^2dv = 1.$$

Equation (15) can be used to determine the velocity of the atoms that are in state $|2\rangle$ after time τ has elapsed. From the requirement

$$|b_2(v_{res},\tau)|^2 = |b_1(v_{res},0)|^2 \tag{16}$$

it follows that the resonant velocity has two values:

$$kv_{res} = \Omega + \omega_R \mp \frac{2\sqrt{2}gm}{\sqrt{n^2 - m^2}},$$

$$n > m;\quad n = 0, \pm 2, \pm 4, \ldots;\quad m = \pm 1; \pm 3, \ldots \tag{17}$$

or

$$n > m;\quad n = \pm 1; \pm 3, \ldots;\quad m = 0, \pm 2, \pm 4, \ldots$$

To each velocity defined by (17) corresponds a specific length of the π pulse (14) for which the probability of discovering an atom in state $|2\rangle$ with the given velocity is close to unity [Eq. (16)], and

$$\tau = 2\pi n/\Delta_0, \tag{18}$$

where $\Delta_0^2 = 8g^2 + (\Omega + \omega_R - kv_{\rm res})^2$. Thus, when a beam of Λ atoms is scattered by the field of traveling waves that propagate in the same direction, the atoms in the beam are selected by velocity. As the interaction time grows, an increasing number of atoms with different velocities participate in the scattering process, and as $t \to +\infty$ atoms with any initial velocity find themselves on level $|2\rangle$ (Fig. 3).

Figure 4 depicts the formation of a narrow velocity structure (an atomic jet) in the scattering process. It shows the probability of state $|2\rangle$ becoming populated after the first π pulse, that is, after the time interval $\tau = \pi/(\sqrt{2}g)$ has elapsed. According to Eqs. (15), the velocity width of such an atomic jet is determined solely by the Rabi frequency,

$$\delta v \approx \sqrt{2}g/k = \sqrt{2}v_R g/\omega_R, \tag{19}$$

and for $g \ll \omega_R$ the width δv is much smaller than the recoil velocity, v_R ($\delta v \ll v_R$).

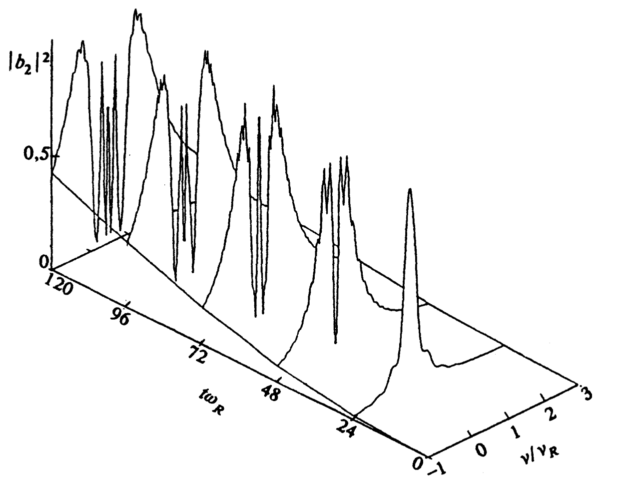

FIG. 3. The temporal evolution of the velocity distribution of the population of level $|2\rangle$ in the case of light waves propagating in the same direction. Initially the entire population is on level $|1\rangle$ with a Gaussian velocity distribution centered at $v_0 = v_R$ and having a width $\Delta v(t=0) = 3v_R$, $g_1 = g_2 = 0.1\omega_R$, and $\Omega_1 = \Omega_2 = 0$.

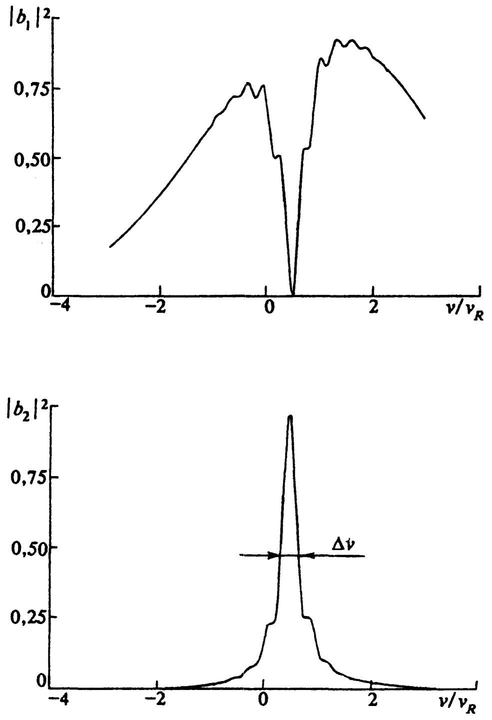

FIG. 4. Velocity selection of Λ atoms in waves propagating in the same direction. The interaction parameters correspond to the first π pulse: $t = 22.2\omega_R^{-1}$, $g_1 = g_2 = 0.1\omega_R$, and $\Omega_1 = \Omega_2 = 0$.

b) Light waves propagating in opposite directions ($k_1 \uparrow\downarrow k_2$)

The picture of resonant scattering of the wave packet of a three-level atom by (two) oppositely directed waves and the way in which state $|2\rangle$ is populated differ considerably from the case of waves propagating in the same direction. The reason lies primarily in the special status of the point $v_{\rm res} = (\Omega_1 - \Omega_2)/2k$ of two-photon resonance on the velocity scale. We demonstrate this status by an example that is important for practical purposes: $\Omega_1 = \Omega_2 \equiv \Omega$, $g_1 = g_2 \equiv g$, $(\Omega + \omega_R)^2 \gg g^2$, and $(\Omega + \omega_R)^2 \gg (kv)^2$. Then the probability of discovering the atom in state $|2\rangle$ is given by Eqs. (10), specifically

$$|b_2(v,t)|^2 \approx |b_1(v,0)|^2 \frac{g^2}{(\Omega+\omega_R)^2} \left\{ \frac{g^2[1-\cos(2kvt)]}{2(kv)^2} + \frac{2g^2 \sin(kvt)}{(\Omega+\omega_R)(kv)} \sin[(\Omega+\omega_R)t] \right\}, \tag{20}$$

where we have put $k_1 \approx k_2 = k$.

We see that level $|2\rangle$ becomes populated most effectively at $v_{\rm res} = 0$ and that for other velocities the probability $|b_2(v,t)|^2$ decreases, according to Eq. (20), in proportion to v^{-2}, as it does for the scattering of a two-level atom. Hence, at every instant in time on level $|2\rangle$ there exists a velocity peak centered at $v_{\rm res}$. This fact is well illustrated by Fig. 5, where we have displayed the temporal evolution of the population of level $|2\rangle$ at $\Omega_1 = \Omega_2 = 0$. The width of the atomic jet formed in this case,

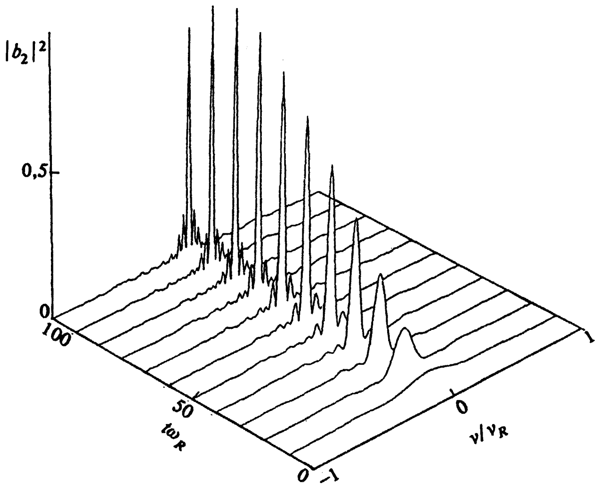

FIG. 5. The temporal evolution of the velocity distribution of the population of level $|2\rangle$ in the case of oppositely propagating light waves. The other parameters are the same as in Fig. 3.

$$\delta v = \frac{\sqrt{3}}{t}\left[1 - \frac{1-\cos(xt)}{x^2t^2}\right]^{1/2} k^{-1},$$

$$x = 2g^2/(\Omega + \omega_R) \quad (21)$$

is primarily determined by the interaction time and depends only weakly on Rabi frequencies and the detunings of the applied fields. Note that for $t \gg \omega_R^{-1}$, where $\omega_R = kv_R/2$ is the recoil frequency, with $\omega_R \approx 10^4$–10^5 Hz for metal atoms in optical transitions, we have $\delta v \ll v_R$.

The intensity of the velocity peak centered at $v_{\rm res} = 0$ oscillates in time like

$$|b_2(v=0,t)|^2 \approx |b_1(v=0,t=0)|^2[1-\cos(xt)] \quad (22)$$

and assumes its maximum value at moments

$$\tau_n = \pi n|\Omega + \omega_R|/(2g^2), \quad n = 1, 2, \ldots, \quad (23)$$

which correspond to the π pulses that link levels $|1\rangle$ and $|2\rangle$. Thus, the highest velocity peak with a width $\delta v \ll v_R$ is obtained by choosing interaction parameters that satisfy the condition

$$\tau \approx \pi n|\Omega + \omega_R|/(2g^2) \gg \omega_R^{-1},$$

or

$$\frac{2g^2}{\pi n|\Omega + \omega_R|} \ll \omega_R.$$

For $\Omega \lesssim \omega_R$ this condition reduces to the requirement that the Rabi frequency be small, $g \ll \omega_R$, as it does in the case of waves propagating in the same direction. On the other hand, for large detuning values, $\Omega \gg \omega_R$, the condition for the existence of a narrow ($\delta v \ll v_R$) and high velocity peak in state $|2\rangle$ assumes the form $g^2/(\Omega\omega_R) \ll 1$, with the result that such a structure can also be observed for Rabi frequencies considerably higher than ω_R. It was precisely this that made possible successful velocity selection involving a beam of sodium atoms and oppositely propagating light waves.[1] Here the appreciable detuning of the laser beams $[\Omega \gg \omega_R, g, (kv)]$ is extremely important since, according to the above estimate, this allows us, on the one hand, to obtain narrow ($\delta v \ll v_R$) velocity distributions of the selected atoms for $g \gg \omega_R$ and, on the other, to ensure that the velocity selection method is applicable.

We note also that Refs. 2 and 3 analyze some aspects of the scattering of a three-level atom by the field of oppositely propagating waves.

In addition to the central peak at $v_{\rm res}$, the velocity distribution of atoms in state $|2\rangle$ contains side peaks (Fig. 5) whose parameters are determined chiefly by the size of the "generalized detuning" $(\Omega + \omega_R)$. Equation (20) also implies that the side peaks emerge at times

$$\tau = \pi n/|\Omega + \omega_R| \quad (n = 1, 2, \ldots) \quad (24)$$

at the following points on the velocity scale:

$$v_{mn} = \frac{2m+1}{2n}|\Omega + \omega_R| \quad (m = 0, \pm 1, \pm 2, \ldots). \quad (25)$$

The height of these peaks is

$$|b_2|^2_{mn} = |b_1(v_{mn}, 0)|^2 \frac{4n^2}{(2m+1)^2} \frac{g^4}{(\Omega + \omega_R)^4},$$

and the width is

$$\delta v \approx \frac{(2m+1)}{2nk}|\Omega + \omega_R| = \frac{\pi(2m+1)}{2\tau k}. \quad (26)$$

Last, we consider one more case of scattering of a Λ atom by the field of oppositely propagating waves when a π pulse selective in velocity can be realized. This corresponds to equal Rabi frequencies, $g_1 = g_2 \equiv g$, and "mirror" detuning $\Omega_1 + \omega_R = -(\Omega_2 + \omega_R)$. With such parameters the population dynamics in the Λ-system drastically simplifies. For instance, if initially all atoms are in state $|1\rangle$, the probability of discovering an atom in state $|2\rangle$ that is moving with velocity is

$$|b_2(v,t)|^2 = |b_1(v,0)|^2 \frac{g^4}{\varepsilon^4}[1 - \cos(\varepsilon t)]^2, \quad (27)$$

with $\varepsilon^2 = 2g^2 + (\Omega_1 + \omega_R)^2$. Hence, the velocity distribution of the atoms in state $|2\rangle$, at times

$$\tau = (2n+1)\pi/(\sqrt{2}g) \quad (n = 0, 1, 2, \ldots), \quad (28)$$

corresponding to the π pulse, will acquire peaks with a width

$$\delta v \approx \sqrt{2}gk^{-1}, \quad (29)$$

and centered at $v_{\rm res} = (\Omega_1 + \omega_R)k^{-1}$. Formation of an atomic jet in the case of "mirror" detuning in oppositely directed waves is shown in Fig. 6.

c) Double radio-optical resonance ($k_2 \ll k_1$)

Now we consider the velocity selection of atoms in the case of double radio-optical resonance (DROR). It is assumed that $|1\rangle$–$|3\rangle$ is an optical transition and $|2\rangle$–$|3\rangle$ is a radio-frequency transition. Atoms that have the required level configuration are, for instance, Zn, Ca, Mg, Cd, and Hg, with the frequency of the optical intercombination transition belonging to the blue band of the visual spectrum and the frequency of the radio-frequency transition amounting

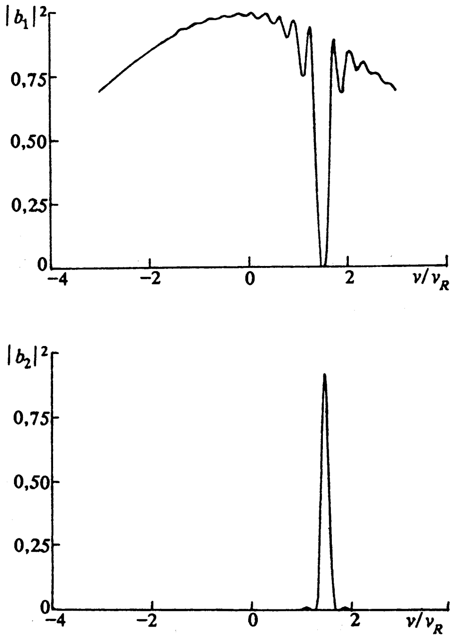

FIG. 6. Velocity selection of Λ atoms in oppositely propagating waves in the case of "mirror" detuning: $\Omega_1 = \omega_R$ and $\Omega_2 = -3\omega_R$. The other parameters are the same as in Fig. 3.

to several terahertz. Here $k_2 \ll k_1$, whence the condition for a two-photon resonance is met at $v_{\text{res}} = (\Omega_1 - \Omega_2 + \omega_{R1})/k_1$. Note that as with oppositely directed light waves, the state $|2\rangle$ becomes populated here most effectively only at a single value of the resonant velocity v_{res}. At $g_1 = g_2 \equiv g$ and $\Omega_1 = \Omega_2 = 0$ Eqs. (10) yield the following expression for the population of state $|2\rangle$:

$$|b_2(v,t)|^2 = |b_1(v,0)|^2 \left[\frac{g^2}{3\Delta_*^2(3\sin^2\varphi - \cos^2\varphi)}\right]^2 \times \{[\cos(\sqrt{3}\Delta_* t\cos\varphi) - \cos(3\Delta_* t\sin\varphi)]^2 + [\sqrt{3}\sin(\sqrt{3}\Delta_* t\cos\varphi)\operatorname{tg}\varphi - \sin(3\Delta_* t\sin\varphi)]^2\}; \qquad (30)$$

where

$$\Delta_*^2 = \frac{1}{3}\left[2g^2 + \frac{1}{3}(\omega_{R1} - k_1 v)^2\right],$$

$$\varphi = \frac{1}{3}\operatorname{arctg}\left\{\sqrt{\frac{3}{2}}\frac{(\omega_{R1} - k_1 v)[g^2 - \frac{2}{9}(\omega_{R1} - k_1 v)^2]}{4g[g^2 + \frac{3}{16}(\omega_{R1} - k_1 v)^2]}\right\}.$$

At $v = v_{R1}/2$ Eq. (30) yields

$$\left|b_2\left(\frac{v_{R1}}{2}, t\right)\right|^2 = \frac{1}{4}\left|b_1\left(\frac{v_{R1}}{2}, 0\right)\right|^2 [1 - \cos(\sqrt{2}gt)]^2$$

and at time

$$\tau = (2n+1)\pi/(\sqrt{2}g) \quad (n = 0, 1, \ldots) \qquad (31)$$

all atoms with a velocity $v_{\text{res}} = v_{R1}/2$ find themselves on level $|2\rangle$. The width of the atomic jet formed at these times is determined by the Rabi frequency:

$$\delta v \approx 2\sqrt{2}gk_1^{-1}.$$

Here for atoms with velocities $v \neq v_{\text{res}}$ the population probability of level $|2\rangle$ decreases in proportion to v^{-2} [see Eq. (30)]. The temporal evolution of the velocity distribution $|b_2(v,t)|^2$ for the case of double radio-optical resonance is depicted in Fig. 7.

Note that a special feature of velocity selection in the event of double radio-optical resonances is the use of only one laser field for forming atomic jets.

We have shown, therefore, that narrow velocity structures form in the lower quantum states of a Λ atom for different configurations of the light field (waves propagating in the same direction and in opposite directions). Moreover, velocity selection of atoms can also occur in the case of double radio-optical resonance, which broadens the class of objects for which this method of forming narrow velocity distributions may be employed. We have established the width of the velocity structures and the time interval during which such structures form and found the conditions for obtaining narrow velocity distributions with widths $\delta v \ll v_R$.

4. VELOCITY SELECTION OF ATOMS WITH A CLOSED EXCITATION CONTOUR

Let us now consider the formation of narrow velocity distributions for systems of atomic levels in which a closed excitation contour can be specified.[7] By way of an example we study atomic jet formation in one of the lower states of what is known as a double Λ-system. The excitation pattern represents two Λ-systems whose lower levels are common and the transitions interacting with the field constitute a closed contour: $|1\rangle - |3\rangle - |2\rangle - |4\rangle - |1\rangle$ (Fig. 8).

It is known[7] that the population dynamics in systems with a closed interaction contour is solely determined by the value of the total phase Φ of the atomic contour. Naturally, the value of Φ must also affect the formation of atomic jets in a double Λ-system. Assuming that the system interacts with the field of four pairwise oppositely directed light waves,

$$\mathbf{E} = \sum_{\substack{m=1,2\\ l=3,4}} \mathbf{e}_{ml}E_{ml}\cos[\omega_{lm}t + (-1)^m k_{lm}z + \eta_{lm}], \qquad (32)$$

where $\mathbf{e}_{ml}$ is the unit polarization vector, E_{ml} is the amplitude ω_{lm} is the frequency, η_{lm} is the phase of the wave that is in resonance with the $|m\rangle - |l\rangle$ transition with a frequency ω_{lm}^0, and the k_{lm} are the lengths of the wave vectors of the fields ($m = 1,2$ and $l = 3,4$), we can write the following system of equations for the probability values in the momentum representation:

$$i\dot{b}_1 = (\Omega_{31} + \omega_R - kv)b_1 + g_{13}^0 b_3 + g_{14}^0 b_4 \exp(-i\Phi),$$
$$i\dot{b}_2 = (\Omega_{32} + \omega_R + kv)b_2 + g_{23}^0 b_3 + g_{24}^0 b_4,$$
$$i\dot{b}_3 = g_{23}^0 b_2 + g_{13}^0 b_1,$$
$$i\dot{b}_4 = (\Omega_{32} - \Omega_{42})b_4 + g_{14}^0 b_1 \exp(i\Phi) + g_{24}^0 b_2. \qquad (33)$$

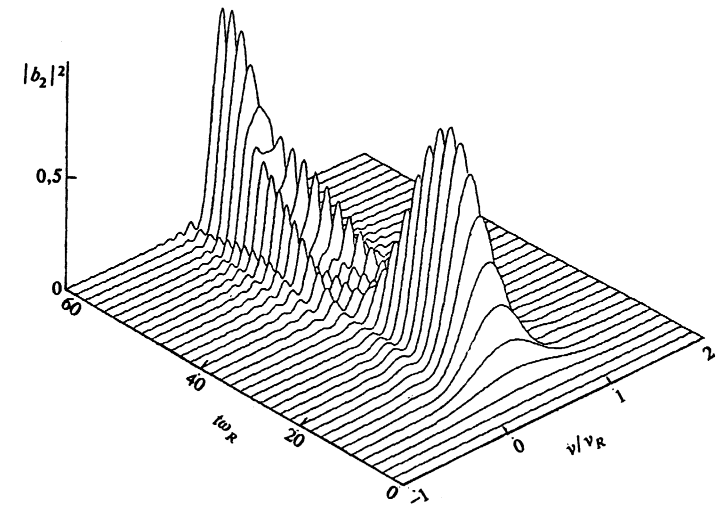

FIG. 7. The temporal evolution of the velocity distribution of the population of level $|2\rangle$ in the case of double radio-optical resonance. The other parameters are the same as in Fig. 3.

Here the $g^0_{ml} = (2\hbar)^{-1} V^0_{ml}$ are the Rabi frequencies of the applied fields, and the amplitudes V^0_{ml} of the matrix elements of the operator of the dipole interaction with the field are specified via the relation

$$V_{lm} \equiv \langle l|\hat{d}\mathbf{E}|m\rangle = V^0_{lm}\exp\{i[\Omega_{lm}t + (-1)^m k_{lm}z + \varphi_{lm}]\},$$

where $\Omega_{ml} = \omega_{ml} - \omega^0_{ml}$, $\varphi_{ml} = \theta_{ml} + \chi_{ml}$ (θ_{ml} is the phase of the matrix element of the operator of the atomic dipole moment), and the total phase of the atomic contour is defined as $\Phi = \varphi_{13} - \varphi_{23} + \varphi_{24} - \varphi_{41}$. It is also assumed that the wave vectors of the exciting waves have the same length, $k_{lm} = k$, and that the condition for multiphoton resonance is met:

$$\Omega_{31} - \Omega_{32} = \Omega_{41} - \Omega_{42}.$$

Note that the procedure that led to the system (33) is similar to that discussed in Sec. 3 in connection with a three-level Λ atom.

Equations (33) clearly show that the population dynamics in a double Λ system and hence the formation of atomic jets in the lower quantum state caused by a π pulse selective in velocity depend essentially on the total phase Φ of the atomic contour. Figure 9 depicts the population $|b_2(v,t)|^2$ of level $|2\rangle$ as a function of atomic velocity at the time corresponding to the first π pulse. Here it is assumed that initially the population is concentrated entirely on level $|1\rangle$ and has a Gaussian atomic-velocity distribution. The reader can see (Fig. 9a) that on level $|2\rangle$ at $\Phi = 0$ a narrow

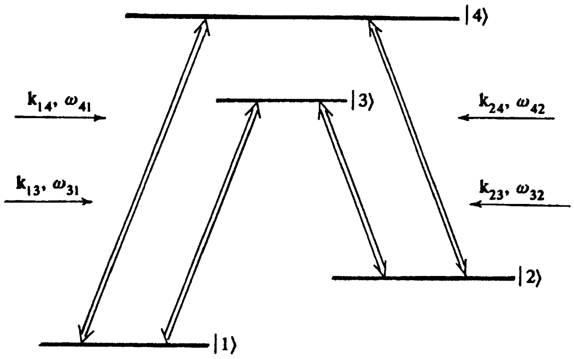

FIG. 8. A double Λ-system. The closed contour is formed by the $|1\rangle-|3\rangle$, $|3\rangle-|2\rangle$, $|2\rangle-|4\rangle$, and $|4\rangle-|1\rangle$ transitions resonantly interacting with the fields. The fields interacting with the $|1\rangle-|3\rangle$ and $|1\rangle-|4\rangle$ transitions are propagating along the z axis in the positive direction, and the fields interacting with the $|2\rangle-|3\rangle$ and $|2\rangle-|4\rangle$ transitions are propagating along the z axis but in the negative direction.

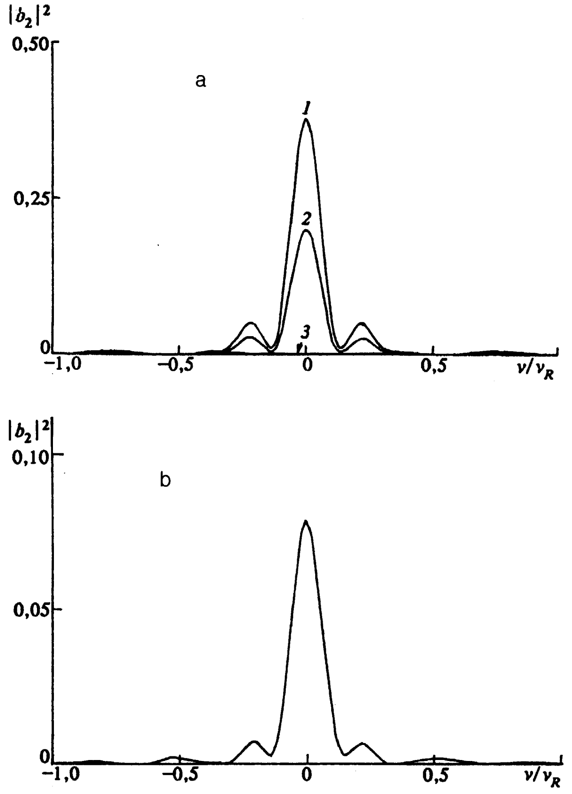

FIG. 9. The velocity distribution of the level $|2\rangle$ population of a double Λ-system. The interaction time t is $22.2\omega_R^{-1}$ and all detunings are the same: $\Omega_1 = \Omega_2 = 0$. The initial distribution is with $v_0 = v_R$ and $\Delta v(t=0) = 3v_R$. (a) $g_{ml} = 0.1\omega_R$ ($m = 1,2$ and $l = 3,4$); curve *1* corresponds to $\Phi = 0$, curve *2* to $\Phi = \pi/2$, and curve *3* to $\Phi = \pi$. (b) $g_{m3} = 0.1\omega_R$, $g_{m4} = 0.2\omega_R$ ($m = 1,2$), and $\Phi = \pi$.

peak appears at $\Phi = 0$ with a width δv that is much smaller than v_R (as in a common Λ-system). At $\Phi = \pi/2$ level $|2\rangle$ has a smaller population and at $\Phi = \pi$ level $|2\rangle$ has none at all ($|b_2|^2 = 0$). This feature of excitation of a double Λ-system at $\Phi = \pi$ is caused by the destructive interference of the $|1\rangle \to |3\rangle \to |2\rangle$ and $|1\rangle \to |4\rangle \to |2\rangle$ excitation channels. Since both channels have the same intensity (all Rabi frequencies are equal) and are antiphased ($\Phi = \pi$), population oscillations between levels $|1\rangle$ and $|2\rangle$ are completely suppressed.

If the excitation via the channels $|1\rangle \to |3\rangle \to |2\rangle$ and $|1\rangle \to |4\rangle \to |2\rangle$ is made asymmetric (Fig. 9b), a narrow population peak forms on level $|2\rangle$ at $\Phi = \pi$, too. In this case there is no suppression of population oscillations between levels $|1\rangle$ and $|2\rangle$.

Note, finally, that the effect of the phase of the atomic contour on velocity selection suggests using atoms with a closed excitation contour for atomic interferometers, where the splitting of the wave packet of an atom caused by velocity selection[11] is controlled, say, by the phase of the optical fields.

5. CONCLUSION

We have thus discussed in detail the formation of narrow velocity structures in one of the quantum states when atoms are scattered coherently by the field of traveling waves. We have demonstrated that such a method of velocity selection can work for both two-level atoms and three-level atoms excited by means of stimulated Raman transitions. Moreover, we have illustrated the possibility of velocity selection in a four-level double Λ-system, which further broadens the range of application of this method.

Note that the width of the narrow structures formed is determined only by the Rabi frequencies of the applied fields and the interaction time, and does not depend on the width of the initial velocity distribution of the atoms [see Eqs. (3), (19), (21), and (29)]. Approximating the initial and selected distributions by rectangular ones, we can estimate the number of atoms that form an atomic jet via the formula $N_s \simeq N_0(\delta v_s/\delta v_0)$, where N_0 is the initial number density of the atoms, N_s is the number density of the selected atoms, and δv_0 and δv_s are the widths of the respective velocity distributions. For instance, if atoms in state $|2\rangle$ with a distribution of width $\delta v_s \approx 10^{-2} v_R$ are selected from an initial distribution of width $\delta v_0 \approx 10^2 v_R$ (this corresponds to an initially cooled atomic beam), the intensity of the beam of selected atoms is lower than that of the initial beam by a factor of 10000 (the same values were observed in the experiments of Kasevich *et al.*[1]) Such a small number of atoms imposes stringent requirements on the experimental technique. The selected atoms can be accumulated in magnetic storage rings[8] because they are all in the same quantum state with a definite magnetic moment.

The calculations conducted in this paper apply to a rectangular field envelope, which means that the edges of the π pulses are fairly sharp. For smooth envelopes the population dynamics in an atom is determined by the parameters

$$\theta_i(t) = \int_{-\infty}^{t} E_i(t') \frac{\mu_i}{\hbar} dt',$$

where the μ_i are the dipole moments of the acting transitions in the atom, which expresses the well-known area theorem (see, e.g., Ref. 12). Needless to say, in this case in the atomic systems considered here there can be a π pulse selective in velocity. The width of the atomic jets formed is determined by the peak value of the amplitude of the applied field and is much smaller than the atomic recoil velocity for $E_i(t)\mu_i/\hbar \ll \omega_{Ri}$ (for all t and i).

Finally, we note that the method of velocity selection discussed here can be used not only to obtain ultranarrow velocity distributions but also as an extremely precise instrument in atomic optics and in atomic[11] and molecular[13] interferometry.

[1]M. Kasevich, D. S. Weiss, E. Rüs, K. Moler, S. Kasapi, and S. Chu, Phys. Rev. Lett. **66**, 2297 (1991).

[2]E. A. Korsunskiĭ, D. V. Kosachev, B. G. Matisov, and Yu. V. Rozhdestvenskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **55**, 313 (1992) [JETP Lett. **55**, 311 (1992)].

[3]K. Moler, D. S. Weiss, M. Kasevich, and S. Chu, Phys. Rev. A **45**, 342 (1992).

[4]P. E. Moskowitz, P. L. Gould, S. R. Atlas, and D. E. Pritchard, Phys. Rev. Lett. **51**, 370 (1983); A. P. Kazantsev, G. A. Ryabenko, G. I. Surdutovich, and V. P. Yakovlev, Phys. Rep. **129**, 75 (1985); A. P. Kazantsev, G. I. Surdutovich, D. O. Chudesnikov, and V. P. Yakovlev, J. Opt. Soc. Am. B **6**, 2130 (1989); S. Glasgow, P. Meystre, M. Wilkens, and E. M. Wright, Phys. Rev. A **43**, 2455 (1991).

[5]A. Aspect, E. Arimondo, R. Kaiser, N. Vansteenkiste and C. Cohen-Tannoudji, Phys. Rev. Lett. **61**, 826 (1988); J. Opt. Soc. Am. B **6**, 2112 (1989); D. V. Kosachev, B. G. Matisov, and Yu. V. Rozhdestvenskiĭ, Kvantovaya Elektron. (Moscow) **19**, 713 (1992) [Sov. J. Quantum Electron. **22**, 663 (1992)]; Yu. V. Rozhdestvenskiĭ and N. N. Yakobson, Zh. Eksp. Teor. Fiz. **99**, 1679 (1991) [Sov. Phys. JETP **72**, 936 (1991)].

[6]J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B **6**, 2023 (1989).

[7]S. J. Buckle, S. M. Barnett, P. L. Knight, M. A. Lauder, and D. T. Pegg, Opt. Acta **33**, 1129 (1986); D. V. Kosachiov, B. G. Matisov, Yu. V. Rozhdestvensky, J. Phys. B **25**, 2473 (1992).

[8]V. S. Letokhov and V. G. Minogin, Opt. Commun. **35**, 199 (1980).

[9]V. G. Minogin and V. S. Letokhov, *Pressure of Laser Radiation on Atoms*, Nauka, Moscow (1986) (in Russian).

[10]P. R. Hemmer, G. P. Ontai, and S. Ezekiel, J. Opt. Soc. Am. B **3**, 219 (1986).

[11]M. Kasevich and S. Chu, Phys. Rev. Lett. **67**, 181 (1991).

[12]L. Allen and J. H. Eberly, *Optical Resonance and Two-Level Atoms*, Wiley, New York (1975).

[13]B. M. Garraway and S. Stenholm, Phys. Rev. A **46**, 1413 (1992).

Translated by Eugene Yankovsky