

# Temporal evolution of atomic populations in three-level systems

E. A. Korsunskii, B. G. Matisov, and Yu. V. Rozhdestvenskii

*St. Petersburg State Technical University*

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The temporal evolution of the atomic populations in a three-level system is investigated under coherent dragging conditions. An analytic expression is obtained for the time dependences of the population and the intensities at which coherent dragging can be observed is determined. The role of spontaneous relaxation in the establishment of the effect is also shown.

## 1. INTRODUCTION

The nonstationary response of a medium to the resonant action of a laser field is one of the subjects of nonlinear optics.<sup>1</sup> The character of the temporal evolution of quantum systems interacting with electromagnetic fields is in itself of interest because it presents the most complete picture of the interaction processes. In addition, nonstationary phenomena such as nutation and free-polarization damping or photon echo are exact and convenient experimental spectroscopy methods.<sup>1</sup> The two-level atom model usually suffices to describe these phenomena. By using a multilevel model (three-level in the simplest case), however, it is usually possible not only to generalize known results, but also to observe new phenomena such as quantum beats, Raman scattering, and coherent dragging of populations.

In general there are two, usually on a par, approaches to the study of nonstationary processes—investigation of the time dependence of the complex susceptibility of the medium (of the absorption coefficient) and of the temporal variation of the atom-level populations in the medium.

The time-dependent dynamics of populations in three-level systems was investigated earlier in Refs. 2–6. One of the early studies<sup>4</sup> dealt with the dynamics of the populations in a cascade system of levels under conditions of two-photon resonance. A numerical analysis has shown that such a two-level system is subject to temporal population oscillations of the Rabi-flopping type. We note also that in Ref. 4 the temporal evolution of the populations was investigated for comparable decay rates  $\gamma$ , ( $\gamma_2$  and  $\gamma_1$  are respectively the rates of decay from the ground to the intermediate level and from the intermediate to the ground level; this is precisely why the populations become equalized when steady state is reached in the system. A cascade level scheme was also considered in Refs. 5 and 6, but with decay of the upper level to unobservable states in a continuous spectrum (ionization). The analytic and numerical solutions in Refs. 5 and 6 point to damped oscillations of the populations, with a period determined by the Rabi frequency and with a damping rate that depends on the rate of loss to ionization.

We report here an investigation of the evolution of the atomic populations in three-level systems with coherent dragging of the populations (CDP). The CDP effect consists physically of formation, under conditions of two-photon resonance, of a coherent superpositional two-level state that captures (drags) practically the entire system population. The general level is then practically unpopulated and it is impossible to increase its degree of population despite the presence of strong resonant fields.<sup>7</sup>

The first and actually the only investigation of population dynamics in the establishment of CDP are reported in Refs. 8 and 9, analyzing a three-level systems with population decay from an intermediate level out of the system. It is shown there that when CDP sets in the level-population probabilities undergo damped Rabi oscillations. When the fields are detuned from two-level resonance, the evolution of the populations is more complicated and reflects interference of two one-photon transitions; there is no population drag in this case. It is noted in Ref. 9 that the fluctuations of the applied laser fields can substantially influence the evolution of the populations for CDP in three-level systems. The coherent dragging vanishes in the presence of fluctuations, but it can be fully restored if the field fluctuations are cross-correlated positively in a  $\Lambda$  system and negatively in a cascade system.<sup>10</sup> The subsequent research was aimed at finding singular CDP states that are not acted upon by light fields when the two-photon resonance condition is met.<sup>11,12</sup> Thus, Ref. 12 deals with the behavior of a nonrelaxing  $V$  system under CDP conditions. It turns out that a system can actually have a state from which it is not excited at all, and the populations are independent of time. Similar singular states were obtained also in Ref. 13 for interaction between light waves with different polarizations and a two-level system that is degenerate in angular momentum in the ground as well as in excited states.

We have investigated the dynamics of atomic populations when CDP sets in a  $\Lambda$  or a cascade ( $\Xi$ ) system interacting with stationary light fields. We use first simple solutions of the Schrödinger equation for three-level systems, with an aim at revealing the main distinguishing features of the excitation. We show that it is possible to point from the outset to the initial conditions under which there is no population of the common level at all. The evolution of the populations is next considered on the basis of solutions of the equations for the density matrix of a three-level system. It becomes possible then to determine the conditions that the light-field intensities must satisfy in order for the common level to be abruptly depopulated (coherent dragging) in both  $\Lambda$  and  $\Xi$  systems. This is of principal significance for the understanding of the coherent dragging process itself. We determine the role of spontaneous relaxation in the establishment of the CDP state.

In the Conclusion we discuss the operation of optical modulators based on CDP. The population dynamics in the operation of these modulators is calculated in real time. It is shown the operating speed of such systems is determined by the rate of spontaneous decay of the common level.

## 2. FEATURES OF EXCITATION OF THREE-LEVEL QUANTUM SYSTEMS

We consider the main features of the excitation of three-level system using as examples  $\Lambda$  and  $\Xi$  systems interacting with classical field

$$\mathbf{E} = E_1 \mathbf{e}_1 \exp(i\omega_1 t) + E_2 \mathbf{e}_2 \exp(i\omega_2 t) + \text{c.c.}, \quad (1)$$

where  $E_m$  and  $\omega_m$  are the amplitudes and frequencies of the optical field, and  $\mathbf{e}_m$  are the polarization unit vectors.

Let us calculate the probability of observing the system on the common level  $|3\rangle$  (Fig. 1). We write for this purpose the system wave function in the form

$$\Psi = \sum_{m=1}^3 a_m(t) \psi_m(\mathbf{r}) \exp\left(-\frac{i}{\hbar} E_m t\right), \quad (2)$$

where  $\psi_m$  are the wave functions of the system stationary states, and  $|a_m|^2$  is the time-dependent probability of populating the level  $m$ . Substituting next Eq. (2) in the nonstationary Schrödinger equation and, considering the case of exact resonance between the waves with frequencies  $\omega_m$  and transitions with frequencies  $\omega_{3m}$  ( $m = 1, 2$ ):

$$\Omega_m = \omega_m - \omega_{3m} = 0 \quad (3)$$

(i.e., the case of zero detunings  $\Omega_m$ ), we obtain a set of equations for the probability amplitudes  $a_m(t)$  of the  $\Lambda$  system:

$$\begin{aligned} \dot{a}_1 - ig^* a_3 &= 0, \\ \dot{a}_2 - ig^* a_3 &= 0, \end{aligned} \quad (4a)$$

$$\dot{a}_3 - ig(a_1 + a_2) = -\gamma_0 a_3,$$

and of the  $\Xi$  system

$$\begin{aligned} \dot{a}_1 - ig^* a_3 &= 0, \\ \dot{a}_2 - ig a_3 &= -2\gamma_2 a_2, \end{aligned} \quad (4b)$$

$$\dot{a}_3 - ig a_1 - ig^* a_2 = -2\gamma_1 a_3.$$

We assume here that the excited levels of the  $\Lambda$  and  $\Xi$  systems decay to unobservable levels outside the system (Fig. 1);  $\gamma_m$  ( $m = 0, 1, 2$ ) are the corresponding spontaneous-decay rates, and  $g = dE/2\hbar$  is the Rabi frequency, which is the same for both optical transitions.

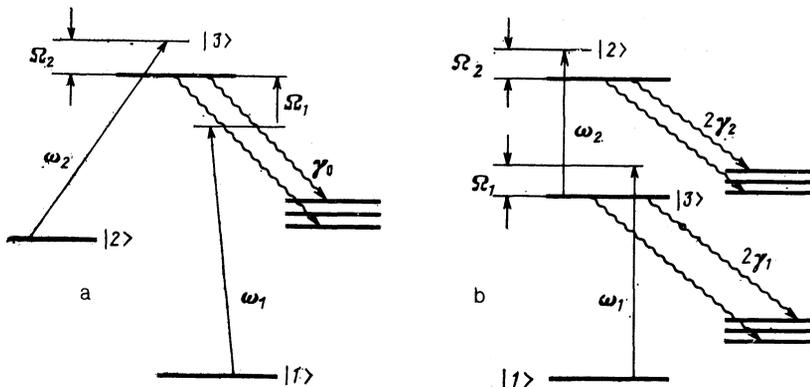


FIG. 1. Types of three-level systems considered in Sec. 2: (a)  $\Lambda$  system, b)  $\Xi$  (cascade) system. The wavy lines with arrows denote spontaneous decay to unobservable levels.

We write now the solutions of Eqs. (4) under the natural initial conditions

$$a_1 = C_1, a_2 = C_2, a_3 = 0 \quad \text{for } t=0. \quad (5)$$

For the  $\Lambda$  system, the probability of observation in the upper state is then

$$|a_3|^2 = \begin{cases} \frac{1}{2}(C_1 + C_2)^2 \exp(-\gamma_0 t) (1 - \cos^2 x), & g \gg \gamma_0, \\ (g^2/2\gamma_0^2) (C_1 + C_2)^2 \exp(-\gamma_0 t) (\text{ch } 2\gamma_0 t - 1), & g \ll \gamma_0, \end{cases} \quad (6a)$$

$$(6b)$$

and for the  $\Xi$  system the probability of populating the intermediate level in the  $\gamma_2 \ll \gamma_1$  is

$$|a_3|^2 = \begin{cases} \frac{(C_1 + C_2)^2}{g^2} \exp\left(-\frac{4}{3} \gamma_1 t\right) \\ \times \left[ \frac{2}{9} \gamma_1^2 (1 - \cos x)^2 + \frac{g^2}{2} \sin^2 x \right. \\ \left. - \frac{2^{1/2}}{12} g \gamma_1 (2 \sin x - \sin 2x) \right], & g \gg \gamma_1, \\ \frac{g^2 (C_1 + C_2)^2}{4\gamma_1^2} [\exp(-4\gamma_1 t) - \exp(-27\gamma_2 t)]^2, & g \ll \gamma_1, \end{cases} \quad (7a)$$

$$(7b)$$

where  $x = 2^{1/2} gt$ .

The solutions (6) and (7) describe completely the common-level population dynamics in  $\Lambda$  and  $\Xi$  systems under initial conditions (5). It is clear directly from (6) and (7) that under the initial conditions  $C_1 = -C_2$  for  $a_{1,2}$  the intermediate level is not populated at all, regardless of the Rabi frequency and of the decay rate  $\gamma_m$ . The initial populations remain unchanged,  $|a_1|^2 = |a_2|^2$ ,  $|a_3|^2 = 0$ , attesting to the existence of a system state in which it no longer "feels" the influence of the resonant fields (1).

At initial values  $C_1 \neq -C_2$  the temporal evolution of the population of level  $|3\rangle$  is via damped oscillations whose period is determined by the Rabi frequency, and  $|a_3|^2 \rightarrow 0$  as  $t \rightarrow \infty$ . The rate at which this state is reached is determined by  $\gamma_m$ , as in Refs. 8 and 9. A distinction must be made here between the high (6) and low (7) Rabi frequencies. Whereas in the former case periodic damped oscillations exist, in the second the population damping  $|a_3|^2$  is aperiodic.

We note here two other circumstances: first—the character of the dynamics remains in principle constant also in the case of detuning from resonance. All that is required here is satisfaction of the two-photon resonance condition, given in our case for the  $\Lambda$  system by

$$\Omega_1 - \Omega_2 = 0 \quad (8a)$$

and for the  $\Xi$  system by

$$\Omega_1 + \Omega_2 = 0. \quad (8b)$$

The second circumstance is that for a three-level atom under conditions (8) part of the atom population is always trapped by the extreme levels, so that the population remains in the system regardless of the decays outside the system. This differs substantially from the analogous treatment for a two-level atom (see Ref. 1, p. 55).

### 3. POPULATION DYNAMICS IN A SYSTEM

We continue a separate investigation of the onset of coherent dragging in the  $\Lambda$  system. We use for this purpose the density-matrix formalism, in which the fullest account can be taken of relaxation processes. As before, we are interested here in the population of level  $|3\rangle$ , i.e., the diagonal element  $\rho_{33}$  of the density matrix.

The field with which the  $\Lambda$  system interacts will be specified in the form (1) and, assuming that the transition  $|1\rangle - |2\rangle$  is dipole-forbidden, we write the equations for the elements  $\rho_{ik}$  of the density matrix:<sup>14</sup>

$$\begin{aligned} i\dot{\rho}_{11} &= -g_1 \exp(i\Omega_1 t) \rho_{31} + \text{c.c.} + 2i\gamma_1 \rho_{33}, \\ i\dot{\rho}_{22} &= -g_2 \exp(i\Omega_2 t) \rho_{32} + \text{c.c.} + 2i\gamma_2 \rho_{33}, \\ i\dot{\rho}_{33} &= g_1 \exp(i\Omega_1 t) \rho_{31} + g_2 \exp(i\Omega_2 t) \rho_{32} - \text{c.c.} - 2i\gamma \rho_{33}, \\ i\dot{\rho}_{13} &= -g_1 \exp(i\Omega_1 t) (\rho_{33} - \rho_{11}) + g_2 \exp(i\Omega_2 t) \rho_{12} - i\gamma \rho_{13}, \\ i\dot{\rho}_{23} &= -g_2 \exp(i\Omega_2 t) (\rho_{33} - \rho_{22}) + g_1 \exp(i\Omega_1 t) \rho_{21} - i\gamma \rho_{23}, \\ i\dot{\rho}_{12} &= -g_1 \exp(i\Omega_1 t) \rho_{32} + g_2 \exp(-i\Omega_2 t) \rho_{13} - i\Gamma \rho_{12}. \end{aligned} \quad (9)$$

Here  $2\gamma_1$  and  $2\gamma_2$  are the partial rates of decay from the level  $|3\rangle$  to the levels  $|1\rangle$  and  $|2\rangle$ ,  $\gamma = \gamma_1 + \gamma_2$ , and  $\Gamma$  is the relaxation rate of the low-frequency coherence  $\rho_{12}$ .

In the case of exact resonance (3),  $\gamma_1 = \gamma_2$ ,  $g_1 = g_2 = g$  and  $\Gamma \ll \gamma$  we have for initial conditions at  $t = 0$ :

$$\begin{aligned} \rho_{ik} &= 0 \quad (i \neq k = 1, 2, 3), \quad \rho_{mm} = 1/2 \quad (m = 1, 2), \\ \rho_{33} &= 0, \end{aligned} \quad (10)$$

and the expression for  $\rho_{33}$  from (9) is

$$\rho_{33} = \begin{cases} \frac{\Gamma}{2\gamma} (1 - e^{-2\gamma t}) + \frac{2g}{\gamma} e^{-2\gamma t} \left[ 2^{1/2} \left( \frac{9}{4} + \frac{2g^2}{\gamma^2} \right) \right]^{-1} & (11a) \\ \times \left[ 2^{1/2} \frac{g}{\gamma} + e^{-\gamma t/2} \left( \frac{1}{2} \sin x - 2^{1/2} \frac{g}{\gamma} \cos x \right) \right] & g^2 \gg \Gamma\gamma, \\ (g^2/\gamma^2) (1 + e^{-2\gamma t} - 2e^{-3\gamma t}), & g^2 \ll \Gamma\gamma. \end{cases} \quad (11b)$$

It is evident from (11) that the behavior of the population  $\rho_{33}(t)$  in a  $\Lambda$  system depends both on the intensity of the light-waves and on the relaxation rates  $\Gamma$  and  $\gamma$ . The character of the upper-level population depends on the ratio of  $g^2$  and  $\Gamma\gamma$ . Let us compare (a) with the solution (6a) obtained using the state-vector formalism. It is possible here to estimate the extent to which this approach is not rigorous. Thus, for example, at high Rabi frequencies the population of the third level in (6a) tends in the stationary state not to the constant value  $\Gamma/2\gamma$ , but to zero, owing to the impossibility of taking transverse relaxation into account in the state-amplitude formalism.

We emphasize that allowance for low-frequency coherence relaxation  $\rho_{1,2}$  in Eqs. (9) is essential, since on the one hand  $\Gamma$  determines the magnitude of the CDP effect (the population of the level  $|3\rangle$ ), and on the other it identifies the light-wave intensities for which the effect itself is possible. Usually  $\Gamma$  is determined by the degree of stabilization of the optical fields (1), and is rarely lower than  $10^5$  Hz. To observe CDP we need therefore fields with Rabi frequencies  $g > (\Gamma\gamma)^{1/2} \sim 10^6 \text{ s}^{-1}$  at  $\gamma \sim 10^7 \text{ s}^{-1}$ , which corresponds to optical transitions, for example, in alkali-metal atoms.

Note that equations similar to (9) were numerically analyzed in Refs. 8 and 9 for different cases of stochastic exciting fields.

Having the solutions (6) and (11), it is interesting to assess the role of spontaneous relaxation when coherent dragging sets in. If spontaneous relaxation exists in the system, a transition from an upper to a lower state transforms the system into a static mixture of pure lower-level states. It is then always probable that the system lands in a coherent state defined by the condition  $a_1 = -a_2$ , after which all excitation ceases. If the  $\Lambda$  system does not land in a coherent state as a result of one spontaneous decay, it is again excited to an upper state until the next spontaneous decay results in a coherent superposition of lower states. It is just such "dropping" of the atoms out of interaction with the field at every new decay which lead to damped dependences of the type (6a) and (11a). At the end of the settling process, the system reaches a state with

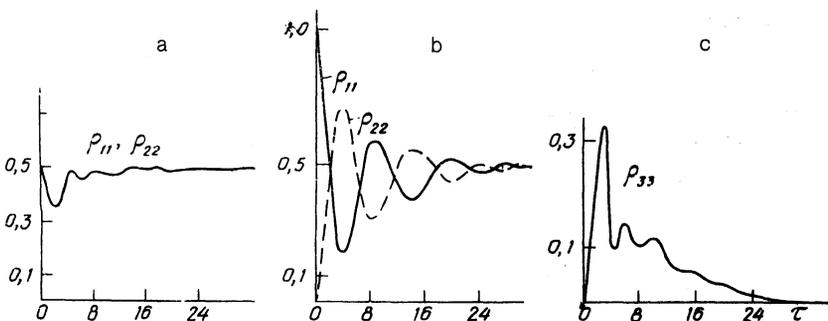


FIG. 2. Temporal evolution of populations in  $\Lambda$  system under coherent-dragging conditions ( $\Omega_1 = \Omega_2 = 0$ ) at  $g = 2\gamma$ ,  $\gamma = 2 \cdot 10^7 \text{ s}^{-1}$ ,  $\Gamma = 10^3 \text{ s}^{-1}$ ,  $\tau = 64 \gamma t$ : a) plots of  $\rho_{11}(t)$  and  $\rho_{22}(t)$  for initial conditions  $\rho_{11}(0) = \rho_{22}(0) = 0.5$ ,  $\rho_{12}(0) = 0$ ; b) the same but for initial conditions  $\rho_{11}(0) = 1.0$ ,  $\rho_{22}(0) = 0$ ,  $\rho_{12}(0) = 0$ ; c) plot of  $\rho_{33}(t)$  (the same for the initial conditions a) and b)).

$$\rho_{mm} \approx 1/2 \quad (m=1, 2), \quad \rho_{13} = \rho_{23} = 0, \quad \rho_{33} \approx 0; \quad \rho_{12} \approx -1/2,$$

which really indicates the existence of a coherent superposition of lower levels ( $\rho_{12} \neq 0$ ) for CDP.

Figures 2 and 3 show the dependences of the populations in a system on the observation time. The plots of Figs. 2 and 3 correspond to the exact-resonance condition (3) and to high Rabi frequencies  $g^2 \gg \Gamma\gamma$ . Interestingly, various initial conditions influence strongly the onset of a population  $\rho_{mm}$  ( $m=1, 2$ ) (Figs. 2a and 2b) but not of  $\rho_{33}$  (Fig. 2c). If the two-photon resonance condition (8) is not met, complicated  $\rho_{33}(t)$  oscillations are observed and attest to interference of two one-photon transitions.

#### 4. POPULATION DYNAMICS IN $\Xi$ SYSTEM

We consider now the onset of coherent dragging in a  $\Xi$  system of levels interacting with the field (1). The equations for the atomic density-matrix elements are<sup>15</sup>

$$\begin{aligned} i\dot{\rho}_{11} &= -g_1 \exp(i\Omega_1 t) \rho_{31} + \text{c.c.} + 2i\gamma_1 \rho_{33}, \\ i\dot{\rho}_{22} &= g_2 \exp(i\Omega_2 t) \rho_{23} + \text{c.c.} - 2i\gamma_2 \rho_{22}, \\ i\dot{\rho}_{33} &= g_1 \exp(i\Omega_1 t) \rho_{31} - g_2 \exp(i\Omega_2 t) \rho_{23} - \text{c.c.} - 2i\gamma_1 \rho_{33} + 2i\gamma_2 \rho_{22}, \\ i\dot{\rho}_{13} &= -g_1 \exp(i\Omega_1 t) (\rho_{33} - \rho_{11}) + g_2 \exp(i\Omega_2 t) \rho_{12} - i\gamma_1 \rho_{13}, \\ i\dot{\rho}_{32} &= g_2 \exp(i\Omega_2 t) (\rho_{22} - \rho_{33}) + g_1 \exp(i\Omega_1 t) \rho_{12} - i(\gamma_1 + \gamma_2) \rho_{32}, \\ i\dot{\rho}_{12} &= -g_1 \exp(i\Omega_1 t) \rho_{32} - g_2 \exp(i\Omega_2 t) \rho_{13} - i\gamma_2 \rho_{12}. \end{aligned} \quad (12)$$

Here  $2\gamma_1$  is the rate of decay from  $|3\rangle$  to level  $|1\rangle$  and  $2\gamma_2$  is the rate of decay from  $|2\rangle$  to  $|3\rangle$ .

Consider first the stationary solution of the system (12) in the case of exact resonance (3) ( $g_1 = g_2 \equiv g$ ):

$$\rho_{33}(\infty) = g^2 \gamma_2 / [\gamma_2 (\gamma_1^2 - 2g^2) + g^2 \gamma_1]. \quad (13)$$

As seen from (13), only under the conditions  $\gamma_2 \ll \gamma_1$  and  $g^2 \gg \gamma_1 \gamma_2$  does  $\rho_{33}$  become small and independent of the intensities of the applied fields, thus attesting to the onset of CDP in the system. For  $\gamma_2 \gtrsim \gamma_1$  and the populations in the system become equalized at laser intensities  $g^2 \gg \gamma_1 \gamma_2$ .<sup>4</sup>

We present therefore the solution of the system (12) under the condition  $\gamma_2 \ll \gamma_1$ , for equal Rabi frequencies

$g_i = g$  ( $i=1, 2$ ) and in the case of the resonance (3). The probability of observing the system in the intermediate state  $|3\rangle$  is then

$$\rho_{33} = \begin{cases} \frac{\gamma_2 + \exp(-2\gamma_1 t)}{\gamma_1} \left[ \frac{1}{4} (1 - \cos x) - \frac{\gamma_2}{\gamma_1} \left( 1 + \frac{2^{1/3} \gamma_1}{4g} \sin x \right) \right], & g^2 \gg \gamma_1 \gamma_2 \\ \frac{g^2}{\gamma_1^2} \left( 1 + \exp(-\gamma_1 t) - 2 \exp\left(-\frac{\gamma_1}{2} t\right) \right), & g^2 \ll \gamma_1 \gamma_2, \end{cases} \quad (14a)$$

$$(14b)$$

where, as before,  $x = 2^{1/2} gt$ .

Evidently, the character of the temporal evolution of the cascade-scheme intermediate-level population is close to the behavior of  $\rho_{33}(t)$  for the  $\Lambda$  system. The decisive role is played here, however, by the ratio of the Rabi frequencies of the applied fields to the rates of spontaneous relaxation in the system. At high Rabi frequencies (14a),  $\rho_{33}$  oscillates and approaches as  $t \rightarrow \infty$  the value  $\gamma_2/\gamma_1$ , which is independent of the field intensities and is an intrinsic property of the given quantum system. In the case of low Rabi frequencies (14b), the population of the level  $|3\rangle$  does not oscillate and in the stationary state we have  $\rho_{33} = g^2/\gamma_1^2$ , i.e., it is proportional to the intensity of the fields interacting with the atom. While the system is then populated, albeit weakly, on the level  $|3\rangle$  (here  $g^2 \ll \gamma_1 \gamma_2$ ), no CDP sets in, since  $\rho_{32} \propto g^2$ , meaning that the system remains field-sensitive as before. The character of the temporal evolution of the populations  $\rho_{ii}$  ( $i=1, 2, 3$ ) is illustrated in Fig. 4, which shows plots of the numerical solution of the system (12) under conditions corresponding to those for which the analytic solutions (14a) have been derived.

The presence of field detuning from the exact resonance (3) leaves the time behavior of the cascade system practically unchanged. If, however, condition (8b) is not met, no CDP takes place despite the sufficient laser intensity, the intermediate level population reaches values comparable with  $\rho_{11}$ , and the upper level is populated weakly. Thus, at  $\Omega_1 = 0$  and  $\Omega_2 \neq 0$  (Fig. 5) the cascade system behaves as a two level system  $|1\rangle - |2\rangle$  that is weakly coupled to the upper level  $|2\rangle$ —its populations oscillates between the levels  $|1\rangle$  and  $|3\rangle$ .

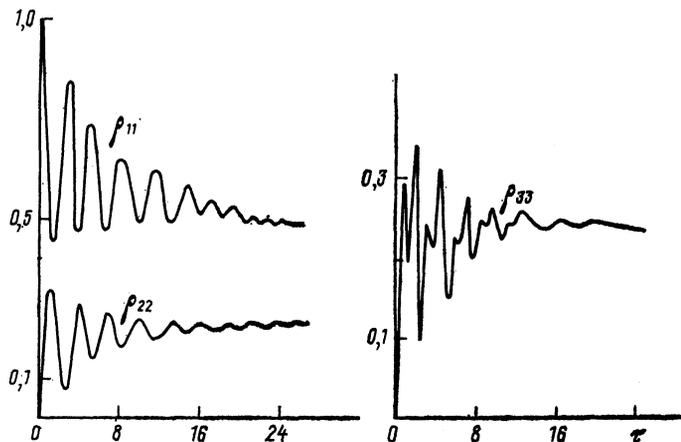


FIG. 3. Temporal evolution of population in a  $\Lambda$  system in the absence of coherent-dragging conditions,  $\Omega_1 = 5\gamma$ ,  $\Omega_2 = 0$ ,  $g = 2.5\gamma$  (the remaining parameters are the same as in Fig. 2).

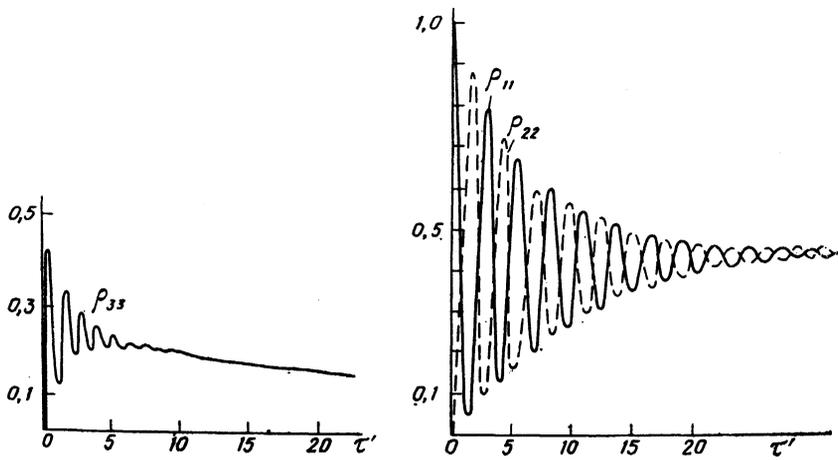


FIG. 4. Temporal evolution of populations in  $\Xi$  system under exact-resonance conditions (3)  $\Omega_1 = \Omega_2 = 0$  at  $\tau' = 2 \gamma_1 t$ ,  $\gamma_2 = 0.1 \gamma_1$ ,  $g = 20 \gamma_1$ ,  $\gamma_1 = 5 \cdot 10^6 \text{ s}^{-1}$  under the initial conditions  $\rho_{11}(0) = 1, \rho_{22}(0) = 0, \rho_{12}(0) = 0$ .

### 5. OPTICAL MODULATORS BASED ON COHERENT POPULATION DRAGGING

We consider here one application of coherent population dragging, namely, the possibility of developing an effective and rapid optical modulator. Such a modulator can be implemented by using an optically dense three-level medium or by simultaneous passage through the medium of two light beams with frequencies  $\omega_m$  ( $m = 1, 2$ ). It is known<sup>14</sup> that weakly linear absorption in such a medium sets in only under conditions of two-photon resonance, and when this condition is met the medium absorbs the radiation exponentially. Since the width of the absorption dip due to the coherent dragging is defined as

$$\Delta \approx g^2 / \gamma,$$

modulation of the field frequencies at a depth larger than  $\Delta$  can lead to amplitude modulation of the light passing through the medium.

An example of how the modulation of the frequency of one of the fields by a square-wave pulse acts on a  $\Lambda$  system is shown in Fig. 6a. Let the system be initially in a CDP state. At the instant  $t$  the frequency of one field changes by  $g^2 / \gamma$ —the atom goes rapidly out of the CDP state, since it begins to interact with the applied fields and the upper level is popu-

lated (the light is consequently absorbed by the medium). At the instant  $t_2$  the frequencies return to the two-photon-resonance condition and CDP sets in with a characteristic time  $\gamma^{-1}$  which determines in fact the operating speed of such a device ( $\gamma = 10^7 \text{ s}^{-1}$ ).

One can propose a converter of another type, using simultaneously three fields that interact with a three-level  $\Lambda$  system. In Ref. 16 there is considered a nonrelaxing  $\Lambda$  system and it is shown that the dynamics of the populations in such a system depends substantially on the phase of the resonant rf field applied to the lower levels of the  $\Lambda$  system, whereas the two upper transitions are acted upon by optical fields with fixed tunings and phases.

A numerical analysis of the system (9) with a resonant rf field turned on shows that the parameter which determines the population dynamics is the phase difference

$$\Phi = \varphi_2 - \varphi_1 + \varphi_r,$$

where  $\varphi_{1,2}$  are the phases of the optical fields with frequencies  $\omega_{1,2}$ , and  $\varphi_r$  is the phase of rf field of frequency  $\omega_{12}$ . If  $\Phi = \pm \pi/2$  or  $\pm 3\pi/2$ , no population dragging takes place in a coherent superposition of levels  $|1\rangle$  and  $|2\rangle$ . If, on the contrary,  $\Phi = 0, \pm \pi, \dots$ , CDP exists as before. This dependence on the phase  $\Phi$  can be used to control the onset of coherent dragging. Figure 6b shows how an initially established coherent dragging is “destroyed” by a  $\pi/2$  change of

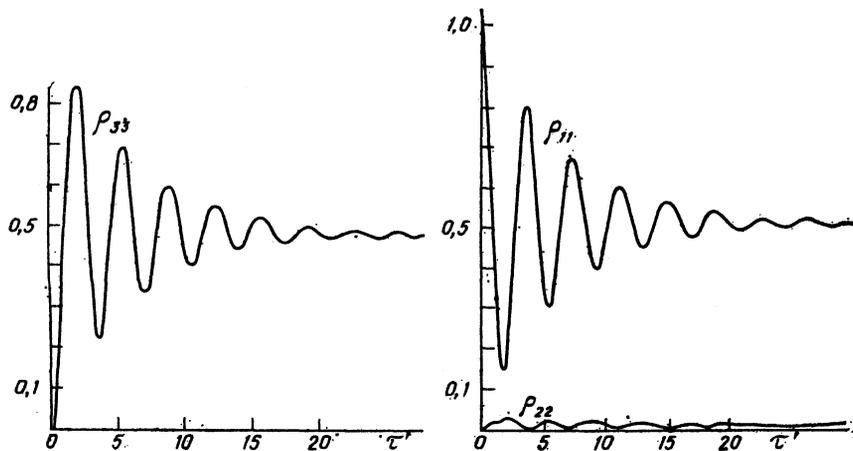


FIG. 5. Temporal evolution of populations in  $\Xi$  system for detunings from exact resonance (3)  $\Omega_1 = 0, \Omega_2 = 60 \gamma_1$  and  $g = 10 \gamma_1$  (the remaining parameters are the same as in Fig. 4).

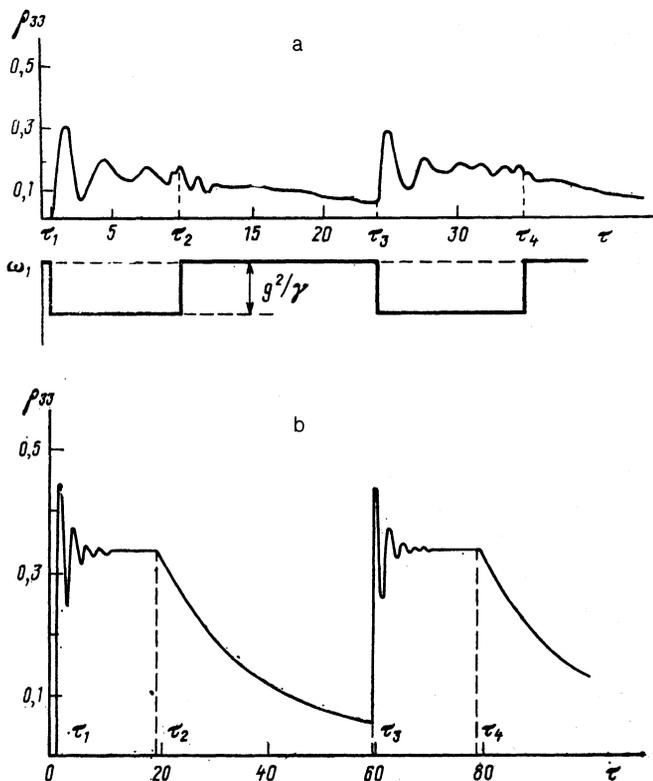


FIG. 6. Time dependence of the upper-level population in a  $\Lambda$  system: a) for light-frequency  $\omega_1$  modulated by square-wave pulses of amplitude  $g^2/\gamma$ , while the light of frequency  $\omega_2$  is at resonance  $\Omega_2 = 0$ , for parameter values  $g = 10 \gamma$ ,  $\gamma = 10^7 \text{ s}^{-1}$ ,  $\Gamma = 0$  and initial conditions  $\rho_{11}(0) = \rho_{22}(0) = 0.5$ ,  $\rho_{12}(0) = -0.5$ ; b) in the case of variation of the total phase of the fields applied to the  $\Lambda$  system. The optical fields are at exact resonance,  $\Omega_1 = \Omega_2 = 0$ , all the Rabi frequencies are equal,  $g_m = 10 \gamma$  ( $m = 1, 2, 3$ ),  $\gamma = 10^7 \text{ s}^{-1}$ ,  $\Gamma = 0$  (the initial conditions are the same as in a)).

the phase  $\Phi$  at the instant of time  $t_1 = \tau_1/2\gamma$ . At the instant  $t_2$  the phase  $\Phi$  is again shifted by  $\pi/2$ , and the coherent dragging state is restored. It is evident from Fig. 6b that the collapse is even faster than the onset. The onset, as noted above, is a damping process in which the atoms "drop" gradually out of the interaction process by being trapped into a coherent superposition of states  $|1\rangle$  and  $|2\rangle$ . The CDP disintegrates because the atom enters into interaction with the applied fields; this interaction sets in over the phase loses the value needed for the CDP.

The described character of the dynamics of a three-level system acted upon by three fields suggests a device in which passage of light is governed by the phases of the field. In this case one can obtain either a phase-amplitude converter with steep switching fronts (if, for example, the laser-field phases are made equal,  $\varphi_1 = \varphi_2$ , the phase of the rf field will control the amplitude of the laser field), or a coincidence system of signals with equal phases (if, for example, the rf phase is

made equal to zero ( $\varphi_r = 0$ ), the CDP sets in, and light passes only at  $\varphi_1 = \varphi_2$ ).

Note that the field phases can be modulated by a variety of methods. This gives grounds for expecting a promising method of controlling the passage of light through a medium.

## 6. CONCLUSION

We conclude by formulating again the main deductions.

1. Analytic expressions (11) and (14) were obtained for the population of the intermediate level of a three-level system. The expressions describe the evolution of quantum system during the onset of CDP. It was shown that the character of the temporal evolution of the atomic populations for  $\Lambda$  and  $\Xi$  systems depends decisively on the ratio of the Rabi frequencies of the applied fields and on the relaxation rates in the system. Thus, under conditions of two-photon resonance (8) in the cases (11a) and (14a) of high Rabi frequencies the populations execute damped oscillations and are almost completely captured in a stationary state on the initial and final levels, while the intermediate level is depleted. The oscillation period is determined by the Rabi frequency, and the damping rate by the rate of spontaneous decay of the intermediate level. In the case of low Rabi frequencies there is no CDP regardless of the satisfaction of the two-photon resonance condition [see solutions (11b) and (14b)].

2. It was found that there exists a linear combination of initial states, in which an atom does not interact with the applied fields, and the level populations do not change at all. This is precisely the state responsible for the CDP. The role of spontaneous relaxation in the establishment of the CDP state is explained.

3. The properties of certain optical devices based on the use of the CDP phenomenon have been investigated. It has been shown that the operating speed of such devices is determined by the decay rate of the common level.

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