Investigation of vortex dynamics in high-T_c superconducting films

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We investigate the dynamics of viscous flow of vortices in high-temperature superconducting films by studying the generation of harmonics in a superconducting film placed in a sinusoidal magnetic field with frequency ~ 10 kHz. In order to discuss the experimental data in the hydrodynamic approximation, we calculated the distribution of vortices along the film in an external AC magnetic field. We show that the field dependence of the third harmonic allows us to determine the coefficient of viscous friction and the pinning force in the superconductor.

1. INTRODUCTION

The investigation of the I-V characteristics of high-temperature superconductor films has been the subject of a number of experimental papers; however, complete understanding of the loss mechanism in the resistive state has not been achieved up to now. Therefore there is considerable interest in using various methods for investigating the I-V characteristics. The majority of papers on experimental investigation of I-V characteristics have been carried out using the conventional scheme, where the current and voltage are applied to the sample by fabricating special ohmic contacts. In order to bound the value of the total current, it is necessary to create narrow superconducting bridges on the film. However, this does not eliminate the problem of heating of these bridges by the current. The fact that up to now no one has succeeded in clearly identifying a regime of viscous vortex flow in HTSC films is probably associated with this problem. Contactless methods can eliminate these inadequacies; however, their widespread use has been hampered by the complicated task of interpreting the experimental results.

In this paper, we investigate the I-V characteristics of YBaCuO films by a contactless method based on generation of harmonics of a given frequency by a superconducting film placed in a sinusoidal AC magnetic field. As a rule, the I-V characteristics are nonlinear; therefore the existence of such effects is obvious. The film's efficiency in generating third harmonics was investigated in Ref. 1 at various temperatures. However, the dependence of the third-harmonic amplitude on the amplitude of the external AC magnetic field was not discussed there. In this paper, we show that we can obtain information from this data about the parameters that characterize the viscous motion of vortices under the action of an external force, and specifically the pinning force and coefficient of viscous friction.

2. EXPERIMENT

In our experimental setup, an AC magnetic field with a frequency on the order of several kilohertz was generated by using a coil fed from an audio-frequency generator. The value of the amplitude of the AC magnetic field, which could be as large as several tens of oersteds, was monitored by measuring the current flowing through the coil.

The film under study was a disk of radius 0.25 and thickness $d \sim 2 \cdot 10^{-5}$ cm grown on a substrate of strontium titanate. It was placed in the center of the coil in such a way that the magnetic field was normal to its surface. A detector coil was placed in the immediate vicinity of the film, and oriented end-on to it. The voltage induced in this coil was fed to a selective amplifier tuned to the third harmonic of the specified frequency. All of the measurements were taken at liquid nitrogen temperatures. In determining the dependence of the third harmonic amplitude on the amplitude of the AC magnetic field in the exciting coil, as a first step we checked whether or not the signal was associated with the film in the superconducting state. As we passed through the critical temperature (~ 92 °K), the third harmonic disappeared.

Figures 1 and 2 show the experimental results for one of the films at two different external field frequencies, 15 kHz and 10 kHz. Along the y-axis we plot the quantity U_3/H_1 in relative units, where U_3 is the voltage at frequency 3ω measured at the output of the selective amplifier, and H_1 is the amplitude of the AC field at frequency ω. Along the x-axis we plot the quantity $H^* = H_1 2\pi f_0/\omega$, where $f_0 = 15$ kHz. In what follows, we will show that this specific ratio H_1/ω arises naturally in the theory. The character of the curves

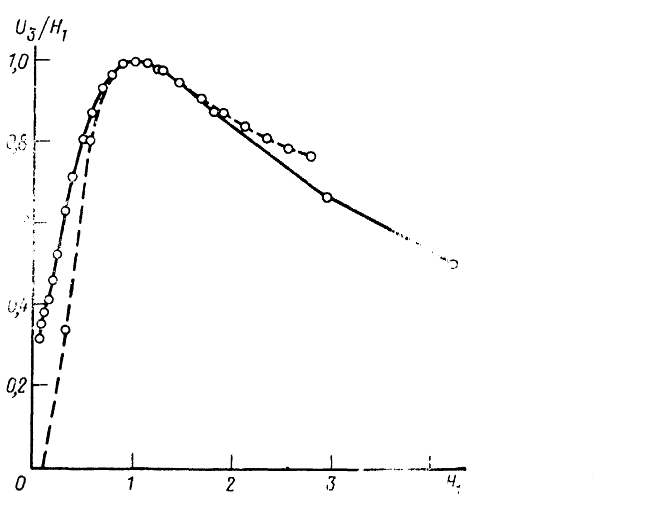

FIG. 1. Dependence of U_3/H_1 on the amplitude of the AC field H_1 for $\omega = 15$ kHz. The solid curve is experiment and the dashed curve is theory.

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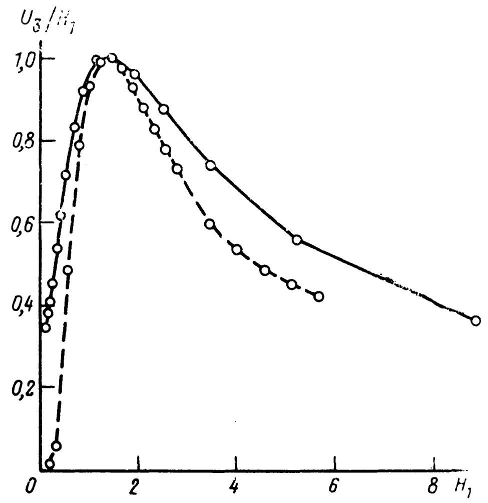

FIG. 2. Dependence of U_3/H_1 on H_1 for $\omega = 10$ kHz. The solid curve is experiment, the dashed curve is theory.

depends somewhat on the fabrication technology for the film; therefore, for different samples these curves differ somewhat from one another. In practice, a constant field on the order of 100 Oe does not influence the harmonic amplitude.

3. THEORY

In order to describe the experimental data, it is necessary to calculate the distribution of vortices throughout the film in an external AC magnetic field. Ignoring the structure of the vortex lattice, we will describe the vortices in the hydrodynamic approximation by the mean density $n(r)$. Then the equation of continuity has the form

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{V}) = 0, \qquad (1)$$

where $\mathbf{V}$ is the vortex velocity. This can be described phenomenologically by

$$\mathbf{V} = \frac{\mathbf{f}}{\eta(f)}, \qquad (2)$$

where $\eta(f)$ is the viscosity coefficient, which generally depends on the force (or upon velocity). In the presence of a pinning force, we can assume that $\eta = \infty$ holds for $f < f_{\text{pin}}$, where f_{pin} is the pinning force. For a cylindrical geometry, assuming that the film lies in the plane $z = 0$ and the vortex velocity is directed along r, we obtain from (1)

$$\frac{\partial}{\partial t} H_z + \frac{1}{r}\frac{\partial}{\partial r}(rVH_z) = 0, \qquad (3)$$

where $H_z = n\Phi_0$ and Φ_0 is the magnetic flux quantum. If we compare (3) with the Maxwell equation

$$\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} + \operatorname{rot}\mathbf{E} = 0 \qquad (4)$$

it is obvious that we obtain the well-known relation $E_\varphi = V/cH_z$. If we use $\eta(f) = \text{const}$ in (2), then Eq. (3) is nonlinear in this case as well, because the force f in (2) depends on H_z. Thus, even when the force acting on the vortex is linearly related to its velocity, harmonics of the fundamental frequency will be generated.

In order to determine the force exerted on a vortex by an external field and the force of interaction between vortices, we must calculate the free energy. We will assume that the thickness of the film satisfies $d \ll \lambda$, where λ is the London penetration depth, so that the variation along the film thickness of all quantities that characterize the vortex can be neglected. In the real experiment, this inequality is not fulfilled. However, it will become clear in what follows that what is important is the interaction between vortices at large distances $r > d$, for which the condition $\lambda > d$ is unimportant.

The vector potential of the field is determined from the following equation (see Ref. 2):

$$-\nabla^2\mathbf{A} + \frac{1}{\lambda_2}\mathbf{A}\delta(z) = \frac{\delta(z)}{\lambda_2}\sum_i \Phi(r - r_i) + \frac{4\pi}{c}\mathbf{j}^s, \qquad (5)$$

where $\lambda_2 = \lambda^2/d$ and r_i are the two-dimensional coordinates of the core of the ith vortex in the film. The vector $\Phi(r)$ in the (r,θ,z) cylindrical coordinate system has only a θ-component $\Phi_\theta = \Phi_0/2\pi r$; here $\mathbf{j}^S$ is the external current that creates the external magnetic field. In what follows, we will assume that the external magnetic field H_0 directed along the z-axis is created by a solenoid of radius R; then $\mathbf{j}^S$ has only a θ component

$$j_\theta{}^s = \frac{cH_0}{4\pi}\delta(r - R).$$

The solution to (5) is easy to find in the Fourier representation

$$\mathbf{A}(r, z) = \sum \mathbf{A}_{k\mathbf{q}} \exp i(kz + \mathbf{qr}) \qquad (6)$$

and has the form

$$\mathbf{A}_{k\mathbf{q}} = \left\{\frac{2q}{1+2q\lambda_2}\sum_i \Phi_\mathbf{q}(r_i) + \frac{4\pi}{c}\left(\mathbf{j}_{k\mathbf{q}}{}^s - \frac{2q}{1+2q\lambda_2}\mathbf{Y}_\mathbf{q}{}^s\right)\right\}\frac{1}{k^2+q^2}, \qquad (7)$$

where

$$\Phi_\mathbf{q}(r) = i\Phi_0\frac{[\mathbf{q}\mathbf{z}_0]}{q^2}e^{-i\mathbf{q}r},$$

$$\mathbf{Y}_\mathbf{q}{}^s = \int\frac{dk}{2\pi}\frac{\mathbf{j}_{k\mathbf{q}}{}^s}{k^2+q^2},$$

$\mathbf{z}_0$ is a unit vector along the z-axis. For the superconducting current in the film,

$$\mathbf{I} = \frac{cd}{4\pi\lambda^2}\left(\sum_i \Phi(r - r_i) - \mathbf{A}\right) \qquad (8)$$

we obtain, by using (7),

$$\mathbf{I}_\mathbf{q} = \left(\frac{c}{4\pi}\sum_i \Phi_\mathbf{q}(r_i) - \mathbf{Y}_\mathbf{q}{}^s\right\}\frac{2q}{1+2q\lambda_2}. \qquad (9)$$

The free energy $\widetilde{F}$ can be expressed in the form of a sum of the kinetic energy of the currents

$$E_k = \frac{2\pi\lambda_2}{c^2}\sum I_q I_{-q} \qquad (10)$$

and the field energy

$$E_f = \int dV\left\{\frac{B^2}{8\pi} - \frac{\mathbf{BH}}{4\pi}\right\}, \qquad (11)$$

where $\mathbf{H} = \mathbf{B} - 4\pi\mathbf{M}$, with $\mathbf{M}$ the film magnetization. In the case under discussion here, this magnetization has only a Z-component, and is easily found from the equation $c\,\mathrm{rot}\,\mathbf{M}=\mathbf{j}$:

$$M_{zq}=\frac{2}{d(1+2q\lambda_2)}\left(\frac{\Phi_0}{4\pi q}\sum e^{-i\mathbf{q}\mathbf{r}_i}-Y_q^s\right). \qquad (12)$$

Assembling all the terms in (10) and (11), and integrating over k and the angle of the vector $\mathbf{q}$, we obtain

$$F=\frac{\Phi_0^2}{8\pi^2}\sum\int dq\frac{J_0(q|r_i-r_j|)}{1+2q\lambda}-\frac{H_0\Phi_0R}{2\pi}\sum\int\frac{dq}{q}\frac{J_1(qR)J_0(qr_i)}{1+2q\lambda_2}, \qquad (13)$$

where J_0, J_1 are Bessel functions. In (13) we have omitted terms which do not depend on the vortex coordinate r_i, and have included the fact that

$$Y_q^s=\frac{cH_0R}{2q^2}J_1(qR). \qquad (14)$$

The first term in (13) is the interaction energy between vortices; for $r_i-r_j\gg\lambda_2$ it has the well-known form

$$\frac{\Phi_0^2}{8\pi}\sum_{ij}\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}, \qquad (15)$$

while the second term gives the interaction energy between a vortex and the external magnetic field H_0. Note that this energy is proportional to R, i.e., the macroscopic size of the film. The latter fact is not astonishing, because, in the film the density of supercurrent at large distances from the core of the vortex is[1]

$$j(r)=\frac{c\Phi_0}{4\pi^2r^2d}$$

and the magnetic moment of the vortex is

$$M_0=\frac{\pi d}{c}\int j(r)r^2\,dr\sim\frac{c\Phi_0R_1}{4\pi},$$

where R_1 is a quantity on the order of the linear dimensions of the film. Then the interaction energy is $M_0H_0\propto R_1$.

Differentiating (13) with respect to r_i, we find the force acting on a vortex

$$\mathbf{f}_i=-\frac{\Phi_0^2}{4\pi^2}\sum_j\frac{\partial}{\partial\mathbf{r}_i}\frac{1}{|\mathbf{r}_i-\mathbf{r}_j|}-\frac{H_0\Phi_0R\mathbf{e}_r}{2\pi}\int_0^\infty dq\frac{J_1(qR)J_1(qr_i)}{1+2q\lambda_2}, \qquad (16)$$

where $\mathbf{e}_r$ is the unit vector directed along the radius. For $\lambda_2\ll R$, the integral in (16) equals (for $r_i<R$)

$$\frac{2}{\pi r_i}\left\{K\left(\frac{r_i}{R}\right)-E\left(\frac{r_i}{R}\right)\right\}, \qquad (17)$$

where K, E are the complete elliptic integrals of the first and second kind. The logarithmic divergence in (17) as $r_i\to R$ is cut off at scales $r_i-R\sim\lambda_2$.

It is clear from (16) that the interaction forces between vortices are long-range, and the resultant force due to all of the vortices is determined by integration of the density $n(r)$ over the entire film. From this it also follows that these results will remain valid for the case of comparatively thick films as well, with $d>\lambda$. In the Appendix, we show that for a film with $d>\lambda$ the interaction energy between vortices at scales $r>d$ is determined by the expression (15).

For an axisymmetric distribution $n(r)$, the resultant force will be directed along the radius, and will equal ($r<R$):

$$f(r)=-\frac{\Phi_0^2}{\pi^2}\int_0^r r'n(r')\frac{\partial}{\partial r}\left[\frac{K(r'/r)}{r}\right]dr'-\frac{\Phi_0^2}{\pi^2}\int_r^\infty r'n(r')\frac{\partial}{\partial r}\left[\frac{K(r/r')}{r'}\right]dr'-\frac{H_0\Phi_0R}{\pi^2r}\{K(r/R)-E(r/R)\}. \qquad (18)$$

For $r\ll R$, the expression for the force exerted by the magnetic field acquires the form

$$-\frac{H_0\Phi_0r}{4\pi R}. \qquad (19)$$

If we treat this force as a Lorentz force $f_1=\Phi_0jd/c$ exerted by the Meissner currents excited in the film by the magnetic field, then for the current density we obtain the expression

$$j=\frac{cH_0r}{4\pi Rd}. \qquad (20)$$

However, this approach does not incorporate the special physics of the problem, because as $d\to0$ for j it is easy to obtain currents that considerably exceed the value of the depairing current.

4. GENERATION OF HARMONICS BY A SUPERCONDUCTING FILM

In order to find the efficiency of harmonic generation by a film placed in an AC magnetic field of frequency ω, we must solve Eqs. (1) and (2) and find the time dependence of the magnetic moment of the film. These equations can only be solved numerically. We will assume that the film is a disk of radius R, and that the external magnetic field has the form $H(t)=H_0+H_1\sin\omega t$. If we neglect the edge barrier, and assume that the thickness of the film d is not too large, we can assume that the magnetic field at the edge of the film is equal to the external field, and that the density of vortices at the edge satisfies $n(R,t)=|H(t)|/\Phi_0$.

Let us introduce n_+, n_-, i.e., the densities of vortices oriented in the positive and negative directions with respect to the z axis. Eq. (1) can then be written in the obvious form

$$\frac{\partial}{\partial t}(rn_+)+\frac{\partial}{\partial r}(rV_+n_+)=-\frac{n_+n_-}{\tau},$$
$$\frac{\partial}{\partial t}(rn_-)+\frac{\partial}{\partial r}(rV_-n_-)=-\frac{n_+n_-}{\tau}, \qquad (21)$$

where V_+, V_- are the velocities of the "positive" and "negative" vortices. It is obvious that $V_+=-V_-\equiv V$, and that τ is the annihilation time for oppositely directed vortices, which we will assume to be small; we will be interested in the limit $\tau\to0$. The quantity with physical meaning is $\delta n=n_+-n_-$. From (21) we have

$$\frac{\partial}{\partial t}g(r)+\frac{\partial}{\partial r}(rV_+n_+)-\frac{\partial}{\partial r}(rV_-n_-)=0, \qquad (22)$$

where $g(r)=r\delta n(r)$. As $\tau\to0$, we have $n_+=\delta n$, $n_-=0$ for $\delta n>0$, and $n_+=0$, $n_-=-\delta n$ for $\delta n<0$. For the numerical solution of Eq. (22) we use the Lelevier method, approximating the derivatives in (22) by finite differences according to the following scheme:

$$\frac{\partial}{\partial r}(Vf)=\begin{cases}\dfrac{V(r)f(r)-V(r-\Delta r)f(r-\Delta r)}{\Delta r}, & V(r)>0,\\[2ex] \dfrac{V(r+\Delta r)f(r+\Delta r)-V(r)f(r)}{\Delta r}, & V(r)<0,\end{cases} \qquad (23)$$

where by f we mean $rn_\pm$, and Δr is the integration step along r.

In dimensionless variables,

$$\tilde V = V\frac{\pi^2 R d\omega\eta_0}{\Phi_0(H_0+H_1)}, \quad \tilde g = g\frac{\Phi_0}{R(H_0+H_1)}, \qquad (24)$$

where η_0 is the average viscosity coefficient; then Eq. (22) has the form

$$\frac{\partial}{\partial \tilde t}\tilde g+\alpha_1\frac{\partial}{\partial x}\left\{\frac{\tilde f x}{\tilde\eta(\alpha_2\tilde f)}(n_+-n_-)\right\}=0, \qquad (25)$$

where $\tilde f$ is the dimensionless force

$$\tilde f=[\tilde H_0+\tilde H_1\sin\tilde t]\frac{E(x)}{x}-\frac{1}{x^2}\int_0^x x'K\left(\frac{x'}{x}\right)\frac{\partial\tilde g}{\partial x'}dx'-\frac{1}{x}\int_x^1 K\left[\frac{x}{x'}\right]\frac{\partial\tilde g}{\partial x'}dx', \qquad (26)$$

$\tilde\eta=\eta/\eta_0$ is the dimensionless viscosity, and

$$\alpha_1=\frac{\Phi_0(H_0+H_1)}{\pi^2 dR\omega\eta_0}, \quad \alpha_2=\frac{\Phi_0(H_0+H_1)}{\pi^2}. \qquad (27)$$

Expression (26) for the force is obtained from (18) after substituting δn in place of n in (18) and integrating by parts. In doing so, we have taken into account $\Phi_0\delta n(R,t)=H(t)$. For a uniform distribution of vortices $\delta n(r)=H(t)/\Phi_0$, the force satisfies $\tilde f=0$, as we find from (26) after evaluating the corresponding integrals. Therefore, $\delta n=\text{const}$ is the steady-state solution of Eq. (25).

The results of the numerical solution of Eq. (25) for $\tilde\eta=1$ are shown in Fig. (3). The maximum in the dependence of U_3/H_1 on H_1 is obtained for $\alpha_1\approx 2$. Physically, $\alpha_1\approx 1$ implies that for these values of the parameters a vortex from the edge of the film successfully manages to move to the film center within a single period. By adopting this interpretation of the maximum in the experimental dependence of the third-harmonic amplitude on the amplitude of the AC field shown in Figs. 1 and 2, we obtain a value for the average viscosity on the order of $4\cdot 10^{-7}$ dyne·sec/cm^2. The measurement results shown in Fig. 4 for a sample in the form of a superconducting ring with inner diameter of 3 mm and outer diameter of 5 mm provide qualitative confirmation of the correctness of these considerations. It is clear that in this case the maximum is shifted toward smaller fields compared to the maximum for a film in the form of a disc with radius 5 mm. The experiment shows that the coefficient of viscosity in these experiments cannot be treated as a constant. This follows from the fact that the experimental curves for various frequencies differ somewhat. For $\eta=\text{const}$, the only parameter of the problem would be the ratio H_1/ω. We made an attempt to describe the experimental data by using a step-function dependence for $\eta(f)$, assuming $\tilde\eta(f)=\infty$ for $f<f_{\text{pin}}$ and $\tilde\eta(f)=1$ for $f>f_{\text{pin}}$.

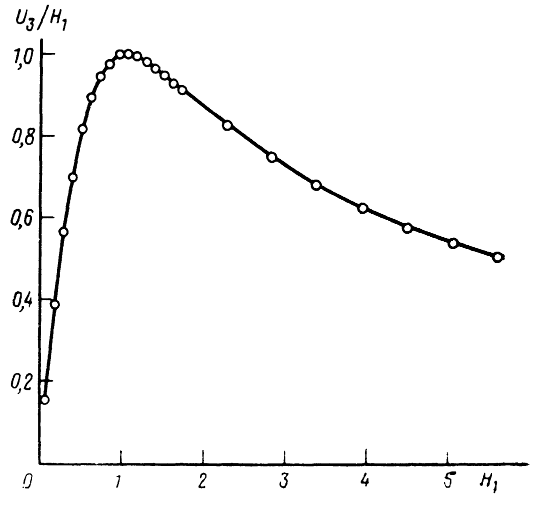

FIG. 3. Results of a numerical solution of Eq. (25) for $\tilde\eta=1$.

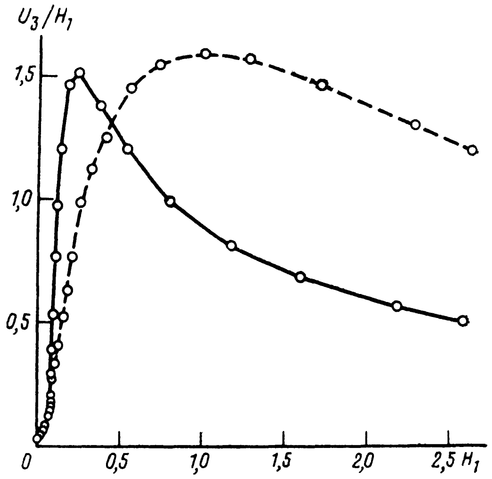

FIG. 4. Results of measurements for $\omega=10$ kHz: the solid curve is for a solid film of diameter 5 mm, the dashed for a superconducting ring with inner diameter 3 mm, outer diameter 5 mm.

The results are shown in Figs. 1 and 2 (the dashed curves). We have used the scales along the x and y axes as fitting parameters. Once we had obtained satisfactory agreement between the theoretical and experimental curves for 15 kHz we did the same calculation for 10 kHz, and found that it was unnecessary to change the scale along the x-axis. The difference between the theoretical and experimental curves is obviously associated with the fact that treating the viscosity as a simple step function is a very rough approximation.

We found that a more precise value of α_1 at the maximum of the dependence of U_3/H_1 on H_1 is 0.9, with the dimensionless pinning force $\tilde f=5.5$. From Eq. (27) we find that $\eta_0=6\cdot 10^{-6}$ dyne·sec/cm^2, while for the pinning force we have $5\cdot 10^{-3}$ dynes, which corresponds to a critical current on the order of $2.5\cdot 10^5$ A/cm^2.

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APPENDIX

In this appendix, we sill consider a vortex in a film of thickness $d \approx \lambda$. Let the film occupy the region $|z| < d/2$. We seek the solution of the vector potential equation for a single vortex located at $r = 0$ in the form

$$\mathbf{A} = \sum \mathbf{A}_q(z) e^{i\mathbf{q}\mathbf{r}}, \tag{A1}$$

where r is the two-dimensional vector in the plane xy. The vector potential in the film has the form

$$\mathbf{A}_q(z) = \frac{\Phi_q}{\lambda^2 q_1^2} \frac{\operatorname{ch} q_1 d/2 + (q_1/q) \operatorname{sh} q_1 d/2 - \operatorname{ch} qz}{\operatorname{ch} q_1 d/2 + (q_1/q) \operatorname{sh} q_1 d/2},$$

$$|z| < d/2, \tag{A2}$$

where $q_1^2 = q^2 + 1/\lambda^2$.

After an inverse Fourier transform and integration over the film thickness, we obtain for the average θ-component of the current density

$$j_\theta(r) = \frac{c\Phi_0}{4\pi\lambda^2} \int_0^\infty \frac{dq}{2\pi} J_1(qr)$$

$$\times \left\{ 1 - \frac{\operatorname{ch} q_1 d/2 + (q_1/q) \operatorname{sh} q_1 d/2 + 2/q_1 d}{\operatorname{ch} q_1 d/2 + (q_1 d/q) \operatorname{sh} q_1 d/2} \right\}. \tag{A3}$$

As $d \to 0$, (A3) coincides with the solution to the problem for the limiting case of a thin film. As $r \to \infty$, the primary contribution is given by the harmonic with $q \to 0$, and the integral in (A3) can be written in the following form:

$$j_\theta(r) = \frac{c\Phi_0}{4\pi\lambda^2 r^2} \int_0^\infty \frac{dx}{2\pi} J_1(x) \frac{2x\lambda_2}{1 + 2x(\lambda_2/r)(d/2\lambda) \operatorname{cth} d/2\lambda}. \tag{A4}$$

After integrating, we obtain the following expression for the current density:

$$j_\theta(r) = \frac{c\Phi_0}{4\pi^2 dr^2}, \tag{A5}$$

which is correct at scales $r > \lambda \coth(d/2\lambda)$. This latter expression (A5) coincides with the expression for the current density in a thin film.

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