Nonlinear dynamics and generation of nonclassical light in two laser fields

G. Yu. Kryuchkyan and K. V. Kheruntsyan

Institute of Physics Research, Armenian Academy of Sciences, Erevan

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The quantum theory of parametric four-photon mixing in a nonlinear medium under the action of two laser fields is developed. The conditions for the generation of three modes of the radiation field in a resonant cavity, with frequencies equal to the frequencies of the driving laser fields and to their half-sum, are considered. Three types of stationary, stable solutions are found for the intensities and phases of these modes, corresponding to three regimes of generation. The spectra of the squeezing and dispersion of the fluctuations of the quadrature amplitudes are calculated, and from these there follows a new possible way of obtaining single-mode squeezed light at the frequencies of each of the modes in the above-threshold regime of generation. An effect is discovered that consists in the suppression of quantum fluctuations of the sum of the intensities of the modes at the frequencies of the perturbing fields below the coherence level.

1. INTRODUCTION

In recent years considerable advances have been achieved in the theoretical and experimental study of nonclassical states of the electromagnetic field, including squeezed states of light.[1)]

In one of the popular schemes for the generation of squeezed light, realized in the experiment of Ref. 4, nondegenerate four-wave mixing (FWM) in a resonant cavity under the action of a monochromatic laser field is used. The quantum theory of nondegenerate FWM in a medium of two-level atoms and in the regime below the generation threshold is given in Refs. 5–7. The spectrum of the squeezing of the two-mode field at the output from the cavity for the system indicated is calculated in Ref. 6, and, in a phenomenological description of the nonlinear medium, in Refs. 8 and 9.

In Refs. 10 and 11 it is suggested that it is possible to obtain nonclassical light in FWM in a resonant atomic medium under the action of a bichromatic laser field containing two components with equal amplitudes and with frequencies $\omega_{at} - \delta$ and $\omega_{at} + \delta$ that are symmetric about the atomic-transition frequency ω_{at}. In this process, in contrast to the standard scheme of nondegenerate FWM, single-mode squeezed light at the frequency ω_{at} is obtained. As the calculations of Ref. 10 show, in this case, as a consequence of the specific form of the perturbing field, with symmetric detunings from resonance, the coefficient of the parametric coupling between the conjugate modes is equal to zero, and the squeezing effect is determined entirely by the correlators of the spontaneous noise. In addition, the results of these papers were obtained in the regime below threshold in the approximation of a classical laser field without allowance for its exhaustion.

In the present paper we demonstrate the possibility of the generation of nonclassical intense light during parametric FWM in a cavity in the regimes below and above threshold under the action of two monochromatic laser fields with frequencies ω_1 and ω_2. The conditions for excitation of three cavity modes, with frequencies ω_1 and ω_2 equal to the frequencies of the perturbing fields and with frequency $\omega_0 = (\omega_1 + \omega_2)/2$, are considered. The nonlinear medium is described phenomenologically by the third-order susceptibility $\chi^{(3)}$. In particular, an atomic beam or gas can be such a medium. In fact, the spectral lines of the radiation in coherent transitions of an atom in a two-component field with frequencies ω_1 and ω_2 have frequencies equal to $(\omega_1 + \omega_2)/2 + q(\omega_2 - \omega_1)/2$, where $q = 0, \pm 1, \pm 2, ...$ (Refs. 12, 13). The three indicated modes are realized for $q = 0, \pm 1$ and can be selected by choosing the parameters of the cavity in an arbitrary manner: $\omega_1 = \omega_{at} - \delta_1$, $\omega_2 = \omega_{at} + \delta_2$, $\omega_0 = \omega_{at} + (\delta_2 - \delta_1)/2$. It should be borne in mind, however, that the phenomenological model of the medium certainly cannot describe the case with $\delta_1 = \delta_2$, considered in Refs. 10 and 11, since here, as already noted, the coefficient of the coupling between the conjugate modes is equal to zero.

In this paper all three modes of the radiation field are described quantum-mechanically in the framework of the stochastic equations of motion (see, e.g., Ref. 14), and the phenomenon of exhaustion of the pump fields is taken into account. It is important to note that for this problem, unlike that of nondegenerate FWM with monochromatic pumping, there exist three types of stable stationary solutions for the amplitudes and phases of the three modes, and this makes it possible to linearize and solve the stochastic equations of motion describing the dynamics of the fluctuations of the modes in the cavity. An important outcome of these calculations is the conclusion that, in the system under consideration in the above-threshold regime, generation of intense single-mode squeezed light can occur in the regions of each of the frequencies ω_0, ω_1, ω_2.

Another nonclassical effect, described theoretically[15–18] and confirmed experimentally for a nondegenerate parametric oscillator[15] and in nondegenerate four-wave mixing,[19] consists in the suppression of quantum fluctuations of the difference of the intensities of the two generated correlated modes below the shot-noise level. In the nonlinear system that we are considering another manifestation of the phenomenon of intermode correlation is discovered. It is found that suppression of the fluctuations occurs for the sum

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of the intensities of the fields at the pump frequencies ω_1 and ω_2 in the regime above the threshold of generation.

The plan of the article is as follows. In Sec. 2 we give the stochastic equations of motion, and in Sec. 3 their stationary stable solutions are obtained. In Sec. 4 the results in the regime below the generation threshold are given. Section 5 is devoted to an analysis of the quantum fluctuations in the above-threshold regime, while Sec. 6 is devoted to a calculation of the squeezing spectra for all three modes at the output from the cavity in the above-threshold region. In Sec. 7 the dispersions of the fluctuations of the quadrature amplitudes of the modes inside the cavity are calculated. In Sec. 8 the results for the quantum fluctuations of the mode intensities are obtained.

2. THE NONLINEAR SYSTEM AND STOCHASTIC EQUATIONS OF MOTION

We shall consider the following model of the parametric four-wave interaction in a $\chi^{(3)}$ medium. The nonlinear medium, placed in a ring cavity with normal-mode frequencies ω_1, ω_2, and ω_0, realizes collinear mixing of the pump modes with frequencies ω_1 and ω_2 with the signal mode of frequency ω_0 such that $\omega_0 = (\omega_1 + \omega_2)/2$, with fulfillment of the synchronism condition $\mathbf{k}_1 + \mathbf{k}_2 = 2\mathbf{k}_0$ between the wave vectors of the modes. The pump modes are driven by two external coherent fields with frequencies ω_1 and ω_2, and the mode ω_0 is excited spontaneously. We take into account the damping of the modes on account of the cavity mirrors, and assume for simplicity that the nonlinear medium is transparent and that the cavity detunings can be neglected. All three modes in the cavity are specified quantum mechanically by means of boson creation operators a_j^+ and annihilation operators a_j $(j = 0,1,2)$. Such a system can be described by the following Hamiltonian:

$$H=\sum_{n=1}^{4} H_n,$$

$$H_1=\sum_{j=0}^{2}\hbar\omega_j a_j^+ a_j,$$

$$H_2={}^1/_2 i\hbar\chi[a_1a_2(a_0^+)^2-a_1^+a_2^+a_0^2],$$

$$H_3=\sum_{j=0}^{2}(a_j^+\Gamma_j+a_j\Gamma_j^+),$$

$$H_4=i\hbar\sum_{k=1,2}(E_k e^{-i\omega_k t}a_k^+-E_k^* e^{i\omega_k t}a_k). \qquad (1)$$

The term H_1 represents the free part of the Hamiltonian, while H_2 describes the effective interaction of the modes in the nonlinear medium, with coupling constant $\chi/2$ proportional to the nonlinear susceptibility $\chi^{(3)}$. The term H_3 takes into account in the standard way (see, e.g., Ref. 20) the damping of the three modes ω_j $(j = 0,1,2)$ in the cavity, by means of the operators Γ_j and Γ_j^+ of the reservoirs corresponding to them, which determine the damping rates γ_0, γ_1, and γ_2 of the modes ω_0, ω_1, and ω_2, respectively. The term H_4 describes the driving of the pump modes ω_1 and ω_2 by external coherent fields, where E_1 and E_2 are the amplitudes of the driving fields in the cavity.

To describe the dynamics of the modes of the radiation field in the cavity we shall use the method of stochastic equations that has been developed in recent years in quantum optics.[14] First, using the standard procedure in the Born and Markov approximations, we obtain the equation in the interaction picture for the reduced density matrix of the three modes of the radiation field in the cavity:

$$\frac{\partial\rho}{\partial t}=\frac{\chi}{2}[a_1a_2(a_0^+)^2-a_1^+a_2^+a_0^2,\rho]+\sum_{k=1,2}[E_ka_k^+-E_k^*a_k,\rho]$$
$$+\sum_{j=0}^{2}\gamma_j(2a_j\rho a_j^+-\rho a_j^+a_j-a_j^+a_j\rho). \qquad (2)$$

Here we have neglected thermal fluctuations and have implemented the following change to operators that vary slowly in time:

$$a_j(t)\to a_j\exp(-i\omega_j t),\quad a_j^+(t)\to a_j^+\exp(i\omega_j t).$$

Equation (2) is then transformed into a Fokker–Planck equation in the space of c-numbers, by means of the positive P-representation[21] of the density matrix:

$$\rho=\int\frac{|\alpha\rangle\langle\alpha^+|}{\langle\alpha^+|\alpha\rangle}P(\alpha,\alpha^+)d\alpha\, d\alpha^+,$$

where the $|\alpha\rangle$ are coherent states, and the quantities $\alpha\equiv(\alpha_0, \alpha_1, \alpha_2)$ and $\alpha^+\equiv(\alpha_0^+, \alpha_1^+, \alpha_2^+)$ are independent complex variables, there being a correspondence between the c-numbers α_j, α_j^+ and the Bose operators a_j, a_j^+. The Fokker–Planck equation for the distribution $P(\alpha,\alpha^+)$ is obtained from (2) by means of well known operator identities,[14,21] and has the following form:

$$\frac{\partial}{\partial t}P(\alpha,\alpha^+)=\Big[\frac{\partial}{\partial\alpha_0}(\gamma_0\alpha_0-\chi\alpha_1\alpha_2\alpha_0^+)+\sum_{k=1,2}\frac{\partial}{\partial\alpha_k}$$
$$\times(\gamma_k\alpha_k+\frac{\chi}{2}\alpha_0^2\alpha_k^+-E_k)$$
$$+\frac{1}{2}\Big(\frac{\partial^2}{\partial\alpha_0^2}\chi\alpha_1\alpha_2-\frac{\partial^2}{\partial\alpha_1\,\partial\alpha_2}\chi\alpha_0^2\Big)$$
$$+\text{c.c.}\Big]P(\alpha,\alpha^+), \qquad (3)$$

where by "c.c." we mean terms in which the following changes of independent variables have been made: $\alpha_j\rightleftarrows\alpha_j^+$ and $E_k\to E_k^*$. Equation (3), which has the following standard form:

$$\frac{\partial}{\partial t}P(\alpha)=\Big[\frac{\partial}{\partial\alpha^\mu}A^\mu(\alpha)+\frac{1}{2}\frac{\partial^2}{\partial\alpha^\mu\,\partial\alpha^\nu}D^{\mu\nu}(\alpha)\Big]P(\alpha),$$

where $\mu, \nu = 1, 2, ..., 6$, $\alpha\equiv(\alpha,\alpha^+) = (\alpha^{(1)}, \alpha^{(2)}, ..., \alpha^{(6)}) = (\alpha_0, \alpha_0^+, \alpha_1, \alpha_1^+, \alpha_2, \alpha_2^+)$, is equivalent to the following system of stochastic differential equations:

$$\frac{\partial\alpha^\mu}{\partial t}=-A^\mu(\alpha)+R^\mu(\alpha,t),$$

where $R^\mu\equiv(R_0, R_0^+, R_1, R_1^+, R_2, R_2^+)$ are Gaussian noise terms with zero mean value and with correlation functions determined by the elements $D^{\mu\nu}(\alpha)$ of the diffusion matrix:

$$\langle R^\mu(\alpha,t)R^\nu(\alpha,t')\rangle=D^{\mu\nu}(\alpha)\delta(t-t').$$

In accordance with what has been said, the stochastic equations of motion of the system that we are considering can be written in explicit form as follows:[2)]

$$\dot{\alpha}_0(t)=-\gamma_0\alpha_0+\chi\alpha_1\alpha_2\alpha_0^{+}+R_0(t),$$
$$\dot{\alpha}_1(t)=-\gamma_1\alpha_1-{}^1/_2\chi\alpha_0^2\alpha_0^{+}+E_1+R_1(t),$$
$$\dot{\alpha}_2(t)=-\gamma_2\alpha_2-{}^1/_2\chi\alpha_0^2\alpha_1^{+}+E_2+R_2(t), \quad (4)$$

together with the corresponding equations for the quantities α_0^+, α_1^+, and α_2^+.

The nonzero correlators of the noise terms are equal to

$$\langle R_0(t)R_0(t')\rangle=\chi\alpha_1\alpha_2\delta(t-t'),$$
$$\langle R_0^{+}(t)R_0^{+}(t')\rangle=\chi\alpha_1^{+}\alpha_2^{+}\delta(t-t'),$$
$$\langle R_1(t)R_2(t')\rangle=\langle R_2(t)R_1(t')\rangle=-{}^1/_2\chi\alpha_0^2\delta(t-t'),$$
$$\langle R_1^{+}(t)R_2^{+}(t')\rangle=\langle R_2^{+}(t)R_1^{+}(t')\rangle=-{}^1/_2\chi\alpha_0^{+2}\delta(t-t'). \quad (5)$$

The subsequent analysis is based on the well known correspondence[21,22] between quantum-statistical averages of normally ordered operators a_j, a_j^+ and statistical averages of the c-numbers α_j, α_j^+ with distribution function $P(\alpha,\alpha^+)$.

The radiation field at the output from the cavity generally speaking, has a continuous frequency spectrum; in the Markov approximation, however, it can be represented in the form of three components in the neighborhood of ω_j, each of which can be expressed in terms of the operators a_j of the modes of the radiation field inside the cavity (see, e.g., Refs. 20 and 23). For the case when the input and output are realized on one of the mirrors of the ring cavity, we have

$$b_j=(2\gamma_j)^{1/2}a_j-c_j,$$
$$E_{1,2}=(2\gamma_{1,2})^{1/2}\langle c_{1,2}\rangle, \quad (6)$$

where the b_j and c_j are, respectively, the operators of the amplitudes of the fields at the output and input in the neighborhood of ω_j, and satisfy the commutation relations

$$[b_i(t),\ b_j^{+}(t')]=\delta_{ij}\delta(t-t'),\quad [c_i(t),\ c_j^{+}(t')]=\delta_{ij}\delta(t-t').$$

The average numbers of photons per unit time for these fields are respectively equal to

$$N_j=\langle b_j^{+}b_j\rangle,\quad n_j^{in}=\langle c_j^{+}c_j\rangle. \quad (7)$$

Below we consider the case of equal pump-mode damping constants ($\gamma_1=\gamma_2=\gamma$) and equal amplitudes and arbitrary phases of the driving fields: $E_{1,2}=E\exp(i\Phi_{1,2})$.

3. THE STATIONARY STABLE SOLUTIONS

The system of equations (4) is solved by the method of linearization about their semiclassical stationary solutions $\alpha_j^0=|\alpha_j^0|\exp(i\psi_j^0)$, $(\alpha_j^0)^*=(\alpha_j^0)^+$, which are obtained from (4) for $\dot{\alpha}_j=R_j=0$. Here it is necessary that the stationary solutions found be stable against small fluctuations. Analysis of the stationary solutions, together with the conditions for their stability, which are derived on the basis of the linearized equations of motion (see Secs. 4 and 5), leads to three possible generation regimes.

a) In the region below the generation threshold ($\varepsilon<1$), where

$$\varepsilon=E/E_{th}\ E_{th}=\gamma(\gamma_0/\chi)^{1/2}, \quad (8)$$

(E_{th} is the threshold value of E), the stable stationary solution of the system (4) is

$$\alpha_0^0=0,\ |\alpha_1^0|=|\alpha_2^0|=E/\gamma,\ \psi_{1,2}^0=\Phi_{1,2}. \quad (9)$$

In the region above the generation threshold ($\varepsilon>1$), where the stationary amplitude of the mode ω_0 being generated is nonzero, we must distinguish two types of stationary solutions.

b) For one of these the intensities of the pump modes (in units of the photon numbers) are equal to each other: $|\alpha_1^0|^2=|\alpha_2^0|^2$, and the solution has the following form:

$$|\alpha_0^0|=[(2\gamma/\chi)(\varepsilon-1)]^{1/2},\ |\alpha_1^0|=|\alpha_2^0|=(\gamma_0/\chi)^{1/2},$$
$$\psi_{1,2}^0=\Phi_{1,2},\ 2\psi_0^0=\Phi_1+\Phi_2. \quad (10)$$

This solution is stable in the region $1<\varepsilon<2$.

c) Other solutions with unequal pump intensities are

$$|\alpha_0^0|=(2\gamma/\chi)^{1/2},\ \psi_{1,2}^0=\Phi_{1,2},\ 2\psi_0^0=\Phi_1+\Phi_2, \quad (11)$$
$$|\alpha_1^0|={}^1/_2(\gamma_0/\chi)^{1/2}[\varepsilon+(\varepsilon^2-4)^{1/2}],$$
$$|\alpha_2^0|={}^1/_2(\gamma_0/\chi)^{1/2}[\varepsilon^2(\varepsilon^2-4)^{1/2}], \quad (11')$$

or

$$|\alpha_1^0|={}^1/_2(\gamma_0/\chi)^{1/2}[\varepsilon-(\varepsilon^2-4)^{1/2}],$$
$$|\alpha_2^0|={}^1/_2(\gamma_0/\chi)^{1/2}[\varepsilon+(\varepsilon^2-4)^{1/2}], \quad (11'')$$

and the system in this case is stable in the region $\varepsilon>2$. The values $\varepsilon=1,2$ at which the stationary solutions are unstable are points of instability of the system.

We note that for all the stationary amplitudes above the generation threshold the following relation is fulfilled:

$$|\alpha_1^0||\alpha_2^0|=E_{th}^2/\gamma^2\quad(\varepsilon>1),$$

i.e., the product of the amplitudes of the pump modes in the cavity reaches saturation. For comparison, in the regime below the threshold we have

$$|\alpha_1^0||\alpha_2^0|=E^2/\gamma^2\quad(\varepsilon<1).$$

We shall analyze the dependence of the stationary values of the intensities $|\alpha_j^0|^2$ of the three modes in the cavity on the ratio $\varepsilon=E/E_{th}$. We see that with increase of E, starting from the threshold value, the number of photons in the mode ω_0 increases linearly, and the number of photons in the pump modes remains unchanged. When the parameter $\varepsilon=2$, the average number $\gamma_0|\alpha_0^0|^2$ of photons in the mode ω_0 reaches the sum of the corresponding average numbers of photons of the two pump modes:

$$\gamma_0|\alpha_0^0|^2=\gamma|\alpha_1^0|^2+\gamma|\alpha_2^0|^2.$$

In the region $\varepsilon>2$ the stationary amplitudes of the pump modes ω_1 and ω_2 are asymmetric and, according to (11′) and (11″), have a bistable character. In this region the production of photons in the mode ω_0 is compensated by the losses in the cavity and by the inverse process of absorption of two photons with frequency ω_0 with the emission of a pair of photons with frequencies ω_1 and ω_2, and, therefore, the intensity of the signal mode ω_0 does not change. Then for each pair of solutions (11′), (11″), the following relation is fulfilled:

$$|\alpha_1^0|+|\alpha_2^0|=E/\gamma\quad(\varepsilon>2).$$

For the coherent cavity-exit field components corresponding to the semiclassical stationary solutions we obtain, taking into account that the mode ω_0 is generated spontaneously

$$b_{1,2}^{0}=\langle b_{1,2}\rangle=(2\gamma)^{1/2}\alpha_{1,2}^{0}-\langle c_{1,2}\rangle,\qquad b_0^{0}=(2\gamma_0)^{1/2}\alpha_0^{0}. \qquad (12)$$

The requirement that the fluxes of energy per unit time for the fields incident on the cavity and emerging from it be equal can be written as follows:

$$\omega_1 n_1^{in}+\omega_2 n_2^{in}=\omega_0|b_0^0|^2+\omega_1|b_1^0|^2+\omega_2|b_2^0|^2.$$

It is easily verified that this relation is fulfilled for all the generation regimes (a)–(c).

We must also draw attention to the phase relations in the stationary solutions obtained. In the above-threshold regime of generation the stationary phases of all three modes are known. This fact essentially distinguishes the process under consideration from the process of nondegenerate FWM with a monochromatic driving field, in which, in the above-threshold regime, only the phase of the pump mode and the sum of the phases of the modes being generated are known. The difference of the phases of the modes being generated, and each of these phases separately, are undetermined, and, therefore, the procedure of linearization about them is not applicable. Nevertheless, analysis of the quantum fluctuations in this case, and also in the case of nondegenerate parametric generation, is possible if additional assumptions are made about the time dependence of the phase difference between the modes being generated.[24,25]

4. RESULTS IN THE REGIME BELOW THE GENERATION THRESHOLD

We turn to the linearization of the equations of motion in the regime below the generation threshold. Introducing small fluctuations

$$\Delta\alpha_j(t)=\alpha_j(t)-\alpha_j^0,\quad \Delta\alpha_j^+(t)=\alpha_j^+(t)-(\alpha_j^0)^* \qquad (13)$$

about the stationary solutions (9), in the linear approximation we obtain the following equations:

$$\Delta\dot{\alpha}_k(t)=-\gamma\Delta\alpha_k(t),\quad \Delta\dot{\alpha}_k^+(t)=-\gamma\Delta\alpha_k^+(t),\quad k=1,\ 2. \qquad (14)$$

For the signal ω_0 mode, $\Delta\dot{\alpha}_0(t)=\alpha_0(t)$ and $\Delta\alpha_0^+(t)=\alpha_0^+(t)$, and we obtain

$$\frac{\partial}{\partial t}\begin{pmatrix}\alpha_0(t)\\ \alpha_0^+(t)\end{pmatrix}=-A\begin{pmatrix}\alpha(t)\\ \alpha_0^+(t)\end{pmatrix}+\begin{pmatrix}R_0^0(t)\\ R_0^{0+}(t)\end{pmatrix}, \qquad (15)$$

where the matrix A is equal to

$$A=\begin{pmatrix}\gamma_0 & -c\\ -c^* & \gamma_0\end{pmatrix},$$

with $c\equiv\chi\alpha_1^0\alpha_2^0$, and the nonzero correlators of the noise have the form

$$\langle R_0(t)R_0(t')\rangle=c\delta(t-t'),\quad \langle R_0^+(t)R_0^+(t')\rangle=c^*\delta(t-t'). \qquad (16)$$

The stationary solution (9) is stable provided that the real parts of the eigenvalues of the matrix A of the linearized equations are positive. By determining these eigenvalues we can verify that the stationary solution (9) is stable for $\varepsilon<1$.

As follows from the equations (14), (15) obtained, below the threshold the fluctuations of the pump modes are not related to the fluctuations of the mode ω_0. The equations of motion for the mode ω_0 correspond to the approximation of unexhausted pumping, and coincide in form with the well known equations of motion for degenerate parametric generation and degenerate FWM (see, e.g., Refs. 9 and 22, and the citations therein). In analogy with these papers, we give the final results.

For the average number of photons (constituting purely spontaneous noise) in the mode ω_0 per unit time at the output of the cavity we obtain

$$N_0=2\gamma_0\langle\alpha_0^+(t)\alpha_0(t)\rangle=\gamma_0\varepsilon^4/(1-\varepsilon^4). \qquad (17)$$

The dispersion of the fluctuations of the quadrature amplitude of the mode ω_0 in the cavity in the normally ordered form is equal to

$$V_0(\theta_0)=\langle(\Delta X_0(\theta_0,t))^2\rangle=1+\langle:(\Delta X_0(\theta_0,t))^2:\rangle,$$

where

$$X_0(\theta_0,t)=a_0(t)e^{-i\theta_0}+a_0^+(t)e^{i\theta_0}$$

is the operator of the quadrature amplitude and we have used the notation $\langle:A,B:\rangle=\langle:AB:\rangle-\langle A\rangle\langle B\rangle$. For the minimum value of this quantity, realized when $2\theta_0=\Phi_1+\Phi_2+\pi$, we obtain

$$V_0=1-\varepsilon^2/(1+\varepsilon^2), \qquad (18)$$

which tells us that the mode ω_0 is in a squeezed state $(0<V_0<1)$.

The corresponding fluctuation spectrum of the quadrature amplitude of the field external to the cavity in the neighborhood of ω_0 is given by the following expression:

$$S_0(\theta_0,\omega)=1+2\gamma_0\int_{-\infty}^{\infty}d\tau e^{i\omega\tau}\langle:X_0(\theta_0,t),X_0(\theta_0,t+\tau):\rangle.$$

The minimum value of this quantity is found to be equal to

$$S_0(\omega)=1-4\varepsilon^2/[(1+\varepsilon^2)^2+(\omega/\gamma_0)^2]. \qquad (19)$$

The expression (19) describes the squeezing spectrum of the mode ω_0 $(0<S_0(\omega)<1)$, which can reach 100% in the limit $E\to E_{th}$ at zero frequency. It should be borne in mind, however, that in the immediate neighborhood of the threshold $E=E_{th}$ the magnitude of the vacuum fluctuations becomes infinitely large, and the linearization procedure and the results (17)–(19) are not applicable.

5. ANALYSIS OF QUANTUM FLUCTUATIONS IN THE ABOVE-THRESHOLD REGION

For the analysis of the system of equations (4) above the threshold it is convenient to separate out the phases ψ_j of the modes and to change to new variables:

$$n_j=\alpha_j\alpha_j^+,\quad \psi_j=-{}^1/_2 i\ln(\alpha_j/\alpha_j^+). \qquad (20)$$

In the variables the system of equations of motion takes the following form:

$$\dot n_0(t)=-2\gamma_0 n_0+2\chi n_0(n_1n_2)^{1/2}\cos\psi+F_0(t),$$
$$\dot n_1(t)=-2\gamma n_1-\chi n_0(n_1n_2)^{1/2}\cos\psi+2n_1^{1/2}E\cos(\Phi_1-\psi_1)+F_1(t),$$
$$\dot n_2(t)=-2\gamma n_2-\chi n_0(n_1n_2)^{1/2}\cos\psi+2n_2^{1/2}E\cos(\Phi_2-\psi_2)+F_2(t),$$
$$\dot\psi_0(t)=\chi(n_1n_2)^{1/2}\sin\psi+f_0(t), \quad (21)$$

$$\dot\psi_1(t)=\frac{\chi}{2}n_0\left(\frac{n_2}{n_1}\right)^{1/2}\sin\psi+\frac{E}{n_1^{1/2}}\sin(\Phi_1-\psi_1)+f_1(t),$$

$$\dot\psi_2(t)=\frac{\chi}{2}n_0\left(\frac{n_1}{n_2}\right)^{1/2}\sin\psi+\frac{E}{n_2^{1/2}}\sin(\Phi_2-\psi_2)+f_2(t), \quad (22)$$

where $\psi=\psi_1+\psi_2-2\psi_0$, and the noise terms are equal to

$$F_j=\alpha_j^+R_j+\alpha_jR_j^+,\quad f_j=R_j/2i\alpha_j-R_j^+/2i\alpha_j^+. \quad (23)$$

The stationary solutions in the new notation have the following form:

$$\text{b)}\quad 1<\varepsilon<2:\ n_0^0=(2\gamma/\chi)(\varepsilon-1),\ n_1^0=n_2^0=\gamma_0/\chi, \quad (24)$$
$$\text{c)}\quad \varepsilon>2:\quad n_0^0=2\gamma/\chi, \quad (25)$$

$$n_1^0=\frac{\gamma_0}{2\chi}[\varepsilon^2-2+\varepsilon(\varepsilon^2-4)^{1/2}],\qquad n_2^0=\frac{\gamma_0}{2\chi}[\varepsilon^2-2-\varepsilon(\varepsilon^2-4)^{1/2}] \quad (25')$$

or

$$n_1^0=\frac{\gamma_0}{2\chi}[\varepsilon^2-2-\varepsilon(\varepsilon^2-4)^{1/2}],\qquad n_2^0=\frac{\gamma_0}{2\chi}[\varepsilon^2-2+\varepsilon(\varepsilon^2-4)^{1/2}]. \quad (25'')$$

For both regimes (b) and (c) the stationary values of the phases are the same and equal to

$$\psi_{1,2}^0=\Phi_{1,2},\qquad 2\psi_0^0=\Phi_1+\Phi_2. \quad (26)$$

Determining the fluctuations of the photon numbers and the phases about their stationary values:

$$\Delta n_j(t)=n_j(t)-n_j^0,\quad \Delta\psi_j(t)=\psi(t)-\psi_j^0, \quad (27)$$

under the condition that they are small, we obtain the following systems of linearized equations of motion in matrix form:

$$\Delta\dot n(t)=-A\Delta n(t)+F^0(t) \quad (28)$$

$$\Delta\dot\psi(t)=\bar A\Delta\psi(t)+f^0(t), \quad (29)$$

where

$$\Delta n=(\Delta n_0,\ \Delta n_1,\ \Delta n_2)^T,\ \Delta\psi=(\Delta\psi_0,\ \Delta\psi_1,\ \Delta\psi_2)^T,$$

$$F^0=(F_0^0,\ F_1^0,\ F_2^0)^T,\qquad f^0=(f_0^0,\ f_1^0,\ f_2^0)^T$$

and the superscript T denotes the operation of taking the transpose. These equations describe the dynamics of the quantum fluctuations and are valid for both generation regimes (b) and (c). The matrixes A and $\bar A$ are equal to

$$A=\begin{vmatrix} 0 & -\dfrac{\gamma_0 n_0^0}{n_1^0} & -\dfrac{\gamma_0 n_0^0}{n_2^0} \\ \gamma_0 & \gamma & \dfrac{\gamma_0 n_0^0}{2n_2^0} \\ \gamma_0 & \dfrac{\gamma_0 n_0^0}{2n_1^0} & \gamma \end{vmatrix},$$

$$\bar A=\begin{vmatrix} 2\gamma_0 & -\gamma_0 & -\gamma_0 \\ \dfrac{\gamma_0 n_0^0}{n_1^0} & \gamma & -\dfrac{\gamma_0 n_0^0}{2n_1^0} \\ \dfrac{\gamma_0 n_0^0}{n_2^0} & -\dfrac{\gamma_0 n_0^0}{2n_2^0} & \gamma \end{vmatrix}, \quad (30)$$

while the noise terms F_j^0 and f_j^0 are obtained from (23) by substitution of the corresponding stationary solutions (b) or (c). The nonzero correlators of the noise are equal to

$$\langle F_0^0(t)F_0^0(t')\rangle=2\gamma_0 n_0^0\delta(t-t'),$$

$$\langle F_1^0(t)F_2^0(t')\rangle=-\gamma_0 n_0^0\delta(t-t'), \quad (31)$$

$$\langle f_0^0(t)f_0^0(t')\rangle=-\frac{\gamma_0}{2n_0^0}\delta(t-t'),$$

$$\langle f_1^0(t)f_2^0(t')\rangle=\frac{\chi^2 n_0^0}{4\gamma_0}\delta(t-t'). \quad (32)$$

The linearization method used is valid if the stationary solutions are stable, i.e., if the eigenvalues of the matrices A and $\bar A$ have positive real parts. Using the Hurwitz criterion, we can show that, as was noted previously, the stationary solution (b) is stable in the region $1<\varepsilon<2$, while the solutions (c) are stable in the region $\varepsilon>2$. The values $\varepsilon=1,2$ are points of unstable equilibrium (instability).

The linearized equations (28), (29) obtained are convenient of the investigation of various two-time correlation functions of the amplitudes, intensities, or phases of the three modes, including for the calculation of the dispersions of the fluctuations of the quadrature amplitudes. For the analysis of the spectral correlation functions of the fields at the output from the cavity it is more convenient to use the equations of motion in the spectral representation.

Introducing the Fourier components for the fluctuations:

$$\Delta n(\omega)=\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}dt\,e^{i\omega t}\Delta n(t),$$

$$\Delta\psi(\omega)=\frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{\infty}dt\,e^{i\omega t}\Delta\psi(t),$$

and analogously for the noise terms, from Eqs. (28) and (29) we obtain

$$(A-i\omega I)\Delta n(\omega)=F^0(\omega), \quad (33)$$

$$(\bar A-i\omega I)\Delta\psi(\omega)=f^0(\omega), \quad (34)$$

where I is the unit matrix, and the nonzero correlators of the Fourier components of the noise terms are equal to

$$\langle F_0^0(\omega)F_0^0(\omega')\rangle=-2\langle F_1^0(\omega)F_2^0(\omega')\rangle=2\gamma_0 n_0^0\delta(\omega+\omega'), \quad (35)$$

$$\langle f_0^0(\omega)f_0^0(\omega')\rangle=-\frac{\gamma_0}{n_0^0}\delta(\omega+\omega'),$$

$$\langle f_1^0(\omega)f_2^0(\omega')\rangle=\frac{\chi^2 n_0^0}{4\gamma_0}\delta(\omega+\omega'). \quad (36)$$

We shall give the results for the numbers of photons per unit time at the output from the cavity, to within terms of second order in the fluctuations: $N_j=|b_j^0|^2$. From Eqs. (12)

in the region $1<\varepsilon<2$ we obtain

$$N_0=\frac{4\gamma\gamma_0}{\chi}(\varepsilon-1),\quad N_1=N_2=\frac{2\gamma\gamma_0}{\chi}\left(1-\frac{\varepsilon}{2}\right)^2. \tag{37}$$

and in the region $\varepsilon>2$

$$N_0=\frac{4\gamma\gamma_0}{\chi},\quad N_1=N_2=\frac{2\gamma\gamma_0}{\chi}\left(\frac{\varepsilon^2}{4}-1\right). \tag{38}$$

We must draw attention to the fact that in the entire region above threshold, including also the region $\varepsilon>2$, the light intensities at the output from the cavity in the regions of the frequencies ω_1 and ω_2 are equal to each other, even though inside the cavity the occupation numbers n_1^0 and n_2^0 of the modes differ for $\varepsilon>2$. This is due to the interference of the amplitudes $\alpha_{1,2}^0$ and $\langle c_{1,2}\rangle$ in the quantities $N_{1,2}$, the contribution of the interference being different for the two modes. The difference between N_1 and N_2 in the region $\varepsilon>2$ is manifest when higher-order fluctuations are taken into account, as follows from the results of Secs. 6–8. It is also easy to see that, as the instability point is approached ($\varepsilon\to2$), both from below and from above, for both pump modes we have $\langle c_{1,2}\rangle=(2\gamma)^{1/2}\alpha_{1,2}^0$ and the coherent components $b_{1,2}^0$ at the output (and, consequently, $N_{1,2}$ as well) vanish.

6. SQUEEZING SPECTRA IN THE ABOVE-THRESHOLD REGION

The aim of this section and Sec. 7 is a detailed analysis of questions concerning the generation of squeezed states of light in the regime above threshold. The nonclassical squeezing effect for the system under consideration here consists in the suppression of quantum fluctuations of the quadrature amplitudes

$$X_j(\theta_j,t)=a_j(t)e^{-i\theta_j}+a_j^+(t)e^{i\theta_j} \tag{39}$$

for each of the modes ω_j ($j=0,1,2$). This effect is manifested, in particular, in the fluctuation spectrum of the photocurrent in the method of optical heterodyning via the squeezing spectrum. This quantity, for the field at the output from the cavity in the neighborhood of the frequency ω_j, is defined as follows:

$$S_j(\theta_j,\omega)=1+2\gamma_j\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle:X_j(\theta_j,t),X_j(\theta_j,t+\tau):\rangle. \tag{40}$$

Using the correspondence between normally ordered averages for the slowly varying amplitude operators a_j and a_j^+ and time-ordered averages for the c-numbers α_j and α_j^+ in the P-representation that we are using,[21,22] it is not difficult to show that, in lowest order in the quantum fluctuations (27), the minimum value of $S_j(\theta_j,\omega)$ is realized at $\theta_j=\psi_j^0+\pi/2$ and is equal to

$$S_j(\omega)=1+8\gamma_j n_j^0\int_{-\infty}^{\infty}d\tau\, e^{i\omega\tau}\langle\Delta\psi_j(t)\Delta\psi_j(t+\tau)\rangle. \tag{41}$$

The first term in the right-hand side of (41) corresponds to the level of the vacuum fluctuations; the squeezing effect occurs for $S_j(\omega)<1$, and the value $S_j(\omega)=0$ corresponds to absolute (100%) squeezing.

In Fourier components the expression (41) can be written as

$$S_j(\omega)=1+8\gamma_j n_j^0\int_{-\infty}^{\infty}d\omega'\langle\Delta\psi_j(\omega')\Delta\psi_j(\omega)\rangle, \tag{42}$$

whence it can be seen that, to calculate the squeezing spectra for the modes ω_j, it is sufficient to calculate the averages $\langle\Delta\psi_j(\omega')\Delta\psi_j(\omega)\rangle$, i.e., to confine ourselves to solving the system of algebraic equations (34). These solutions have the following form:

$$\Delta\psi_0(\omega)=\frac{1}{D(\omega)}\left\{\left[(\gamma-i\omega)^2-\gamma_0^2\frac{(n_0^0)^2}{4n_1^0n_2^0}\right]f_0^0(\omega)+\left[\gamma_0(\gamma-i\omega)+\gamma_0^2\frac{n_0^0}{2n_2^0}\right]f_1^0(\omega)+\left[\gamma_0(\gamma-i\omega)+\gamma_0^2\frac{n_0^0}{2n_1^0}\right]f_2^0(\omega)\right\},$$

$$\Delta\psi_{1(2)}(\omega)=\frac{1}{D(\omega)}\left\{-\frac{n_0^0}{n_{1(2)}^0}\left[\gamma_0(\gamma-i\omega)+\gamma_0^2\frac{n_0^0}{2n_{2(1)}^0}\right]f_0^0(\omega)+\left[(2\gamma_0-i\omega)(\gamma-i\omega)+\gamma_0^2\frac{n_0^0}{n_{2(1)}^0}\right]f_{1(2)}^0(\omega)+\frac{n_0^0}{2n_{1(2)}^0}[\gamma_0(2\gamma_0-i\omega)-2\gamma_0^2]f_{2(1)}(\omega)\right\}, \tag{43}$$

where

$$D(\omega)=\det(\bar{A}-i\omega I)=(2\gamma_0-i\omega)(\gamma-i\omega)^2-\gamma_0^2(2\gamma_0-i\omega)\frac{(n_0^0)^2}{4n_1^0n_2^0}+\gamma_0^2(\gamma-i\omega)\left(\frac{n_0^0}{n_1^0}+\frac{n_0^0}{n_2^0}\right)+\frac{\gamma_0^3(n_0^0)^2}{n_1^0n_2^0}. \tag{44}$$

a) Squeezing spectrum for the signal mode

First of all we shall calculate the squeezing spectrum for the field in the neighborhood of the signal-mode frequency ω_0. The solution (43) and the correlators (36) lead to the following result for the second-order average:

$$\langle\Delta\psi_0(\omega')\Delta\psi_0(\omega)\rangle=\left[2q_0^2\frac{1+q_1q_2+q_1+q_2+r^2(\omega/\gamma_0)^2}{\gamma_0n_0^0d(\omega)}-\frac{[q_1q_2-1+r^2(\omega/\gamma_0)^2]^2+4r^2(\omega/\gamma_0)^2}{2\gamma_0n_0^0d(\omega)}\right]\delta(\omega+\omega'), \tag{45}$$

in which, for convenience, we have used the following dimensionless parameters:

$$r=\frac{\gamma_0}{\gamma},\quad q_0=\frac{\chi n_0^0}{2\gamma},\quad q_{1,2}=\frac{\gamma_0n_0^0}{2\gamma n_{1,2}^0}, \tag{46}$$

and the quantity $d(\omega)$ is equal to

$$d(\omega)=\frac{|D(\omega)|^2}{r^2\gamma^6}=4\left[1+q_1q_2+q_1+q_2-(r+r^2)\left(\frac{\omega}{\gamma_0}\right)^2\right]^2+\left(\frac{\omega}{\gamma_0}\right)^2\left[q_1q_2-2r(q_1+q_2)-4r-1+r^2\left(\frac{\omega}{\gamma_0}\right)^2\right]^2.$$

We note that $d(\omega)$ is nonzero in the entire region $\varepsilon>1$.

Substituting the expression (45) into (42), we obtain for the minimum value of the squeezing spectrum of the mode ω_0

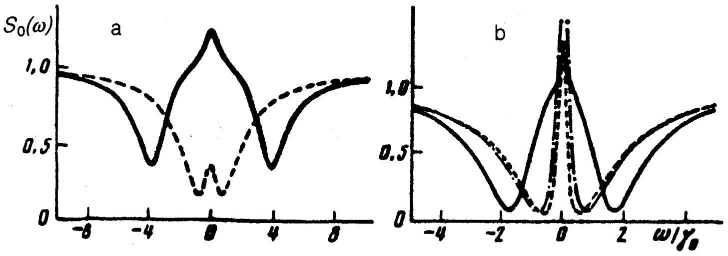

FIG. 1. Squeezing spectrum $S_0(\omega)$ for the signal field as a function of ω/γ_0 for various values of the parameters ε and r: a) $\varepsilon = 1.1$, $r = 2$ (dashed curve); $\varepsilon = 4$, $r = 2$ (solid curve); b) $\varepsilon = 1.5$, $r = 10$ (dashed curve); $\varepsilon = 2.1$, $r = 10$ (dashed-dotted curve); $\varepsilon = 4$, $r = 10$ (solid curve).

$$S_0(\omega)=1+\frac{16q_0^2}{d(\omega)}\left[1+q_1q_2+q_1+q_2+r^2\left(\frac{\omega}{\gamma_0}\right)^2\right]-\frac{4}{d(\omega)}\left\{\left[q_1q_2-1+r^2\left(\frac{\omega}{\gamma_0}\right)^2\right]^2+4r^2\left(\frac{\omega}{\gamma_0}\right)^2\right\}. \qquad (47)$$

b) Squeezing spectra for the pump modes

Analogous calculations on the basis of the solutions (44) lead to the following result for the correlation function of the fluctuations of the phases of the pump modes ω_k ($k = 1, 2$):

$$\langle\Delta\psi_k(\omega')\Delta\psi_k(\omega)\rangle=\left[\frac{q_0^2(1+2r)(\omega/\gamma)^2}{\gamma n_k^0 r^2 d(\omega)}-\frac{(q_1q_2+q_k)^2+q_k^2(\omega/\gamma)^2}{\gamma n_k^0 q_k d(\omega)}\right]\delta(\omega+\omega'). \qquad (48)$$

Finally, for the squeezing spectra of the fields at the frequencies ω_1 and ω_2, we obtain from Eq. (42)

$$S_k(\omega)=1+\frac{8q_0^2}{d(\omega)}\left(\frac{1}{r^2}+\frac{2}{r}\right)\left(\frac{\omega}{\gamma}\right)^2-\frac{8q_k}{d(\omega)}\left[\left(1+\frac{q_1q_2}{q_k}\right)^2+\left(\frac{\omega}{\gamma}\right)^2\right]. \qquad (49)$$

The results (47) and (49) have been presented in general form, and describe the phenomenon of the suppression of quantum fluctuations of the quadrature amplitudes of each of the modes ω_0, ω_1, ω_2 below the vacuum level in both the above-threshold generation regimes (b) and (c). In each of these regimes the values of the quantities n_j^0 are different, and the dimensionless parameters are equal to

b) $1<\varepsilon<2$: $q_0=q_1=q_2=\varepsilon-1$,
c) $\varepsilon>2$: $q_0=1$, $q_{1,2}=2/[\varepsilon^2-2\pm\varepsilon(\varepsilon^2-4)^{1/2}]$

for the solutions (25′), and to

$$q_{1,2}=2/[\varepsilon^2-2\mp\varepsilon(\varepsilon^2-4)^{1/2}]$$

for the solutions (25″). Thus, the choice of the solution (25′) or (25″) leads only to a change of the indices $1\rightleftarrows 2$ in the expression (49).

As is shown by analysis of the expressions (47) and (49), which are represented in graphical form in Figs. 1–4, in the above-threshold regime effective squeezing occurs both for the generated mode ω_0 and for each of the pump modes ω_1 and ω_2. This is an extremely interesting feature of the FWM process under the action of two driving fields. We recall that in nondegenerate FWM with a monochromatic driving field two-mode squeezed states are formed for the generated modes, while squeezing effects involving all of the modes, including the pump mode, are absent.

The squeezing spectrum (47) for the mode ω_0 is given in Fig. 1 for various values of the parameters ε and r. Near the threshold $\varepsilon = 1$ and for small r the spectrum has one dip at zero frequency $\omega = 0$. The value of $S_0(\omega)$ at $\omega = 0$ depends only on ε, and a large value of the squeezing (100%) occurs in the limit $\varepsilon \to 1$. As we move away from the threshold, with increase of ε, the spectrum acquires two minima, symmetrically placed about zero frequency, and a large magnitude of the squeezing is reached for relatively large values of r. Figure 2 shows the dependence of the quantity $S_0(\omega_{\rm opt})$ (the squeezing spectrum $S_0(\omega)$ at the points $\omega = \omega_{\rm opt}$ of its minimum values) on the parameter ε. It is easy to see that for values of r that are already $\gtrsim 10$ it is possible to achieve suppression of the fluctuations that is close to absolute (100%) in the entire above-threshold region $\varepsilon > 1$.

The pump-mode squeezing spectra described by Eq. (49) are given in Fig. 3. In the region $1 < \varepsilon < 2$, in which the intensities of the two pump modes in the cavity are equal, the squeezing spectra also coincide $[S_1(\omega) = S_2(\omega)]$, and have one minimum at zero frequency. The maximum squeezing (50%) is reached here near the point $\varepsilon = 2$, irrespective of the value of r. In the region $\varepsilon > 2$ the intensities of the pump modes in the cavity differ, and the corresponding squeezing spectra are also different [$q_1 \neq q_2$ in Eq. (49)]. Effective squeezing in the entire range $\varepsilon > 2$ occurs only for that pump mode (for definiteness, the mode ω_2) whose intensity in the cavity decreases with increase of ε. We recall that the corresponding intensity at the output increases in this case. The maximum effect is reached at side frequencies in the regions of the two minima of the spectra for definite values of r. The dependence of $S_2(\omega_{\rm opt})$ (the squeezing spectrum $S_2(\omega)$ at the points $\omega = \omega_{\rm opt}$ of the minima) on r for various values of ε is shown in Fig. 4. For the pump mode that increases in intensity in the region $\varepsilon > 2$ the squeezing is effective (~50%) at zero frequency near the value $\varepsilon = 2$, and vanishes with increase of ε.

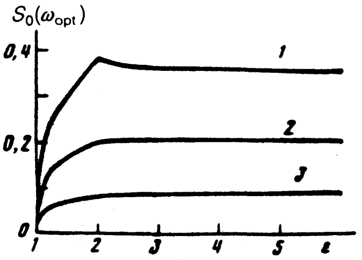

FIG. 2. Dependence of the quantity $S_0(\omega_{\rm opt})$ on ε for various values of r: 1) $r = 2$; 2) $r = 4$; 3) $r = 10$.

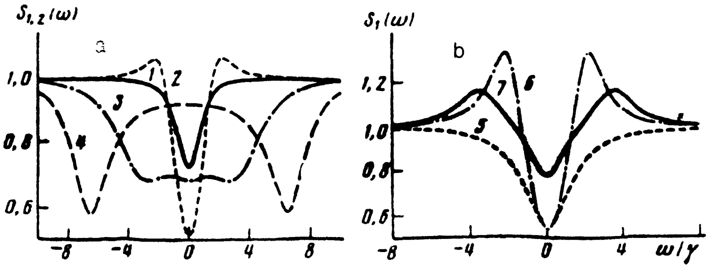

FIG. 3. Squeezing spectra $S_{1,2}(\omega)$ of the pump fields as a function of ω/γ. In the region $1<\varepsilon<2$ the spectra are the same ($S_1=S_2$), and the curves 1 ($\varepsilon=1.2, r=1$) and 2 ($\varepsilon=1.7, r=1$) in Fig. (a) correspond to them. The curves 3 ($\varepsilon=2.5, r=2$) and 4 ($\varepsilon=5, r=1$) pertain to that pump mode whose intensity in the cavity decreases in the region $\varepsilon>2$. Figure (b) corresponds to the pump mode (for definiteness, ω_1) that increases in intensity: Curve 5) $\varepsilon=2.1, r=10$; curve 6) $\varepsilon=2.1, r=1$; curve 7) $\varepsilon=3, r=1$.

7. TEMPORAL ANALYSIS AND VARIANCES OF THE QUANTUM FLUCTUATIONS

To complete our analysis, it is necessary to give the results of the temporal analysis of the quantum fluctuations above the generation threshold. The solution of this problem, on the one hand, makes it possible to elucidate a number of questions pertaining to effects involving two-time photon correlations, and, on the other, simplifies the calculation of the dispersions of the fluctuations of the quadrature amplitudes. We shall discuss this aspect in more detail.

We shall give the formula for the dispersions of the quantum fluctuations of the quadrature amplitudes of the modes ω_j inside the cavity:

$$V_j(\theta_j)=1+\langle:(\Delta X_j(\theta_j,t))^2:\rangle,$$

in lowest order in the fluctuations about the stationary values of the photon numbers and phases. For the minimum value of this quantity for $\theta_j=\psi_j^0+\pi/2$ we can obtain the following result:

$$V_j=1+4n_j^0\langle\Delta\psi_j(t)^2\rangle. \quad (50)$$

As can be seen directly from comparison of Eqs. (41) and (50), there is an obvious relationship between the quantities defining the integral squeezing and spectral squeezing of the fluctuations of the quadrature amplitudes:

$$2\gamma_j(V_j-1)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\omega[S_j(\omega)-1]. \quad (51)$$

From a practical point of view, however, for calculation of the quantities V_j Eq. (50), based on the temporal picture, is more convenient.

We return to the systems of equations (28), (29). In general matrix form their solutions for large times $t\gg\gamma_j^{-1}$ can be written in the following form:

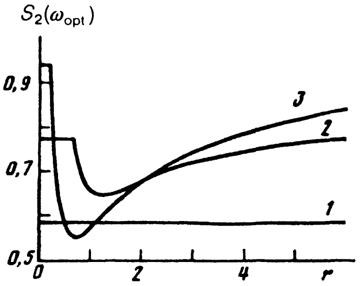

FIG. 4. Dependence of the quantity $S_2(\omega_{\rm opt})$ on r: 1) $\varepsilon=2.2$; 2) $\varepsilon=3$; 3) $\varepsilon=6$.

$$\Delta n(t)=\int_{-\infty}^{t}dt'\sum_{k=1}^{3}\exp[\lambda_k(t'-t)]\frac{\prod_{i\neq k}(A-\lambda_i I)}{\prod_{i\neq k}(\lambda_k-\lambda_i)}F^0(t'), \quad (52)$$

$$\Delta\psi(t)=\int_{-\infty}^{t}dt'\sum_{k=1}^{3}\exp[\bar\lambda_k(t'-t)]\frac{\prod_{i\neq k}(\bar A-\bar\lambda_i I)}{\prod_{i\neq k}(\bar\lambda_k-\bar\lambda_i)}f^0(t'), \quad (53)$$

where λ_k and $\bar\lambda_k$ ($k=1,2,3$) denote the eigenvalues of the matrices A and $\bar A$, respectively.

a) The region below the instability point ($1<\varepsilon<2$)

We shall give the results for the generation regime (b) with the stationary intensities (24) and phases (26). In this case the eigenvalues of the matrices A and $\bar A$ are equal to

$$\lambda_{1,2}=[\gamma\varepsilon\pm(\gamma^2\varepsilon^2-8\chi\gamma_0 n_0^0)^{1/2}]/2,\quad \lambda_3=2\gamma-\gamma\varepsilon, \quad (54)$$

$$\bar\lambda_{1,2}=\{2\gamma_0+2\gamma-\gamma\varepsilon\pm[(2\gamma_0+2\gamma-\gamma\varepsilon)^2-8\gamma_0\gamma\varepsilon]^{1/2}\}/2,\quad \bar\lambda_3=\gamma\varepsilon. \quad (55)$$

From the eigenvalues λ_k and $\bar\lambda_k$ written out in explicit form it can be seen directly that, as was noted previously on the basis of the Hurwitz criterion, the stationary solution (b) is stable in the region $1<\varepsilon<2$.

The solutions (53), with use of the correlators (32), make it possible to obtain expressions for the two-time correlation functions of the phases. Omitting the intermediate calculations, we shall give the final results for the dispersions of the fluctuations of the quadrature amplitudes of the modes ω_j in the cavity from Eq. (50).

For the signal mode ω_0 the dispersion is equal to

$$V_0=\frac{1}{2}+\frac{\varepsilon-1}{2+2r-\varepsilon}. \quad (56)$$

This result describes the phenomenon of squeezing of the mode ω_0 ($0<V_0<1$), which can reach 50% a values of E close to the threshold value or at large values of the parameter $r=\gamma_0/\gamma$. The dependence of the dispersion V_0 on ε for various values of r is presented in Fig. 5.

For the dispersions of the fluctuations of the pump modes we obtain

$$V_1=V_2=1-\frac{(\varepsilon-1)(1+r-\varepsilon)}{2\varepsilon(1+r-\varepsilon/2)}. \quad (57)$$

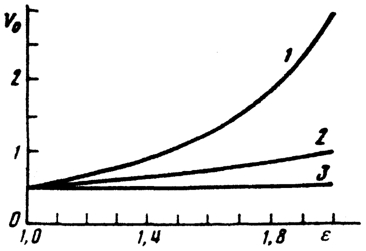

FIG. 5. Dependence of the dispersion V_0 of the fluctuations of the quadrature amplitude of the mode ω_0 in the cavity on ε: 1) $r = 0.2$; 2) $r = 1$; 3) $r = 10$.

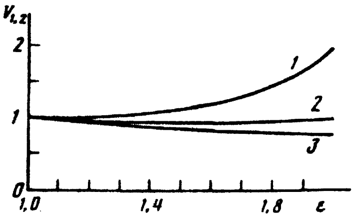

FIG. 6. Dependence of the dispersions $V_{1,2}$ of the fluctuations of the quadrature amplitudes of the pump modes on ε: 1) $r = 0.2$; 2) $r = 1$; 3) $r = 10$.

This result is evidence of the suppression of quantum fluctuations below the vacuum level for both pump modes in regime (b), if the inequality $1 + r - \varepsilon > 0$ is fulfilled. For large values of the parameter $r \gg 1$ we obtain

$$V_{1,2} \approx 1 - (\varepsilon - 1)/2\varepsilon,$$

i.e., the squeezing of the modes ω_1 and ω_2 ceases to depend on r, and can reach 25% as $\varepsilon \to 2$. The dependence of the dispersions (57) on ε is presented graphically in Fig. 6.

b) The region above the point of instability ($\varepsilon > 2$)

We shall give the results for regime (c), in which the intensities of the two pump modes in the cavity have an asymmetric dependence on ε for the case when one of the intensities is much greater than the other. Suppose, for definiteness, that $n_1^0 \gg n_2^0$. We note that for $\varepsilon = 4$ the ratio $n_1^0/n_2^0 \approx 200$. In this case, in the matrices (30) we can neglect elements proportional to $1/n_1^0$ in comparison with elements $\sim 1/n_2^0$. Then the equations of third degree for the eigenvalues λ_k and $\bar{\lambda}_k$ of the matrices A and $\bar{A}$ factorize, and we obtain

$$\lambda_{1,2} = [\gamma \mp (\gamma^2 - 4\gamma_0^2 n_0^0/n_2^0)^{1/2}]/2, \quad \lambda_3 = \gamma. \tag{58}$$

$$\bar{\lambda}_{1,2} = (\gamma_0 + \gamma/2) \pm [(\gamma_0 + \gamma/2)^2 - 2\gamma_0\gamma - \gamma_0^2 n_0^0/n_2^0]^{1/2}, \quad \bar{\lambda}_3 = \gamma. \tag{59}$$

Omitting the intermediate calculations, which are analogous to those described above, we shall give the final expressions for the dispersions (50). For the mode ω_0 the dispersion of the fluctuations of the quadrature amplitude is equal to

$$V_0 = 1 + \frac{2}{1 + 2r + 2r/f(\varepsilon)} - \frac{1}{2 + 1/r}\left[1 + \frac{1}{2r + 4r/f(\varepsilon)}\right], \tag{60}$$

where $f(\varepsilon) = \varepsilon^2 - 2 - \varepsilon(\varepsilon^2 - 4)^{1/2}$. In the region $\varepsilon^2 \gg 1$ we have $f(\varepsilon) \approx 2\varepsilon^2$, while for $r\varepsilon^2 \gg 1$ we obtain the simple expression

$$V_0 \approx 1 - \frac{1}{2 + 1/r}, \tag{61}$$

from which it follows that the dispersion ceases to depend on ε, and, for $r \gg 1$, tends to its minimum value 1/2 (50% squeezing).

For the pump mode (for definiteness, the mode ω_2) whose intensity in the cavity decreases with increase of ε, the dispersion of the fluctuations is found to be equal to

$$V_2 = 1 + \frac{1}{1 + 2r + 2r/f(\varepsilon)} - \frac{2}{[2 + f(\varepsilon)](2 + 1/r)}. \tag{62}$$

For $\varepsilon^2 \gg 1$ this expression is simplified, and for $r\varepsilon^2 \gg 1$ it takes the form (61), indicating that the maximum possible squeezing (50%) for this mode is reached at large values of r. As regards the pump mode that increases in intensity, $V_1 = 1$ for it in the approximation under consideration. We note that, although there is no integral squeezing for this pump mode, in the corresponding fluctuation spectrum at the output, in the region $\varepsilon \gtrsim 4$ (in which $n_1^0 \gg n_2^0$), there is a dip below the vacuum level at zero frequency. However, the amount by which the magnitude of the fluctuations exceeds the vacuum level at the side frequencies of the spectrum cancels this suppression, leading to absence of integral squeezing.

The results of Secs. 6 and 7 have been obtained in the linearized theory, and are valid provided that the fluctuations are small:

$$|\langle \Delta\varphi_j(t)^2 \rangle| \ll 1, \tag{62a}$$

$$|\langle \Delta n_j(t)^2 \rangle| \ll (n_j^0)^2 \tag{62b}$$

It is clear from physical considerations that these conditions are certainly fulfilled for large mode intensities $n_j^0 \gg 1$. The derivation of more-exact conditions requires a special analysis. For example, the conditions (62a) with allowance for Eq. (50) can be written as $|V_j - 1| \ll 4n_j^0$ and can be made more detailed using results of a calculation of the dispersions. The conditions (62b) can be analyzed on the basis of the results of Sec. 8 [see Eqs. (76) and (77)].

8. SUPPRESSION OF QUANTUM FLUCTUATIONS OF THE SUM OF THE INTENSITIES

The results obtained in the preceding sections can also be used in the investigation of other effects of a nonclassical nature. In particular, Eqs. (28) and their solutions (52) describe phase-independent effects involving correlations between instantaneous fluctuations of photon numbers of interacting modes. These effects can be manifested in various physical quantities containing interference of photon numbers.

The phenomenon of mutual correlation of fluctuations of the intensities of two light beams obtained in parametric processes leads to lowering of the level of the fluctuations of the difference of their intensities below the coherence level. This phenomenon was discovered for the radiation field of a nondegenerate parametric oscillator and in nondegenerate four-wave mixing,[15-19] and has been discussed in connection with possible applications, including in ultraprecise optical measurements, in spectroscopic absorption.

As shown by the calculations presented below, the phenomenon of the correlation of fluctuations of the photon numbers in FWM in two laser fields displays a number of new properties. These are discussed in application to the following experimental scheme. Two photodetectors measure the intensities of the two pump modes at the output from the cavity, and the fluctuations of their sum or difference are investigated by analyzing the fluctuation spectrum

$$P_\pm(\omega)=2\int_0^\infty d\tau\cos\omega\tau\langle i_\pm(t), i_\pm(t+\tau)\rangle \tag{63}$$

of the sum or difference of the corresponding photocurrents $i_\pm = i_1 \pm i_2$.

The average photodetection currents from each of the pump modes at the output from the cavity are equal to

$$\langle i_{1,2}\rangle = Q\eta N_{1,2},$$

where Q is the total charge of the current pulses and η is the dimensionless efficiency of the detector $(0<\eta\leqslant 1)$, chosen to be the same for the two modes.

According to the standard theory of photodetection,[26] for the correlation function of the photocurrent we have

$$\langle i_\pm(t), i_\pm(t+\tau)\rangle = \langle i(t)i(t+\tau)\rangle_{sh} + (Q\eta)^2\langle :N_\pm(t), N_\pm(t+\tau):\rangle. \tag{64}$$

The first term here determines the shot contribution, proportional to the intensities of the two modes and the same for the difference and sum of the currents:

$$\langle i(t)i(t+\tau)\rangle_{sh} = \langle i_1(t)i_1(t+\tau)\rangle_{sh} + \langle i_2(t)i_2(t+\tau)\rangle_{sh} = \eta Q^2\theta(\tau_0-\tau)\frac{\tau_0-\tau}{\tau_0^2}(N_1+N_2), \tag{65}$$

where τ_0 is the duration of the current pulses and θ is the Heaviside function, the appearance of which is related to the discrete character of the absorption at the photodetector. The second term, in which the operators $N_\pm$ are equal to

$$N_\pm(t) = b_1^+(t)b_1(t) \pm b_2^+(t)b_2(t), \tag{66}$$

describes the fluctuations of the sum or difference of the intensities of the two fields in the neighborhood of the pumping frequencies at the cavity output.

Using the expression (6), and confining ourselves to the second order of smallness in the fluctuations (27), we obtain

$$b_k^+(t)b_k(t) = (b^0)^2 + b^0(2\gamma n_k^0)^{1/2}\times\left[\frac{\Delta n_k(t)}{n_k^0} - \frac{\Delta n_k(t)^2}{4(n_k^0)^2} - \Delta\psi_k(t)^2\right] + 2\gamma n_k^0\left[\frac{\Delta n_k(t)^2}{4(n_k^0)^2} - \Delta\psi_k(t)^2\right], \quad k=1,2, \tag{67}$$

where $b^0 = |b_1^0| = |b_2^0|$. This quantity contains fluctuations of both the photon numbers and the phases of the modes. In the calculation of the correlation function $\langle :N_\pm(t), N_\pm(t+\tau):\rangle$ with the aid of (66) and (67), in second order the contribution of the phase fluctuations cancels and the final result acquires the following form:

$$\langle :N_\pm(t), N_\pm(t+\tau):\rangle = 2\gamma(b^0)^2\left[\sum_{k=1,2}\frac{\langle\Delta n_k(t)\Delta n_k(t+\tau)\rangle}{n_k^0} \pm \frac{\langle\Delta n_1(t)\Delta n_2(t+\tau)\rangle + \langle\Delta n_2(t)\Delta n_1(t+\tau)\rangle}{(n_1^0 n_2^0)^{1/2}}\right]. \tag{68}$$

a) Variances of fluctuations of the photon numbers

We shall give the results of the calculations for the above-threshold region $1<\varepsilon<2$, in which the average photon numbers in the pump modes in the cavity are equal: $n_1^0 = n_2^0$.

Using the expressions (64) and (68) we obtain for the variances $\langle(\Delta i_\pm(t))^2\rangle = \langle i_\pm(t)^2\rangle - \langle i_\pm(t)\rangle^2$ of the sum or difference of the photocurrents

$$\langle(\Delta i_\pm(t))^2\rangle = \langle(\Delta i)^2\rangle_{sh}[1+2\gamma\eta\tau_0(D_\pm/2n_1^0-1)]. \tag{69}$$

This result is written in terms of the variances of the fluctuations of the sum or difference of the photon numbers in the cavity:

$$D_\pm = \langle(a_1^+a_1 \pm a_2^+a_2)^2\rangle - (\langle a_1^+a_1\rangle \pm \langle a_2^+a_2\rangle)^2 = 2n_1^0 + \langle(\Delta n_\pm(t))^2\rangle, \tag{70}$$

where $\Delta n_\pm = \Delta n_1 \pm \Delta n_2$.

In the region $1<\varepsilon<2$ under consideration, as follows from the solutions (52), the quantities $\Delta n_\pm(t)$ are equal to

$$\Delta n_+(t) = \frac{2\gamma_0}{\lambda_1-\lambda_2}\int_{-\infty}^t dt'\{\exp[\lambda_1(t'-t)] - \exp[\lambda_2(t'-t)]\}F_0^0(t') + \frac{1}{\lambda_1-\lambda_2}\int_{-\infty}^t dt'\{(\gamma\varepsilon-\lambda_2)\exp[\lambda_1(t'-t)] + (\gamma\varepsilon-\lambda_1)\exp[\lambda_2(t'-t)]\}\times[F_1^0(t')+F_2^0(t')]. \tag{71}$$

$$\Delta n_-(t) = \int_{-\infty}^t dt'\exp[\lambda_3(t'-t)][F_1^0(t')-F_2^0(t')], \tag{72}$$

where the eigenvalues λ_k are given by the expressions (54).

Using Eqs. (31) for the correlators of the noise, and calculating the second-order averages $\langle\Delta n_\pm(t)^2\rangle$, after algebraic transformations we obtain

$$D_+ = 2n_1^0\left(1-\frac{\varepsilon-r-1}{\varepsilon}\right), \tag{73}$$

$$D_- = 2n_1^0\left(1+\frac{\varepsilon-1}{2-\varepsilon}\right), \tag{74}$$

where $n_1^0 = n_2^0 = \gamma_0/\chi$. Suppression of fluctuations of the photocurrents below the shot-noise level occurs with decrease of the dispersions $D_\pm$ of the quantum fluctuations below the coherence level: $D_\pm < 2n_1^0$. From the results (73) and (74) it is easy to see that this effect is realized for the sum of the photon numbers of the pump modes when the condition $\varepsilon - r - 1 > 0$ is fulfilled, and the suppression can reach

50% for $r \ll 1$ near the instability point $\varepsilon = 2$.

We shall also give the results of calculations of the variances of the fluctuations of the actual photon numbers of the three modes in the cavity:

$$D_j = \langle (a_j^+ a_j)^2 \rangle - \langle a_j^+ a_j \rangle^2 = n_j^0 + \langle \Delta n_j(t)^2 \rangle. \tag{75}$$

Using the general formula (52) and the correlators (31), for regime (b) we obtain

$$\langle \Delta n_0(t)^2 \rangle = n_0^0 \frac{\varepsilon^2 + 4(\varepsilon-1)(1+r-\varepsilon)}{4\varepsilon(\varepsilon-1)},$$

$$\langle \Delta n_1(t)^2 \rangle = \langle \Delta n_2(t)^2 \rangle = n_1^0 \frac{r(2-\varepsilon) + 2(\varepsilon-1)^2}{2\varepsilon(2-\varepsilon)}. \tag{76}$$

Analogous calculations for the generation regime (c) with asymmetric behavior of the pump intensities in the cavity in the approximation $n_1^0 \gg n_2^0$ give

$$\langle \Delta n_0(t)^2 \rangle = n_0^0 [r + f(\varepsilon)/4],$$

$$\langle \Delta n_1(t)^2 \rangle = n_1^0 \frac{4 + 2r^2 f(\varepsilon) + 8r + r[2 - f(\varepsilon)]^2}{g(\varepsilon)[2r + f(\varepsilon)]},$$

$$\langle \Delta n_2(t)^2 \rangle = n_2^0 r, \tag{77}$$

where

$$f(\varepsilon) = \varepsilon^2 - 2 - \varepsilon(\varepsilon^2 - 4)^{1/2}, \quad g(\varepsilon) = \varepsilon^2 - 2 + \varepsilon(\varepsilon^2 - 4)^{1/2}.$$

The results (76) and (77) show that the level of the quantum fluctuations of the intensities of each of the modes exceeds the coherence noise level $\langle n_j \rangle = n_j^0$ in both above-threshold generation regimes. Calculations of the variances of the fluctuations of the sum and difference of the pump intensities in regime (c), for $n_1^0 \gg n_2^0$, which can be performed on the basis of the general solutions (52), also indicate the absence of suppression of these fluctuations below the coherence level.

b) Spectra of the fluctuations of the photocurrents

Equations (63)–(65) and (68) lead to the following expression for the fluctuation spectra of the sum or difference of the photocurrents:

$$P_\pm(\omega) = P_{sh}[1 + \eta\gamma C_\pm(\omega)], \tag{78}$$

in which

$$G_\pm(\omega) = \int_{-\infty}^{\infty} d\omega' \left[\sum_{k=1,2} \frac{\langle \Delta n_k(\omega') \Delta n_k(\omega) \rangle}{n_k^0} \pm \frac{\langle \Delta n_1(\omega') \Delta n_2(\omega) \rangle + \langle \Delta n_2(\omega') \Delta n_1(\omega) \rangle}{(n_1^0 n_2^0)^{1/2}} \right], \tag{79}$$

and we have used the following frequency-independent simplified expression for the shot noise:

$$P_{sh} = \eta Q^2 (N_1 + N_2), \tag{80}$$

which is valid for $\omega\tau_0 \ll 1$.

The Fourier components $\Delta n_{1,2}(\omega)$ of the fluctuations, as the solutions of the system of equations (33), have the following form:

$$\Delta n_{1(2)}(\omega) = \frac{1}{B(\omega)} \left\{ \left[\frac{\gamma_0^2 n_0^0}{2 n_{2(1)}^0} - \gamma_0(\gamma - i\omega) \right] F_0^0(\omega) + \left[\frac{\gamma_0^2 n_0^2}{n_{2(1)}^0} - i\omega(\gamma - i\omega) \right] F_{1(2)}^0(\omega) - \frac{\gamma_0 n_0^0}{2 n_{2(1)}^0} (2\gamma_0 - i\omega) F_{2(1)}^0(\omega) \right\}, \tag{81}$$

where

$$B(\omega) = \det(A - i\omega I) = \gamma_0^2 \gamma \left(\frac{n_0^0}{n_1^0} + \frac{n_0^0}{n_2^0} \right) - \frac{\gamma_0^3 (n_0^0)^2}{n_1^0 n_2^0} - 2\gamma\omega^2 - i\omega \left[\gamma_0^2 \left(\frac{n_0^0}{n_1^0} + \frac{n_0^0}{n_2^0} \right) - \frac{\gamma_0^2 (n_0^0)^2}{4 n_1^0 n_2^0} + \gamma^2 - \omega^2 \right].$$

By means of these solutions and the noise correlators (35) we can calculate the averages $\langle \Delta n_i(\omega') \Delta n_j(\omega) \rangle$ and then the quantities $C_\pm(\omega)$. Omitting the intermediate results, we give the final expression for the normalized fluctuation spectrum of the sum and difference of the pump photocurrents:

$$P_\pm(\omega)/P_{sh} = 1 + \frac{4\eta}{b(\omega)} \left\{ r^2 [p(1+q) + 4q(p-1)] + [pr^2 + 2(1-2r)q] \left(\frac{\omega}{\gamma} \right)^2 \right\} \pm \frac{4\eta q^{1/2}}{b(\omega)} \left\{ 2r^2 [1 - p - 3q] + [2r^2 + 2rp - q - 1] \left(\frac{\omega}{\gamma} \right)^2 - \left(\frac{\omega}{\gamma} \right)^4 \right\}, \tag{82}$$

where

$$b(\omega) = \frac{|B(\omega)|^2}{\gamma^6} = 4\left[r(p - 2q) - \left(\frac{\omega}{\gamma} \right)^2 \right]^2 + \left(\frac{\omega}{\gamma} \right)^2 \left[1 + 2rp - q - \left(\frac{\omega}{\gamma} \right)^2 \right]^2$$

and we have used the notation:

$$r = \frac{\gamma_0}{\gamma}, \quad p = \frac{\gamma_0}{2\gamma} \left(\frac{n_0^0}{n_1^0} + \frac{n_0^0}{n_2^0} \right), \quad q = \frac{\gamma_0^2 (n_0^0)^2}{4\gamma^2 n_1^0 n_2^0}.$$

This result has been written in general form for application to both regimes [(b) and (c)]. In each of the regimes the stationary photon numbers are different, and the parameters p and q are respectively equal to

b) $1 < \varepsilon < 2$: $p = 2(\varepsilon - 1)$, $q = (\varepsilon - 1)^2$.

c) $\varepsilon > 2$: $p = \varepsilon^2 - 1$, $q = 1$.

In Fig. 7 the result (82) for the sum of the photocurrents is presented in graphical form for the case of ideal photodetectors with $\eta = 1$. A decrease of the fluctuations below the shot-noise level ($0 < P_+(\omega)/P_{sh} < 1$) occurs in the regions of the side frequencies of the spectrum and is absent at zero frequency. Here, the greatest effect is close to 100% for $\varepsilon \approx 2$ and for small values of the parameter $r = \gamma_0/\gamma$.

In the region $\varepsilon > 2$ the effect decreases with increase of ε and vanishes for $\varepsilon^2 \gg 1$.

Analysis of the expression (82) for $P_-(\omega)$ shows that for the difference of the photocurrents suppression of fluctuations is absent in the entire range of the spectrum.

The suppression of the fluctuations in the sum of the intensities can be explained qualitatively as follows. For two

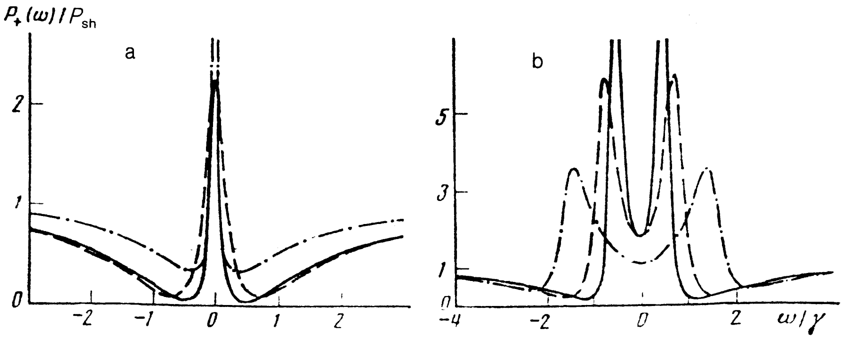

FIG. 7. Normalized spectrum $P_+(\omega)/P_{sh}$ as a function of ω/γ: a) $\varepsilon = 1.3$, $r = 0.05$ (dashed-dotted curve); $\varepsilon = 1.8$, $r = 0.05$ (solid curve); $\varepsilon = 1.8$, $r = 0.1$ (dashed curve); b) $\varepsilon = 3$, $r = 0.05$ (solid curve); $\varepsilon = 3$, $r = 0.1$ (dashed curve); $\varepsilon = 6$, $r = 0.05$ (dashed-dotted curve).

coherent fields at the input of the cavity, as is well known, correlation of the instantaneous fluctuations of the photon numbers is absent: $\langle \Delta n_1^{\text{in}}(t) \Delta n_2^{\text{in}}(t) \rangle = 0$. As a consequence of the nonlinear four-photon interaction in the cavity the two beams acquire the correlated statistical properties that are characteristic for two-photon absorption. As a result, the correlation of the fluctuations of the photon numbers in the pump modes becomes negative: $\langle \Delta n_1(t) \Delta n_2(t) \rangle < 0$, and, as is easily seen from (79), quantum fluctuations are found to be suppressed in the sum of the photon numbers. This circumstance can also be seen from Eqs. (28), if we draw attention to the fact that a decrease of the fluctuations of one of the coupled modes (say, Δn_1) leads to an increase of Δn_2 as a consequence of the fact that in (30) the corresponding matrix element $A_{23} > 0$.

Analogous predictions concerning the suppression of fluctuations of the sum of the intensities have been made recently for such processes as FWM with monochromatic pumping,[24] second-harmonic generation,[27] and two-photon absorption bistability.[28]

9. CONCLUSION

An important distinctive feature of the nonlinear system considered has turned out to be the fact that it permits an analytical treatment above the generation threshold in the framework of linearization of the stochastic equations of motion. As a result, in the semiclassical approximation we have found the stationary values of the occupation numbers and phases of the modes in the cavity and the corresponding intensities of the radiation fields at the output from the cavity, and have also carried out a quantum analysis of the fluctuations of the intensities and phases of each of the modes. The results obtained convince us that the process of FWM in two laser fields is extremely promising for the generation of single-mode squeezed light with suppressed quantum fluctuations of the quadrature amplitudes. This process leads also to the formation of a nonclassical two-mode field on the frequencies of the pump fields; for this field the fluctuations of the intensities of the two modes are mutually anticorrelated.

The light-squeezing effect in the region of the signal-mode frequency ω_0 can reach $\sim 100\%$ in the entire above-threshold region $\varepsilon > 1$ and for $r \gtrsim 10$, but the intensity is bounded by the value $N_0 = 4\gamma_0\gamma/\chi$. Generation of more intense light in the squeezed state occurs for pump fields with $\varepsilon > 2$. The corresponding intensities at the output from the cavity are not bounded, and grow with increase of the intensities of the fields at the input:

$$N_1 = N_2 = \frac{2\gamma_0\gamma}{\chi}\left(\frac{\varepsilon^2}{4} - 1\right),$$

although the maximum squeezing can be 50%. We shall give some numerical results. For $\varepsilon = 5$ the intensities $N_{1,2} = 4.4\gamma_0\gamma/\chi$, and the maximum squeezing for $r = 0.8$ amounts to 43% in the region of spectral frequencies $\omega_{\text{opt}}/\gamma = \pm 5.8$. For $\varepsilon = 10$ we have $N_{1,2} = 48\gamma_0\gamma/\chi$, and the maximum squeezing for $r = 0.6$ amounts to 48% for $\omega_{\text{opt}}/\gamma = \pm 10.7$.

For the other nonclassical effect—the lowering of the level of the quantum noise in the sum of the photocurrents to below the shot level—the optimum values are values of ε near the instability point $\varepsilon = 2$ and small values of r, for which the suppression of the fluctuations is close to 100%.

Another feature of this nonlinear system, requiring special consideration, is the bistable behavior of the intensities of the pump modes in the cavity in the region $\varepsilon > 2$.

[1)] See, e.g., the special journal issues[1,2] devoted to this problem, and the review Ref. 3.
[2)] Henceforth we assume the Stratonovich form for the stochastic equations and integrals,[14] in which the usual rules of analysis are preserved in the calculations.

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