Radiation by a monoenergetic beam of noninteracting ultrarelativistic charged particles in a scattering medium

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The bremsstrahlung of a monoenergetic beam of charged particles which are classically fast and which do not interact with each other is analyzed for the case in which these particles undergo repeated elastic collisions with randomly distributed atoms of a medium. A systematic kinetic analysis of the radiation process in the medium yields the spectral density of the bremsstrahlung emitted by these particles. The spectrum found here differs from that in the case of an isolated radiator, which has been studied previously, in being very nonmonotonic and in having at least one extremum. The value of the radiation energy at the extremum, the position of this extremum, and its width all depend on parameters which characterize the initial beam of radiating particles and the scattering medium. A detailed study is made of the radiation by a collimated monoenergetic beam of charged particles and also of the bremsstrahlung of a highly anisotropic point source of ultrarelativistic radiators.

1. INTRODUCTION

The bremsstrahlung of a classically fast charged particle in a scattering medium was first studied by Landau and Pomeranchuk.[1,2] They derived expressions for the spectral energy density of the bremsstrahlung. They pointed out that the bremsstrahlung intensity was suppressed at low frequencies by repeated elastic collisions of the carriers with the atoms of the medium (the Landau-Pomeranchuk effect). Migdal[3] derived a quantitative theory for the bremsstrahlung of such a particle by averaging the spectrum of the radiation energy over all possible carrier trajectories in an amorphous medium. The method proposed by Migdal[3] for calculating the spectrum of the bremsstrahlung of a classically fast particle in a medium was developed further[4–7] in research on how the dispersion properties of the scattering medium,[4,6] its boundaries,[5] and inelastic processes which occur in the medium[6,7] affect the frequency distribution of the bremsstrahlung.

However, only the radiation of an individual particle was studied in Refs. 1–7. In many cases (Refs. 8 and 9, for example), the source of the bremsstrahlung as fast carriers move through a scattering medium is a set of radiating particles. In addition, there is general physical interest in a study of the bremsstrahlung of a system of charged particles in a medium, since in this case an interference mechanism as well as the collisional mechanism shapes the radiation spectrum. As a result, the frequency distribution of the bremsstrahlung and the dependence of the bremsstrahlung intensity on the thickness of the medium and on parameters characterizing the scattering of the particles in the medium are markedly different from those in the case of an individual radiator.

In the present paper we examine the bremsstrahlung of a system of monoenergetic, classically fast charged particles which do not interact with each other but which do undergo repeated elastic scattering by randomly positioned atoms of the medium. We derive the spectrum of the bremsstrahlung of such particles through a systematic kinetic analysis of the radiation process in the medium. The spectrum found differs from that in the case of a individual radiator[1–3] in being very nonmonotonic and in having at least one extremum, which results from interference of the waves emitted by the individual particles. In the limit of very low frequencies, the bremsstrahlung of a system of noninteracting carriers in a medium is formed under conditions corresponding to complete coherence of the individual radiators, while in the extreme short-wave part of the spectrum the bremsstrahlung intensity is proportional to the number of particles.

We analyze in detail the radiation by a collimated beam of charged particles with a δ-function momentum distribution and also the bremsstrahlung of a highly anisotropic point source of ultrarelativistic radiators. We show that in these cases, even if there are no spatially distributed radiation sources along the bremsstrahlung propagation direction, the bremsstrahlung spectrum of the system of particles in the medium always has a maximum, and this maximum is unique. The value of the radiation energy at this maximum and also the shape of this maximum depend strongly on the characteristics of the scattering medium and also on parameters which specify the initial beam of ultrarelativistic particles.

2. STATEMENT OF THE PROBLEM; TWO-TIME DISTRIBUTION FUNCTION IN THE *k* REPRESENTATION

Let's study the system of charged ultrarelativistic ($E \gg m$) particles which do not interact with each other ($E \gg \omega$) is a radiation frequency; E, m, and e are the energy mass and the charge of each particles. These particles enter a homogeneous, semi-infinitive, amorphous scattering media. In the initial period $t = 0$, particles are located in the points $\mathbf{r}_{01}, \mathbf{r}_{02}, ..., \mathbf{r}_{0N}$ and are of the velocity of $\mathbf{v}_{01}, \mathbf{v}_{02}, ..., \mathbf{v}_{0N}$, equal to $v_0 = [1 - (m/E)^2],^{1/2}$ and they are directed under the $|\Delta\mu| \ll 1$, $\mu = 1, ..., N$ angle to the $\mathbf{e}_z$ vector (vector of the inward normal to the boundary of the medium).

The spectral energy density radiated by these particles is[10]

$$\frac{dE_\omega}{d\omega} = \frac{e^2\omega^2}{4\pi^2} \sum_{\mu,\nu=1}^{N} \int d\Omega_\mathbf{n} \int dt_1 \exp(-i\omega t_1) \int dt_2 \exp(i\omega t_2)$$

$$\times [\mathbf{n}\times\mathbf{j}^{\mu}_{i\to f}(\mathbf{k}, t_1)] [\mathbf{n}\times(\mathbf{j}^{\nu}_{i\to f}(\mathbf{k}, t_2))^*], \qquad (1)$$

where N is the number of particles, $\mathbf{k}$ is the wave vector of the

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radiation field, $d\Omega_{\mathbf{n}}$ is an element of solid angle in the direction $\mathbf{n}=\mathbf{k}/k$, $k=\omega$, and $\mathbf{j}^{\mu}_{i-f}(\mathbf{k},t)$ is a matrix element of the current of the transition between the states i and f in the momentum representation. The integration in (1) is over the time spent by the particle in the medium.

If we ignore the interaction between particles, the function $\mathbf{j}^{\mu}_{i-f}(\mathbf{k},t)$ is proportional to the Fourier component of the one-particle density matrix $\rho^{\mu}(\mathbf{r}_{\mu},\mathbf{r}'_{\mu};\mathbf{R}_1,\mathbf{R}_2,...,\mathbf{R}_L)$, which depends on the coordinates of particle μ and also on the radius vectors $\mathbf{R}_1$, $\mathbf{R}_2$,..., $\mathbf{R}_L$. The latter specify the positions of the scattering centers in the medium (L is the number of scatterers).

To calculate the spectral energy density $d\varepsilon_{\omega}/d\omega$ of the bremsstrahlung which would be observed in the medium, we should average expression (1) over all possible carrier trajectories in the scattering medium.[3] To do this we need to find the expectation value (over all $\mathbf{R}_1,\mathbf{R}_2,...,\mathbf{R}_L$) of a bilinear combination of the density matrices $\rho^{\mu}(\mathbf{r}_{\mu},\mathbf{r}'_{\mu};\mathbf{R}_1,\mathbf{R}_2,...,\mathbf{R}_L)$ and $\rho^{\nu}(r_{\nu},r'_{\nu};\ \mathbf{R}_1,\ \mathbf{R}_2,...,\mathbf{R}_L)$. Multiplying the equations of motion for the operators $\rho^{\mu}(\mathbf{r}_{\mu},\mathbf{r}'_{\mu};\mathbf{R}_1,...,\mathbf{R}_L)$ and $\rho^{\nu}(\mathbf{r}_{\nu},\mathbf{r}'_{\nu};\mathbf{R}_1,...,\mathbf{R}_L)$ from the right and left by the matrices $\rho^{\nu}(\mathbf{r}_{\nu},\mathbf{r}'_{\nu},\mathbf{R}_1,...,\mathbf{R}_L)$ and $\rho^{\mu}(\mathbf{r}_{\mu},\mathbf{r}'_{\mu};\mathbf{R}_1,...,\mathbf{R}_L)$, respectively, and then summing the results, we find the following equations for the operator $\hat{\mathscr{P}}$, which is the product of ρ^{μ} and ρ^{ν}:

$$i\frac{\partial\hat{\mathscr{P}}}{\partial\tau}=[\hat{H}^{\mu},\hat{\mathscr{P}}(\mathbf{r}_{\mu},\mathbf{r}_{\mu}';\mathbf{r}_{\nu},\mathbf{r}_{\nu}';\mathbf{R}_1,\ldots,\mathbf{R}_L)],$$
$$i\frac{\partial\hat{\mathscr{P}}}{\partial t}=[\hat{H}^{\mu\nu};\hat{\mathscr{P}}(\mathbf{r}_{\mu},\mathbf{r}_{\mu}';\mathbf{r}_{\nu},\mathbf{r}_{\nu}';\mathbf{R}_s,\ldots,\mathbf{R}_L)]. \quad (2)$$

The Hamiltonian $\hat{H}^{\mu}$ acts on the variables $\mathbf{r}_{\mu}$, the Hamiltonian $\hat{H}^{\mu\nu}$ acts on the variables $\mathbf{r}_{\mu}$, and $\mathbf{r}_{\nu}$, and we have $\tau=t_1-t_2$ and $t=t_2$. In the problem of the radiation by a charged particle, the quantity τ is the time scale of the photon formation (the coherence time), and t is the time at which the photon is emitted.[3]

We expand the operator $\hat{\mathscr{P}}$ in (2) in a complete set of plane waves $u^{\lambda}_{\mathbf{p}}\cdot\exp(i\mathbf{pr})$ ($\mathbf{p}$ is the momentum of the particle, and λ is the spin variable[10]), and we take an average over the positions of the scatterers in the medium in the resulting equations. Ignoring the "mixing" of the spin components of the wave functions caused by a scattering center (this simplification is legitimate for ultrarelativistic particles[11]), we then find the following expressions for $F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)$, the coefficients in the expansion of the operator $\mathscr{P}$ (we are omitting the spinor indices):

$$i\frac{\partial}{\partial\tau}\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle-(E_{\mathbf{p}_1}-E_{\mathbf{p}_2})\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$$
$$=\sum_{\mathbf{g}}\{\langle V(\mathbf{g})F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t,t+\tau)\rangle-\langle V(\mathbf{g})F_{\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t,t+\tau)\rangle\}, \quad (3)$$

$$i\frac{\partial}{\partial t}\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle-(E_{\mathbf{p}_1}-E_{\mathbf{p}_2}+E_{\mathbf{p}_3}-E_{\mathbf{p}_4})\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$$
$$=\sum_{\mathbf{g}}\{\langle V(\mathbf{g})F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t,t+\tau)\rangle-\langle V(\mathbf{g})F_{\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t,t+\tau)\rangle$$
$$+\langle V(\mathbf{g})F_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3+\mathbf{g},\mathbf{p}_4}(t,t+\tau)\rangle-\langle V(\mathbf{g})F_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4-\mathbf{g}}(t,t+\tau)\rangle\}, \quad (4)$$

where

$$V(\mathbf{g})=\sum_{\lambda=1}^{L}U(\mathbf{g})\exp(-i\mathbf{g}\mathbf{R}_{\lambda}),$$

$U(\mathbf{g})$ is a Fourier component of the potential of the scattering center, $E_{\mathbf{p}}$ is the energy of a particle with a momentum $\mathbf{p}$, and the angle brackets mean an average over the positions of the scatterers.

Equations (3) and (4) constitute systems of integrodifferential equations which are not closed with respect to the unknown function $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$, which depends on two time variables. The latter circumstance makes a calculation of the correlation function on the right sides of Eqs. (3) and (4) and also the derivation of $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ far more complicated than in the case of ordinary one-time problems of kinetic theory.[12] However, by virtue of the very formulation of the problem of the emission by a charged particle in a medium, the times t and τ satisfy the inequality $\tau\ll t$: The photon must manage to be emitted during the time the particle spends in the medium. To first order in the parameter $\tau/t\ll1$ we can then ignore the dependence of the function $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ on the variable τ in Eq. (4), since the time scale of the variation in $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ specified by this equation is of order $t\gg\tau$, while the derivatives of the function $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ with respect to the variable t are fairly smooth functions of t by virtue of the homogeneity of the medium.

Setting $\tau=0$ in (4), we can then successively construct equations[12] for the functions of the type $\langle V(\mathbf{g})F_{\mathbf{p}_1\pm\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t)\rangle$ which appear on the right side of (4). Substituting the solutions of the latter equations into (4), with $\tau=0$, and using the standard rules[12] for splitting up the correlation functions of the type

$$\langle V(\mathbf{g}_1)V(\mathbf{g}_2)F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle=\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle\langle V(\mathbf{g}_1)V(\mathbf{g}_2)\rangle,$$

which arise in the process, we find an equation for $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle$. Proceeding in the same way, we find an equation for the function $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ from (3) (but in this case with $\tau\neq0$). Expanding the collision integrals in the equations found for $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle$ and $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ in the small momentum transfer $\mathbf{g}$ and also in $\omega/E\ll1$—this is a legitimate step in the case of ultrarelativistic, classically fast particles—we find the following equations for the functions $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle$ and $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ [a detailed derivation of Eqs. (5) and (6) is given in the Appendix]:

$$\frac{\partial F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,\tau)}{\partial\tau}-i\mathbf{k}\mathbf{v}_{\mu}(\boldsymbol{\eta})F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,\tau)=\frac{q}{4}\frac{\partial^2}{\partial\boldsymbol{\eta}^2}F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,\tau), \quad (5)$$

$$\frac{\partial F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,0)}{\partial t}-i\mathbf{k}(\mathbf{v}_{\mu}(\boldsymbol{\eta})-\mathbf{v}_{\nu}(\boldsymbol{\zeta}))F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,0)=\frac{q}{4}\left[\frac{\partial}{\partial\boldsymbol{\eta}}+\frac{\partial}{\partial\boldsymbol{\zeta}}\right]^2F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,0). \quad (6)$$

Here $F_{\mathbf{k}}(\mathbf{v}_{\mu},\mathbf{v}_{\nu},t,\tau)$ is the two-time distribution function in the k representation,[1)] which is found from $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,t+\tau)\rangle$ by making the change of variables $\mathbf{p}_{1,2}=\mathbf{p}_{\mu}\mp\mathbf{k}/2$, $\mathbf{p}_{3,4}=\mathbf{p}_{\nu}\pm\mathbf{k}/2$. This distribution function completely determines the kinetics of the radiation process in the medium. The quantities $\mathbf{v}_\mu$, $\mathbf{v}_\nu$, $\mathbf{p}_\mu$, and $\mathbf{p}_\nu$ are the velocities and momenta of particles μ and ν; q is the mean square value of the multiple-scattering angle per unit path length;[13] and $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ are angular vectors which satisfy

$$\mathbf{v}_\mu=v_0\mathbf{e}_z(1-\boldsymbol{\eta}^2/2)+v_0\boldsymbol{\eta},\quad \mathbf{e}_z\boldsymbol{\eta}=0,\quad |\boldsymbol{\eta}|\ll 1,$$
$$\mathbf{v}_\nu=v_0\mathbf{e}_z(1-\boldsymbol{\zeta}^2/2)+v_0\boldsymbol{\zeta},\quad \mathbf{e}_z\boldsymbol{\zeta}=0,\quad |\boldsymbol{\zeta}|\ll 1. \tag{7}$$

Here $\mathbf{e}_z$ is the unit vector along the inward normal to the boundary of the medium.

Expanding the scalar products $\mathbf{kv}_\mu$ and $\mathbf{kv}_\nu$ on the left sides of (5) and (6) in the small quantities $|\boldsymbol{\eta}|$, $|\boldsymbol{\zeta}|$, $|\boldsymbol{\theta}_k|\ll 1$ [$\boldsymbol{\theta}_k$ is an angular vector associated with the radiation angle θ_k and the wave vector $\mathbf{k}$ by equations like (7) (with $v_0\to k$)], we find the following expression for the function $F_\mathbf{k}(\mathbf{v}_\mu,\mathbf{v}_\nu,t,\tau)$ from Eqs. (5) and (6):

$$F_\mathbf{k}(\boldsymbol{\eta},\boldsymbol{\zeta},t,\tau)=(\pi qt)^{-1}\int d^2\boldsymbol{\eta}'\,d^2\boldsymbol{\eta}''G_\mathbf{k}(\boldsymbol{\eta}-\boldsymbol{\eta}'',\tau)$$
$$\times\varphi_\mathbf{k}(\boldsymbol{\eta}',\boldsymbol{\eta}'-\boldsymbol{\eta}''+\boldsymbol{\zeta})$$
$$\times\exp\{-iv_0t\mathbf{k}(\boldsymbol{\eta}-\boldsymbol{\zeta})-ikv_0t(\boldsymbol{\eta}''-\boldsymbol{\zeta})(\boldsymbol{\eta}'-{}^1/_2(\boldsymbol{\eta}''-\boldsymbol{\zeta})]$$
$$-(\boldsymbol{\eta}''-\boldsymbol{\eta}')^2(qt)^{-1}$$
$$-(i/2)(\boldsymbol{\eta}''-\boldsymbol{\eta}')(\boldsymbol{\eta}-\boldsymbol{\zeta})kv_0t-qk^2v_0^3t^3(\boldsymbol{\eta}''-\boldsymbol{\zeta})^2\}/48. \tag{8}$$

Here $\varphi_\mathbf{k}(x,y)$ is the Fourier component of the particle distribution function at the time of entrance into the medium ($t=\tau=0$), and $G_\mathbf{k}(\eta-\eta'',\tau)$ is the Green's function of Eq. (6), given by

$$G_\mathbf{k}(\boldsymbol{\eta}-\boldsymbol{\eta}'',\tau)$$
$$=\frac{a}{\pi\,\mathrm{sh}\,a\tau}\exp\left\{-\frac{a(\boldsymbol{\eta}-\boldsymbol{\eta}'')^2}{q\,\mathrm{th}\,a\tau}-\frac{2a}{q}\,\mathrm{th}\frac{a\tau}{2}(\boldsymbol{\theta}_\mathbf{k}-\boldsymbol{\eta}'')^2\right.$$
$$\left.+\frac{2a}{q}\,\mathrm{th}\frac{a\tau}{2}(\boldsymbol{\theta}_\mathbf{k}-\boldsymbol{\eta}'')(\boldsymbol{\eta}-\boldsymbol{\eta}'')+ikv_0\tau\right\}, \tag{9}$$
$$a=(ikv_0q/2)^{1/2}.$$

We then find the following result for the expectation value (over the positions of the scattering centers) of the bilinear combination of matrix elements of the transition current in the expression for the bremsstrahlung energy density in the medium:

$$\langle[\mathbf{n}\times\mathbf{j}^{\mu}_{i\to f}(\mathbf{k},t)][\mathbf{n}\times(\mathbf{j}^{\nu}_{i\to f}(\mathbf{k},t))^*]\rangle$$
$$=\{\boldsymbol{\theta}_k^2+\boldsymbol{\eta}\boldsymbol{\zeta}-\boldsymbol{\eta}\boldsymbol{\theta}_k-\boldsymbol{\zeta}\boldsymbol{\theta}_k\}F_\mathbf{k}(\boldsymbol{\eta},\boldsymbol{\zeta},t,\tau), \tag{10}$$

where $F_\mathbf{k}(\boldsymbol{\eta},\boldsymbol{\zeta},t,\tau)$ is determined by (7)–(9).

3. SPECTRAL ENERGY DISTRIBUTION OF THE BREMSSTRAHLUNG OF A SYSTEM OF NONINTERACTING, CLASSICALLY FAST CHARGED PARTICLES IN A SCATTERING MEDIUM

We assume that the system of classically fast charged particles consists of N carriers which at the time $t=0$ are in the plane coinciding with the boundary of the medium and have the coordinates $\mathbf{r}_{01}$, $\mathbf{r}_{02},\ldots,$ $\mathbf{r}_{0N}$; $(\mathbf{r}_{01})_z=(\mathbf{r}_{02})_z=\ldots=(\mathbf{r}_{0N})_z=0$. The velocities of the particles and the time at which they enter the medium are

$$v_\mu=v_0\mathbf{e}_z(1-\boldsymbol{\Delta}_\mu^2/2)+v_0\boldsymbol{\Delta}_\mu,\quad \mathbf{e}_z\boldsymbol{\Delta}_\mu=0,$$
$$|\boldsymbol{\Delta}_\mu|\ll 1,\quad \mu=1,\ldots,N. \tag{10a}$$

We can then write the following expression for the function $\varphi_\mathbf{k}(\boldsymbol{\Delta}_\mu,\boldsymbol{\Delta}_\nu)$, which determines the state of the system of particles at the time $t=\tau=0$:

$$\varphi_\mathbf{k}(\boldsymbol{\Delta}_\mu,\boldsymbol{\Delta}_\nu)=\delta(\boldsymbol{\eta}-\boldsymbol{\Delta}_\mu)\delta(\boldsymbol{\zeta}-\boldsymbol{\Delta}_\nu)\exp(i\mathbf{kd}_{\mu\nu}),\quad \mathbf{d}_{\mu\nu}=\mathbf{r}_{0\mu}-\mathbf{r}_{0\nu}. \tag{11}$$

Substituting the expectation value found for the bilinear combination of matrix elements of the transition current [see (10)] into (1), and integrating over the variables $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ in the resulting expression [allowing for (8), (9), and (11)], we then find the following expression for the spectral energy distribution of the bremsstrahlung which would be observed in the medium, $d\varepsilon_\omega/d\omega=\langle dE\omega/d\omega\rangle$:

$$\frac{d\varepsilon_\omega}{d\omega}=-\frac{e^2\omega\xi^2}{\pi}\,\mathrm{Im}\left\{\sum_{\mu,\nu=1}^{N}\int_0^T dt\exp\left\{\frac{i\omega v_0t}{2}\mathbf{b}_{\mu\nu}^2-\frac{q\omega^2v_0^2t^3}{48}\mathbf{b}_{\mu\nu}^2\right.\right.$$
$$\left.-\frac{qt\omega^2}{4}\left(\mathbf{d}_{\mu\nu}+\frac{v_0t}{2}\mathbf{b}_{\mu\nu}\right)^2+i\omega\mathbf{d}_{\mu\nu}\boldsymbol{\Delta}_\nu\right\}$$
$$\times\left\{\left[\left[\int_0^\infty\frac{ds}{\mathrm{th}\,s}\exp\left[-\frac{(1+i)s}{2\varkappa}-\gamma\,\mathrm{cth}\,s\right]\right.\right.\right.$$
$$\left.+i\frac{\pi}{4}(\delta_{\mu\nu}+\delta_{\gamma 0})\right]-a\xi^{-2}\int_0^\infty\frac{ds}{\mathrm{sh}^2s}\exp\left[-\frac{(1+i)s}{2\varkappa}-\gamma\,\mathrm{cth}\,s\right]$$
$$\left.\left.\times\mathbf{b}_{\mu\nu}(\mathbf{d}_{\mu\nu}+v_0t\mathbf{b}_{\mu\nu})\right\}\right\}, \tag{12}$$

where $\varkappa=\xi^{-2}(q/\omega)^{1/2}$, $\gamma=q\omega^2/4a(\mathbf{d}_{\mu\nu}+v_0t\mathbf{b}_{\mu\nu})^2$, $\xi=m/E$, $\mathbf{b}_{\mu\nu}=\boldsymbol{\Delta}_\mu-\boldsymbol{\Delta}_\nu$, $a=(i\omega qv_0/2)^{1/2}$, $k=\omega$, $\delta_{\mu\nu}$ is the Kronecker delta, and T is the total time spent by the particles in the medium (the thickness of the layer of the medium).[2)]

Let us analyze the frequency distribution given by (12). In the extreme long-wavelength part of the spectrum ($q\xi^{-4}\gg\omega\to 0$), we find the following result by setting the frequency ω equal to zero and by replacing tanh z by unity in the integral over the variable z:

$$\frac{d\varepsilon_\omega}{d\omega}=N^2\frac{e^2(q\omega)^{1/2}T}{\pi}. \tag{13}$$

In other words, at sufficiently small values of ω, where the Landau-Pomeranchuk effect occurs, the bremsstrahlung of a system of noninteracting charged particles in the medium is formed under conditions of complete coherence of the radiators ($d\varepsilon_\omega/d\omega\propto N^2$), and the bremsstrahlung energy

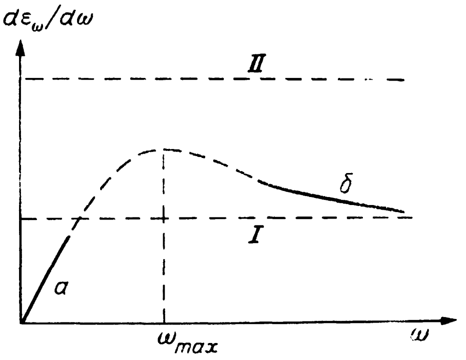

FIG. 1. Energy spectrum of the bremsstrahlung of a beam of fast charged particles with a δ-function momentum distribution in a medium (ω is the frequency of the radiation). Level I) The value $V\cdot le^2qT/3\pi^2\xi^2$; level II) the value $V^2\cdot le^2qT/3\pi^2\xi^2$. a—$\propto N^2(q\omega)^{1/2}\cdot T$; b—$\propto\Sigma^\infty_{\mu+\nu=l}\exp(-\omega\xi d_{\mu\nu})(\omega\xi d_{\mu\nu})^{-1/2}$.

does not depend on the relative positions of the particles. The reason is that in the limit $\omega \to 0$ the wavelength of the radiation is long in comparison with the difference between the path lengths of bremsstrahlung photons coming from any two individual radiators.

For very high frequencies, $\omega \gg \max\{q\xi^{-4}; (qT)^{-1/2} \times [\min(d_{\mu\nu}; v_0 t b_{\mu\nu})]^{-1}\}$, we expand the preexponential coefficient and the argument of the exponential function in powers of the small quantity $z \ll 1$ in the integrals over the variable z. From (12) we then find

$$\frac{d\varepsilon_\omega}{d\omega} = N\frac{2e^2 qT}{3\pi\xi^2} + \frac{e^2 q}{(2\pi\xi)^{1/2}} \sum_{\mu\neq\nu=1}^{N} \int_0^T dt \exp\left\{-\frac{q\omega^2 t^3}{48}\mathbf{b}_{\rho\nu}^2 - \frac{q\omega^2 t}{4}\left(\mathbf{d}_{\mu\nu} + \frac{v_0 t}{2}\mathbf{b}_{\mu\nu}\right)^2\right\} \omega^{1/2} \left|\mathbf{d}_{\mu\nu} + \frac{v_0 t}{2}\mathbf{b}_{\mu\nu}\right|^{3/2}$$
$$\times \exp\{-\omega\xi|\mathbf{d}_{\mu\nu} + v_0 t\mathbf{b}_{\mu\nu}|\} \underset{\omega\to\infty}{\longrightarrow} N\frac{2e^2 qT}{3\pi\xi^2} \equiv \left(\frac{d\varepsilon_\omega}{d\omega}\right)_{\rm BH}. \qquad (14)$$

We thus see that at sufficiently high values of ω the term in the spectrum (14) which stems from the interference of the bremsstrahlung photons emitted by different particles tends toward zero with increasing frequency, while the quantity $d\varepsilon_\omega/d\omega$ becomes equal to $(d\varepsilon_\omega/d\omega)_{\rm BH}$, i.e., the Bethe-Heitler bremsstrahlung energy for N independent radiators.

Since the second term on the right side of (14) is nonnegative at any ω, it follows from the asymptotic expressions in (13) and (14) that the frequency distribution of the bremsstrahlung of a system of noninteraction charged particles always has at least one extremum (Fig. 1). This result stands in contrast with the result for an individual radiator,[1-3] in which case the bremsstrahlung energy spectrum in the medium is a monotonically increasing function of ω. In the case $\mathbf{d}_{\mu\nu}\mathbf{\Delta}_\nu \neq 0$, there is generally more than one extremum, because the system of ultrarelativistic radiators has a nonzero dimension along the radiation propagation direction.

Below we will study in detail the cases in which either $\mathbf{\Delta}_\mu = 0$ for $\mu = 1,...,N$ (this is the case of a plane, collimated beam of emitting particles) or $|\mathbf{d}_{\mu\nu}| = 0$ for $|\Delta_\mu| \neq 0$, $\mu,\nu = 1,...,N$ (this is the case of a highly anisotropic point source of ultrarelativistic radiators). These situations have been singled out because in the case $\mathbf{\Delta}_\mu = 0$ or $|\mathbf{d}_{\mu\nu}| = 0$, even if there is no ordinary interference of waves from sources which are spatially separated along the radiation propagation direction, the bremsstrahlung energy spectrum of the system of particles in the medium always has an extremum because of the transverse coherence of the radiating particles ("transverse" with respect to the radiation propagation direction).

4. SPECTRUM OF THE RADIATION EMITTED BY A COLLIMATED BEAM OF ULTRARELATIVISTIC CHARGED PARTICLES IN A MEDIUM

Since the initial ($t = 0$) distances between the beam particles, $|\mathbf{d}_{\mu\nu}|$, and the angle at which they are moving apart, $|\mathbf{b}_{\mu\nu}| = \mathbf{\Delta}_\mu - \Delta_\nu$, and at $t = 0$ are generally random quantities, we need to average expression (12) over all possible values of the vectors $\mathbf{d}_{\mu\nu}$ and $\mathbf{b}_{\mu\nu}$ in order to find the spectral energy density of the radiation which would be observed experimentally. Setting $\mathbf{\Delta}_\mu = 0, \mu = 1,...,N$, and finding the average of $\mathbf{d}_{\mu\nu}$ over the cross section of the collimated beam (we approximate this cross section as a circle of diameter D), we find

$$\frac{d\varepsilon_\omega}{d\omega} = N\left(\frac{d\varepsilon_\omega}{d\omega}\right)_M - N(N-1)\frac{4e^2\omega\xi^2}{\pi D^2}\,{\rm Im}\int_0^T dt\int_0^\infty \frac{ds}{{\rm th}\, s} \times \exp\left[-\frac{(1+i)s}{2\varkappa}\right]$$
$$\times \frac{4\{1-\exp[-(qt\omega^2 + q\omega^2/a\,{\rm th}\,s)/16D^2]\}}{qt\omega^2 + q\omega^2/a\,{\rm th}\,s}, \qquad (15)$$

where $(d\varepsilon_\omega/d\omega)_\mu$ is the spectral energy density of the bremsstrahlung of an individual radiator in a medium, which was found by Migdal.[3]

In the low-frequency region, $\omega \lesssim q\xi^{-4}$, we set $\tanh s \to 1$ in the last expression, and we use the relations $t \sim T \gg \tau_0 = \varepsilon_q$ [$\tau_q = (q\omega)^{-1/2}$ is the formation time of a bremsstrahlung photon under the conditions corresponding to the Landau-Pomeranchuk effect[2)]]. We find

$$\frac{d\varepsilon_\omega}{d\omega} = N\frac{e^2(q\omega)^{1/2}T}{\pi} + \frac{16e^2N(N-1)}{(q\omega)^{1/2}\omega\pi D^2}\left\{\ln\frac{T}{\tau_q} - E_1\left(\frac{q\omega^2\tau_q D^2}{16}\right) + E_1\left(\frac{q\omega^2 TD^2}{16}\right)\right\}, \qquad (16)$$

where $E_n(s)$ is the integral exponential function.[14]

From (16) we find

$$\frac{d\varepsilon_\omega}{d\omega} = \begin{cases} N^2\dfrac{e^2(q\omega)^{1/2}T}{\pi} - \dfrac{N(N-1)e^2}{64\pi}q^{3/2}\omega^{5/2}T^2D^2, \\ \omega \ll \min\{q\xi^{-4}; (qTD^2)^{-1/2}\} \\ N\dfrac{e^2(q\omega)^{1/2}T}{\pi} + \dfrac{16e^2N(N-1)}{(q\omega)^{1/2}\omega\pi D^2}\ln\left(\dfrac{T}{\tau_q}\right); \\ (qD^4)^{-1/3} \ll \omega \ll q\xi^{-4}. \end{cases} \qquad (16a)$$

At frequencies $\omega \lesssim q\xi^{-4}$ the derivative $d\varepsilon_\omega/d\omega$ is thus an increasing function of ω. As the frequency increases, the interference term in (16) increases more slowly as a function of the time T than the term responsible for the intrinsic bremsstrahlung of the individual radiators.

In the short-wavelength part of the spectrum, $\omega \gtrsim q\xi^{-4}$, we expand the preexponential coefficient and the argument of the exponential function in powers of the small

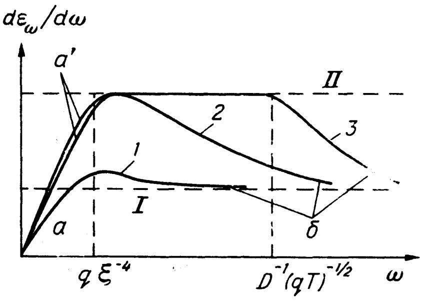

FIG. 2. Energy spectrum of the bremsstrahlung of a collimated beam of fast charged particles in a medium (ω is the frequency of the radiation). 1—$qD\xi^{-3} \gg 1$; 2—$qD\xi^{-3} \lesssim \xi(qT)^{-1/2} \ll 1$; 3—$qD\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$. Levels I, II) The same as in Fig. 1. a: $\propto N\omega^{1/2}$. a': $\propto N^2\omega^{1/2}$. b: $N(N-1)\omega^{-2}$.

quantity $s \ll 1$ in the integrals over the variable s in expression (15). Note that the relations $t \sim T \gg \tau_0 = \tau_e$ are typical.[2)] We find

$$\frac{d\varepsilon_\omega}{d\omega} = N\frac{2e^2qT}{3\pi\xi^2} + \frac{4e^2N(N-1)}{3\pi}\left\{\frac{8\ln(T/\tau_\xi)}{(D\omega\xi)_2} - K_2\left(\frac{\omega\xi D}{2}\right)\right.$$
$$\left.\times\left[E_1\left(\frac{q\omega^2D^2\tau_\xi}{16}\right) - E_1\left(\frac{q\omega^2D^2T}{16}\right)\right]\right\}, \quad (17)$$

where $\tau_\xi = (\tau_e)_{\max} = q^{-1}\xi^2$, and $K_\nu(s)$ is the modified Bessel function.[15]

Using asymptotic expansions of the functions $E_n(s)$ and $K_\nu(s)$ in the cases $s \gg 1$ and $s \ll 1$ (Refs. 14 and 15), we find from (17)

$$\frac{d\varepsilon_\omega}{d\omega} = \begin{cases} N^2\dfrac{2e^2qT}{3\pi\xi^2} - \dfrac{N(N-1)e^2}{96\pi}\dfrac{(q\omega DT)^2}{\xi^2}, \\ q\xi^{-4} \lesssim \omega \ll (qTD^2)^{-1/2} \\ N\dfrac{2e^2qT}{3\pi\xi^2} + N(N-1)\dfrac{32e^2\ln(T/\tau_\xi)}{3\pi D^2\omega^2\xi^2}, \\ \omega \gg \max\{q\xi^{-4}; (D\xi)^{-1}\}. \end{cases} \quad (18)$$

The results in (17) and (18) show that $d\varepsilon_\omega/d\omega$ is an increasing function of the frequency at $\omega \lesssim q\xi^{-4}$, and at $\omega \gtrsim q\xi^{-4}$ the derivative $d\varepsilon_\omega/d\omega$ decreases with increasing ω. It follows that the spectral energy density of the bremsstrahlung of a collimated beam in a medium has a maximum, and this maximum is unique. If $qD\xi^{-3} \gg 1$, then $\omega_{\max} \sim q\xi^{-4}$, and the bremsstrahlung energy $d\varepsilon_\omega/d\omega$ at $\omega \approx \omega_{\max}$ is on the same order of magnitude as the "background" due to Bethe-Heitler radiation (Fig. 2). In the case $qD\xi^{-3} \lesssim \xi(qT)^{-1/2} \ll 1$, the maximum of the $d\varepsilon_\omega/d\omega$ spectrum is again at the frequency $\omega_{\max} \sim q\xi^{-4}$, but in this case we have a ratio $(d\varepsilon_\omega/d\omega)_{\max}/(d\varepsilon_\omega/d\omega)_{\rm BH} \approx N$ (N is the number of radiating particles). If, on the other hand, the conditions $qD\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$ hold, the maximum is a plateau with a width equal in order of magnitude to $D^{-1}(qT)^{-1/2}$ (Fig. 2), and we have $(d\varepsilon_\omega/d\omega)_{\max}/(d\varepsilon_\omega/d\omega)_{\rm BH} \approx N$.

The reason for the existence of a maximum in the bremsstrahlung energy spectrum is the following: As the frequency ω decreases, the particles radiate more coherently, so $d\varepsilon_\omega/d\omega$ increases. As ω increases, on the other hand, $d\varepsilon_\omega/d\omega$ for an individual radiator increases.[3] These two opposite tendencies in the frequency dependence of $d\varepsilon_\omega/d\omega$ give rise to the maximum which we have been discussing. This result differs from that in the case of an individual radiator, in which case the spectral energy density of the bremsstrahlung in the medium is a monotonic function of ω.

5. BREMSSTRAHLUNG ENERGY SPECTRUM OF A HIGHLY ANISOTROPIC POINT SOURCE OF ULTRARELATIVISTIC RADIATORS IN A MEDIUM

In (12) we set $\mathbf{d}_{\mu\nu} = 0$ (i.e., $|\mathbf{d}_{\mu\nu}| \ll |\mathbf{\Delta}_\mu - \mathbf{\Delta}_\nu|\tau_0$). We then take an average over all possible values of the angular vectors $\mathbf{b}_{\mu\nu}$ in a circle of radius $\chi_0 \ll 1$ in the resulting expression (χ_0 is a characteristic value of the cone vertex angle which includes the initial velocities of the particles). We then find the following expression for the bremsstrahlung energy spectrum of a highly anisotropic point source of radiators:

$$\frac{d\varepsilon_\omega}{d\omega} = N\left(\frac{d\varepsilon_\omega}{d\omega}\right)_M + \frac{e^2\xi^2N(N-1)}{\pi\chi_0^2}\,\mathrm{Re}\left\{i\omega\int_{\tau_0}^{T}dt\int_0^\infty\frac{ds}{\mathrm{th}\,s}\right.$$
$$\times\exp\left[-\frac{(1+i)s}{2\varkappa}\right]\left\{1-\exp\left[-\chi_0^2\left(\frac{q^2\omega^2t^3}{12}\right.\right.\right.$$
$$\left.\left.\left.+\frac{q\omega^2t^2}{4a\,\mathrm{th}\,s}\right)\right]\right\}\left(\frac{q\omega^2t^3}{12}\right.$$
$$\left.+\frac{q\omega^2t^2}{4a\,\mathrm{th}\,s}\right)^{-1} + (1+i)\left(\frac{\omega}{q}\right)^{1/2}\int^{T}dt\int^{\infty}\frac{ds}{\mathrm{th}\,s}\exp\left[-\frac{(1-i)s}{2\varkappa}\right]$$
$$\times\left\{1-\exp\left[-\chi_0^2\left(\frac{q\omega^2t^3}{12}+\frac{q\omega^2t^2}{4a\,\mathrm{th}\,s}\right)\right]\right\}$$
$$\left.\times\left(\frac{q\omega^2t^3}{12}+\frac{q\omega^2t^2}{4a\,\mathrm{th}\,s}\right)^{-1}\right\}, \quad (19)$$

where τ_0 is the formation time of a bremsstrahlung photon in the medium, and[2] $\tau_0 \ll t \sim T$.

In the long-wave part of the spectrum, for $\omega \lesssim q\xi^{-4}$ and $\tau_0 = \tau_q = (q\omega)^{-1/2}$, we find from (19)

$$\frac{d\varepsilon_\omega}{d\omega} = N\frac{e^2(q\omega)^{1/2}T}{\pi} + N(N-1)\frac{12e^2}{\pi\chi_0^2\omega^2}\left(\frac{\omega}{q}\right)^{1/2}$$
$$\times\int_{\tau_q}^{T}\frac{dt}{t^3}\left[1-\exp\left(-\frac{q\omega^2t^3\chi_0^2}{12}\right)\right]. \quad (20)$$

It follows, in particular, from this expression that for $\omega \lesssim q\xi^{-4}$ the spectral energy density of the bremsstrahlung in (19) is an increasing function of the frequency. At very small values of ω ($q\xi^{-4} \gg \omega \to 0$) the bremsstrahlung spectrum in the medium is formed under conditions of a complete coherence of the radiating particles.

At sufficiently high frequencies, $\omega \gtrsim q\xi^{-4}$, on the other hand, we find the following expression by expanding the preexponential coefficient and the argument of the exponential function in powers of the small quantity $s \ll 1$ in the integrand of the integral over the variable s in (19):

$$\frac{d\varepsilon_\omega}{d\omega} = N\frac{2e^2qT}{3\pi\xi^2} + \frac{8e^2N(N-1)}{\pi\chi_0^2\xi^2\omega^2}\int_{\tau_\xi}^{T}\frac{dt}{t^3}\left\{1 - \frac{\chi_0^2t^2\omega^2\xi^2}{2}\right.$$
$$\left.\times K_2(\omega t\xi\chi_0)\exp\left(-\frac{q\omega^2t^3\chi_0^2}{12}\right)\right\}. \quad (21)$$

Here $\tau_\xi = q^{-1}\xi^2$ is the maximum formation time of a bremsstrahlung photon for $\omega \gtrsim q\xi^{-4}$.

Using the asymptotic expansion[15] of the function $K_\nu(z)$, we find

$$\frac{d\varepsilon_\omega}{d\omega} = \begin{cases} N^2\dfrac{2e^2qT}{3\pi\xi^2} - \dfrac{N(N-1)e^2q^2\omega^2\chi_0^2T^4}{144\pi\xi^2}, \\ q\xi^{-4} \lesssim \omega \ll (qT^3\chi_0^2)^{-1/2} \\ N\dfrac{2e^2qT}{3\pi\xi^2} + \dfrac{4e^2N(N-1)}{\pi\chi_0^2\xi^2\omega^2\tau_\xi^2}, \\ \omega \gg \max\{q\xi^{-4}; q\xi^{-3}\chi_0^{-1}\}. \end{cases} \quad (22)$$

Analysis of (20) and (22) shows that the bremsstrahlung energy spectrum of a highly anisotropic point source of ultrarelativistic radiators in a medium always has a maximum, and this maximum is unique. If the characteristic cone vertex angle χ_0 which includes the velocities of the emitting particles is such that the relations $q\chi_0 T\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$ hold, the maximum is a plateau with a width on the order of $(\chi_0 T)^{-1}(qT)^{-1/2}$. The ratio of $(d\varepsilon_\omega/d\omega)_{\max}$ to $(d\varepsilon_\omega/d\omega)_{\rm BH}$ is roughly equal to N. In the opposite limit, $q\chi_0 T\xi^{-3} \gg 1$, we find $(d\varepsilon_\omega/d\omega)_{\max} \simeq (d\varepsilon_\omega/d\omega)_{\rm BH}$.

6. SPECTRAL ENERGY DENSITY OF THE BREMSSTRAHLUNG OF A BEAM OF NONINTERACTING ULTRARELATIVISTIC CHARGED PARTICLES IN A MEDIUM

Let us generalize the results derived above to the case of a beam of ultrarelativistic radiators which has a finite size $l_B \ll T$ in the direction in which the particles are moving.

We assume that a beam of fast charged particles, which do not interact with each other, starts to enter a scattering medium at $t = 0$ and continues to enter it for a time $t_B = l_B$. The particles are initially $(t = 0)$ at the points with the coordinates $\mathbf{r}_{01}, \mathbf{r}_{02}, \ldots \mathbf{r}_{0N}$ $[(\mathbf{r}_{0\mu})_z \neq 0)]$ and have velocities $\mathbf{v}_{01}, \mathbf{v}_{02}, \ldots, \mathbf{v}_{0N}$, given by Eqs. (10a). The energy spectrum of the bremsstrahlung of such particles is given by (1). Since the integration over the variables t_1 and t_2 in (1) is to be carried out over the time spent by the particles in the medium, the component of $d\varepsilon_\omega/d\omega$ which comes from the terms with $\mu = \nu$ is exactly the same as the energy of the intrinsic radiation $(\mu = \nu)$ of the particles of a beam with a δ-function momentum distribution [the terms with $\mu = \nu$ in (12); the times t_1 and t_2 refer to the same particle]. To calculate the interference terms $(\mu \neq \nu)$ in the spectrum (1), we would generally need to allow for the circumstance that terms with $\mu \neq \nu$ are zero except in the case in which both particles (particle μ and particle ν) have entered the scattering medium (we are ignoring effects which are related to the time at which the particles enter the medium, since they are small quantities, on the order of $\tau/T \ll 1$). For a compact beam $(l_B \ll T)$, on the other hand, consideration of the size of the beam in the longitudinal direction at the time at which the radiating particles enter the medium leads to small corrections (on the order of $l_B/T \ll 1$) to the energy of the bremsstrahlung which these particles emit over the time T spent by the particles in the scattering medium.

With these thoughts in mind, we substitute into (1) the average value of the bilinear combination of current matrix elements found above, (10), which is expressed in terms of the two-time distribution function $F_{\mathbf{k}}(t,\tau;\boldsymbol{\eta},\boldsymbol{\zeta})$, in which we have $(\mathbf{d}_{\mu\nu})_z \neq 0$, in contrast with the case of a beam with a δ-function momentum distribution. Integrating over all $\boldsymbol{\eta}$ and $\boldsymbol{\zeta}$ in the resulting expression, we find the following result for the bremsstrahlung energy spectrum of the beam of ultrarelativistic charged particles:

$$\frac{d\varepsilon_\omega}{d\omega} = N\left(\frac{d\varepsilon_\omega}{d\omega}\right)_M + \frac{e^2\omega^2 v_0^2}{2\pi q} \times \mathrm{Re}\sum_{\mu\neq\nu=1}^{N}\int_0^T dt\int_0^\infty \frac{ds\exp[-(1+i)s/2\varkappa]}{a\,\mathrm{ch}^2 s} \times \frac{\exp\{C-\mathbf{B}^2/4[(qt)^{-1}+i\omega a\,\mathrm{th}\,s(2Aq)^{-1}(\mathbf{d}_{\mu\nu})_z]\}}{[(qt)^{-1}+i\omega a(\mathbf{d}_{\mu\nu})_z\,\mathrm{th}\,s/2Aq]A^2}$$
$$\times\left\{1-\frac{\omega^2}{4A}\left\{[(\mathbf{d}_{\mu\nu})_\perp+v_0 t\mathbf{b}_{\mu\nu}]^2-\frac{i}{A}(\mathbf{d}_{\mu\nu})_z\mathbf{B}[(\mathbf{d}_{\mu\nu})_\perp+v_{0t}\mathbf{b}_{\mu\nu}]+\frac{(\mathbf{d}_{\mu\nu})_z^2}{A}\left(1-\frac{\mathbf{B}^2}{4}\right)\right\}+i\frac{\omega}{2}\left[\mathbf{b}_{\mu\nu}(\mathbf{d}_{\mu\nu}+v_0 t\mathbf{b}_{\mu\nu})-\frac{i}{2A}(\mathbf{d}_{\mu\nu})_z\mathbf{B}\mathbf{b}_{\mu\nu}\right]\right\}, \quad (23)$$

where

$$\mathbf{d}_{\mu\nu}=(\mathbf{d}_{\mu\nu})_\perp+\mathbf{e}_z(\mathbf{d}_{\mu\nu})_z,\quad \mathbf{b}_{\mu\nu}=\boldsymbol{\Delta}_\nu-\boldsymbol{\Delta}_\mu,\quad a=(iq\omega v_0/2)^{1/2},$$
$$A=\frac{a}{q}\,\mathrm{th}\,s+\frac{i\omega}{2}(\mathbf{d}_{\mu\nu})_z;\quad \mathbf{B}=-\frac{i\omega^2}{2A}(\mathbf{d}_{\mu\nu})_z[(\mathbf{d}_{\mu\nu})_\perp+v_0 t\mathbf{b}_{\mu\nu}]-\frac{2i\boldsymbol{\Delta}_\nu}{qt}+\omega\left[(\mathbf{d}_{\mu\nu})_\perp+\frac{v_0 t\mathbf{b}_{\mu\nu}}{2}\right],$$
$$C=-\frac{\omega^2}{4A}[(\mathbf{d}_{\mu\nu})_\perp+v_0 t\mathbf{b}_{\mu\nu}]^2-\frac{\boldsymbol{\Delta}_\nu^2}{qt}-\frac{i\omega v_0 t\mathbf{b}_{\mu\nu}\boldsymbol{\Delta}_\mu}{2}-\frac{q\omega^2 v_0^2 t^3\mathbf{b}_{\mu\nu}^2}{48}+i\omega(\mathbf{d}_{\mu\nu})_z;$$

and $(d\varepsilon_\omega/d\omega)_M$ is the energy of the bremsstrahlung of an individual radiator.[3]

If the characteristic longitudinal dimensions (in the direction in which the particles are moving) are such that the relation $\min\{(\mathbf{d}_{\mu\nu})_z\} \gg \max\{\omega^{-1};\tau\}$ holds [but, of course, $\max\{(\mathbf{d}_{\mu\nu})_z\} \sim l_B \ll T$], the terms in the sum on the right side of (23) are periodic functions of the frequency ω. The following inequality holds at any value of ω:

$$\left|\left(\frac{d\varepsilon_\omega}{d\omega}\right)_{\max(\min)}-N\left(\frac{d\varepsilon_\omega}{d\omega}\right)_M\right|\Big/N\left(\frac{d\varepsilon_\omega}{d\omega}\right)_M \leqslant (N-1)\frac{\tau}{T} \ll 1. \quad (24)$$

For a sufficiently large beam of radiating particles, the interference effects are thus highly suppressed, and the bremsstrahlung energy spectrum $d\varepsilon_\omega/d\omega$ has basically[3)] the same dependence on the frequency ω as $(d\varepsilon_\omega/d\omega)_M$ in the case of an individual radiator.[3]

In the opposite limit, $(\mathbf{d}_{\mu\nu})_z/\tau \ll \xi^2 \ll 1$, expression (23) becomes the bremsstrahlung energy spectrum in (12), for a beam of ultrarelativistic charged particles with a δ-function momentum distribution.

7. CONCLUSION

We have derived a systematic kinetic theory of the radiation of a system of monoenergetic, classically fast charged particles which do not interact with each other but which are elastically scattered repeatedly by randomly placed atoms of a medium. We have found the bremsstrahlung spectrum of such particles. This spectrum differs from that of an individual radiator[1-3] in being very nonmonotonic and in having at least one extremum, which is a consequence of interference of the waves emitted by the individual particles. The value of the bremsstrahlung at the extremum, the position of the extremum, and its width all depend substantially on parameters which specify the initial beam of radiating particles and also on the characteristics of the scattering medium.

We have carried out a detailed study of the radiation emitted by a collimated, monoenergetic beam $(\min\{\omega^{-1},\tau\} \gg \max(\mathbf{d}_{\mu\nu})_z \to 0)$ of fast charged particles with a δ-function momentum distribution and of the bremsstrahlung of a highly anisotropic point source of ultrarelativistic radiators, i.e., the radiation emitted by systems of particles in which there is initially no spatial distribution of the bremsstrahlung sources along the radiation propagation direction. It has been shown that in these cases the bremsstrahlung spectrum in the medium always has a maximum, and this maximum is unique. If the parameters D and χ_0, which characterize the initial (unscattered) beam of radiating particles, are such that the conditions $qD\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$

and $q\chi_0 T\xi^{-3} \ll \xi(qT)^{-1/2} \ll 1$ hold, the maximum in the bremsstrahlung energy spectrum is a plateau, with a width which depends strongly on D and χ_0, for both a collimated beam of charged particles and an anisotropic point source of ultrarelativistic radiators. The ratio of $(d\varepsilon_\omega/d\omega)_{\max}$ to the background level [the energy of the Bethe-Heitler radiation, $(d\varepsilon_\omega/d\omega)_{BH} = 2Ne^2qT/3\pi\xi^2$] is approximately equal to N, the number of radiating particles. As the parameters D and χ_0 increase χ_0 $[qD\xi^{-3} \lesssim \xi(qT)^{-1/2}$ and $q\chi_0 T\xi^{-3} \lesssim \xi(qT)^{-1/2}]$, this plateau converts into a "strict" maximum. As before, we have $(d\varepsilon_\omega/d\omega)_{\max}/(d\varepsilon_\omega/d\omega)_{BH} \approx N$. If, on the other hand, we have both $qD\xi^{-3} \gg 1$ and $q\chi_0 T\xi^{-3} \gg 1$, then the quantities $(d\varepsilon_\omega/d\omega)_{\max}$ and $(d\varepsilon_\omega/d\omega)_{BH}$ become the same in order of magnitude.

In the opposite limit, of a fairly large beam of radiating particles, $(\mathbf{d}_{\mu\nu})_z \sim l_B \gg \max\{\omega^{-1},\tau\}$, interference effects are suppressed, and the bremsstrahlung energy spectrum is basically the same as the plot of $d\varepsilon_\omega/d\omega$ versus ω for an individual radiator.

It follows, in particular, that by producing compact beams [with a small longitudinal dimension $\max(\mathbf{d}_{\mu\nu})_z \sim l_B \ll \omega^{-1}$] of charged particles in accelerators, and by varying their parameters (N, D, and χ_0), one can achieve a prespecified bremsstrahlung energy in the desired frequency interval.

The range of applicability of the results derived above is limited by the various approximations which have been used. The condition $E \gg m, \omega$ is a customary condition in the analysis of topics of this type. It holds fairly accurately, for example, in cases in which the bremsstrahlung spectrum is due to fast charged particles in the cosmic rays.[8] The upper limit $\omega \ll E$ on the radiation frequency has no substantial effect on the results found here (the presence of at least one extremum in the bremsstrahlung spectrum of the system of particles in a medium), since for $\omega \sim E$ the spectral energy density of the bremsstrahlung is an increasing function of the frequency, tending toward zero as $\omega \to E$ (Ref. 10). It is legitimate to ignore the interaction between the radiating particles during the emission of bremsstrahlung photons in a medium if the curvature of the particle trajectories caused by this interaction is small in comparison with the effect of multiple elastic collisions with the atoms of the medium on the motion of the particles. Quantitatively, the latter condition means

$$(\Delta \mathbf{v}_e)^2 = \frac{e^2}{md^2} \ll v_0^2 qT = (\Delta \mathbf{v}_q)^2,$$

where $(\Delta \mathbf{v}_q)^2$ and $(\Delta \mathbf{v}_e)^2$ are the squares of the transverse velocities (transverse with respect to the initial direction) acquired by a particle as a result of multiple scattering in the medium and as a result of the interaction with other radiating particles, respectively. In addition, T is the thickness of the slab of medium, and d is the characteristic distance between particles. If the particles undergo Coulomb collisions with the atoms of the medium, and the mean square value of the multiple-scattering angle per unit path length is $q = 4\pi n_0 z^2 (e^2/m) \log(180 z^{-1/3})$, this inequality becomes

$$T \gg [4\pi n_0 d (e^2/m) Z^2 \lg (180 Z^{-1/3})]^{-1},$$

where n_0 is the density of scattering centers, and Ze is the charge of each.

I wish to thank S. P. Andreev for a discussion of questions concerning the bremsstrahlung of individual radiators in a medium.

APPENDIX

Following Ref. 11, we write equations for the functions on the right side of (4) (for the case $\tau = 0$). We use the standard rules[12] for breaking up the correlation functions of the type $\langle V(\mathbf{g}) V(\mathbf{g}') F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t, t+\tau)\rangle$ which arise in the process:

$$\langle V(\mathbf{g}) V(\mathbf{g}') F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t, t+\tau)\rangle = n_0 |U(\mathbf{g})|^2 \delta_{\mathbf{g},-\mathbf{g}'} \langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t, t+\tau)\rangle,$$

where n_0 is the number of scattering centers per unit volume.

Solving the equations which result (Ref. 11, for example) under the initial condition $\langle V(\mathbf{g}) F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(0,0)\rangle = \langle V(\mathbf{g})\rangle \langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(0,0)\rangle = 0$ (this condition means that there are not correlations at the time $t = \tau = 0$), we find the functions $\langle V(\mathbf{g}) F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t,t)\rangle$, $\langle V(\mathbf{g}) F_{\mathbf{p}_1\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t,t)\rangle$, $\langle V(\mathbf{g}) F_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3+\mathbf{g},\mathbf{p}_4}(t,t)\rangle$, $\langle V(\mathbf{g}) F_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4-\mathbf{g}}(t,t)\rangle$. Substituting the expressions for the latter functions into the right side of Eq. (4) with $\tau = 0$, we find

$$i\frac{\partial}{\partial t}\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle - (E_{\mathbf{p}_1}+E_{\mathbf{p}_3}-E_{\mathbf{p}_2}-E_{\mathbf{p}_4})\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle$$
$$= -in_0 \sum_{\mathbf{g}} |U(\mathbf{g})|^2 \int_{-t}^{0} dt' \{\exp[it'(E_{\mathbf{p}_1+\mathbf{g}}-E_{\mathbf{p}_2}+E_{\mathbf{p}_3}-E_{\mathbf{p}_4})]$$
$$\times [\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t+t')\rangle + \langle F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3-\mathbf{g},\mathbf{p}_4}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2+\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4+\mathbf{g}}(t+t')\rangle] + \exp[it'(E_{\mathbf{p}_1}+E_{\mathbf{p}_3+\mathbf{g}}-E_{\mathbf{p}_2}-E_{\mathbf{p}_4})]\cdot$$
$$\times [\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t+t')\rangle + \langle F_{\mathbf{p}_1-\mathbf{g},\mathbf{p}_2,\mathbf{p}_3+\mathbf{g},\mathbf{p}_4}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1,\mathbf{p}_2+\mathbf{g},\mathbf{p}_3+\mathbf{g},\mathbf{p}_4}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3+\mathbf{g},\mathbf{p}_4+\mathbf{g}}(t+t')\rangle] - \exp[it'(E_{\mathbf{p}_1}+E_{\mathbf{p}_3}-E_{\mathbf{p}_2-\mathbf{g}}-E_{\mathbf{p}_4})]\cdot$$
$$\times \{\langle F_{\mathbf{p}_1-\mathbf{g},\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t+t')\rangle + \langle F_{\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3-\mathbf{g},\mathbf{p}_4}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4+\mathbf{g}}(t+t')\rangle] - \exp[it'(E_{\mathbf{p}_1}+E_{\mathbf{p}_3}-E_{\mathbf{p}_2}-E_{\mathbf{p}_4-\mathbf{g}})]$$
$$\times [\langle F_{\mathbf{p}_1-\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4-\mathbf{g}}(t+t')\rangle + \langle F_{\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3-\mathbf{g},\mathbf{p}_4-\mathbf{g}}(t+t')\rangle$$
$$-\langle F_{\mathbf{p}_1,\mathbf{p}_2+\mathbf{g},\mathbf{p}_3,\mathbf{p}_4-\mathbf{g}}(t+t')\rangle - \langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t+t')\rangle]\}. \quad (25)$$

Proceeding in a similar way with the correlation functions which appear on the right side of Eq. (3), we find

$$i\frac{\partial}{\partial \tau}\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,\tau)\rangle - (E_{\mathbf{p}_1}-E_{\mathbf{p}_2})\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,\tau)\rangle$$
$$= -in_0 \sum_{\mathbf{g}} |U(\mathbf{g})|^2 \int_{-\tau}^{0} d\tau' \{\exp[i\tau'(E_{\mathbf{p}_1+\mathbf{g}}-E_{\mathbf{p}_2})]$$
$$\times [\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,\tau+\tau')\rangle$$
$$-\langle F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2+\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t,\tau+\tau')\rangle] - \exp[i\tau'(E_{\mathbf{p}_1}-E_{\mathbf{p}_2-\mathbf{g}})]$$
$$\times [\langle F_{\mathbf{p}_1-\mathbf{g},\mathbf{p}_2-\mathbf{g},\mathbf{p}_3\mathbf{p}_4}(t,\tau+\tau')\rangle - \langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,\tau+\tau')\rangle]\}$$
$$+ \sum_{\mathbf{g}} \{\langle V(\mathbf{g}) F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t)\rangle - \langle V(\mathbf{g}) F_{\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t)\rangle\}. \quad (26)$$

Inhomogeneities arise on the right side of (26) because of the requirement that the correlation functions of the type $\langle V(\mathbf{g}) F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t,\tau)\rangle$ be continuous as functions of the variable τ at the time $\tau = 0$:

$$\langle V(\mathbf{g})F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t,\tau)\rangle|_{\tau=0} = \langle V(\mathbf{g})F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t)\rangle.$$

Since the time scales of the interaction of a particle with a scattering center, τ_0, are small in comparison with t and τ, we can extend the integration over the variables t and τ in (25) and (26) to $-\infty$, and in the zeroth approximation in $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t+t';\tau+q')\rangle$ we can replace the functions $\tau_0/\tau \ll 1$ and $\tau_0/t \ll 1$ by $\langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,\tau)\rangle$. Introducing a two-time distribution function in the k representation,

$$F_\mathbf{k}(\mathbf{p}-\mathbf{k}/2;\ \mathbf{p}+\mathbf{k}/2;\ \mathbf{p}'+\mathbf{k}/2;\ \mathbf{p}'-\mathbf{k}/2;\ t,\ \tau) \equiv \langle F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t,\ \tau)\rangle,$$

we then find the following result from Eqs. (25) and (26):

$$\frac{\partial}{\partial\tau}F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4;t,\tau)+i(E_{\mathbf{p}_1}-E_{\mathbf{p}_2})F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4;t,\tau)$$
$$=-n_0\sum_\mathbf{g}|U(\mathbf{g})|^2\{\delta_-(E_{\mathbf{p}_1+\mathbf{g}}-E_{\mathbf{p}_2})[F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4;t,\tau)$$
$$-F_\mathbf{k}(\mathbf{p}_1+\mathbf{g},\mathbf{p}_2+\mathbf{g},\mathbf{p}_3,\mathbf{p}_4;t,\tau)]$$
$$-\delta_-(E_{\mathbf{p}_1}-E_{\mathbf{p}_2-\mathbf{g}})[F_\mathbf{k}(\mathbf{p}_1-\mathbf{g},\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4;t,\tau)$$
$$-F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4;t,\tau)]\}-i\sum_\mathbf{g}\{\langle V(\mathbf{g})F_{\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4}(t,\tau=0)\rangle$$
$$-\langle V(\mathbf{g})F_{\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4}(t,\tau=0)\rangle\}, \qquad (27)$$

$$\frac{\partial}{\partial t}\{F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,t,0)\}$$
$$+i(E_{\mathbf{p}_1}-E_{\mathbf{p}_2}+E_{\mathbf{p}_3}-E_{\mathbf{p}_4})F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,t,0)$$
$$=-n_0\sum_\mathbf{g}|U(\mathbf{g})|^2\{\delta_-(E_{\mathbf{p}_1+\mathbf{g}}-E_{\mathbf{p}_2}+E_{\mathbf{p}_3}-E_{\mathbf{p}_4})$$
$$\times[F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,t,0)$$
$$+F_\mathbf{k}(\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3-\mathbf{g},\mathbf{p}_4,t,0)-F_\mathbf{k}(\mathbf{p}_1+\mathbf{g},\mathbf{p}_2+\mathbf{g},\mathbf{p}_3,\mathbf{p}_4,t,0)$$
$$-F_\mathbf{k}(\mathbf{p}_1+\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4+\mathbf{g},t,0)]+\delta_-(E_{\mathbf{p}_1}-E_{\mathbf{p}_2}+E_{\mathbf{p}_3+\mathbf{g}}-E_{\mathbf{p}_4})$$
$$\times[F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,t,0)+F_\mathbf{k}(\mathbf{p}_1-\mathbf{g},\mathbf{p}_2,\mathbf{p}_3+\mathbf{g},\mathbf{p}_4,t,0)$$
$$-F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2+\mathbf{g},\mathbf{p}_3+\mathbf{g},\mathbf{p}_4,t,0)-F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3+\mathbf{g},\mathbf{p}_4+\mathbf{g},t,0)]$$
$$+\delta_-(E_{\mathbf{p}_1}-E_{\mathbf{p}_2-\mathbf{g}}+E_{\mathbf{p}_3}-E_{\mathbf{p}_4})$$
$$\times[F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,t,0)+F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4+\mathbf{g},t,0)$$
$$-F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2-\mathbf{g},\mathbf{p}_3-\mathbf{g},\mathbf{p}_4,t,0)-F_\mathbf{k}(\mathbf{p}_1-\mathbf{g},\mathbf{p}_2-\mathbf{g},\mathbf{p}_3,\mathbf{p}_4,t,0)]$$
$$+\delta_-(E_{\mathbf{p}_1}-E_{\mathbf{p}_2}+E_{\mathbf{p}_3}-E_{\mathbf{p}_4-\mathbf{g}})$$
$$\times[F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4,t,0)+F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2+\mathbf{g},\mathbf{p}_3,\mathbf{p}_4-\mathbf{g},t,0)$$
$$-F_\mathbf{k}(\mathbf{p}_1-\mathbf{g},\mathbf{p}_2,\mathbf{p}_3,\mathbf{p}_4-\mathbf{g},t,0)-F_\mathbf{k}(\mathbf{p}_1,\mathbf{p}_2,\mathbf{p}_3-\mathbf{g},\mathbf{p}_4-\mathbf{g},t,0)]\}, \qquad (28)$$

Here $\delta_-(x)$ is a one-sided δ-function,[15] $\mathbf{p}_{1,2}=\mathbf{p}\mp\mathbf{k}/2$, and $\mathbf{p}_{3,4}=\mathbf{p}'\pm\mathbf{k}/2$.

Equations (27) and (28) describe the kinetics of the emission of bremsstrahlung photons in a medium both in the definitely classical case ($E \gg \omega$) and in the quantum-mechanical case, with $E\sim\omega$. In this case the function $F_\mathbf{k}(p,p',t,\tau)$ also depends on the spin variables $\lambda_1, \lambda_2, \lambda_3$ and λ_4.

Taking the classical limit $E\gg\omega$, and expanding the functions $F_\mathbf{k}(\mathbf{p}_1\pm\mathbf{g},\mathbf{p}_2\pm\mathbf{g},\mathbf{p}_3\pm\mathbf{g},\mathbf{p}_4\pm\mathbf{g},t,\tau)$ on the right sides of Eqs. (27) and (28) in the small quantity $|\mathbf{g}|\ll|\mathbf{p}_0|$ (p_0 is the momentum of the particle as it enters the medium), we find the following result in the small-angle diffusion approximation:[13]

$$\frac{\partial F_\mathbf{k}(\mathbf{v},\mathbf{v}',t,\tau)}{\partial\tau}-i\mathbf{k}\mathbf{v}F_\mathbf{k}(\mathbf{v},\mathbf{v}',t,\tau)$$
$$=\frac{q}{4}\frac{\partial^2}{\partial\boldsymbol{\eta}^2}\{F_\mathbf{k}(\mathbf{v},\mathbf{v}',t,\tau)\}, \qquad (29)$$
$$\frac{\partial F_\mathbf{k}(\mathbf{v},\mathbf{v}',t,\tau)}{\partial\tau}-i\mathbf{k}(\mathbf{v}-\mathbf{v}')F_\mathbf{k}(\mathbf{v},\mathbf{v}',t,\tau)$$
$$=\frac{q}{4}\left[\frac{\partial}{\partial\boldsymbol{\eta}}+\frac{\partial}{\partial\boldsymbol{\xi}}\right]^2F_\mathbf{k}(\mathbf{v},\mathbf{v}',t,0), \qquad (30)$$

Here $\mathbf{v}=\mathbf{p}/E$, $\mathbf{v}'=\mathbf{p}'/E$, $q=2n_0p_0^2\Sigma_\mathbf{g}|U(\mathbf{g})|^2\times\delta(E_\mathbf{p}-E_{\mathbf{p}+\mathbf{g}})\mathbf{g}^2$ is the mean square multiple-scattering angle per unit path length,[13] and the angular vectors η and ξ, in which we have $\mathbf{v}_\mu\equiv\mathbf{v}$, $\mathbf{v}_\nu\equiv\mathbf{v}'$, are given by (7). In deriving Eq. (29) we noted that the terms proportional to correlation functions of the type $\langle V(\mathbf{g})F_{\mathbf{p}_1\mathbf{p}_2\mathbf{p}_3\mathbf{p}_4}(t)\rangle$ [see (27)] have been discarded, since they are small quantities on the order of $\tau/t\ll1$ with respect to the other terms in the equation.

1) The function $F_\mathbf{k}(\mathbf{v}_\mu,\mathbf{v}_\nu,t,\tau)$ is related to the standard two-time distribution function[12] by a Fourier transformation in the coordinates of the particles. In the case $\mu=\nu$ the function $F_\mathbf{k}(\mathbf{v}_\mu,\mathbf{v}_\nu,t,\tau)$ is the Fourier component of a conditional probability density.

2) In deriving (12) we made use of the relations $t\sim T\gg\tau_0$ (where τ_0 is the time scale for the formation of a bremsstrahlung photon in the medium), which follows from the very formulation of this problem of the radiation by a charged particle in a medium. If $\omega\lesssim q\xi^{-4}$, then $\tau_0=(q\omega)^{-1/2}$, while if $\omega\gg q\xi^{-4}$ the quantity τ_0 is $\omega^{-1}\xi^{-2}$.

3) In general, $d\varepsilon_\omega/d\omega$ is an oscillatory function of the frequency ω, but the value of the radiation energy at the extremum is small [see (24)] in comparison with the background created by the system of N independent radiators.

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