Multiple spontaneous oscillating states in a bistable semiconductor cavity with nonlinearity competition

A.V. Grigor'yants and I.N. Dyuzhikov

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Results are presented of an experimental investigation of a variety of spontaneous-oscillation states in a bistable semiconductor cavity with competition between the electronic and temperature nonlinear-dispersion mechanisms. Stability-loss bifurcations of various types are obtained. Hysteretic transitions between vibrational and stable states as well as between different oscillation regimes are recorded. Generation of single optical pulses in a slaved regime is observed. Good agreement with a theoretical model is obtained.

INTRODUCTION

McCall has shown¹ that spontaneous oscillations (SO) of the output radiation can be observed in stable optical cavities in the presence of two competing nonlinear-dispersion mechanisms with different characteristic times. These oscillations, which are due to competition between the electronic and thermal nonlinearity mechanisms, were observed experimentally in certain semiconductor cavities excited near the edge of the band gap.²⁻⁴ Only the observation of these oscillations was reported there, however, without any investigation of the possible oscillatory regime, the conditions for their existences, and the dependences of their characteristics on external parameters. A theory proposed in Refs. 5-9 describes the spatiotemporal dynamics of a semiconductor cavity in the presence of negative electronic and positive thermal linear refraction due to photogeneration of the carriers and recombination heating of the semiconductor. Account was also taken of the presence of growing absorption due to a shift of the band gap edge with change of temperature, and transverse current and heat diffusion leading to the existence of various spatial dissipative structures.^{5,6,9} It was shown that even in the spatially homogeneous case the behavior of the system can be considerably more varied compared with Refs. 1-4, and that various SO states can be realized in conjunction with bi- and multistability.

For SO to set in it is necessary that to exceed a certain analytically determined input intensity and to choose correspondingly the ratio of the characteristic thermal and recombination times.⁵⁻⁷ When both conditions are met, the system stationary point, determined by intersection of the zero-isoclines of the homogeneous differential equations describing the dynamics of the photocarrier density N and of the crystal temperature T, is unstable and is surrounded by a discontinuous limit cycle. It was shown that for a corresponding choice of parameter there can exist several oscillatory and stable states which correspond to different orders of interference and between which hysteretic transitions are possible. It was also demonstrated that SO exist in a bounded range of input power P_0 having a width that depends on the initial cavity phase deviation δ and the characteristic thermal time τ_T (Ref. 8). In addition, a possibility was predicted of generating single optical pulses in response to a small additional perturbation.

It was shown thus that the dynamic behavior of a relatively simple system, a plane-parallel semiconductor slab pumped by coherent radiation in the region of the intrinsic absorption edge can be quite complicated and varied.

The present paper reports the first detailed experimental investigation, based on the theory of Refs. 5–9, of the variety of spontaneous-oscillation regimes caused by nonlinearity competition in a bistable semiconductor cavity.

EXPERIMENTAL RESULTS AND DISCUSSION

The experiments were performed on an InSb cavity excited by a CO laser of wavelength $\lambda = 5.6-6 \,\mu\text{m}$ at a temperature T = 80-100 K. The sample was a polished planeparallel InSb slab of thickness $l = 500 \,\mu\text{m}$ and area $5 \times 5 \,\text{mm}^2$, with a reflecting coating of gold sputtered on the rear face into a spot $\approx 400 \,\mu\text{m}$ in diameter. By varying the ways of securing the sample in the cryostat it was possible to change the heat-transfer conditions and hence τ_T .

Figure 1 shows the quasistationary dependences of the reflected radiation P_R on the incident power P_0 , obtained from an x-y plotter at various cavity detunings δ (the value of δ was varied by changing the initial non-illuminated temperature T_0 of the sample). The $P_R(P_0)$ plots show the thermal instability regions and the SO region caused by the competition between the electronic and thermal nonlinearities. The location of these regions and their P_0 range vary substantially with change of δ . Depending on this parameter, the output characteristic can either begin with a hysteresis followed by SO when P_0 increases or be subject to SO at small P_0 and bistability at large input powers. At certain values of δ the oscillatory and stable states are realized at one and the same value of P_0 with a hysteretic transition between them, i.e., the cavity can be optically switched between stable and oscillatory states.

Figure 2 shows the SO and bistability regions plotted using the set of output $P_R(P_0)$ characteristics as P_0 and δ are varied. The intersection of the bistability and SO regions corresponds to simultaneous existence of oscillatory and stable states at fixed P_0 and δ . The periodic repetition of the bistability and SO states when δ is varied is due to the periodic dependence of the radiation inside the cavity on the phase shift.

We observed SO in three orders of interference when λ and T_0 were properly chosen (Fig. 3). The SO amplitude decreases with increase of the order of interference, owing to the temperature rise and the increased absorption in the specimen, in accord with theoretical results.^{6,7} Hysteretic transitions were also recorded between different oscillatory states were also recorded (Fig. 4). In Fig. 4, as P_0 increases



FIG. 1. Quasistationary dependences of reflected power P_R on the incident power P_0 , obtained at various initial cavity phase deviations δ : a $-T_0 = 89$ K, $\delta = 0.8\pi$; b $-T_0 = 90$ K, $\delta = -0.5\pi$; c $-T_0 = 92$ K, $\delta = 0$; d $-T_0 = 94$ K, $\delta = 0.5\pi$. The time to change P_0 from zero to the maximum was ≈ 5 min. The numbers I-6 on the characteristic (c) denotes the P_0 levels corresponding to oscillograms with the corresponding numbers in Fig. 5.

from zero, the output characteristic goes in succession through a region of stable states, an SO region, again a stability region, and then is switched to another oscillatory mode with different period and oscillation amplitude. This switching corresponds to a sample temperature jump to a highertemperature isocline branch in the phase plane of the system.⁵⁻⁸ As P_0 decreases from its maximum to zero, the oscillation region intersects the stable and oscillatory sections on the branch with lower T. At a certain value of P_0 the stationary state becomes stable. With further decrease of P_0 a jumplike transition to the initial oscillatory regime takes place and is accompanied by a jumplike decrease of T. Thus, a single characteristic exhibits the various hysteretic transitions between stable and oscillatory states, and also between different oscillatory states, and the system exhibits steady



FIG. 2. Regions of variation of the input power P_0 and of the initial cavity detuning δ , corresponding to spontaneous oscillations (1, dashed line) and bistability 2, solid line). The intersection of regions 1 and 2 correspond to an oscillatory stable regime (dashed). The triangles mark the maximum P_0 level limiting the measurements.

vibrational as well as bivibrational behavior. The optical hysteresis is accompanied by a temperature hysteresis.

Simultaneously with plotting the output characteristic with an automatic plotter having a time constant 0.01 s/cm, the oscilloscope screen showed the dynamics of the reflected radiation (the time constant of the Ge:Au photoresistor was $\approx 1 \,\mu$ s). The oscillograms make it possible to determine the form, period, and amplitude of the SO at different P_0 (Fig. 5).

Depending on the initial cavity detuning δ , we recorded in one and the same order of interference two different types of bifurcation of the SO production with variation of P_0 , soft and abrupt [different from the classical hard one (Figs. 1c and 5)]. Namely, at a certain input power $P_0^{\rm cr}$ noise fluctuations with a characteristic time ~ 0.1 ms appear against the background of a constant reflected signal, due to enhancement of the noise in the input radiation near the bifurcation point. Further increase of P_0 leads to ordering of the signal and to the appearance of more stable nearly harmonic oscillations with a period $t_{so} \approx 0.3-0.5 \ \mu s$ and with large amplitude. Further increase of P_0 causes a constant growth of the SO period up to several seconds (i.e., to a purely optical retuning of the SO period in the range from 10^{-4} to 3 s) and to an increase of their amplitude. An increase of the stability of the SO and a change of their form from nearly harmonic to pronouncedly relaxed (Fig. 5) are simultaneously observed.



FIG. 3. Spontaneous oscillations in three interference order (the return characteristic obtained by decreasing P_0 is not shown).



FIG. 4. Hysteresis of the spontaneous oscillations in various orders of interference. The characteristics a and b were obtained respectively for increasing and decreasing P_0 . The temperature hysteresis (c) was obtained using a thermocouple that averaged out the *T* oscillations. The stability and spontaneous-oscillation regimes are designated *I* and *2*, respectively.

Figures 6 and 7 show the dependence of the SO period and amplitude on the values of P_0 above critical at a soft loss of stability. The oscillation amplitude is proportional in the entire measurement range, accurate to several percent, to the



FIG. 5. Evolution of temporal dynamics of reflected signal with increase of input power, for the soft stability loss case shown in Fig. 1c. The oscillogram numbers correspond to the P_0 levels marked on Fig. 1c. The time scale for oscillograms is 20 ms/div for oscillograms 1-3, 0.2 s/div for 4, and 1 s/div for 5 and 6.

square root of the above-critical P_0 , viz., $A = \text{const} \cdot [P_0 - P_0^{\text{cr}}]^{1/2}$. The use of logarithmic scales yields the linear relation

 $\lg A = \operatorname{const} + 0.5 \lg (P_o - P_o^{er})$

(see insert in Fig. 7). This character of the amplitude variation agrees with the general result of oscillation theory for the case of soft instability loss.^{10,11}

In the case of abrupt stability loss (Figs. 1a and 1b), when a certain value P_0 is exceeded a jump is observed from a stable state into relaxation oscillations, with finite amplitude and with a period on the order of a second, near the initially stable state. The oscillation period and amplitude likewise increase with P_0 in this case, which is reminescent of the classical hard stability loss^{10,11} but in contrast to the latter the system behaves, just as in the soft regime, reversibly, i.e., SO appear or vanish at one and the same value of P_0 (without hysteresis) when P_0 is increased or decreased.

The difference between the soft and hard stability losses is illustrated by the position of the zero-isocline system in the



FIG. 6. Dependences of the period (a) and amplitude (b) of SO on the above-critical value of the input power for the case of soft stability loss.



FIG. 7. Mathematical-model⁵⁻⁷ zero-isocline placement illustrating the soft (a) and abrupt (b) stationary-state stability loss upon increase of the input power: $P_0^{\prime} < P_0^{2} < P_0^{3} < P_0^{4}$. The unstable section of the nonmonoclinic isocline is shown dashed, the limit cycles are tagged by arrows, \bullet —stable stationary states, O—unstable states corresponding to SO, \triangle —unstable states of saddle type.

NT phase plate at different initial cavity detuning δ (Fig. 8). (The isoclines are drawn in Fig. 8 approximately, on the basis of numerous computer calculations made jointly with Yu. A. Rzhanov). In the soft case, which is realized in experiment at δ close to zero, the stable stationary point for small P_0 is located in the region of the S-shaped section of a nonmonotonic isocline that depends on P_0 (the straight isocline is determined by the heat transfer from the sample and is independent of P_0). When P_0 approaches P_0^{cr} the point is located in the middle part of the S-shaped section. At $P = P_0^{\rm cr}$ this isocline section reverses slope and loses stability.⁵ A small limit cycle is produced at this instant and increases with increase of P_0 , in accord with the gradual increase of the amplitude and period of the SO. At above-critical P_0 the limit cycle contains slow-motion sections located on an isocline and describing simultaneous variation of N and T, with characteristic time τ_T , and fast jumps of N without noticeable change of T. It is evident from Fig. 8a that at large input powers $(P_0 = P_0^4)$ there exists beside the limit cycle also a stable stationary state on a higher-temperature branch of the isocline, i.e., the experimentally obtained (Fig. 1c) vibrationally stable regime is realized.

At a sufficiently large initial cavity detuning, the stationary point at small P_0 is far from the middle part of the Sshaped section of the isocline and remains stable at input powers exceeding the value $P_0^{\rm cr}$ corresponding to reversal of the sign of the isocline slope. In this case the stationary state lands on the unstable section shown dashed in Fig. 8 at larger P_0 . The limit cycle produced in this case is finite and has a pronounced discontinuous character, in accord with the abrupt onset of the oscillations. Just as in the soft regime, the onset of the limit cycle is not connected here with the jumplike transition of a stationary point into another region of the phase plane, as in the classical case of strong stability loss,^{10,11} and the system behaves reversibly, i.e., the reverse transition from SO to the stable state is without hysteresis.

Creation of SO in higher-order interference (Figs. 3 and 4) has always a classical hard character due to the jumplike transition of a stationary state to a higher low-temperature isocline branch already containing an unstable section at the instant of the jump. In these cases the reverse transition to the initial stable or oscillatory regime is accompanied by temperature and optical hystersis.

Let us compare the experimental critical power P_0^{cr} corresponding to soft loss of stability with the theoretically predicted value given in Ref. 5:

$$P_{0}^{cr} = -\frac{\lambda\hbar\omega S(1-R_1R_2e^{-2\alpha t})^2}{16\sigma\tau\eta (R_1R_2)^{\frac{1}{2}}e^{-\alpha t}(1-R_1)(1-e^{-\alpha t})(1+R_2e^{-\alpha t})}$$

where λ and $\hbar\omega$ are the wavelength and the radiation-quantum energy, η is the quantum yield, σ is a coefficient descriptive of the nonlinear carrier-generated increment to the *n* refractive index, $\Delta n_e = \sigma N$, α is the absorption coefficient, R_1 and R_2 are respectively the reflection coefficients of the front and rear faces of the semiconductor cavity, and S is the area of the illuminated spot.

The relation above was obtained from the condition that the middle part of the S-shaped section of the isocline reverse sign when linearly approximated, and should overestimate P_0^{cr} compared with the exact function describing the isocline.⁵ Using the experimental parameter values corresponding to Fig. 1c, namely $\lambda = 5.6 \mu m$, $R_1 = 0.36$, $R_2 = 0.9$, $S = 1.3 \cdot 10^{-3} \text{ cm}^2$, $l = 500 \mu m$, $\eta = 1$, and $\tau = 9 \cdot 10^{-8} \text{ s}$ (Ref. 12) and our earlier measured data¹³ $\sigma(T = 86-104 \text{ K}) = -3.6 \cdot 10^{-18} \text{ cm}^3$ and $\alpha(T = 92 \text{ K}) = 11 \text{ cm}^{-1}$, we get $P_0^{cr} = 29.4 \text{ mW}$. The experimental value of the critical power for a soft stability loss [Fig. 1c] is $P_0^{cr} = 22 \text{ mW}$. We have thus not only qualitative but also good quantitative agreement (with allowance for the linear approximation made in Ref. 5, which overestimates P_0^{cr}) between experiment and the theory developed in Refs. 5–9.

The character of the zero-isocline and correspondingly the behavior of the phase trajectories in the NT plane for different input parameters can be determined by recording the transient processes. Figure 9 shows oscillograms of P_R



FIG. 8. Transitions to stable (a) and spontaneous-oscillation (b) regimes, obtained by stepwise application of P_0 , and the corresponding qualitative placement of the zero-isoclines (on the right). The grace 2 on the oscillograms is a continuation in time of trace 1.



FIG. 9. Transition to stable state (left) corresponding to the slaved regime, and generation of a single pulse (tagged by an arrow) due to fluctuation of the input power P_0 . The corresponding placement of the zeroisoclines is shown on the right.

obtained with a storage oscilloscope after turning on P_0 stepwise for two different initial detunings δ , and the corresponding qualitative character of the isocline placement. The initial jump corresponds to a rapid (with characteristic electron-system time τ) jump of the representative point from the origin to the nonmonotonic isocline obtained from the condition $\partial N / \partial t = 0$. Further smooth variation of P_R is described by slow motion of the point along the isocline with a thermal time $\tau_T(\tau_T \gg \tau)$. An abrupt jump of P_R is seen on both oscillograms and corresponds to a rapid (electronic) transition between stable branches of the isocline in the lower order of interference. This switching is a purely dynamic effect and is absent from the stationary $P_R(P_0)$ plots obtained for the same δ . The subsequent slow motion along the isocline is terminated by formation of a stable state (Fig. 9a) or an oscillatory regime (Fig. 9b).

As shown theoretically in Refs. 5-7 and 9, with proper choice of the parameters the stationary point of the system can be located near an unstable section of the isocline, on its right or left. In this case an insignificant above-threshold perturbation of the maintaining input power, of either sign, should lead to generation of a single standard output-radiation pulse, corresponding to one passage of the representative point around the unstable section and a return to the stable state (slaved regime). A similar pulsed activity is produced in nonlinear dynamic systems of varying types and is qualitatively analogous to the behavior of a neuron.^{14,15} At sufficiently large transverse dimensions of the system, local excitation of a pulse in the slaved regime can lead to a selfsustaining spatial traveling pulse propagating in the plane of the system as a result of nonlinearity diffusion (in our case, diffusion of N and T, Refs. 5, 6, and 9). The effect predicted was observed by us experimentally under nonlocal excitation. Figure 9 shows a transition of this type to a stable state of the described type, obtained at a power P_0 close to the threshold of the abrupt onset of oscillations. The single pulse P_R tagged by an arrow is due in this case to natural fluctuation of the laser-output radiation, which serves as the priming perturbation. Similar pulse generation in the slaved regime was obtained also in response to weak pulsed modulation of the sustaining power P_0 .

All the results above were obtained for weak heat transfer from the sample, when the crystal was mounted on a copper cold finger through a thermal-insulation liner. Enhancement of the heat dissipation from the sample (i.e., a decrease of τ_T) decreases the slope of the straight isocline relative to the N axis and increases accordingly the input power needed for intersection with the unstable section of the S-shaped isocline. For good thermal contact between the sample and the cold finger, stable relaxation SO of the reflected radiation, with a period 10–30 ms, were observed at an input intensity $I_0 \approx 120 \text{ W/cm}^2$ (at $I_0 = 10-20 \text{ W/cm}^2$ in the case of weak heat transfer) in a narrow range of I_0 (~5%).

CONCLUSION

We have thus observed experimentally, for the first time ever, the possible existence of various types of spontaneousoscillation states in bistable semiconductor cavities with nonlinearity competition. We have investigated the laws governing the onset of SO and obtained the regions where they exist. We have demonstrated the possibilities of soft, abrupt, and classical hard stationary-state stability losses. We have investigated the changes of the amplitude and period of the oscillations as functions of the input intensity and demonstrated the possibility of purely optical tuning of the spontaneous-oscillation period over four orders of magnitude. Output characteristics corresponding to bistable vibrationally-stable and bivibrational behavior were obtained for various cavity detunings. Hysteretic transitions were recorded between stable and vibrational states, and also between different oscillation regimes. We investigated the onset of stable and oscillatory stationary states and observed generation of single optical pulses in the slaved regime. Good qualitative and quantitative agreement with a previously developed theoretical model was obtained.

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