Three-dimensional, field-theoretic equation for the pion–nucleon scattering problem

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We consider a field-theoretical three-dimensional spectral decomposition for the amplitudes of low-energy πN scattering, which, in contrast to the familiar quadratically-nonlinear Low equations for this problem contains not only pions but also nucleons off the mass shell. Correspondingly, the particle-exchange part of the inhomogeneous term of the proposed equations consists of a sum of s- and t-channel πN scattering diagrams and does not contain the familiar u-channel crossing nonlinearity. We linearize these equations and then reduce them to a Lippman–Schwinger equation, whose potential is unambiguously related to the inhomogeneous term of the original Low equations. Based on these fully linear equations we reproduce the P_{33} resonant πN scattering phase shift.

1. INTRODUCTION

The field-theoretic approach to the problem of pion-nucleon (πN) scattering in the region of low and intermediate energies (up to 1–2 GeV) permits reproduction of the microscopic picture of the interaction (the exchange of nucleons and heavy mesons), including the requirements of special relativity, chirality and crossing symmetry. Such an approach to πN scattering gives hope that also other important properties of strong interactions that follow from QCD can be included in the problem.

The relativistic description of πN scattering usually employs the three-dimensional linear integral equations of the Lippman–Schwinger type.[1–10] Such equations are often specified from the very beginning including relativistic kinematics,[1–4] but they can be derived by three-dimensional reduction of the Bethe–Salpeter equations[5–7] or after linearization of the field-theoretic Low equations.[8–10] In the approach based on the Low equations,[8–14] starting from the celebrated Chew–Low model,[12] there appears explicitly the nonlinear u-channel term, which is obtained by crossing of pions in the corresponding s-channel term with a pion-nucleon intermediate state (Fig. 1a and 1b). Just such a nonlinear term is also present in the dispersion relations for πN scattering,[15] thus taking into account the important contribution of the so-called left-hand cut. A similar nonlinearity, which appears as a result of crossing the external pions in the s-channel term, arises also in the Bethe–Salpeter equation after inclusion of disconnected diagrams with the $\pi\pi NN$ intermediate state.[9] Further, as was shown in Refs. 8 and 9, a similar nonlinearity arises in any field-theoretic formulation of the πN scattering problem which takes explicitly into account the crossing symmetry of the pions, due to the essentially nonpotential nature of the crossing symmetry. The presence of such nonlinear terms significantly complicates practical evaluation of the integral equations. For this reason they are often neglected, or included approximately, which however violates the crossing symmetry of the desired amplitude for πN scattering.

Another difference in the approach based on the Low equations or on the field-theoretic spectral decomposition of the scattering amplitude is connected with the historical tradition of the Chew–Low model,[12] according to which in Refs. 9, 12, and 14 the P_{33} resonance is reproduced by solving these equations without the introduction of any additional parameters which could be related to the properties of a "bare" Δ particle. In particular, in the approach based for example, on the quasi-potential equations of Refs. 5–7, the Δ resonance is viewed as an independent degree of freedom. The necessary connection between these two approaches was established in Ref. 16, where it was shown for the case of πN scattering in the bag model in the static approximation, that the "physical" solution of the Low equation contains a Castillejo–Dalitz–Dyson pole, which is due to the existence in this model of a state corresponding to the "bare" Δ isobar.

The essential difference between the approaches based on the Bethe–Salpeter and Low equations consists in the different virtual and off-mass-shell behavior of the amplitudes that solve these equations. Thus the t-matrix, which appears in the Bethe–Salpeter equation, is usually treated on the energy shell, while all the particles in the intermediate states are off the mass shell. In contrast to this, all the particles on whose momenta the t-matrix that figures in the Low equations depends, are on the mass shell. In particular, in the amplitude that is the solution of the Low equation for πN scattering,[8–14,16] both nucleons and one of the pions are on the mass shell while the second pion, to which one assigns a 4-momentum calculated from the energy-momentum conservation law, is considered to be off the mass shell (Fig. 2a). It is relevant that the t-matrix in the Low equation does not

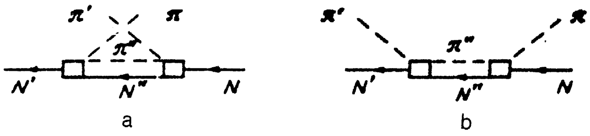

FIG. 1. a) The nonlinear u-channel crossing term in the Low equation, related to the s-channel term by crossing of the pion lines. b) The s-channel term in the Low equation, corresponding to the time-ordered product of the two transition amplitudes $\pi N \to \pi'' N''$ and $\pi'' N'' \to \pi' N'$.

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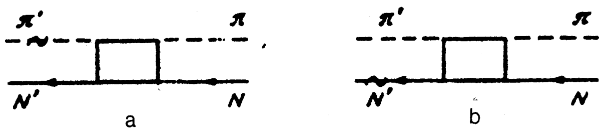

FIG. 2. The πN-scattering amplitude a) with the pion off the mass shell $\langle N'(\mathbf{p}'s')\mathbf{j}_{i'}(0)|N(\mathbf{p}s),\ \pi(\mathbf{q}i);\ in\rangle$ and b) with the nucleon off the mass shell $\langle \pi'(\mathbf{q}'i')|\eta_{p's'}(0)|N(\mathbf{p}s),\ \pi(\mathbf{q}i);\ in\rangle$.

depend on the 4-momentum of this off-mass-shell pion. Contrariwise, if we view such a pion as being on the mass shell, then the scattering t-matrix under consideration can be viewed as prescribed off the energy shell. Below we shall follow the second interpretation, in close analogy with the potential theory of scattering. However, in agreement with the generally accepted terminology we shall use for the t-matrix determined in Sec. 2 the name "off-mass-shell t-matrix." Namely we consider all the incoming and outgoing particles to be on the mass shell, while the total energies of the πN system in the in- and out-states, generally speaking, do not coincide, i.e., the t-matrix that appears in the Low equation represents the off-energy-shell t-matrix. The connection between the amplitudes that are the solutions of the Bethe–Salpeter and Low equations is considered in more detail in Ref. 17.

The potential in the Low equations consists of two parts. The first part is determined by the equal-time commutators of the field operators of two particles, while the second part corresponds to particle-exchange diagrams with particles from intermediate states on the mass shell. The particle-exchange part considered in Refs. 9, 10, 13 and 14 contained exchange of particles in the s-, u-, $\bar{s}$- and $\bar{u}$-channels, while the equal-time commutator, calculated by means of the simplest model of a πN Lagrangian, corresponded to single-particle exchange of σ and ρ mesons. Further, in the equations of Refs. 9, 10, 13 and 14 the pions in the initial and final states were off the mass shell. In this work we derive a different version of the Low equation, which contains off-mass-shell effects not only for the pion but also for the nucleon. Correspondingly, the particle-exchange potential in the proposed equation is described by πN interaction diagrams in the s-, t-, $\bar{s}$- and $\bar{t}$-channels, while the equal-time commutator in the model with simplest πN Lagrangians is determined by the one-nucleon-exchange diagram in the u-channel. Using the linearization procedure for these quadratically nonlinear integral equations, which was proposed in our previous paper,[9] we obtain linear integral Lippman-Schwinger equations. In the proposed equation the πN-interaction potential, in contrast to the potential of Ref. 9, does not contain the nonlinear u-channel term, shown in Fig. 1a. The price that must be paid for this total linearization of the Low equation for πN scattering is that the potential of this equation contains vertex functions of the meson-nucleon system with off-mass-shell contributions not only for the pion but also for one of the nucleons. Moreover, the proposed equation is not manifestly crossing-symmetric.

We further show that the numerical solution of this totally linear equation can reproduce the P_{33} resonance, provided use is made of the accepted parametrization of the vertex functions.

2. THE LOW-TYPE EQUATION WITH OFF-MASS-SHELL NUCLEON

To proceed further we need two kinds of πN scattering amplitudes: 1) the πN amplitude with nucleons in the in- (N) and out- (N') states on the mass shell and the pion π from the in-state, also on the mass shell:

$$t^{(1)}_{a'a}(\mathbf{p}',\mathbf{p})=\langle \mathbf{p}'s'\,|\,j_{i'}(0)\,|\,\mathbf{p}s,\mathbf{q}i;\,in\rangle, \qquad (1a)$$

and 2) the πN amplitude in which the particles π' (out), π (in) and N (in) are on the mass shell:

$$t^{(2)}_{a'a}(\mathbf{p}',\mathbf{p})=-\langle \mathbf{q}'i'\,|\,\eta_{\mathbf{p}'s'}(0)\,|\,\mathbf{p}s,\mathbf{q}i;\,in\rangle, \qquad (1b)$$

where $\mathbf{p}s$, $\mathbf{q}i$ and $\mathbf{p}'s'$, $\mathbf{q}'i'$ denote the 3-momentum, spin and isospin of the nucleon and pion respectively in the initial and final state. We consider the amplitudes in the center-of-mass frame, i.e., $\mathbf{p}=-\mathbf{q}$ and $\mathbf{p}'=-\mathbf{q}'$. The indices a and a' have been introduced for an abbreviated description of the spin-isospin indices of the πN system in the initial and final states. Here $j_1(x)$ and $\eta_{ps}(x)$ denote the operators for the source of the meson and nucleon fields, which are expressed in terms of the Heisenberg field of the pion $\Phi_i(x)$ and nucleon $\psi(x)$ as follows:

$$j_i(x)=(\Box_x+m_\pi^2)\Phi_i(x), \quad \eta_{\mathbf{p}s}(x)=\bar{u}(\mathbf{p}s)(i\hat{\nabla}_x-M)\psi(x).$$

Everywhere in what follows we use the notation and normalization from the book by Itzykon and Zuber.[18] The particles that appear in the state vectors in expressions (1a) and (1b) are taken to be on the mass shell, i.e., $p^0=\omega_N(\mathbf{p})=(M^2+\mathbf{p}^2),^{1/2}$ $q^0=\omega_\pi(\mathbf{q})=(m_\pi^2+\mathbf{q}^2)^{1/2}$ and $p'^0=\omega_N(\mathbf{p}')=(M^2+\mathbf{p}'^2)^{1/2}$ in (1a) and $q'^0=\omega_\pi(\mathbf{q}')=(m_\pi^2+\mathbf{q}'^2)^{1/2}$ in (1b). Moreover the expression (1a) does not depend on the pion 3-momentum $\mathbf{q}'$, and the expression (1b) depends on the nucleon 3-momentum $\mathbf{p}'$ only through the spinor function $\bar{u}(\mathbf{p}'s')$. It follows from the explicit form of the expressions (1a) and (1b) that we can consider the 4-momentum of the pion in (1a) and the 4-momentum of the nucleon in (1b) as given in terms of the 4-momenta of the remaining particles: $q'_\mu=p_\mu+q_\mu-p'_\mu$ and $p'_\mu=p_\mu+q_\mu-q'_\mu$, respectively. Further, the mass-shell conditions $q'^2_\mu=m_\pi^2$ and $p'^2_\mu=M^2$ are satisfied only on the mass shell, when $|\mathbf{p}|\neq|\mathbf{p}'|$ in the center-of-mass frame. Consequently we can view the amplitude $t^{(1)}$ (1a) as corresponding to the pion π' in the out-state being off the mass shell, and the amplitude $t^{(2)}$ (1b) as corresponding to the nucleon N' in the out-state being off the mass shell. These amplitudes are shown graphically in Figs. 2a–2b. In these figures, and in those that follow, the lines describing particles off the mass shell are marked by a wavy line.

According to the LSZ formulation of S-matrix field theory,[18] the relation between the pion–nucleon scattering matrix S_{fi} and the t-matrices (1a) and (1b) has the form:

$$S_{fi}=\hat{1}_{fi}+(2\pi)^4 i\delta^{(4)}(P_f-P_i)\,t^{(\alpha)}, \quad \alpha=1,2, \qquad (2)$$

where $P_{f(i)}$ is the total momentum of the πN system. Further, making use of the LSZ reduction formalism in the center-of-mass system, where $\mathbf{p}=-\mathbf{q}$ and $\mathbf{p}'=-\mathbf{q}'$, we have

$$t^{(1)}_{a'a}(\mathbf{p}',\mathbf{p}) = \langle\, \mathbf{p}'s' \,|\, j_{i'}(0)\, b_{in}{}^{+}(\mathbf{p}s)\,|\,\mathbf{q}i\rangle$$

$$=\langle \mathbf{p}'s' \,|\,(-[j_{i'}(0),\, b_{ps}{}^{+}(0)]$$

$$-i\int d^4x e^{-ipx} T(j_{i'}(0)\,\eta_{ps}(x)))\,|\,\mathbf{q}i\rangle, \qquad (3a)$$

$$t^{(2)}_{a'a}(\mathbf{p}',\mathbf{p}) = -\langle \mathbf{q}'i' \,|\, \eta_{p's'}(0)\, a_{in}{}^{+}(\mathbf{q}i)\,|\,\mathbf{p}s\rangle$$

$$=\langle \mathbf{q}'i' \,|\,([\eta_{p's'}(0),\, a_{qi}{}^{+}(0)]$$

$$-i\int d^4x e^{-iqx} T(\eta_{p's'}(0)\, j_i(x)))\,|\,\mathbf{p}s\rangle, \qquad (3b)$$

where $b^{+}_{\rm in}(\mathbf{p}s)$ and $a^{+}_{\rm in}(\mathbf{q}i)$ denote the nucleon and pion creation operators in the in-state, while the operators $b^{+}_{ps}(x^0)$ and $a^{+}_{qi}(x^0)$ are expressed in terms of the Heisenberg field operators of the nucleon and pion as follows:

$$b_{ps}{}^{+}(x^0) = \int d^3\mathbf{x}\, e^{-ipx} \bar{\psi}(x)\gamma^0 u(\mathbf{p}s), \qquad (4a)$$

$$a_{qi}{}^{+}(x^0) = -i\int d^3\mathbf{x}\, e^{-iqx} \frac{\overleftrightarrow{\partial}}{\partial x^0} \Phi_i(x). \qquad (4b)$$

The relations (3a) and (3b) provide the basis for the derivation of the Low equation. Thus, if we introduce into the second terms on the right-hand sides of relations (3a) and (3b) the completeness condition between the source operators and integrate over x, using the integral representation of the step function, we obtain similarly to the Low equations in Refs. 9, 11, and 13:

$$t^{(\alpha)}_{a'a}(\mathbf{p}',\mathbf{p}) = Y^{(\alpha)}_{a'a}(\mathbf{p}',\mathbf{p}) + V^{(\alpha)}_{a'a}(\mathbf{p}',\mathbf{p})$$

$$+\sum_{a''}\int d\tilde{p}'' \frac{t^{(\alpha)}_{a'a''}(\mathbf{p}',\mathbf{p}'')\, t^{(\beta)*}_{aa''}(\mathbf{p},\mathbf{p}'')}{E_{p''}-E_p-i0}, \qquad \alpha,\beta=1,2, \quad \alpha\neq\beta, \qquad (5)$$

where $d\tilde{p}'' \equiv d^3\mathbf{p}''[M/(2\pi)^3 2\omega_\pi(\mathbf{p}'')\omega_N(\mathbf{p}'')]$, $E_p = \omega_\pi(\mathbf{p}) + \omega_N(\mathbf{p})$ is the total energy of the πN system in the center-of-mass frame, $Y^{(\alpha)}$ $(\alpha = 1,2)$ denote the equal-time commutators:

$$Y^{(1)}_{a'a}(\mathbf{p}',\mathbf{p}) = -\langle \mathbf{p}'s' \,|\,[j_{i'}(0),\, b_{ps}{}^{+}(0)]\,|\,\mathbf{q}i\rangle, \qquad (6a)$$

$$Y^{(2)}_{a'a}(\mathbf{p}',\mathbf{p}) = \langle \mathbf{q}'i' \,|\,[\eta_{p's'}(0),\, a_{qi}{}^{+}(0)]\,|\,\mathbf{p}s\rangle, \qquad (6b)$$

and $V^{(\alpha)}$ $(\alpha = 1,2)$ correspond to the n-particle-exchange interaction, when all the particles in the intermediate state, in accordance with the completeness condition, are on the mass shell:

$$V^{(1)}_{a'a}(\mathbf{p}',\mathbf{p}) = (2\pi)^3 \sum_{n\neq\pi N} \frac{\langle \mathbf{p}'s' \,|\, j_{i'}(0)\,|\, n;\, in\rangle_c \delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{P}_n)\langle n;\, in\,|\,\bar{\eta}_{ps}(0)\,|\,\mathbf{q}i\rangle_c}{P_n{}^0-\omega_\pi(\mathbf{p})-\omega_N(\mathbf{p})-i0}$$

$$+(2\pi)^3 \sum_{m=\sigma,\rho,\ldots} \frac{\langle \mathbf{p}'s' \,|\, \bar{\eta}_{ps}(0)\,|\, m;\, in\rangle_c \delta^{(3)}(\mathbf{p}'-\mathbf{p}-\mathbf{P}_m)\langle m;\, in\,|\, j_i(0)\,|\,\mathbf{q}i\rangle_c}{P_m{}^0+\omega_N(\mathbf{p})-\omega_N(\mathbf{p}')}$$

$$+\bar{V}^{(1)}_{a'a}(\mathbf{p}',\mathbf{p}), \qquad (7a)$$

$$V^{(2)}_{a'a}(\mathbf{p}',\mathbf{p}) = (2\pi)^3 \sum_{n\neq\pi N} \frac{\langle \mathbf{q}'i' \,|\, \eta_{p's'}(0)\,|\, n;\, in\rangle_c \delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{P}_n)\langle n;\, in\,|\, j_i(0)\,|\,\mathbf{p}s\rangle_c}{P_n{}^0-\omega_\pi(\mathbf{p})-\omega_N(\mathbf{p})-i0}$$

$$+(2\pi)^3 \sum_{m=\sigma,\rho,\ldots} \frac{\langle \mathbf{q}'i' \,|\, j_i(0)\,|\, m;\, in\rangle_c \delta^{(3)}(\mathbf{q}'-\mathbf{q}-\mathbf{P}_m)\langle m;\, in\,|\, \eta_{p's'}(0)\,|\,\mathbf{p}s\rangle_c}{P_m{}^0+\omega_\pi(\mathbf{q})-\omega_\pi(\mathbf{q}')}$$

$$+\bar{V}^{(2)}_{a'a}(\mathbf{p}',\mathbf{p}). \qquad (7b)$$

The first two terms in the relations (7a) and (7b) are described by diagrams with on-the-mass-shell n-particle and m-particle intermediate states (see Figs. 3a,b and 4a,b). We denote the total 4-momentum of these intermediate states by $P_n \equiv (P^0_n, \mathbf{P}_n)$ and $P_m \equiv (P^0_m, \mathbf{P}_m)$. Usually such terms are called the s- and t-channel exchange terms of the potential of the corresponding equations.[15,18] The last terms in the relations (7a) and (7b) arise after cluster decomposition of the

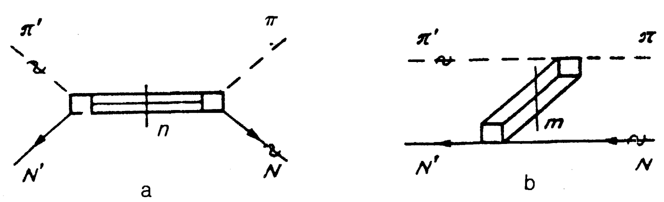

FIG. 3. The πN-scattering diagrams a) in the s channel and b) in the t channel with all possible $n = N, \pi N, \pi\pi N, \ldots$ and $m = \sigma, \rho, 2\pi, N\bar{N}, \ldots$ intermediate states.

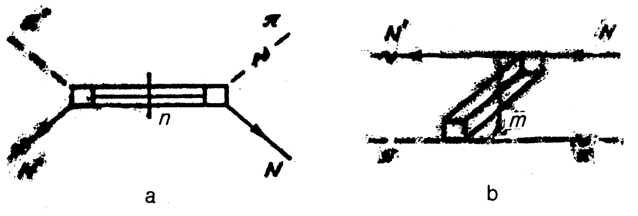

FIG. 4. Same as Fig. 3, but with N' and π off the mass shell instead of π' and N. This difference leads to a change from the temporal sequence of the processes $\pi\to\pi'+m$, $N+m\to N'$ to the sequence $N\to m+N'$, $\pi+m\to\pi'$.

s- and t-channel terms,[13,19] which corresponds to the inclusion of disconnected diagrams in the passage to the n-particle or m-particle state. After this procedure only connected matrix elements remain in the potential term of the Low equation. These matrix elements are denoted by the subscript c.

$$\begin{aligned}\tilde V^{(1)}_{a'a}(\mathbf{p}',\mathbf{p}) =\ & (2\pi)^3 \sum_{l=\pi N,\,\rho N,\,\dots} \frac{\langle \mathbf{p}'s'|j_{i'}(0)|\mathbf{q}i,l;in\rangle_c\,\delta^{(3)}(\mathbf{p}-\mathbf{P}_l)\langle l;in|\bar\eta_{ps}(0)|0\rangle}{P_l^0-\omega_N(\mathbf{p})} \\ &+(2\pi)^3 \sum_{k=N\bar N,\,\pi\sigma,\,\dots} \frac{\langle 0|j_{i'}(0)|k;in\rangle\,\delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{p}'-\mathbf{P}_k)\langle k,\mathbf{p}'s';in|\bar\eta_{ps}(0)|\mathbf{q}i\rangle_c}{P_k^0+\omega_N(\mathbf{p}')-\omega_\pi(\mathbf{q})-\omega_N(\mathbf{p})-i0} \\ &+(2\pi)^3 \sum_{m=\sigma,\,\rho,\,\dots} \frac{\langle 0|j_{i'}(0)|\mathbf{q}i,m;in\rangle\,\delta^{(3)}(\mathbf{p}-\mathbf{p}'-\mathbf{P}_m)\langle m,\mathbf{p}'s';in|\bar\eta_{ps}(0)|0\rangle}{P_m^0+\omega_N(\mathbf{p}')-\omega_N(\mathbf{p})} \\ &-(2\pi)^3 \sum_{\bar l=\pi\bar N,\,\dots} \frac{\langle 0|\bar\eta_{ps}(0)|\bar l;in\rangle\,\delta^{(3)}(-\mathbf{p}-\mathbf{P}_{\bar l})\langle \bar l,\mathbf{p}'s';in|j_{i'}(0)|\mathbf{q}i\rangle_c}{P_{\bar l}^0+\omega_N(\mathbf{p})} \\ &+(2\pi)^3 \sum_{k=N\bar N,\,\pi\sigma,\,\dots} \frac{\langle \mathbf{p}'s'|\bar\eta_{ps}(0)|\mathbf{q}i,k;in\rangle_c\,\delta^{(3)}(\mathbf{p}'-\mathbf{p}-\mathbf{q}-\mathbf{P}_k)\langle k;in|j_{i'}(0)|0\rangle}{P_k^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{q})-\omega_N(\mathbf{p}')} \\ &-(2\pi)^3 \sum_{\bar l=\bar N,\,\pi\bar N,\,\dots} \frac{\langle 0|\bar\eta_{ps}(0)|\mathbf{q}i,\bar l;in\rangle\,\delta^{(3)}(-\mathbf{q}-\mathbf{p}-\mathbf{P}_{\bar l})\langle \bar l,\mathbf{p}'s';in|j_{i'}(0)|0\rangle}{P_{\bar l}^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{q})}. \end{aligned} \tag{8a}$$

All six terms $\tilde V^{(1)}_{a'a}$ are shown graphically in Fig. 5, in the same order as in Eq. (8a). The negative sign of the fourth and sixth term in (8a) is due to the permutation of nucleon fields. A similar expression for $\tilde V^{(2)}_{a'a}$ is given by Eq. (8b) and shown in Fig. 6.

$$\begin{aligned}\tilde V^{(2)}_{a'a}(\mathbf{p}',\mathbf{p}) =\ & (2\pi)^3 \sum_{l=\pi N,\,\dots} \frac{\langle 0|\eta_{p's'}(0)|l;in\rangle\,\delta^{(3)}(\mathbf{p}+\mathbf{q}-\mathbf{q}'-\mathbf{P}_l)\langle l,\mathbf{q}'i';in|j_i(0)|\mathbf{p}s\rangle_c}{P_l^0-\omega_\pi(\mathbf{q})-\omega_N(\mathbf{p})+\omega_\pi(\mathbf{q}')-i0} \\ &+(2\pi)^3 \sum_{k=N\bar N,\,\pi\sigma,\,\dots} \frac{\langle \mathbf{q}'i'|\eta_{p's'}(0)|\mathbf{p}s,k;in\rangle_c\,\delta^{(3)}(\mathbf{q}-\mathbf{P}_k)\langle k;in|j_i(0)|0\rangle}{P_k^0-\omega_\pi(\mathbf{q})} \\ &+(2\pi)^3 \sum_{m=\sigma,\,\rho,\,\dots} \frac{\langle 0|\eta_{p's'}(0)|\mathbf{p}s,m;in\rangle\,\delta^{(3)}(\mathbf{q}-\mathbf{q}'-\mathbf{P}_m)\langle m,\mathbf{q}'i';in|j_i(0)|0\rangle}{P_m^0-\omega_\pi(\mathbf{q})+\omega_\pi(\mathbf{q}')} \\ &+(2\pi)^3 \sum_{k=N\bar N,\,\dots} \frac{\langle 0|j_i(0)|k;in\rangle\,\delta^{(3)}(-\mathbf{q}-\mathbf{P}_k)\langle k,\mathbf{q}'i';in|\eta_{p's'}(0)|\mathbf{p}s\rangle_c}{P_k^0+\omega_\pi(\mathbf{q})} \\ &-(2\pi)^3 \sum_{\bar l=\pi\bar N,\,\dots} \frac{\langle \mathbf{q}'i'|j_i(0)|\mathbf{p}s,\bar l;in\rangle_c\,\delta^{(3)}(\mathbf{q}'-\mathbf{q}-\mathbf{p}-\mathbf{P}_{\bar l})\langle \bar l;in|\eta_{p's'}(0)|0\rangle}{P_{\bar l}^0+\omega_\pi(\mathbf{q})+\omega_N(\mathbf{p})-\omega_\pi(\mathbf{q}')} \\ &-(2\pi)^3 \sum_{\bar l=\bar N,\,\pi N,\,\dots} \frac{\langle 0|j_i(0)|\mathbf{p}s,\bar l;in\rangle\,\delta^{(3)}(-\mathbf{p}-\mathbf{q}-\mathbf{P}_{\bar l})\langle \bar l,\mathbf{q}'i';in|\eta_{p's'}(0)|0\rangle}{P_{\bar l}^0+\omega_\pi(\mathbf{q})+\omega_N(\mathbf{p})}. \end{aligned} \tag{8b}$$

The main difference between formulas (6a),(7a),(8a) and the corresponding expressions (6b),(7b),(8b) is that π' (out) and N (in) particles are off the mass shell in (6a),(7a),(8a), while N' (out) and π (in) particles are off the mass shell in (6b),(7b),(8b). This difference leads to a different form of the propagators of the particles in the intermediate states in expressions (7a),(8a) and (7b),(8b). Further, as can be seen from a comparison of Figs. 3b and 5

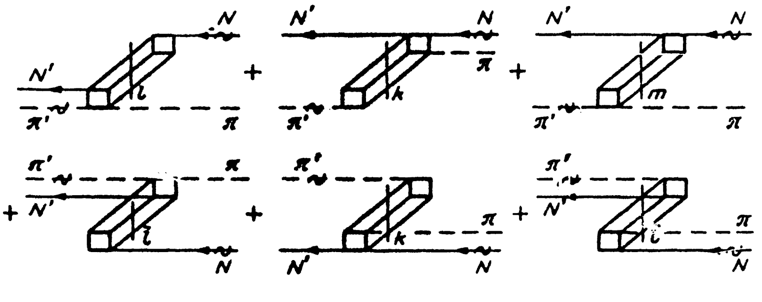

FIG. 5. Graphic representation of the inhomogeneous term $\tilde V^{(1)}$ (8a). The first three terms correspond to an expansion of the s-channel term and the last three—the t-channel term.

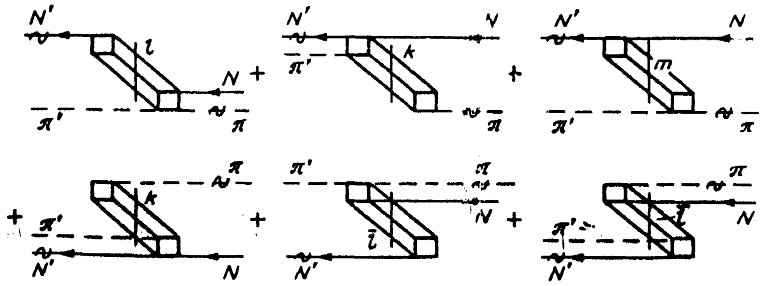

FIG. 6. Graphic representation of the inhomogeneous term $\tilde{V}^{(2)}$ (8b), which is obtained after cluster decomposition of the diagrams of Figs. 4a,4b.

with Figs. 4b and 6, the particle-exchange potentials $V^{(1)}$ (7a) and $V^{(2)}$ (7b) differ in the temporal sequence of absorption of the in-particles π, N and emission of the out-particles π', N' and the interaction of these external particles with the particles in the intermediate states.

The system of integral equations (5) for the amplitude of πN scattering with different off-shell behavior of $t^{(1)}$ (1a) and $t^{(2)}$ (1b) is one of possible versions of the Low equation for the πN-scattering problem. One usually considers Low-type equations for πN scattering with off-shell pions only.[8-14,16] Such a version of the Low equations contains a smaller number of transition matrices in the particle-exchange potential V, whose hermiticity is violated only due to the nonsymmetric form of the propagators of the particles in the intermediate states. However, this potential also contains the nonlinear u-channel term (Fig. 1a), which makes the solution of these equations substantially more difficult. In the formulation of the Low equations (5) considered here such a crossing nonlinearity in the potentials $V^{(1)}$ and $V^{(2)}$ does not occur, the particle-exchange potentials (6a),(6b) and (7a),(7b) contain only s-, t-, $\bar{s}$- and $\bar{t}$-channel terms. In another version of Low-type equations for πN scattering with all the particles off the mass shell, which was considered in Ref. 20, there are also complications connected with the u-channel crossing term (Fig. 1a). Similar complications occur also after the derivation of the Low equation with only the nucleons N' (out) and N (in) off the mass shell, i.e., the equation for the scattering amplitude $t^{(2)}$ (1b).

To conclude this paragraph we write out explicitly the unitarity condition for the sought-for solutions of Eq. (5). It is shown in Appendix A that the following formula is valid for the equal-time commutators (6a),(6b):

$$Y^{(1)}_{a'a}(\mathbf{p}',\mathbf{p}) = Y^{(2)*}_{aa'}(\mathbf{p},\mathbf{p}') + (E_p - E_{p'}) \times \langle \mathbf{p}'s' | [a_{q'i'}(0), \bar{\psi}(0)\gamma_0 u(\mathbf{p}s)] | \mathbf{q}i\rangle. \tag{9}$$

Further, it follows from formulas (3a),(3b) with expression (9) taken into account, that we have on the energy shell $E_p = E'_p$:

$$t^{(1)}_{a'a}(\mathbf{p},\mathbf{p}) - t^{(2)*}_{aa'}(\mathbf{p},\mathbf{p}) = -i\int d^4x \langle \mathbf{p}'s' | e^{-ipx} T(j_{i'}(0)\bar{\eta}_{\mathbf{p}s}(x)) + e^{iq'x} T(j_{i'}(x)\eta_{\mathbf{p}s}(0)) | \mathbf{q}i\rangle. \tag{10}$$

Keeping in mind that the relation (2) between the S matrix for πN scattering and the amplitudes (1a),(1b) implies

$$t^{(1)}_{a'a}(\mathbf{p},\mathbf{p}) = t^{(2)}_{a'a}(\mathbf{p},\mathbf{p}), \tag{11}$$

we obtain from formula (10) the desired unitarity condition for the t matrix of πN scattering on the energy shell:

$$\langle \mathbf{p}'s' | j_{i'}(0) | \mathbf{p}s, -\mathbf{p}i; in\rangle - \langle \mathbf{p}s', -\mathbf{p}i'; in | j_i(0) | \mathbf{p}s\rangle = -(2\pi)^4 i \sum_{n=\pi N,\pi\pi N,\dots} \langle \mathbf{p}'s' | j_{i'}(0) | n; in\rangle \delta^{(4)} \times (p+q-P_n) \langle n; in | \bar{\eta}_{\mathbf{p}s}(0) | \mathbf{q}i\rangle. \tag{12}$$

The same condition of n-particle unitarity is obtained for the amplitude $t^{(1)}$ (1a), which is a solution of the Low equations with only pions off the mass shell.[8-14,16] These equations contain a more symmetric inhomogeneous term.

3. LINEARIZATION OF THE LOW EQUATION

Linearization of the Low Eq. (5) can be achieved analogously to the linearization of the similar equations with only pions off the mass shell, which we considered in Ref. 9. To this end we first replace the propagator of the particles in the intermediate states in Eqs. (7a),(7b) as follows:

$$\frac{1}{P_n^0 - \omega_\pi(\mathbf{p}) - \omega_N(\mathbf{p}) - i0} \to \frac{P_n^0 - E}{[P_n^0 - \omega_\pi(\mathbf{p}) - \omega_N(\mathbf{p}) - i0][P_n^0 - \omega_\pi(\mathbf{p}') - \omega_N(\mathbf{p}') + i0]},$$

$$\frac{1}{P_m^0 + \omega_N(\mathbf{p}) - \omega_N(\mathbf{p}')} \to \frac{P_m^0 + \omega_N(\mathbf{p}) - \omega_N(\mathbf{p}') + E - E_p}{[P_m^0 + \omega_N(\mathbf{p}) - \omega_N(\mathbf{p}')][P_m^0 + \omega_\pi(\mathbf{p}') - \omega_\pi(\mathbf{p})]},$$

$$\frac{1}{P_m^0 + \omega_\pi(\mathbf{p}) - \omega_\pi(\mathbf{p}')} \to \frac{P_m^0 + \omega_\pi(\mathbf{p}) - \omega_\pi(\mathbf{p}') + E - E_p}{[P_m^0 + \omega_\pi(\mathbf{p}) - \omega_\pi(\mathbf{p}')][P_m^0 + \omega_N(\mathbf{p}') - \omega_N(\mathbf{p})]}. \tag{13}$$

It is not hard to see from the expressions (13) that for $E = E'_p = \omega_N(\mathbf{p}') + \omega_\pi(\mathbf{p}')$ these formulas become identities. We also carry out similar transformations in the propagators from the potential $\tilde{V}^{(1)}$ (8a):

$$\frac{1}{P_l^0 - \omega_N(\mathbf{p})} \to \frac{P_l^0 + \omega_\pi(\mathbf{p}) - E}{[P_l^0 - \omega_N(\mathbf{p})][P_l^0 - \omega_N(\mathbf{p}') - \omega_\pi(\mathbf{p}') + \omega_\pi(\mathbf{p}) + i0]} \times \frac{1}{P_k^0 + \omega_N(\mathbf{p}') - \omega_N(\mathbf{p}) - \omega_\pi(\mathbf{p}) - i0}$$

$$\to \frac{P_k^0+\omega_N(\mathbf{p}')-E}{[P_k^0+\omega_N(\mathbf{p}')-\omega_N(\mathbf{p})-\omega_\pi(\mathbf{p})-i0][P_k^0-\omega_\pi(\mathbf{p}')]},$$

$$\frac{1}{P_n^0+\omega_N(\mathbf{p}')-\omega_N(\mathbf{p})}$$

$$\to \frac{P_m^0+\omega_N(\mathbf{p}')+\omega_\pi(\mathbf{p})-E}{[P_m^0+\omega_N(\mathbf{p}')-\omega_N(\mathbf{p})][P_m^0-\omega_\pi(\mathbf{p}')+\omega_\pi(\mathbf{p})]},$$

$$\frac{1}{P_l^0+\omega_N(\mathbf{p})} \to \frac{P_{\bar l}^0-\omega_\pi(p')+E}{[P_{\bar l}^0+\omega_N(\mathbf{p})][P_{\bar l}^0+\omega_\pi(\mathbf{p}')+\omega_N(\mathbf{p}')-\omega_\pi(\mathbf{p})]},$$

$$\frac{1}{P_k^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p})-\omega_N(\mathbf{p}')}$$

$$\to \frac{P_k^0-\omega_N(\mathbf{p}')+E}{[P_k^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p})-\omega_N(\mathbf{p}')][P_k^0+\omega_\pi(\mathbf{p}')]},$$

$$\frac{1}{P_{\bar l}^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p})}$$

$$\to \frac{P_{\bar l}^0+E}{[P_{\bar l}^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p})][P_{\bar l}^0+\omega_N(\mathbf{p}')+\omega_\pi(\mathbf{p}')]}, \tag{14a}$$

and, correspondingly, from $\widetilde{V}^{(2)}$ (8b):

$$\frac{1}{P_l^0-\omega_N(\mathbf{p})-\omega_\pi(\mathbf{p})+\omega_\pi(\mathbf{p}')-i0}$$

$$\to \frac{P_l^0+\omega_\pi(\mathbf{p}')-E}{[P_l^0-\omega_N(\mathbf{p})-\omega_\pi(\mathbf{p})+\omega_\pi(\mathbf{p}')-i0][P_l^0-\omega_N(\mathbf{p}')]},$$

$$\frac{1}{P_k^0-\omega_\pi(\mathbf{p})}$$

$$\to \frac{P_k^0+\omega_N(\mathbf{p})-E}{[P_k^0-\omega_\pi(\mathbf{p})][P_k^0+\omega_N(\mathbf{p})-\omega_N(\mathbf{p}')-\omega_\pi(\mathbf{p}')+i0]},$$

$$\frac{1}{P_m^0-\omega_\pi(\mathbf{p})+\omega_\pi(\mathbf{p}')}$$

$$\to \frac{P_m^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p}')-E}{[P_m^0-\omega_\pi(\mathbf{p})+\omega_\pi(\mathbf{p}')][P_m^0+\omega_N(\mathbf{p})-\omega_N(\mathbf{p}')]},$$

$$\frac{1}{P_k^0+\omega_\pi(\mathbf{p})} \to \frac{P_k^0-\omega_N(\mathbf{p})+E}{[P_k^0+\omega_\pi(\mathbf{p})][P_k^0+\omega_N(\mathbf{p}')+\omega_\pi(\mathbf{p}')-\omega_N(\mathbf{p})]},$$

$$\frac{1}{P_{\bar l}^0+\omega_\pi(\mathbf{p})+\omega_N(\mathbf{p})-\omega_\pi(\mathbf{p}')}$$

$$\to \frac{P_{\bar l}^0-\omega_\pi(\mathbf{p}')+E}{[P_{\bar l}^0+\omega_\pi(\mathbf{p})+\omega_N(\mathbf{p})-\omega_\pi(\mathbf{p}')][P_{\bar l}^0+\omega_N(\mathbf{p}')]},$$

$$\frac{1}{P_{\bar l}^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p})}$$

$$\to \frac{P_{\bar l}^0+E}{[P_{\bar l}^0+\omega_N(\mathbf{p})+\omega_\pi(\mathbf{p})][P_{\bar l}^0+\omega_N(\mathbf{p}')+\omega_\pi(\mathbf{p}')]}. \tag{14b}$$

It is not hard to see that under hermitian conjugation the right-hand sides of formulas (14a) go into the right-hand sides of Eqs. (14b), and for $E=E_p'$ these formulas turn into identities.

We next introduce the notation for the inhomogeneous terms of the system of equations (5):

$$W_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p})=Y_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p})+V_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p}) \tag{15}$$

and define the potentials

$$U_{a'a}^{(1)}(\mathbf{p}',\mathbf{p};E)=Y_{a'a}^{(1)}(\mathbf{p}',\mathbf{p})+v_{a'a}^{(1)}(\mathbf{p}',\mathbf{p};E) \tag{16a}$$
$$+(E-E_{p'})\langle \mathbf{p}'s'|[a_{q'i'}(0),\bar{\psi}(0)\gamma_0 u(ps)]|\mathbf{q}i\rangle,$$

$$U_{a'a}^{(2)}(\mathbf{p}',\mathbf{p};E)=Y_{a'a}^{(2)}(\mathbf{p}',\mathbf{p})+v_{a'a}^{(2)}(\mathbf{p}',\mathbf{p};E)$$
$$+(E-E_{p'})\langle \mathbf{q}'i'|[\bar{u}(p's')\gamma^0\psi(0),a_{qi}^+(0)]|ps\rangle, \tag{16b}$$

where the equal-time commutators $Y_{a'a}^{(\alpha)}$ are determined by Eqs. (6a),(6b), and the potentials $v_{a'a}^{(\alpha)}$, which depend linearly on the energy, are obtained from the expressions (7a),(7b) for the $V_{a'a}^{(\alpha)}$ by replacing the propagators according to the formulas (13) and (14a),(14b). It can be verified from the definition (16a),(16b) of the $U_{a'a}^{(\alpha)}$, the relation (9) and the explicit form of the propagators (13) and (14a),(14b), that these potentials satisfy the following relations:

$$U_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p};E=E_{p'})=W_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p}), \tag{17a}$$

$$U_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p};E=E_p)=W_{aa'}^{(\beta)*}(\mathbf{p},\mathbf{p}'), \tag{17b}$$

$$U_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p};E)=U_{aa'}^{(\beta)*}(\mathbf{p},\mathbf{p}';E), \tag{17c}$$

where $\alpha,\beta=1,2$ and $\alpha\neq\beta$.

We can then show that the solution of the linear integral equation

$$T_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p};E)=U_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p};E)$$
$$+\sum_{a''}\int d\tilde{p}''\frac{U_{a'a''}^{(\alpha)}(\mathbf{p}',\mathbf{p}'';E)\,T_{a''a}^{(\alpha)}(\mathbf{p}'',\mathbf{p};E)}{E_{p''}-E-i0} \tag{18}$$

reproduces the solution of the nonlinear integral equation (5). To this end, following Ref. 9, we can construct an iterative series for Eqs. (5) and (18). After making use of the properties of the linearly energy-dependent potential $U_{a'a}^{(\alpha)}$ (17a)–(17c) and the identity for the product of the linear propagators given in Ref. 9 [Eq. (3.11) from that work], we can show that the solution of the Low equation (5) and the linear integral equation (18) coincide on the energy shell:

$$T_{a'a}^{(\alpha)}(\mathbf{p},\mathbf{p};E_p)=t_{a'a}^{(\alpha)}(\mathbf{p},\mathbf{p}), \tag{19}$$

while on the half-energy-shell the solution of the nonlinear Low equation (5) can be expressed in terms of the solution of the Lippman–Schwinger equation:

$$t_{a'a}^{(\alpha')}(\mathbf{p}',\mathbf{p})=W_{a'a}^{(\alpha)}(\mathbf{p}',\mathbf{p})$$
$$+\sum_{a''}\int d\tilde{p}''\frac{W_{a'a''}^{(\alpha)}(\mathbf{p}',\mathbf{p}'')\,T_{a''a}^{(\alpha)}(\mathbf{p}'',\mathbf{p};E_p)}{E_{p''}-E_p-i0}. \tag{20}$$

Next, the desired πN-scattering amplitude $|\mathbf{p}'s'|j_{i'}(0)|\mathbf{p}s,\ \mathbf{q}i;\ in\rangle$ (1a) can be calculated by solving the Low equations with only pions off the mass shell,[8-14,16] and the Low equations (5) with the pion and nucleon off the mass shell. The solutions of these nonlinear integral equations can be found through the solutions of the corresponding Lippman–Schwinger equations, namely Eq. (3.12) from Ref. 9 and Eq. (18). Here the part of the potential containing the exchange of particles on the mass shell consists in our case of s-, t-, $\bar{s}$- and $\bar{t}$-channel terms, while in the equations of Refs. 9 and 10 this part of the potential consists of s-, u-, $\bar{s}$-

and $\bar{u}$-channel terms and contains the u-channel crossing nonlinearity which is characteristic of the πN scattering problem (Fig. 1a). Moreover, the potentials contain equal-time commutators, whose evaluation requires the explicit form of the πN-interaction Lagrangian. In this approach all the information about the u- and $\bar{u}$-channel interactions is contained in these equal-time commutators $Y^{(\alpha)}$ (6a), (6b). Furthermore, it turns out to be impossible to extract from these terms the nonlinear crossing term (Fig. 1a). These equal-time commutators are evaluated in Appendix A in a model based on the simplest effective πN Lagrangian.

In this manner, Eq. (18), in contrast to the equations of Refs. 9 and 10, can be viewed as the fully linearized version of the Low equations for the πN-scattering problem. However, additional complications arise with Eq. (18), connected with the nonhermiticity of the potential $U^{(\alpha)}_{a'a}$ (16a),(16b), for whose construction we must specify not only the transition matrices with an off the mass shell pion, but also the transition matrices with one nucleon from the in- or out-state off the mass shell.

It is known from the theory of nonlinear integral equations that these equations can have a unique solution, as well as an infinitude of solutions. Thus it was shown for the Chew–Low equation in Refs. 21–23 that, under certain conditions imposed on the interaction constants and the asymptotic behavior of the form factors, this nonlinear equation has a unique solution on a specified class of functions. Apparently similar studies need to be performed also for the more general Low-type equations to clarify the relation of the solution manifold of the Low Eq. (5) and the solution of the Lippman–Schwinger equation (18).

4. MODEL OF SINGLE-PARTICLE EXCHANGE FOR THE πN-INTERACTION POTENTIAL

The description of the particle interaction in the low and intermediate energy region by the mechanism of single-particle exchange is the accepted and simplest model for the theoretical study of these interactions. For πN scattering already in the familiar Chew–Low model[12] nucleon exchange in the s- and u-channel was taken into account. In Refs. 13 and 14 for the Low equation with only pions off the mass shell the need was demonstrated for including in the potential for πN scattering not only the exchange of a nucleon on the mass shell, but also an antinucleon on the mass shell in the $\bar{s}$- and $\bar{u}$-channels. The equal-time commutator in these papers was described by the exchange of a σ meson in the t-channel, while in Refs. 9 and 10 we have also included the exchange of a ρ meson in the t-channel.

Let us consider the potential for Eq. (18) in the approximation where only single-particle intermediate states are included. The complication which occurs in such an approach to the calculation of the s-, t-, $\bar{s}$- and $\bar{t}$-channel potentials (7a),(7b) and, correspondingly, the potentials $U^{(\alpha)}_{a'a}(\mathbf{p}',\mathbf{p};E)$ (16a),(16b), arises because these potentials are not hermitian due to the asymmetric way the transition matrices that determine the particle-exchange potentials (7a),(7b) are taken off the mass shell. It should be noted that the similar potential in Eq. (3.12) of Ref. 9, containing the π(in) and π'(out) pions taken symmetrically off the mass shell, is hermitian. For this reason no difficulties relating to the unitarity condition arise in the construction of the potential in the single-particle exchange approximation in Ref. 9.

In the approximation in which only single-particle intermediate states are included in the inhomogeneous term of the Low equation the unitarity condition (12) reduces to the so-called two-particle unitarity condition, containing in relation (12) only the two-particle πN state. However, although the relation (10) is also valid in the single-particle-exchange model, to derive the identity (11) and hence the very condition of two-particle unitarity additional efforts are needed. We note that if in the construction of the potentials $V^{(\alpha)}$ given by (7a),(7b) one uses in the intermediate states in place of in-states half of the sum of in-states and out-states, then after the partial expansion these potentials will be real. We show below that the reality property of these potentials permits the reproduction of the unitarity condition also in the case when only single-particle intermediate states are included. However, if we use from the very outset the completeness condition $\hat{1}=\frac{1}{2}(\Sigma_{n;\text{in}}+\Sigma_{n;\text{out}})|n\rangle\langle n|$ then in the s-channel term of the Low Eq. (5) the expression $\mathrm{Re}[t^{(\alpha)}t^{(\beta)*}]$ appears in place of $t^{(\alpha)}t^{(\beta)*}$. In Appendix B we demonstrate the equivalence of the two types of Low equations with real potentials. The desired form of the single-particle-exchange potential can be obtained in yet another way. In Appendix C we show that, if we follow the single-particle-exchange approximation, we can obtain directly from the expressions (7a),(7b) with intermediate in-states the desired potential with a half-sum of intermediate in-and out-states.

For $\alpha=1$ the potential $U^{(\alpha)}(E)$ can be represented as follows:

$$
\begin{aligned}
U^{(1)}_{a'a}(\mathbf{p}',\mathbf{p};E)={}&Y^{(1)}_{a'a}(\mathbf{p}',\mathbf{p})+(E-E_{\mathbf{p}'})\\
&\times\langle\mathbf{p}'s'|[a_{\mathbf{q}'i'}(0),\bar{\psi}(0)\gamma_0u(\mathbf{p}s)]|\mathbf{q}i\rangle+\sum_{s_N}\frac{\langle\mathbf{p}'s'|j_{i'}(0)|\langle 0s_N\rangle(M-E)\langle 0s_N|\bar{\eta}_{ps}(0)|\mathbf{q}i\rangle}{[M-\omega_\pi(\mathbf{p})-\omega_N(\mathbf{p})][M-\omega_\pi(\mathbf{p}')-\omega_N(\mathbf{p}')]}\\
&+\sum_{\sigma,\rho(\lambda)}\frac{\langle\mathbf{p}'s'|\bar{\eta}_{ps}(0)|(\mathbf{p}'-\mathbf{p})\lambda\rangle\langle(\mathbf{p}'-\mathbf{p})\lambda|j_{i'}(0)|\mathbf{q}i\rangle}{[\omega_{\sigma(\rho)}(\mathbf{p}'-\mathbf{p})+\omega_N(\mathbf{p})-\omega_N(\mathbf{p}')][\omega_{\sigma(\rho)}(\mathbf{p}'-\mathbf{p})+\omega_\pi(\mathbf{p}')-\omega_\pi(\mathbf{p})]}\;\frac{\omega_{\sigma(\rho)}(\mathbf{p}'-\mathbf{p})-\omega_\pi(\mathbf{p})-\omega_N(\mathbf{p}')+E}{2\omega_{\sigma(\rho)}(\mathbf{p}'-\mathbf{p})}\\
&+\frac{1}{2}\sum_{\sigma,\rho(\lambda)}\frac{\langle 0|j_{i'}(0)|\mathbf{q}i,(\mathbf{p}-\mathbf{p}')\lambda\rangle\langle(\mathbf{p}-\mathbf{p}')\lambda,\mathbf{p}'s'|\bar{\eta}_{ps}(0)|0\rangle}{[\omega_{\sigma(\rho)}(\mathbf{p}-\mathbf{p}')-\omega_N(\mathbf{p})+\omega_N(\mathbf{p}')][\omega_{\sigma(\rho)}(\mathbf{p}-\mathbf{p}')-\omega_\pi(\mathbf{p}')+\omega_\pi(\mathbf{p})]}\;\frac{\omega_{\sigma(\rho)}(\mathbf{p}-\mathbf{p}')+\omega_\pi(\mathbf{p})+\omega_N(\mathbf{p}')+E}{2\omega_{\sigma(\rho)}(\mathbf{p}-\mathbf{p}')}\\
&-\frac{1}{2}\sum_{s_A}\frac{\langle 0|\bar{\eta}_{ps}(0)|\mathbf{q}i,0s_A\rangle(M+E)\langle 0s_A,\mathbf{p}'s'|j_{i'}(0)|0\rangle}{[M+\omega_\pi(\mathbf{p})+\omega_N(\mathbf{p})][M+\omega_\pi(\mathbf{p}')+\omega_N(\mathbf{p}')]},
\end{aligned}
\tag{21}
$$

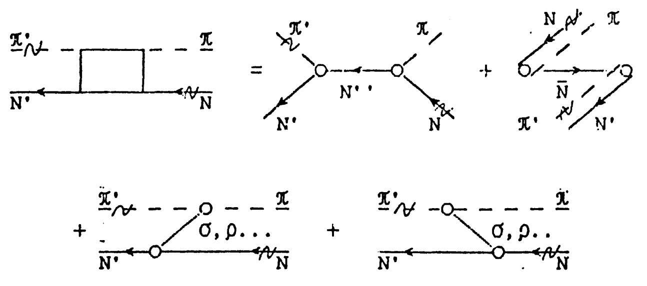

FIG. 7. The potential of the linearized equation (18) in the approximation when only single-particle intermediate states are included.

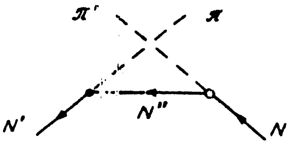

FIG. 8. The equal-time commutator (6a), calculated on the basis of the Lagrangian of the linear σ model. The solid circle denotes the πNN vertex function (22b) and the open circle—the πNN vertex function in the tree approximation.

where (λ) denotes summation over the isospin and polarization variables in the terms corresponding to the exchange of the ρ meson, while in the case of the exchange of the σ meson no summation is carried out. In the last two terms of expression (21) a double summation is performed—over in-states and out-states. The explicit form of the equal-time commutator, calculated using the model of the simplest phenomenological Lagrangian, is given in Appendix A. Expression (21) is depicted graphically in Fig. 7, where the quadrangle in the first diagram corresponds to the equal-time commutator in that expression.

We next write out the single-particle-exchange potential (21) explicitly including only N, $\bar{N}$ and σ as intermediate particles. For the πNN vertex function with the pion off the mass shell we have the well-known general expression, containing a single scalar form factor:

$$\langle \mathbf{p}'s'|j_i(0)|\mathbf{p}s\rangle = i\bar{u}(\mathbf{p}'s')\tau_i\gamma^5 u(\mathbf{p}s)F^\pi((p'-p)^2). \qquad (22a)$$

When the nucleon is off the mass shell the general form of the vertex function contains two independent form factors:[24]

$$-\langle \mathbf{p}'s'|\eta_{ps}(0)|\mathbf{q}i\rangle = i\bar{u}(\mathbf{p}'s')\tau_i\gamma^5\Big[\frac{\hat{p}'-\hat{q}+M}{2M}F_+^N((p'-q)^2) + \frac{\hat{p}'-\hat{q}-M}{2M}F_-^N((p'-q)^2)\Big]u(\mathbf{p}s). \qquad (22b)$$

The invariant structure of the vertex functions for the $\sigma\pi\pi$ and σNN systems is given by the following expressions:

$$\langle \mathbf{k}_\sigma|j_{i'}(0)|\mathbf{q}i\rangle = \delta_{ii'}G_{\sigma\pi\pi}((k_\sigma-q)^2), \qquad (23a)$$

$$-\langle \mathbf{p}'s'|\eta_{ps}(0)|\mathbf{k}_\sigma\rangle = \bar{u}(\mathbf{p}'s')\Big[\frac{\hat{p}'-\hat{k}_\sigma+M}{2M}F_+^\sigma((p'-k_\sigma)^2) + \frac{\hat{p}'-\hat{k}_\sigma-M}{2M}F_-^\sigma((p'-k_\sigma)^2)\Big]u(\mathbf{p}s). \qquad (23b)$$

The vertex functions in formulas (22a), (22b) and (23a), (23b) are defined in the space-like region and are real. However, in the $\bar{s}$- and $\bar{u}$-channel terms in expression (21) the same vertex functions occur in the timelike region, where they are complex. The presence of the half-sum over intermediate in- and out-states in the $\bar{s}$- and $\bar{u}$-channel terms permits, similarly to Ref. 13, the product of two complex form factors to be replaced by the real part of that product.

In this way, after taking into account the explicit form of the equal-time commutators, obtained by us in Appendix A and shown in Fig. 8, we obtained for the potential (21)

$$U^{(1)}_{a'a}(\mathbf{p}',\mathbf{p};E) = -\bar{u}(\mathbf{p}'s')i\tau_i\gamma^5\Big[\frac{\hat{p}'-\hat{q}+M}{2M}F_+^N((p'-q)^2) + \frac{\hat{p}'-\hat{q}-M}{2M}F_-^N((p'-q)^2)\Big]\frac{1}{\hat{p}'-\hat{q}-M}i\tau_{i'}\gamma^5 g_{\pi NN}u(\mathbf{p}s)$$

$$+\frac{M-E}{(M-E_{p'})(M-E_p)}\bar{u}(\mathbf{p}'s')i\tau_{i'}\gamma^5F^\pi((p'-n)^2)\frac{1+\gamma^0}{2}i\tau_i\gamma^5 \times\Big[\frac{M(1+\gamma^0)-\hat{q}}{2M}F_+^N((n-q)^2) + \frac{M(\gamma^0-1)-\hat{q}}{2M}F_-^N((n-q)^2)\Big]$$

$$\times u(\mathbf{p}s)|_{n_\mu=(M,\,0)} - \frac{M+E}{(M+E_{p'})(M+E_p)}\bar{u}(\mathbf{p}'s')i\tau_{i'}\gamma^5 \times\frac{\gamma^0-1}{2}i\tau_i\gamma^5\Big\{\frac{M(1-\gamma^0)-\hat{q}}{2M}$$

$$\times\mathrm{Re}\,[F^\pi((p'+n)^2)F_+^N((n+q)^2)] + \frac{-M(1+\gamma^0)-\hat{q}}{2M}\mathrm{Re}\,[F^\pi((p'+n)^2)F_-^N((n+q)^2)]\Big\}$$

$$\times u(\mathbf{p}s)|_{n_\mu=(M,\,0)}$$

$$+\frac{1}{2\omega_\sigma}\frac{\omega_\sigma-q^0-p'^0+E}{(\omega_\sigma-p'^0+p^0)(\omega_\sigma-q^0+q'^0)} \times\bar{u}(\mathbf{p}'s')\Big\{\frac{\hat{p}'-\hat{l}+M}{2M}F_+^\sigma((p'-l)^2) + \frac{\hat{p}'-\hat{l}-M}{2M}F_-^\sigma((p'-l)^2)\Big\}\delta_{i'i}G_{\sigma\pi\pi}$$

$$\times((l-q)^2)u(\mathbf{p}s)|_{l_\mu=(\omega_\sigma(\mathbf{p}'-\mathbf{p}),\,\mathbf{p}'-\mathbf{p})}$$

$$+\frac{1}{2\omega_\sigma}\frac{\omega_\sigma+q^0+p'^0-E}{(\omega_\sigma+p'^0-p^0)(\omega_\sigma+q^0-q'^0)} \times\bar{u}(\mathbf{p}'s')\delta_{i'i}\Big\{\frac{\hat{p}'+\hat{l}+M}{2M}\mathrm{Re}\,[G_{\sigma\pi\pi}$$

$$\times((l+q)^2)F_+^\sigma((p'+l)^2)] + \frac{\hat{p}'+\hat{l}-M}{2M}\mathrm{Re}\,[G_{\sigma\pi\pi}((l+q)^2) \times F_-^\sigma((p'+l)^2)]\Big\}u(\mathbf{p}s)|_{l_\sigma=(\omega_\sigma(\mathbf{p}-\mathbf{p}'),\,\mathbf{p}-\mathbf{p}')}. \qquad (24)$$

The expression (24) for the potential of Eq. (18) is given in the form of a product of hermitian kinematic factors and real functions, depending on the initial and final 3-momentum and the energy E. Imaginary parts are absent from these scalar functions because the full sum over intermediate states is represented as a half-sum over in- and out-states.

Next we carry out the partial expansion of Eq. (18) for the πN-scattering amplitude. To this end we utilize the procedure and notation from Refs. 13 and 9.

$$U^{(\alpha)}_{a'a}(\mathbf{p}',\mathbf{p};E)=4\pi\sum_{ljI}U^{(\alpha)}_{ljI}(p',p;E)P_l^j(\mathbf{p}'s',\mathbf{p}s)\Pi^I(i'i), \tag{25a}$$

$$T^{(\alpha)}_{a'a}(\mathbf{p}',\mathbf{p};E)=4\pi\sum_{ljI}T^{(\alpha)}_{ljI}(p',p;E)P_l^j(\mathbf{p}'s',\mathbf{p}s)\Pi^I(i'i), \tag{25b}$$

where P^j_l and Π^I denote projection operators in spin and isospin space. It is not hard to see that the potentials $U^{(\alpha)}_{ljI}$ will be real because the scalar functions entering expression (24) are real:

$$U^{(\alpha)*}_{ljI}(p',p;E)=U^{(\alpha)}_{ljI}(p',p;E), \tag{26}$$

while the condition that the potentials $U^{(\alpha)}$ (17b) be hermitian, with the reality of (26) taken into account, gives a relation for the potentials under the exchange of p' and p:

$$U^{(1)}_{ljI}(p',p;E)=U^{(2)}_{ljI}(p,p';E). \tag{27}$$

After this we obtain for the πN-scattering amplitude from the partial expansion of Eq. (18)

$$T^{(\alpha)}_{ljI}(p',p;E)$$
$$=U^{(\alpha)}_{ljI}(p',p;E)-\int d\bar{k}\frac{U^{(\alpha)}_{ljI}(p',k;E)T^{(\alpha)}_{ljI}(k,p;E)}{E-E_k+i0}, \tag{28}$$

where

$$d\bar{k}=\frac{1}{2\pi^2}\frac{M}{\omega_N(\mathbf{k})}\frac{1}{2\omega_\pi(\mathbf{k})}dk.$$

It is easily seen that

$$T^{(1)}_{ljI}(p,p;E_p)=T^{(2)}_{ljI}(p,p;E_p). \tag{29}$$

On the other hand, just as in the derivation of relation (12), the condition that the potentials be hermitian yields for the solutions of Eq. (28)

$$T^{(1)}_{ljI}(p,p;E_p)-T^{(2)*}_{ljI}(p,p;E_p)$$
$$=2\pi i\int d\bar{k}\,T^{(1)}_{ljI}(p,k;E_p)\delta(E_k-E_p)T^{(2)*}_{ljI}(k,p;E_p). \tag{30}$$

Formulas (29) and (30) ensure the desired two-particle unitarity on the energy shell for the solutions of Eq. (28).

5. CHOICE OF PHENOMENOLOGICAL FORM FACTORS AND NUMERICAL CALCULATIONS

In this section we demonstrate that we can, on the basis of the solution of the linear integral equations (28), reproduce the resonant P_{33} πN-scattering phase shift within the framework of the accepted parametrization of the vertex functions (22a),(22b) and (23a),(23b). In order to reduce the number of independent parameters in the theory as much as possible, we have neglected the contribution of the ρ-meson exchange and confined ourselves to just the contribution of the σ-meson exchange. Note that according to Ref. 9 the inclusion of the ρ-meson exchange does not qualitatively change the behavior of the scattering phase shift in the P_{33} wave. In a detailed study of low-energy πN scattering, which presumes also the description of other partial waves, a number of important effects should be included. For example, the exchange of the ρ meson should be included in the description of the P_{11} wave of πN scattering,[9] while for the case of the S-waves the Low equation with a subtraction should be used. We shall not be concerned with these effects in the present work and as a first step in the description of πN scattering we shall only reproduce the experimental behavior of the P_{33} phase shift.

For the pion-nucleon-nucleon vertex functions (22a),(22b) we used the simple single-pole parametrization. [Note that (31b) is compatible in the tree approximation with the absence of pseudovector coupling in the πN Lagrangian, which is satisfied in the linear σ model.]

$$F^\pi(t)=g_{\pi NN}\frac{\mu_N{}^2-m_\pi{}^2}{\mu_N{}^2-t}, \tag{31a}$$

$$F_+{}^N(t)=-F_-{}^N(t)=g_{\pi NN}\frac{\Lambda_N{}^2-M^2}{\Lambda_N{}^2-t}, \tag{31b}$$

where we have used for the πN-interaction constant the value $g_{\pi NN}=14.0$, and for the cut-off parameters μ_N and Λ_N after fitting we found the values $\mu_N=1.195$ GeV and $\Lambda_N=1.171$ GeV.

In the potential $U^{(1)}$ given by Eq. (21) $\sigma\pi\pi$ and σNN vertex functions occur with arguments varying in the space-like, as well as in the time-like, region. Therefore, following Refs. 25 and 26, these form factors must be taken in the form of complex functions:

$$G_{\sigma\pi\pi}(t)$$
$$=g_{\sigma\pi\pi}\frac{\mu_s{}^2-m_\pi{}^2}{\mu_s{}^2-t-(i\gamma/2)[t-(m_\pi+m_\sigma)^2]\theta(t-(m_\pi+m_\sigma)^2)}, \tag{32a}$$

$$F_+{}^\sigma(t)=-F_-{}^\sigma(t)$$
$$=g_{\sigma NN}\frac{\Lambda_s{}^2-M^2}{\Lambda_s{}^2-t-(i\Gamma/2)[t-(M+m_\sigma)^2]\theta(t-(M+m_\sigma)^2)}. \tag{32b}$$

In choosing the constants of the $\sigma\pi\pi$ and σNN interactions, as well as in the construction of the equal-time commutator, we started from the linear σ model. Therefore these constants were determined from the relations $g_{\sigma NN}=g_{\pi NN}$ and $g_{\sigma\pi\pi}=g^2_{\pi NN}m_\sigma/g_{\pi NN}M$, which are valid in the linear σ model in the tree approximation, where the mass m_σ of the σ meson equals 500 MeV. Matching with experiment gives the following values for the form factor masses: $\Lambda_s=1.22$ GeV, $\mu_s=1.0$ GeV. For the parameters γ and Γ we used the same value $\gamma=\Gamma=1.232$ that was used for the similar quantities in Refs. 25 and 26. It was checked that changing the values of these parameters in the rather wide interval $1\leqslant\gamma,\ \Gamma\leqslant1.5$

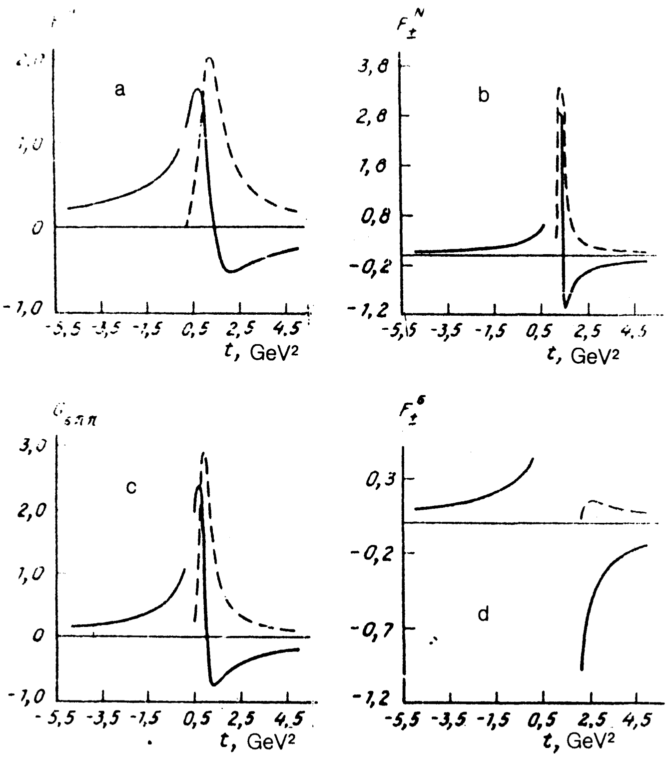

FIG. 9. The meson–nucleon and meson–meson vertex functions $G_H(t)$, normalized to unity on the mass shell. a) the πNN vertex with the pion off the mass shell. b) The πNN vertex with the nucleon off the mass shell. c) The $\sigma\pi\pi$ vertex with the pion off the mass shell. d) The σNN vertex with the nucleon off the mass shell. The vertices are shown for both space-like $[t \leqslant (m_1 - m_2)^2]$ and time-like $[t \geqslant (m_1 + m_2)^2]$ values of the arguments. For the πNN vertex in the time-like region we used a parametrization coinciding in form with (32a)–(32b), with the same values of the parameters $\gamma = \Gamma = 1.232$.

does not substantially change the behavior of the solution in the P_{33} partial wave.

In Fig. 9 we show various meson-nucleon and meson-meson vertex functions, normalized to unity on the mass shell for the fitting parameters given above. As can be seen from Fig. 9 these form factors, as well as the πNN form factors from Refs. 25 and 26, are rapidly varying functions near the mass shell in the time-like region. Similar behavior of the πNN form factor was obtained in Ref. 27 on the basis of the dispersion approach with neglect of inelastic channels, while taking inelasticity into account apparently "smooths out" these rapid oscillations.[27]

In Fig. 10 we show the scattering phase shift in the P_{33} partial wave obtained as a result of solving Eq. (18). The experimental data are taken from Ref. 28. It is seen from Fig. 10 that in the region ~200 MeV below the resonance we obtain a fairly good description of the experiment, while above the resonance the solution of Eq. (18) lies below the experimental points. Apparently this kind of behavior is typical of the P_{33} scattering phase shift, calculated on the basis of relativistic Low-type equations in the single-particle-exchange approximation. Note that the scattering phase shift in the P_{33} wave, obtained from the Low equation with pions off the mass shell, behaves similarly.[9,14]

It should be noted that in the approach of the Low equation with pions and nucleons off the mass shell various terms in the potential are responsible for the resonant behavior of the P_{33} scattering phase shift. For example, the $\bar{u}$-channel z-graph (Fig. 11), which gives the largest contribution to the potential in the equations of Refs. 9 and 14, is absent from the potential of Eq. (18). And conversely, the σ-meson exchange, which plays an important role in this approach, is much less important in the Low equations of Refs. 9 and 14. Evidently the increase in the relative weight of the contributions corresponding to the nucleon pole and σ-exchange to the potential in Eq. (18), in comparison to the similar potential in Ref. 9, arises because the nucleon goes off the mass shell and compensates for the absence of the $\bar{u}$-channel antinucleon pole.

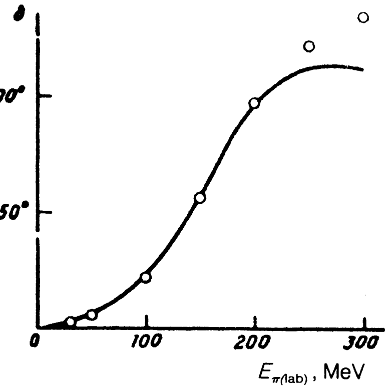

FIG. 10. The P_{33} πN-scattering phase shift.

6. CONCLUSION

We show in this work that we can obtain two forms of the Low equation for one and the same amplitude for πN

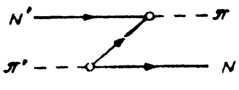

FIG. 11. The $\bar{u}$-channel antinucleon pole (z-graph), which doesn't arise in the inhomogeneous term of the Low equation with the nucleon off the mass shell.

scattering $\langle \mathbf{p}'s'|j_{i'}(0)|\mathbf{p}s,\mathbf{q}i;in\rangle$, (1a) i.e., that there exist two kinds of spectral decomposition of this amplitude. In the ordinary Low equations only the pions π' and π are customarily considered off the mass shell,[9,10,13,14] while in the equation proposed in this work we take off the mass shell the pion π' and the nucleon N. Both types of Low equations reduce to Lippman–Schwinger equations, whose potential is uniquely determined by the inhomogeneous term of the initial nonlinear equations. Further, different choices of the particle reduced from the in-states in these equations significantly changes the form of the corresponding potentials. Thus the potential of Eq. (18) or (28) does not contain the u-channel crossing nonlinearity (Fig. 1a), and the $\bar{u}$-channel z-diagram (Fig. 11), which makes an important contribution to the P-wave πN scattering in the approach of Refs. 9 and 14, does not appear. On the other hand, in the proposed equation with the particles π' and N off the mass shell, the construction of the potential requires a larger number of vertex functions than in the equations with pions off the mass shell.[9,10,13,14]

Another interesting question, which arises in the study of low-energy P-wave πN scattering in the approach based on relativistic equations, is connected with the nature of the Δ resonance. In our opinion the answer to the question of how the P_{33} resonance should be described—as an independent particle on the same footing as the nucleon or as the solution of the corresponding equations—should be looked for in the defining conditions imposed on the vertex functions, in terms of which the πN-interaction potential is determined. Unfortunately such vertex functions are obtained these days, in essence, phenomenologically and their parametrization and explicit form are quite ambiguous.[29] However, the attempts to calculate various meson-nucleon and meson-meson vertex functions within the framework of quark models[6,16,30,31] permits one to hope that this problem will be cleared up. In our opinion such investigations are convenient in the field-theoretical approach of the Low equations, since taking into account quark degrees of freedom does not change the structure of the equations for the scattering amplitudes.[32]

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APPENDIX A

Equal-time commutators

We first derive formula (9). To this end we make use of the relation:

$$-\langle \mathbf{p}'s'|[\dot{a}_{q'i'}(x^0),\ \overset{+}{b}_{ps}(x^0)]|\mathbf{q}i\rangle$$
$$=\langle \mathbf{p}'s'|[a_{q'i'}(x^0),\ \dot{\overset{+}{b}}_{ps}(x^0))|\mathbf{q}i\rangle$$
$$-\langle \mathbf{p}'s'|\frac{d}{dx^0}[a_{q'i'}(x^0), b^+_{ps}(x^0))|\mathbf{q}i\rangle. \qquad (A1)$$

Formula (9) is obtained from (A1) for $x^0=0$ if we make use of the property of translation invariance in the last term and use the following relations:

$$\dot{a}_{q'i'}(x^0)\equiv\frac{d}{dx^0}a_{q'i'}(x^0)=i\int d^3\mathbf{x}\, e^{iq'x}j_{i'}(x),$$
$$(A2)$$
$$\dot{b}^+_{ps}(x^0)\equiv\frac{d}{dx^0}b^+_{ps}(x^0)=i\int d^3\mathbf{x}\, e^{-ipx}\bar{\eta}_{ps}(x).$$

To calculate the equal-time commutators in (9) we make use of the simplest phenomenological Lagrangian of the linear σ model.[18,19] That Lagrangian contains no field derivatives. Therefore, with the help of the canonical commutation relations we obtain:

$$-\langle \mathbf{p}'s'|[j_{i'}(0), b_{ps}^+(0))|\mathbf{q}i\rangle$$
$$=ig_{\pi NN}\langle \mathbf{p}'s'|\bar{\psi}(0)|\mathbf{q}i\rangle\gamma^5\tau_{i'}u(\mathbf{p}s)$$
$$=ig_{\pi NN}\langle \mathbf{p}'s'|\bar{\eta}(0)|\mathbf{q}i\rangle\frac{\hat{p}'-\hat{q}+M}{(p'-q)^2-M^2}\gamma^5\tau_{i'}u(\mathbf{p}s). \qquad (A3)$$

Making use of the expression for the vertex function for the πNN system off the mass shell (23b), we obtain from (A3) the explicit form of the equal-time commutator (6a) in the framework of the linear σ model, which coincides with the first term of the potential $U^{(1)}$.[24] This term corresponds to the nucleon pole diagram in the u-channel (Fig. 8), with one of the vertices, calculated in the tree approximation.

APPENDIX B

Low equations with real nonhermitian potentials

We show below that for a real and nonhermitian potential $W^{(\alpha)}_{ljI}(p',p)$, which is endowed with the following properties

$$U^{(\alpha)}_{ljI}(p',p;E_{p'})=W^{(\alpha)}_{ljI}(p',p),$$
$$U^{(\alpha)*}_{ljI}(p',p;E)=U^{(\alpha)}_{ljI}(p',p;E), \qquad (B1)$$
$$U^{(1)}_{ljI}(p',p;E)=U^{(2)}_{ljI}(p,p';E),$$

the Low equation

$$t^{(\alpha)}_{ljI}(p',p)=W^{(\alpha)}_{ljI}(p',p)+\int d\bar{k}\frac{t^{(\alpha)}_{ljI}(p',k)t^{(\beta)*}_{ljI}(p,k)}{E_k-E_p-i0}$$
$$(\alpha,\ \beta=1,\ 2;\ \alpha\neq\beta) \qquad (B2)$$

has the same solution as the equation

$$t^{(\alpha)}_{ljI}(p',p)=W^{(\alpha)}_{ljI}(p',p)+\int d\bar{k}\frac{\mathrm{Re}[t^{(\alpha)}_{ljI}(p',k)t^{(\beta)*}_{ljI}(p,k)]}{E_k-E_p-i0}$$
$$(\alpha,\ \beta=1,\ 2;\ \alpha\neq\beta). \qquad (B3)$$

To this end we will show that the solution of the Lippman–Schwinger equation (18), which is obtained from Eq. (B2), satisfies also Eq. (B3). This is achieved by making use of the circumstance that the following relation

$$T^{(\alpha)}_{ljI}(p',p;E_p+i0)-T^{(\alpha)}_{ljI}(p',p;E_p-i0)$$
$$=2\pi i\int d\bar{k}\,T^{(\alpha)}_{ljI}(p',k;E_p+i0)\delta(E_k-E_p)T^{(\alpha)}_{ljI}(p,p;E_p-i0)$$
$$(B4)$$

is satisfied by the solution of Eq. (18) with the potential $U^{(\alpha)}_{ljI}(p',p;E)$, which obeys the conditions (B1).

Substituting (B4) into (20) we obtain

$$\mathrm{Im}[t^{(\alpha)}_{lji}(p',p)]=\pi\rho(p)t^{(\alpha)}_{lji}(p',p)t^{(\alpha)*}_{lji}(p,p), \qquad (B5)$$

where

$$\rho(p)=\int d\bar{k}\,\delta(E_k-E_p). \qquad (B6)$$

We can show, making use of (B5), that the solution of Eq. (B2) on the half-energy shell can be represented in the form of the product of the solution on the energy shell and a real function, which becomes unity on the energy shell:

$$\mathrm{Im}\left[\frac{t_{ijl}^{(\alpha)}(p',p)}{t_{ijl}^{(\alpha)}(p,p)}\right]=0. \qquad \text{(B7)}$$

When (B7) is substituted into (B3) it is easily seen that the instruction Re in the integrand of that equation can be omitted. This completes the proof of our assertion.

APPENDIX C

πN potential in the approximation of single-particle exchange

If from the very beginning we use in the completeness relation the sum over in-states, and not the half-sum over in- and out-states, the linearization procedure can be performed directly. As a result we obtain Eq. (18). However in the potential $U(E)$ (21) in the last two terms the half-sum over in- and out-states is replaced by the sum over in-states only.

Further, making use of the following well-known relation between creation and annihilation operators

$$a_{qi}{}^{+}(in)=a_{qi}{}^{+}(out)+i\int d^4x\, e^{-iqx}j_i(x),$$
$$b_{p's'}(in)=b_{p's'}(out)+i\int d^4y\, e^{ip'y}\eta_{p's'}(y), \qquad \text{(C1)}$$

we can transform various matrix elements that enter expression (21). For example, for the product of matrix elements which appears in the last term of expression (21), we have

$$\langle 0|\bar{\eta}_{ps}(0)|\mathbf{q}i,0s_A;in\rangle\langle in;0s_A,\mathbf{p}'s'|j_{i'}(0)|0\rangle$$
$$=\langle 0|\bar{\eta}_{ps}(0)a_{qi}{}^{+}(in)|0s_A\rangle\langle 0s_A|b_{p's'}(in)j_{i'}(0)|0\rangle$$
$$=\langle 0|\bar{\eta}_{ps}(0)\left(a_{qi}{}^{+}(out)+i\int d^4x\,e^{-iqx}j_i(x)\right)|0s_A\rangle$$
$$\times\langle 0s_A|\left(b_{p's'}(out)+i\int d^4y\,e^{ip'y}\eta_{p's'}(y)\right)j_{i'}(0)|0\rangle. \qquad \text{(C2)}$$

It is obvious that the matrix elements $\langle 0|\bar{\eta}_{ps}(0)j_i(x)|0s_A\rangle$ and $\langle 0s_A|\eta_{p's'}(y)j_{i'}(0)|0\rangle$ vanish in the approximation in which only single-particle states and the two-particle πN state are allowed in the full sum over intermediate states. Taking this circumstance into account leads again to the potential $U(E)$ given by expression (21).

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