New type of weak localization of electrons in disordered media

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It is shown that in the inelastic scattering of intermediate-energy electrons through large angles in noncrystalline matter a new coherent effect should exist. It is due to the interference of electron wave fields, each of which is associated with one of the possible realizations of the process of multiple elastic scattering of an electron and electron scattering with excitation of some state of the medium. The effect leads to the appearance of an additional (in comparison with the case of elastic scattering) dependence of the cross section for scattering of electrons by the medium as a whole on their scattering angle. This additional angular dependence corresponds to weak localization of the inelastically scattered electrons. In its physical nature, the new phenomenon is similar to the phenomenon of weak Anderson localization, but differs essentially from it in that the presnece of an act of inelastic scattering is a necessary condition for the existence of the new type of weak localization. Unlike weak localization of the Anderson type, the new phenomenon occurs in a rather wide range of scattering angles of the order of γ/ω, where γ is the frequency of electron collisions and $\hbar\omega$ is the energy lost by the electron.

1. INTRODUCTION

Recent years have been characterized by a sharp growth of interest in the phenomenon of weak localization of waves—a phenomenon manifested in the coherent amplification of waves that are elastically scattered almost precisely backward in disordered media. This has been facilitated by the understanding that has emerged of the generality of effects of this kind in the case of waves of the most varied types (electron, electromagnetic, and even sound) with Anderson localization of electrons in disordered systems.[1–8] The direct experimental confirmation of the existance of weak localization of light[9–16] and the appearance of similar coherent effects in the theory of electronic conduction, including such a beautiful phenomenon as negative anomalous magnetoresistance,[17–23] has made the study of all manifestations of the phenomenon of weak localization an urgent problem. The existence of a similar phenomenon may be suspected even in astrophysics,[24,25] although in this case other explanations of the effect may also exist. Most recently, there has been a tendency to seek effects involving the weak localization of waves of diverse types, including longitudinal electromagnetic waves in artificially created incommensurate layer systems,[26,27] surface waves,[28–31] and so on. The relationship of the weak-localization effect to the symmetry of scattering processes under time reversal was noted in Refs. 7 and 32.

Besides the searches for new systems in which weak localization can exist, work is being done in the direction of a more accurate theoretical description of the phenomenon of weak localization. In the description of the weak localization of fast electrons or light two assumptions are usually made. First, multiple scattering is usually described in a scheme in which the collisions are reduced to multiple forward scattering and single scattering through a large angle,[5,33] or the diffusion approximation is used.[3,21,34–37] Second, the scattering at each individual scatterer is usually assumed to be isotropic. A treatment that went beyond the diffusion approximation was given in Ref. 11 and 38. In the latter paper, an attempt was also made to take into account the influence of inelastic processes on the weak localization of elastically scattered particles in an object of finite thickness. As regards the assumption that each individual elastic scattering is isotropic, an analysis of the real significance of this assumption has not yet been performed, to our knowledge.

All of these papers have been concerned with weak localization of elastically scattered particles. The actual localization in the disordered medium has been attributed to a coherent process in which the next act of elastic scattering begins before the previous one has ended. In the case of backward scattering of electromagnetic waves, this corresponds to double passage of the wave through one and the same region with irregularly positioned scatterers.[6] In the backward scattering of electrons, there is interference of electron waves passing through the same scatterers in a different sequence.[7,17]

Weak localization is manifested principally in an increase of the elastic backward scattering in an extremely narrow range of solid angles the order of λ/l, where λ is the wavelength of the electron wave or light wave and l is mean free path of the electrons or photons. In any case, it has been assumed that the role of inelastic processes in weak localization is secondary and even negative, since it is assumed that the existence of inelastic scattering reduces the probability of coherent processes.

In this paper we show that, in addition to the usual weak localization that has been discussed hitherto, there should exist a new type of weak localization of electrons, which involves an inelastic process in an essential way but for which the origin of the inelasticity is of no importance. We consider three mechanisms of inelastic scattering—excitation of plasmons and of transverse electromagnetic waves by an electron in the medium, and also excitation and ionization of individual atoms. Summation over all final states of the excited medium for a fixed value of the energy $\hbar\omega$ of the excitation is assumed. Certain differences in the localization of electrons for different mechanisms of inelastic scattering exist, but it turns out that, nevertheless, the common fea-

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tures of this effect are dominant. In all cases, the scattering is greatest in the region of scattering angles close to $\pi/2$, and the effect is manifested in a considerably wider range of angles than in the case of ordinary localization. Whereas in the elastic channel the range of such angles is of the order of λ/l, in our case it is of the order of $\gamma/\omega = (\lambda/l)(E/\hbar\omega)$, where γ is the electron-collision frequency and E is the electron energy. Apart from its purely physical interest, the latter circumstance is also important in that it makes the experimental observation of such localization of comparatively fast electrons much more realistic than it is for ordinary localization.

We do not assume isotropy of scattering of electrons by the scattering centers, the role of which, in our case, is played by atoms in a noncrystalline solid. The angular and energy dependence of the inelastic scattering is determined by a realistic interaction Hamiltonian.

The energy of an electron incident on the medium is assumed to be high enough for the possible excitation of, e.g., plasmons or atoms. At the same time, this energy should not be too high, since we are concerned with the interference of electron waves. There exists a region of so-called intermediate energies, for which the scattering of electrons through large angles by the atoms constituting the medium, with a comparatively small loss of energy, occurs in such a way that the electron trajectory is determined by a single scattering of the electron through a large angle and multiple scattering through small angles.[39–41] The corresponding energies lie in the range from one hundred eV to 1.5 keV (Ref. 39). This scheme of the process, in which in the elastic scattering one takes into account multiple collisions of the electron with scattering through small angles and single scattering through large angles is the one we use. It has also been applied in the theory of conventional weak localization.[5]

The final aim of the work of the work is to show that there should exist a new universal (independent of the nature of the inelasticity) phenomenon, which can be regarded as weak localization of electrons in disordered media in the inelastic-scattering channel.

2. PROBABILITY OF PLASMON EMISSION DURING MOTION OF AN INTERMEDIATE-ENERGY ELECTRON IN A DISORDERED MEDIUM

The operator of the energy of interaction of an electron with the medium consists of two terms:

$$\hat{H}_{int}=\hat{H}_a+\hat{H}_i, \tag{1}$$

where $\hat{H}_a$ is the operator of the energy of interaction with the atoms of the medium:

$$\hat{H}_a=\sum_l \hat{V}(\mathbf{r}-\mathbf{r}_l) \tag{2}$$

in which $V(\mathbf{r}-\mathbf{r}_l)$ is the operator of the interaction of the electron with atom l. The sum in (2) is taken over all the atoms. The operator $\hat{H}_i$ describes the interaction with the electrons of the medium, and determines the electric polarization of the medium that arises under the action of the moving electron. In the case of plasmons we have

$$\hat{H}_i=\int d\mathbf{r}\,\hat{\rho}(\mathbf{r})\hat{\varphi}(\mathbf{r}), \tag{3}$$

where $\hat{\varphi}(\mathbf{r})$ is the operator of the potential of the electric field of the electrons capable of taking part in plasma oscillations and $\hat{\rho}(\mathbf{r})$ is the operator of the charge density of the moving electron.

The initial state of the system (moving electron plus medium) is characterized by the wave funcion $|n,\mathbf{p}\rangle$, where n describes the set of quantum numbers of the medium in the initial state and $\mathbf{p}$ is the momentum of the incident electron. The final state of the whole system is described by the wave function $|m,\mathbf{p}-\mathbf{Q}\rangle$, where $\mathbf{Q}$ is the total momentum transferred to the medium.

The probability of a transition of the system in unit time from one state to another with transfer of momentum $\mathbf{Q}$ from the incident electron to the medium and with transition of the medium from state n to state m has the form (in this section, as a rule, we take $\hbar=1$)

$$w_{mn}(\mathbf{Q})=2\pi|T_{mn,\mathbf{Q}}|^2\delta(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{Q}}-E_m+E_n). \tag{4}$$

In (4) we have the T-matrix

$$T_{mn,\mathbf{Q}}=\langle m,\mathbf{p}-\mathbf{Q}|\hat{H}_i+\hat{H}_a|\psi^+_{n,\mathbf{p}}\rangle, \tag{5}$$

in which

$$\psi^+_{n,\mathbf{p}}=|n,\mathbf{p}\rangle+\frac{1}{E_n+E_\mathbf{p}-\hat{H}_0+i\delta}(\hat{H}_a+\hat{H}_i)\psi^+_{n,\mathbf{p}}. \tag{6}$$

Here, $\hat{H}_0$ is the operator of the energy of the electron and medium without allowance for their interaction;

$$\hat{G}_0=[E_n+E_\mathbf{p}-\hat{H}_0+i\delta]^{-1} \tag{7}$$

is the corresponding Green's function. Henceforth, we assume that $E_n=0$.

Bearing in mind that the operator $\hat{H}_i$ the Born approximation is sufficient, while for the operator $\hat{H}_a$ the theory should go beyond the Born approximation, we can write the transition amplitude in the form

$$T_{mn,\mathbf{Q}}=\sum_{\mathbf{p}',\mathbf{p}''}\langle m,\mathbf{p}-\mathbf{Q}|\hat{H}_a\hat{G}_a+\delta_{\mathbf{p}-\mathbf{Q},\mathbf{p}'}|\mathbf{p}',m\rangle\langle m,\mathbf{p}'|\hat{H}_i|n,\mathbf{p}''\rangle \times\langle n,\mathbf{p}''|\delta_{\mathbf{p}'',\mathbf{p}}+\hat{G}_a\hat{H}_a|n,\mathbf{p}\rangle, \tag{8}$$

where $\hat{G}_a$ denotes the Green's function

$$\hat{G}_a=\hat{G}_0+\hat{G}_0\hat{H}_a\hat{G}_a. \tag{9}$$

We introduce the notation $\langle m,\mathbf{p}'|\hat{H}_i|n,\mathbf{p}''\rangle = T_i(mn,\mathbf{q})$, where $\mathbf{q}=\mathbf{p}''-\mathbf{p}'$ is the momentum transferred to the electron system of the medium. Here it has been assumed that $T_i(mn,\mathbf{p}',\mathbf{p}'')=T_i(mn,\mathbf{q})$, which is true in the case of plasmons.

Therefore, the transition amplitude is

$$T_{mn,\mathbf{Q}}=\sum_\mathbf{q} T_i(mn,\mathbf{q})B(\mathbf{p},\mathbf{q},\mathbf{Q},mn), \tag{10}$$

where the function

$$B(\mathbf{p},\mathbf{q},\mathbf{Q},mn)=\delta_{\mathbf{q},\mathbf{Q}}+\langle n,\mathbf{p}-\mathbf{Q}+\mathbf{q}|\hat{G}_a\hat{H}_a|n,\mathbf{p}\rangle +\langle m,\mathbf{p}-\mathbf{Q}|\hat{H}_a\hat{G}_a|m,\mathbf{p}-\mathbf{q}\rangle + \sum_{\mathbf{p}_1}\langle m,\mathbf{p}-\mathbf{Q}|\hat{H}_a\hat{G}_a|m,\mathbf{p}_1\rangle\langle n,\mathbf{p}_1+\mathbf{q}|\hat{G}_a\hat{H}_a|n,\mathbf{p}\rangle \tag{11}$$

describes the scattering of the electron by the atoms. The first term in (11) corresponds to the electron-scattering process in which there is no scattering by atoms and only a plasmon with momentum $\mathbf{q}$ is excited. The second term corresponds to a process that begins with scattering by atoms and ends with excitation of plasmons. The third term corresponds to the process in which the events occur in the opposite order. The fourth term corresponds to a process in which the emission of a plasmon occurs between scatterings of the electron by atoms. If we are not interested in small-angle scattering by the medium as a whole, the first term can be omitted.

It follows from (4) and (10) that the transition probability $dw(\mathbf{Q},\omega)$ per unit time, averaged over the initial and summed over the final states of the medium, has the form

$$dw(\mathbf{Q},\omega)=\frac{m_e V(p^2-2\omega)^{1/2}}{(2\pi)^3}\sum_{\mathbf{q}} w_i(\mathbf{q},\omega)$$

$$\times|B(\mathbf{p},\mathbf{q},\mathbf{Q},\omega)|^2\,d\Omega_{\mathbf{Q}}\,d\omega, \tag{12}$$

where m_e is the electron mass. This formula is valid if for the electrons we take the disorder of the medium into account while the electric field of the plasmon we assume the medium to be entirely uniform. The quantity

$$w_i(\mathbf{q},\omega)=\left(\frac{2e^2}{V}\right)\operatorname{Im}D^R(\mathbf{q},\omega)$$

is due to $T_i(mn,\mathbf{q})T_i^*(mn,\mathbf{q}')$ and is the probability per unit time of emission of a plasmon with momentum $\mathbf{q}$ in the uniform medium and without allowance for the interaction of the electron with the atoms of the medium; $d\Omega_{\mathbf{Q}}$ in (12) is the solid-angle element within which the electron scattered in the medium leaves the medium; $\omega=E_{\mathbf{p}}-E_{\mathbf{p}-\mathbf{Q}}$, and V is the normalization volume.

We have

$$|B|^2=|\langle n,\ \mathbf{p}+\mathbf{q}-\mathbf{Q}|\hat{G}_a\hat{H}_a|n,\ \mathbf{p}\rangle|^2$$
$$+|\langle m,\ \mathbf{p}-\mathbf{Q}|\hat{H}_a\hat{G}_a|m,\ \mathbf{p}-\mathbf{q}\rangle|^2+2\operatorname{Re}\{\langle n,\ \mathbf{p}-\mathbf{Q}$$
$$+\mathbf{q}|\hat{G}_a\hat{H}_a|n,\ \mathbf{p}\rangle\langle m,\ \mathbf{p}-\mathbf{Q}|\hat{H}_a\hat{G}_a|m,\ \mathbf{p}-\mathbf{q}\rangle^*\}+J. \tag{13}$$

The term J contains a term associated with the presence in (11) of the last term, which describes the emission of a plasmon between scatterings of the electron by atoms. One can convince oneself that the quantity J can be omitted if only one collision with scattering through a large angle occurs.

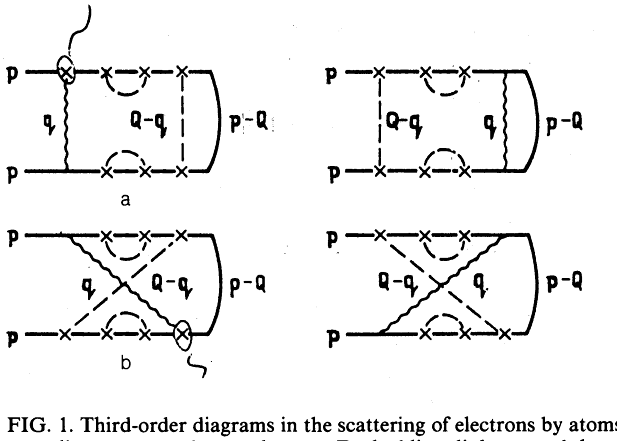

FIG. 1. Third-order diagrams in the scattering of electrons by atoms. A wavy line corresponds to a plasmon. Dashed lines link one and the same atom.

The function (13), and hence the transition probability (12), must be averaged over the positions of the randomly situated atoms. As usual, on the diagrams corresponding to Eq. (12) we shall use a dashed line to link two vertices (denoted by crosses) pertaining to the same atom. The lowest-order diagrams capable of describing weak localization are diagrams of third order in the amplitude with respect to the electron-electron interaction. These diagrams are depicted in Fig. 1. The lower parts of the diagrams correspond to the analytical expression that is the complex conjugate of the expression equivalent to the upper parts. A wavy line corresponds to a plasmon. It can be seen from the way the dashed lines are drawn that the large-angle scattering of an electron by atoms is incoherent.

If the conditions $u\ll N_0^{-1/3}$ and $N_0|u^2|\lambda\ll 1$ are fulfilled (u is the amplitude for scattering by one atom and N_0 is the concentration of atoms), we can disregard diagrams with intersections of dashed lines and with two lines emerging from one vertex.[42] This, in particular, implies that the usual weak localization, corresponding to allowance for the "fan" diagrams[7,17] that describe only elastic scattering, is not taken into account because the regions of angles for the ordinary weak localization and for the new type of localization are essentially nonoverlapping. If we assume that $u\approx 4\pi(p^2+\alpha^2)^{-1}$ (where $\alpha^{-2}\sim 10^{-15}$ cm) and $N_0^{-1/3}\sim 10^{-8}$ cm, then for $\lambda\sim 10^{-9}$ cm (which corresponds to intermediate electron energies) both of the above inequalities are fulfilled. We note, finally, that the diagrams of Figs. 1c and 1d correspond to the principle (valid in the theory of ordinary localization[17]) that a change of the direction of motion of the electron in the lower parts of the diagrams transforms these diagrams into "ladder" diagrams, albeit with "steps" of a different nature.

We note also that from the form of the diagrams and from (13) it follows that the role of the damping in the electron Green's functions should be played by the exact vertex γ.

The dependence of $dw(\mathbf{Q},\omega)$ on the vector $\mathbf{Q}$ (i.e., on the scattering angle χ for scattering of the electron by the medium as a whole) is determined by two factors. First, this dependence is connected directly with the dashed lines linking the upper solid line to the lower solid line in each diagram. These lines give rise to the existence of a factor $|u(\mathbf{Q}-\mathbf{q})|^2$ that is the same for all diagrams. Since we have $Q\gg q$, we may assume that this factor has the form $|u(\mathbf{Q})|^2$. This would also be the angular dependence if just incoherent elastic scattering were present. Second, the electron Green's functions depend on $\mathbf{Q}$. If, after integration over all directions of the vector $\mathbf{q}$ (which corresponds to summation over the final states of the medium), on which the electron Green's functions also depend, the dependence on $\mathbf{Q}$ that arises from the fact that the vector $\mathbf{Q}$ appears in the electron Green's functions remains, then this implies the appearance of an additional angular dependence, existing in the inelastic-scattering channel. Since, as a rule, this additional dependence is nonmonotonic, we may speak of the appearance of weak localization of the scattered electrons in this channel.

The diagrams depicted in Fig. 1 correspond to the function $|B|^2$

$$|B|^2=\frac{N_0}{V}|u(\mathbf{Q}-\mathbf{q})|^2\left[\frac{1}{(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{q}}-\omega)^2+\gamma^2}+\frac{1}{(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{Q}+\mathbf{q}})^2+\gamma^2}+2\,\mathrm{Re}\frac{1}{(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{q}}-\omega-i\gamma)(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{Q}+\mathbf{q}}+i\gamma)}\right], \quad (14)$$

so that the differential transition probability is

$$\frac{dw(\mathbf{Q},\omega)}{d\omega\,d\Omega_\mathbf{Q}}=m_e\frac{N_0(p^2-2\omega)^{1/2}}{(2\pi)^3}\sum_\mathbf{q}|u(\mathbf{Q})|^2w_i(\mathbf{q},\omega)\times\left|\frac{1}{E_\mathbf{p}-E_{\mathbf{p}-\mathbf{q}}-\omega-i\gamma}+\frac{1}{E_\mathbf{p}-E_{\mathbf{p}+\mathbf{q}-\mathbf{Q}}-i\gamma}\right|^2. \quad (15)$$

The first term inside the symbol of the square of the absolute value in (15) corresponds to a process which begins with the excitation of a plasmon and ends with a scattering of the electron through a large angle. The second term corresponds to a process with the opposite order of events. It can be seen from the structure of the diagrams that the Born probability for large-angle scattering of the electron can be replaced by the exact probability.

3. COHERENCE EFFECTS IN THE SCATTERING OF ELECTRONS BY ATOMS AND BY A PLASMON

Neglecting terms of order ω/E, we rewrite Eq. (15) in the form

$$\frac{dw}{d\omega\,d\Omega_\mathbf{Q}}=\frac{m_epN_0}{(2\pi)^3}|u(\mathbf{Q})|^2\times\sum_\mathbf{q}w_i(\mathbf{q},\omega)\left|\frac{1}{\mathbf{qv}-\omega-i\gamma}+\frac{1}{\omega-\mathbf{qv}'-i\gamma}\right|^2. \quad (16)$$

Here, $\mathbf{v}'=(\mathbf{p}-\mathbf{Q})/m_e$ is the velocity of the electron in the final state. We denote by the symbol S (and study in more detail) the factor

$$S(\omega,\chi)=\frac{1}{(2\pi)^3}\sum_\mathbf{q}w_i(\mathbf{q},\omega)\left|\frac{1}{\mathbf{qv}-\omega-i\gamma}+\frac{1}{\omega-\mathbf{qv}'-i\gamma}\right|^2, \quad (17)$$

that multiplies the quantity $N_0m_ep|u(\mathbf{Q})|^2$. We draw attention particularly to the dependence of the quantity S on the electron-scattering angle χ. Going over from summation to integration, we write

$$S(\omega,\chi)=\frac{1}{(2\pi)^3}\int_0^\infty dq\,q^2w_i(q,\omega)\mathcal{G}(q,\omega,\chi), \quad (18)$$

where

$$\mathcal{G}(q,\omega,\chi)=\int d\Omega_\mathbf{q}\left|\frac{1}{\mathbf{vq}-\omega-i\gamma}+\frac{1}{\omega-\mathbf{v}'\mathbf{q}-i\gamma}\right|^2. \quad (19)$$

For what follows, the dependence of (19) on the angle χ has decisive significance.

We consider two limiting ways of describing the probability of excitation of a plasmon in a uniform electron gas. First, we assume that the shape of the plasma-resonance line is determined by the plasmon dispersion. In this case,

$$w_i(q,\omega)=\frac{8\pi e^2\omega_p^2}{q^2V}\delta[\omega^2-\omega_p^2(1+Mq^2)]. \quad (20)$$

This formula differs from the simplest one in that the dispersion of the plasmon frequency has been taken into account in it. The coefficient M has the order of magnitude of v_F/ω_p, where v_F is a velocity of the order of the Fermi velocity and ω_p is the plasma frequency. Substituting (20) into (18), we obtain

$$S(\omega,\chi)=\frac{e^2\omega_p^2\theta(\omega-\omega_p)}{2\pi VM^{1/2}(\omega^2-\omega_p^2)^{1/2}}\mathcal{G}\left(q=\frac{(\omega^2-\omega_p^2)^{1/2}}{\omega_pM^{1/2}},\omega,\chi\right). \quad (21)$$

The presence of the θ symbol here means that $\omega>\omega_p$.

Next, in $\mathcal{G}$, we take the integral over the angles $\Omega_\mathbf{q}$. As a result of the calculations, we obtain the explicit form of the function $\mathcal{G}$

$$\mathcal{G}(q,\omega,\chi)=\frac{4\pi}{qv\gamma}\left\{\mathrm{arctg}\left(\frac{qv+\omega}{\gamma}\right)+\mathrm{arctg}\left(\frac{qv-\omega}{\gamma}\right)-\gamma\,\mathrm{Re}[2\omega_c^2(1-\cos\chi)-q^2v^2\sin^2\chi]^{-1/2}\times\mathrm{Ln}\left[\frac{\omega_c^2-q^2v^2\cos\chi+qv[2\omega_c^2(1-\cos\chi)-q^2v^2\sin^2\chi]^{1/2}}{\omega_c^2-q^2v^2\cos\chi-qv[2\omega_c^2(1-\cos\chi)-q^2v^2\sin^2\chi]^{1/2}}\right]\right\}, \quad (22)$$

In this formula, $\omega_c=\omega-i\gamma$ and $q=(M\omega_p^2)^{-1}(\omega^2-\omega_p^2)^{1/2}$.

The limiting case of the motion of an electron in a uniform electron gas, when the collisions of electrons with atoms do not affect the excitation of plasmons, corresponds to a small value of the parameter γ/ω. In this case, in (22) the term containing the logarithm, which is entirely due to interference, vanishes. The function $\mathcal{G}$ ceases to depend on the electron-scattering angle χ and tends to the limit

$$\lim_{\gamma\to0}\mathcal{G}\to\frac{4\pi}{qv\gamma}\left(\mathrm{arctg}\frac{qv+\omega}{\gamma}+\mathrm{arctg}\frac{qv-\omega}{\gamma}\right). \quad (23)$$

The sum of the arctangents is nonzero only if $qv-\omega>0$, so that, with this condition and in the limit of small γ,

$$\mathcal{G}\to\frac{4\pi^2}{qv\gamma}\theta(qv-\omega). \quad (24)$$

It is obvious that this limit corresponds to those collisions of the electron with atoms and with a plasmon that can be regarded as consecutive, so that a collision begins only after the previous one has ended. A plasmon is then emitted in the Cherenkov manner, as indicated by the condition $qv-\omega>0$.

Figures 2 and 3 depict the function $\mathcal{G}(\chi)$, normalized to its value at $\chi=\pi$, for various values of the parameters $z=qv/\omega$ and $b=\gamma/\omega$. From now on we call this the localization function. For a given electron energy the parameter $z=qv/\omega$ can be regarded as the quantity determining the energy lost by the electron to excitation of a plasmon. For $z\gg1$ Cherenkov emission of a plasmon is possible. If we have $z<1$, a plasmon cannot be emitted in the Cherenkov manner, and the energy loss is due to bremsstrahlung emission of longitudinal electromagnetic waves.[43] The curves in Fig. 5 correspond to this case.

It follows from Fig. 2 that in the deep "pre-Cherenkov"

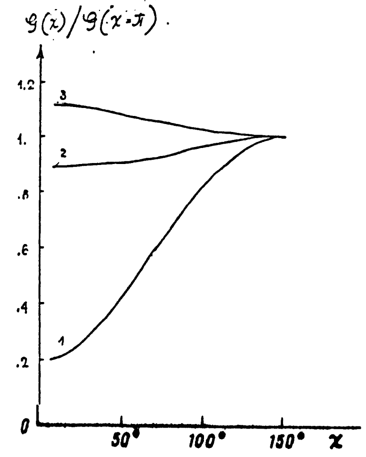

FIG. 2. Localization functions for bremsstrahlung excitation of a plasmon in the case when the dispersion of the plasmon frequency is more important than the plasmon damping ($qv/\omega = 0.5$); $b = \gamma/\omega = 0.1$ (curve 1); $b = 0.3$ (curve 2); $b = 0.5$ (curve 3).

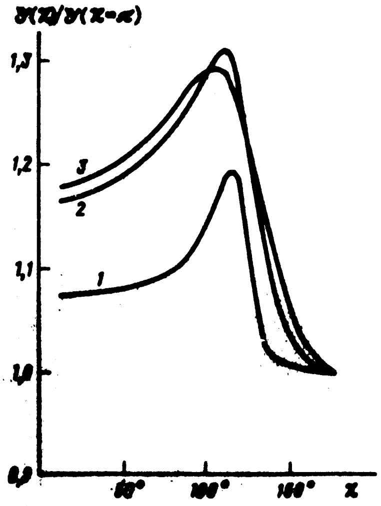

FIG. 4. Localization functions in the region of Cherenkov excitation of a plasmon ($qv/\omega = 2$). The curves, 1, 2, and 3 correspond to the same values of the parameter $b = \chi/\omega$ as in Fig. 3.

region, when the very possibility of emission of a plasmon is due entirely to the fact that the collision with the plasmon begins before the scattering of the electron by atoms has ended (or, on the other hand, ends after the collision of the electron with an atom has begun), localization occurs only for a comparatively small ratio γ/ω. For large values of γ the interference is great, but localization does not set in.

With increase of the ratio qv/ω weak localization becomes more pronounced (Fig. 3), and scattering through small angles is found to be prominent. Weak localization turns out to be most strongly pronounced for $z \geqslant 1$, i.e., at frequencies at which Cherenkov emission of plasmons becomes possible. In addition, it is necessary that γ/ω be smaller than unity. The dominant scattering of electrons in this case occurs in a direction close to $\chi \approx \pi/2$. For $z > 1$ clear weak localization appears in the region of angles of order 100–120°, the localization maximum being reached at $\gamma/\omega \simeq 0.3$ (Fig. 4).

The integral (over the plasmon frequencies) curve $\mathcal{G}(\chi)/\mathcal{G}(\chi = \pi)$ (Fig. 5) also shows weak localization of plasmons in the region of angles $\chi \approx 120$–140°. The degree of coherence, defined as the ratio of the term in (22) containing the logarithm to the sum of the arctangents also depends on the angle χ (Fig. 6).

In the other limiting case, when the shape of the plasma-resonance line is determined by the damping of the plasmon, the probability (20) should be replaced by the quantity

$$w_i(q,\omega) = \frac{A(\omega) q^{-2}}{(\omega^2 - \omega_p^2)^2 + \omega^2 \Gamma^2}, \tag{25}$$

(here, Γ is the plasmon damping). In accordance with (18) and (19), this implies that the localization function can be represented by the integral

$$F_1(\chi) = \int_0^{q_0} \frac{dq}{q} \Big\{ \operatorname{arctg} \frac{qv+\omega}{\gamma} + \operatorname{arctg} \frac{qv-\omega}{\gamma} - \gamma \operatorname{Re} \frac{1}{[2\omega_c^2(1-\cos\chi) - q^2v^2\sin^2\chi]^{1/2}}$$
$$\times \operatorname{Ln} \left[\frac{\omega_c^2 - q^2v^2\cos\chi + qv[2\omega_c^2(1-\cos\chi) - q^2v^2\sin^2\chi]^{1/2}}{\omega_c^2 - q^2v^2\cos\chi - qv[2\omega_c^2(1-\cos\chi) - q^2v^2\sin^2\chi]^{1/2}} \right] \Big\}, \tag{26}$$

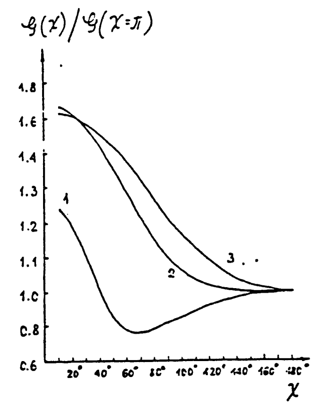

FIG. 3. Localization functions at the threshold for Cherenkov excitation of a plasmon ($qv/\omega = 1$); $b = 0.1$ (curve 1); $b = 0.3$ (curve 2); $b = 0.5$ (curve 3). The dispersion of the plasmon is assumed to be more important than its damping.

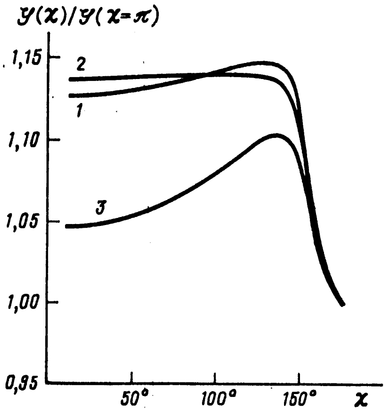

FIG. 5. Integral (over the frequency) localization curve for excitation of a plasmon (in conditions when the dispersion is more important than the damping of the plasmon) and $v/v_a = 5$; $b = \gamma/\omega = 0.3$ (curve 1); $b = 0.5$ (curve 2); $b = 0.8$ (curve 3).

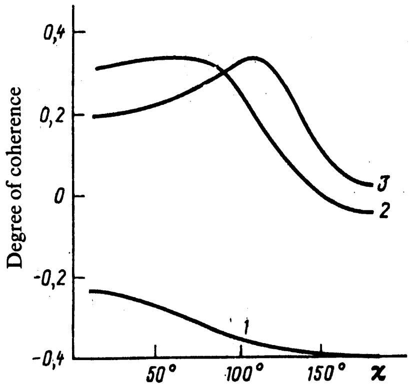

FIG. 6. Degree of coherence as a function of scattering angle, in the case when $v/v_a = 5$ and $\gamma/\omega = 0.5$; $\omega/\omega_p = 1.01$ (curve 1); $\omega/\omega_p = 1.05$ (curve 2); $\omega/\omega_p = 1.1$ (curve 3).

where q_0 is the momentum cutoff at which Landau damping of the plasmons appears. This function of χ, normalized to its value at $\chi = \pi$, has been obtained by numerical integration, and is depicted in Fig. 7. As can be seen, when the electron loses energy equal to the plasmon energy, all the localization function are asymmetric and scattering through the largest angles is suppressed. The maxima of all these functions lie in the range $60° < \chi < 120°$.

4. COHERENCE EFFECT IN THE CASE WHEN THE INELASTICITY IS DUE TO THE GENERATION OF TRANSVERSE ELECTROMAGNETIC WAVES

We wrote the scattering amplitude in the case of generation of plasmons in the form (10). In the case of generation of photons the scattering amplitude takes the form

$$T_{ph,\mathbf{Q}} = \sum_{\mathbf{p}'} T_{ph}(\mathbf{p}-\mathbf{Q},\mathbf{p}')\langle\mathbf{p}'|\hat{G}_a\hat{H}_a|\mathbf{p}\rangle + \sum_{\mathbf{p}'}\langle\mathbf{p}-\mathbf{Q}|\hat{H}_a\hat{G}_a|\mathbf{p}'\rangle T_{ph}(\mathbf{p}',\mathbf{p}), \tag{27}$$

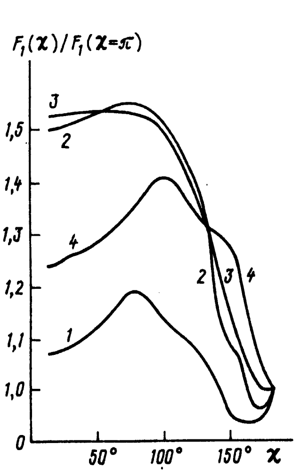

FIG. 7. Localization functions for excitation of a plasmon in conditions when the damping of the plasmon is more important than the dispersion of its frequency. Curve 1) $v/v_a = 3$, $\gamma/\omega = 0.1$; curve 2) $v/v_a = 3$, $\gamma/\omega = 0.3$; curve 3) $v/v_a = 3$, $\gamma/\omega = 0.5$; curve 4) $v/v_a = 7$, $\gamma/\omega = 0.3$.

where $T_{ph}(\mathbf{p}_1,\mathbf{p}_2)$ is the photon-emission amplitude:

$$T_{ph}(\mathbf{p}_1,\mathbf{p}_2) = \frac{e}{m_e c}\langle\mathbf{p}_1|\hat{\mathbf{p}}\hat{\mathbf{A}}|\mathbf{p}_2\rangle = \frac{e}{m_e c}\left(\frac{2\pi c^2\hbar}{\varepsilon\omega V}\right)^{1/2}\mathbf{p}_2\mathbf{e}_{\mathbf{q},\sigma}\delta_{\mathbf{q},\mathbf{p}_2-\mathbf{p}_1} = T_{ph,\mathbf{p}_1,\mathbf{p}_2}\delta_{\mathbf{q},\mathbf{p}_2-\mathbf{p}_1}. \tag{28}$$

Here, $\mathbf{q}$, ω, and $\mathbf{e}_{\mathbf{q},\sigma}$ are the wave vector, frequency, and polarization vector of the emitted photon, $\varepsilon(\omega)$ is the dielectric permittivity of the medium, and V is the normalization volume.

The calculation of the probability dw_{ph} of scattering of an electron with an energy loss equal to the energy of the emitted photon is performed in the same way as was done in the case of emission of a plasmon. After averaging over the positions of the atoms of the medium, we obtain

$$dw_{ph}(\mathbf{Q},\omega) = \frac{2\pi}{\hbar}\frac{2\pi e c^2}{\varepsilon(\omega)\omega V}|u(\mathbf{Q})|^2 \times\left|\frac{(\mathbf{p}-\mathbf{Q}+\mathbf{q})\mathbf{e}_{\mathbf{q},\sigma}/m_e c}{\omega-\mathbf{q}\mathbf{v}'-i\gamma} + \frac{\mathbf{p}\mathbf{e}_{\mathbf{q},\sigma}/m_e c}{\mathbf{q}\mathbf{v}-\omega-i\gamma}\right|^2 \times\delta(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{Q}}-\hbar\omega)\frac{V^2 d^3q\, d^3Q}{(2\pi)^3(2\pi\hbar)^3}. \tag{29}$$

We integrate over the angles of emission of the photon and sum over its polarizations. It is obvious that, as in the case of plasmons, after integration over the angles characterizing the direction of the vector $\mathbf{q}$, the scattering angle χ can appear only in the terms containing the product $(\omega - \mathbf{q}\mathbf{v} + i\gamma)^{-1}(\omega - \mathbf{q}\mathbf{v}' - i\gamma)^{-1}$. Bearing this in mind, we obtain

$$dw_{ph}(\mathbf{Q},\omega) = \frac{2\pi}{\hbar}|u(\mathbf{Q})|^2\frac{e^2\hbar}{\varepsilon(\omega)\omega v_{ph}\pi} \times\left\{\frac{\beta^2-1+b^2}{b\beta}\left[\operatorname{arctg}\frac{1+\beta}{b}+\operatorname{arctg}\frac{\beta-1}{b}\right] +2\frac{b}{\beta}\left(\operatorname{arctg}\frac{b}{1+\beta}+\operatorname{arctg}\frac{b}{\beta-1}\right)+\operatorname{Re}\left[\left(\frac{[\mu^2-\cos^2(\chi/2)]^{1/2}}{\sin(\chi/2)} + \frac{\sin(\chi/2)}{[\mu^2-\cos^2(\chi/2)]^{1/2}}\right)\operatorname{Ln}\frac{[\mu^2-\cos^2(\chi/2)]^{1/2}+\sin(\chi/2)}{[\mu^2-\cos^2(\chi/2)]^{1/2}-\sin(\chi/2)}\right]\right\} \times\delta(E_\mathbf{p}-E_{\mathbf{p}-\mathbf{Q}}-\hbar\omega)\frac{V d\omega\, d^3Q}{(2\pi\hbar)^3}. \tag{30}$$

Here, $b = \gamma/\omega$, $\beta = v/v_{ph}$, $v_{ph} = c/\varepsilon^{1/2}(\omega)$, and $\mu = (1 + ib)/\beta$. In the calculations we have taken into account that $v - v' \ll v$.

An estimate of the quantity dw_{ph} gives

$$dw_{ph}(\mathbf{Q},\omega) \approx \frac{dw_{el}}{\gamma}\frac{e^2 d\omega}{\pi\varepsilon\hbar v_{ph}}, \tag{31}$$

where dw_{el} is the differential probability of elastic scattering of the electron through a large angle.

It follows from (30) that weak localization depends on two parameters: $b = \gamma/\omega$ and the ratio of the velocity of the electron to the phase velocity of the photon. Figure 8 shows curves that demonstrate the angular dependence of the factor in curly brackets in the expression (30). The values of this factor, as in the case of plasmons, are normalized to its value at $\chi = \pi$. From Fig. 8 we can draw the following conclusions:

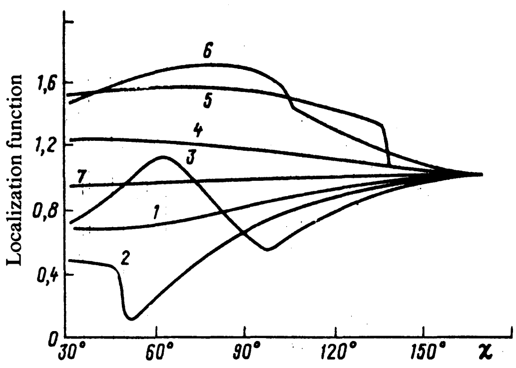

FIG. 8. Localization functions for excitation of a photon. Curves 1 ($\beta = 0.9$), 2 ($\beta = 1.1$), and 3 ($\beta = 1.5$) correspond to $\gamma/\omega = 0.2$, curves 4 ($\beta = 0.9$), 5 ($\beta = 1.1$), and 6 ($\beta = 1.5$) correspond to $\gamma/\omega = 1$, and curve 7 corresponds to $\gamma/\omega = 5$.

1) In the case of strong interference, when $b \sim 1$, localization is suppressed for all values of β.

2) For $b \lesssim 1$, the character of the localization is determined by the quantity β. For $\beta < 1$ the localization function is weakly expressed, while for $\beta \gtrsim 1$ the localization is sharply expressed. The case $\beta \gtrsim 1$ corresponds to fulfillment of the condition for the existence of Cherenkov radiation.

3) It follows from (30) that the angle at which the characteristic inflection appears in the localization function is determined by the condition

$$\mathrm{Re}\,[\mu^2 - \cos^2(\chi/2)] = 0, \tag{32}$$

The condition (32) is equivalent to the equation

$$\cos(\chi/2) = \beta^{-1} = \cos\theta_C, \tag{33}$$

in which θ_C is the Cherenkov angle. Thus, localization corresponds to an electron-scattering angle χ equal to twice the Cherenkov angle. Since the cause of the localization is the interference of processes that can be regarded as a forward and a reverse process, the condition (33) has an intuitive character.

5. COHERENCE EFFECT IN THE CASE WHEN THE INELASTICITY IS DUE TO EXCITATION OF ELECTRONS IN ATOMS AND IONIZATION OF ATOMS

In this case, in accordance with Ref. 44, the probability of inelastic scattering by one individual atom can be written in the form

$$w_i(\mathbf{q}, \omega) = 4N_0 v\left(\frac{e^2 m_e}{\hbar^2}\right)\frac{1}{q^4}\left|\int \sum_a \exp(-i\mathbf{q}\mathbf{r}_a)\Phi_m^* \Phi_n\, d\tau\right|^2 \tag{34}$$

Here, $\mathbf{r}_a$ is the position vector of each individual electron in one atom, Φ is the wave function of this electron, and $d\tau$ is the volume element in which the atomic electrons are found. The summation over a corresponds to summation over the electrons of the atom.

In such electron scattering the principal role is played by collisions leading to scattering of the electron through small angles,[44] when the momentum transfer $q \ll 1/a$, where a is a length of the order of the atomic size. In this case, the matrix element in the atomic wave functions Φ in (34) can be written in the dipole approximation, so that

$$w_i = \frac{4}{3} N_0 v \left(\frac{e m_e}{\hbar^2}\right)^2 \frac{1}{q^2} |\mathbf{d}_{mn}|^2, \tag{35}$$

where $\mathbf{d}_{mn}$ is a matrix element of the operator of the dipole-moment vector.

Substituting (35) into (12) and (17), we see that in this case (18) takes the form

$$S(\omega, \chi) = \frac{1}{(2\pi)^3} \cdot \frac{4}{3} N_0 v \left(\frac{e m_e}{\hbar^2}\right)^2 |\mathbf{d}_{mn}|^2 \int_0^{1/a} dq\, \mathcal{G}(q, \omega, \chi), \tag{36}$$

in which the function $\mathcal{G}$ has, as before, the form (19).

If the energy lost is such that ionization of the atoms is also possible (and, therefore, momentum transfers $q \gtrsim a^{-1}$ must also be taken into account), the square of the absolute value of the matrix element in the atomic wave functions that appears in (35) can be represented conveniently in the form

$$\left|\sum_a \exp(-i\mathbf{q}\mathbf{r}_a)\Phi_m^* \Phi_n\, d\tau\right|^2 = \frac{Zq^2}{q^2 + q_0^2}, \tag{37}$$

in which $q_0^2 = e^2 Z / |d_{xn0}|^2$, where Z is the atomic number. This formula must be regarded as an interpolation formula that ensures the true asymptotic values of the squared magnitude of the matrix element for small and comparatively large q. For the smallest momentum transfers the matrix element (37), in accordance with (35), is proportional to $1/q^2$. For relatively large q it should not depend on q, in accordance with the fact that the differential cross section for Rutherford scattering by each of the atomic electrons has a dependence on the energy loss of the form $d\omega/\omega^2$.

Now, the existence of weak localization should correspond to a dependence of the factor F_2 on the angle χ of the form

$$\begin{aligned}
F_2(\chi) = {} & (4\pi e^2)^2 Z \frac{4\pi}{\gamma v} \int_0^\infty \frac{dq}{q(q^2+q_0^2)} \Big\{ \mathrm{arctg}\,\frac{qv+\omega}{\gamma} + \mathrm{arctg}\,\frac{qv-\omega}{\gamma} \\
& - \gamma\, \mathrm{Re}\, \frac{1}{[2\omega_c^2(1-\cos\chi) - q^2v^2\sin^2\chi]^{1/2}} \\
& \times \mathrm{Ln}\left[\frac{\omega_c^2 - q^2v^2\cos\chi + qv[2\omega_c^2(1-\cos\chi) - q^2v^2\sin^2\chi]^{1/2}}{\omega_c^2 - q^2v^2\cos\chi - qv[2\omega_c^2(1-\cos\chi) - q^2v^2\sin^2\chi]^{1/2}}\right]\Big\} \\
= {} & \mathrm{const}\cdot\Big\{\frac{\pi}{4}\ln\left[\frac{(1-b^2-z_0^2)^2+4b^2}{(1+b^2)^2}\,\frac{(b+z_0)^2+1}{(b-z_0)^2+1}\right] \\
& - \frac{b z_0^2}{\sin(\chi/2)} \int_0^\infty \frac{dz}{z(z^2+z_0^2)} \frac{1}{O^2(z)+P^2(z)} \\
& \times \Big[\frac{O(z)}{2} \ln\left(\frac{P^2(z)+[O(z)+z\sin(\chi/2)]^2}{P^2(z)+[O(z)-z\sin(\chi/2)]^2}\right) \\
& - P(z)\Big(\mathrm{arctg}\,\frac{O(z)+z\sin(\chi/2)}{P(z)} \\
& - \mathrm{arctg}\,\frac{O(z)-z\sin(\chi/2)}{P(z)}\Big)\Big]\Big\}.
\end{aligned} \tag{38}$$

Here, $z = qv/\omega$, $z_0 = q_0 v/\omega \sim v/v_a$, and

$$O(z) = 2^{-1/2}\Big\{1 - b^2 - z^2\cos^2\frac{\chi}{2} + \Big[\Big(1 - b^2 - z^2\cos^2\frac{\chi}{2}\Big)^2 + 4b^2\Big]^{1/2}\Big\}^{1/2}.$$

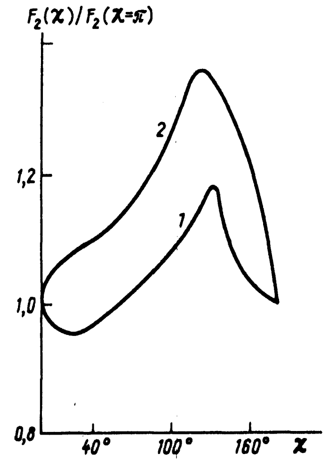

FIG. 9. Localization functions for excitation and ionization of atoms ($q_0v/\omega = 7$); $\gamma/\omega = 0.1$ (curve 1); $\gamma/\omega = 0.5$ (curve 2).

$$P(z)=2^{-1/2}\left\{-1+b^2+z^2\cos^2\frac{\chi}{2}+\left[\left(1-b^2-z^2\cos^2\frac{\chi}{2}\right)^2+4b^2\right]^{1/2}\right\}^{1/2}.$$

The function (38), normalized in the usual way, is depicted in Fig. 9. A characteristic feature of weak localization associated with the excitation and ionization of atoms is the fact that the localization function tends to unity at $\chi = 0$ and $\chi = \pi$.

6. CONCLUSION

It has been found that inelastic collisions not only limit the actual number of elastic scattering events when there is localization in the elastic channel, but also lead to weak localization of a new type in the inelastic-scattering channel. Although the amplitude of the localization curve in the space of the angles is smaller in the case of inelastic scattering than in the case of elastic scattering, the angular width of the region of localization in the inelastic-scattering channel is greater than in the elastic-scattering channel, so that the area under the corresponding curve may be no smaller than in the case of ordinary weak localization.

The new type of localization is universal in the sense that it occurs for any mechanism of electron-energy loss, and the position of the maximum of the localization function depends only little on the type of inelastic process. At the same time, the shape of the localization curve when collective excitations and one-electron excitations of the disordered medium are excited depends on the type of excitation, and can be used as a method for identifying the character of the energy loss. Weak localization in the inelastic-scattering channel must be expected in the case of scattering of intermediate-energy electrons incident obliquely on a disordered medium.

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