Pinning of vortex fluid in high-T_c superconductors

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The vortex fluid in high-T_c superconductors has two temperature ranges with different resistivity vs temperature relations. Pinning is insignificant at high temperatures; the resistivity is equal to ρ_{flow} and manifests a power-law variation with temperature. At low temperatures, the resistivity is proportional to the plastic strain time of the vortex fluid and has an activation-law dependence on temperature. The preexponential multiplier in this law is determined by the pinning force.

1. INTRODUCTION

One of the most interesting properties of high-T_c superconductors is the strong broadening of the resistive transition as the external magnetic field increases.[1,2] The width of the resistive transition in zero magnetic field is ordinarily below 1 K, at the same time that in a field $H \sim 10$ T this width is of the order of 10–20 K for YBaCuO and ~ 40 K for the bismuth superconductors. Such broadening is independent of crystal quality and represents a fundamental property of high-T_c superconductors in the mixed state. The first detailed studies of the resistive transition in a magnetic field[2,3] demonstrated that the resistivity is independent of current and is thermally activated, $\rho \sim \rho_0 \exp(-U_0/T)$, with a characteristic energh U_0 varying from 10^4 K in a magnetic field of $H \approx 10$ T to 10^5 K at a magnetic field $H \approx 0.1$ T for YBaCuO. Subsequent research[4,5] identified a shift of the current-voltage characteristics (CVC) on a certain $T_g(H)$ line on the H–T phase diagram, specifically: the CVC for $T > T_g(H)$ varies linearly with a resistivity, manifesting an exponential temperature depencence; for $T < T_g(H)$ the voltage V has a highly nonlinear dependence on the current.

$$V \propto \exp(-A/j^{\alpha}).$$

This behavior has been attributed[4] to a transition from the vortex fluid state (where no pinning occurs; vortex motion takes the form of viscous flux flow, and the CVC is linear) to the pinned vortex glass state which has a highly nonlinear CVC. The $T_g(H)$ line is close[5] to the so-called "irreversibility line" that can be measured from system responses to a variety of actions (changes in external field; microwave field absorption; mechanical probes, etc.).[6,7]

It is, however, possible to identify two different regions in the "vortex fluid" phase ($T > T_g(H)$) [1–5,8,9]. The resistivity decays rather slowly with temperature down to a certain temperature $T_k > T_g$ ($\rho T_k) \approx (0.1–0.2)\rho_n$, where ρ_n is the resistivity in the normal state. The resistivity decays exponentially between T_k and T_g, although the CVC remains linear: $\rho \sim \rho_0 \exp(-U_0/T)$ where $U_0(T_k) \gg T_k$ (Ref. 9) (this implies $\rho_0 \gg \rho_n$) (Refs. 2,3). Occasionally this transition from one regime to another will manifest itself as an inflection point on the resistivity curves.[5,8]

Melting of the vortex lattice and formation of a vortex fluid has been analyzed in the absence of pinning by a number of authors[10–12] by application of the Lindemann criterion. The high critical temperature of high-T_c superconductors, the large value of the Ginzburg-Landau parameter χ and the strong anisotropy all cause the melting line of the vortex lattice to lie significantly below the $H^0_{c2}(T)$ line calculated in average field theory.

In the absence of pinning, both the vortex lattice and the vortex fluid are driven by the Lorentz force generated by application of electrical current, which produces a flux flow with a resistivity $\rho_{\text{flow}} \approx \rho_n B/H_{c2}$ (Ref. 13). The interaction between the vortices and superconductor defects (pinning) generates energy barriers to vortex motion. Three different cases may arise depending on the size of such barriers: 1) The energy barriers lie below the temperature and can therefore be neglected when $\rho \approx \rho_{\text{flow}}$; 2) the barriers have a characteristic magnitude $U_0 \gg T_0$ which is independent of applied current. This corresponds to the so-called thermally-activated flux flow (TAFF) regime.[14] In this case the resistivity is exponentially small: $\rho \propto \exp(-U_0/T)$ (Ref. 15); 3) the $U(j)$ barriers grow without limit as the current j decreases, while the vortex velocity and the electrical field decay exponentially to zero as the current approaches zero, i.e., the linear resistivity

$$\rho_{lin} = \lim_{j \to 0} \frac{dE(j)}{dj} = 0.$$

Such a state is called the vortex glass state.

It has been argued[16] that a pinned vortex lattice is a vortex glass. The collective creep theory[17] describing the dynamics of the vortex glass resulting from weak disorder predicts that the activation barriers $U(j)$ to lattice motion grow as $U(j) \propto j^{-\alpha}$, which leads to an CVC of the form $V \propto \exp(-A/j^{\alpha})$. The exponent α was calculated for different collective creep regimes.[17]

Vortex hopping over distances shorter than the lattice period has been analyzed in Ref. 17. At high temperatures, strong magnetic fields, and moderately low currents we have $\alpha = 7/9$ in this regime. Hopping over a distance corresponding to the lattice period becomes significant at the lowest currents. Such creep has been analyzed in Ref. 18, which derived a value $\alpha = 1/2$. Both values are in reasonable agreement with experiment.[4]

Elastic strains are considered in the collective creep theory. It has been claimed[7,19] that plastic strains may substantially modify the result. This indeed occurs in the two-dimensional case, when the activation barriers to plastic flow driven by dislocation pair motion are independent of current as $j \to 0$ and hence the linear resistivity is finite at any (nonzero) temperature, and no vortex glass state occurs.[20,21] In the three-dimensional case the dislocation loops lie in the slip plane, cannot convey magnetic flux, and have no effect on the exponent α in $U(j) \propto j^{-\alpha}$ which follows

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from the collective creep theory. There are no infinitely long dislocations at low temperatures, since they have macroscopically large energy. Vortex glass therefore appears to be stable against dislocation formation.

We analyze the effect of disorder on the properties of a vortex fluid in this paper. Some of the results reported here have been presented briefly in Ref. 22. For purposes of simplicity we assume the field H lies along the c axis. We consider disorder with a spatial scale of variability of the random potential that is smaller than the vortex core radius ξ (for example, oxygen vacancies). Weak disorder is assumed.

The present paper shows that weak disorder drives the flux lattice to the vortex glass state with a linear resistivity $\rho_{\rm lin} = 0$. The vortex fluid in the absence of a random potential remains a liquid with a finite $\rho_{\rm lin}$. The vortex fluid contains two temperature regions. At high temperatures $T > T_k$ disorder has no effect on liquid motion and $\rho_{\rm lin} = \rho_{\rm flow}$. With diminishing temperature a transition ensues at $T \sim T_k$ to a partially pinned regime (the TAFF) with a linear resistivity far below $\rho_{\rm flow}$ and one that decays exponentially with temperature. The assumption of weak pinning appears to be more realistic for high-T_c superconductors: The critical current j_c lies far below the depairing current j_0, and recent measurements[23,24] have revealed a significant growth of critical current in irradiated YBaCuO samples, while the position of the irreversibility line remained unchanged.

Two regions with different linear resistivity vs temperature relations have in fact been observed in experiment.[1–5,8,9] This paper will focus on an analysis of the boundary between these regions.

The possibility for vortex fluid pinning by a weak, random potential is not obvious. The naive view is that pinning in general cannot occur in the liquid state: The interaction of the fluxes with an arbitrary potential is much weaker than interflux interaction, and since the latter is relatively small in the liquid state, the arbitrary potential seems quite insignificant. It was therefore determined[19] that the existence of pinning in a vortex fluid above T_g, which is responsible for the exponential decay of resistivity with temperature, is incompatible with the assumption of weak disorder.

We know, on the other hand, that an arbitrary potential will be significant at any temperature to such a linear object as an isolated vortex.[25] It has also been demonstrated that disorder determines vortex motion in a weak field at any temperature.[26] The results of Refs. 25, 26 therefore suggest that arbitrarily large barriers to the motion of an isolated vortex can exist, i.e., a single vortex is always in the vortex glass state. Indeed, if there were to be a finite barrier U_0, an arbitrary potential would have no effect on vortex motion for $T > U_0$. We then have the opposite question: Why are the barriers to vortex fluid motion independent of applied current, at the same time that they grow without limit as $j \to 0$ for individual vortices?

2. QUALITATIVE ANALYSIS

In order to understand the nature of pinning in a vortex fluid, we initially consider the effect of thermal fluctuations on the pinning of an unmelted vortex lattice.[26] Above a certain depinning temperature T_p (an exact expression for this temperature will be provided below) the mean square amplitude of thermal vibrations of the vortex lattice

$$u_{ph} = \langle u^2 \rangle_T^{1/2} \approx \xi (T/T_p)^{1/2}$$

exceeds the core radius ξ while thermal motion of the vortex lines averages the arbitrary interaction potential of the vortex core with the defects over the range $u_{\rm ph}^2$ (the vortices largely interact with the defects by means of their core, where the superconducting order parameter is suppressed). The characteristic averaged scale of the arbitrary potential in this case is approximately $r_j \approx (\xi^2 + u_{\rm ph}^2)^{1/2}$ while the critical current j_c drops off rapidly with increasing temperature.[26,27] In deriving an expression for $j_c(T)$ averaging was initially carried out over the thermal vibrations and only then over the arbitrary potential in Ref. 26. This is a legitimate procedure if the characteristic time of the thermal "phonon" vibrations $t_{\rm ph}$ is much smaller than the characteristic pinning time $t_{\rm pin}$. As demosntrated below, the characteristic pinning time satisfies $t_{\rm pin} \sim r_j/v_c$, where v_c is the viscous flow velocity of the vortex lattice at a current slighly above the critical current: $v_c = j_c$ V/Gs (friction coefficient $\Gamma \approx {\rm BH}_{c2}/\rho_n c^2$ from Ref. 13). Here $t_{\rm ph}$ is determined by the elastic properties of the vortex lattice and is independent of the arbitrary potential. An expression will be derived below for $t_{\rm ph}$; here it turns out that

$$\frac{t_{pin}}{t_{ph}} \propto \frac{j_0}{j_c},$$

where j_0 is the depairing current of the superconductor. We therefore have $t_{\rm pin} \gg t_{\rm ph}$ in the case of weak pinning ($j_c \gg j_0$), and, consequently, the method employed in Ref. 26 has been substantiated. Pinning occurs as a direct result of the inhomogeneous vortex structure. Note here that although the thermal fluctuations serve to substantially smooth over the vortex cores, the vortex lattice will nonetheless retain its periodicity and the interaction between this periodic (i.e., inhomogeneous) structure and the arbitrary potential will produce pinning at a temperature below the melting point T_M.

We now consider the vortex fluid. Note that in an ordinary liquid all characteristic times are quantities of the same order of magnitude, i.e., $\sim t_{\rm ph}$. Therefore by averaging over the thermal fluctuations for a period $t_{\rm pin} \gg t_{\rm ph}$ we obtain a fully smoothed homogeneous vortex structure that cannot be pinned.

This result is radically modified for a highly viscous liquid where there are two time scales $t_{\rm ph}$ and $t_{\rm pl} \gg t_{\rm ph}$ such that the liquid structure is inhomogeneous over times $t < t_{\rm pl}$. If the characteristic smearing time of the structure satisfies $t_{\rm pl} \gg t_{\rm pin}$, thermal averaging will be incomplete over the pinning time $t_{\rm pin}$, while the vortex configuration retains its inhomogeneous structure over such times; such a structure can be effectively pinned by an arbitrary potential.

The exponentially large smearing times of the inhomogeneous structure $t_{\rm pl}$ in the vortex fluid may result from the presence of strong barriers $U_{\rm pl}$ to thermally activated plastic vortex motion. In this case, $t_{\rm pl} \sim t_{\rm ph} \exp(U_{\rm pl}/T)$. The characteristic value of the plastic barriers was estimated in Ref. 27. In the anisotropic case, the characteristic energy takes the form

$$u_{pl} \sim \left(\frac{m}{M}\right)^{1/2} \Phi_0^2 \frac{a}{8\pi^2\lambda^2} \propto \frac{T_c - T}{H^{1/2}}. \quad (1)$$

Here m and M represent the effective masses in the ab plane and along the c axis, respectively; Φ_0 is the flux quantum; $a \approx (\Phi_0/H)^{1/2}$ is the average distance between vortices; and

λ is the London depth of penetration for the $H \| C$ field. The large barriers $U_{\rm pl}$ may also be associated with entanglement of the vortex fluid.[12] Relative flux motion in such a liquid can be initiated either by reptation[12] or by the breakdown and repairing of vortex lines. The latter mechanism would appear to be preferable, since the characteristic relaxation time for the reptations grows very rapidly with increasing sample size L ($\propto L^3$ according to Ref. 12 and even $\propto \exp[(L/a)^6]$ according to Ref. 28). The barriers associated with repairing are also given by Eq. (1) in fields $H \gg H_{c1}$.

Note that the energy (1) is within an order of magnitude of the energy of a vortex segment of length a, rather than ξ as reported in Refs. 12, 28, since in order for the vortices to intersect they must bend and the characteristic scale along c for bending is a quantity of the order of a (in the anisotropic case $\sim (m/M)^{1/2}a$), and here the core interaction energy is a small part ($\sim \xi/a$) of the total energy (1). The energy (1) can be assigned to any plastic strains of the vortex structure with a spatial scale $\sim a$. The Lindemann criterion for melting[11,12] can therefore be written as $T_g \sim 1/2c_L\, U_{\rm pl}$, where $c_L \sim 0.1$–0.2 is the Lindemann constant. The large barriers to vortex motion are retained in a melted liquid given the small value of c_L for $T \sim T_g\, U_{\rm pl} \gg T$.

The structural inhomogeneities in the vortex fluid are significant as long as $t_{\rm pin} \ll t_{\rm pl}$ holds. The characteristic magnitude of the plastic barriers decreases with rising temperature and the transition from the pinned to the unpinned regime occurs at a temperature T_k such that

$$t_{pin} \approx t_{pl} \sim t_{ph} \exp(U_{pl}/T).$$

This transition may appear as an inflection point or a "knee" on the resistivity curves. Note that due to weak pinning we have $t_{\rm pin} \gg t_{\rm ph}$ and, consequently, $U_{\rm pl}(T_k) \gg T_k$, which is in agreement with experiment.[2,3,5,8,9]

3. FUNDAMENTAL EQUATIONS FOR PINNING OF THE VORTEX SYSTEM

We use the dynamic approach developed in Ref. 29 for a vortex lattice in our quantitative analysis of pinning in the vortex system, modifying this approach for the case of vortex motion in the liquid state. In this approach we consider the motion of the vortex structure driven by a constant Lorentz force $\mathbf{j}\times\mathbf{B}/c$ generated by a current $j > j_c$ in the presence of an arbitrary potential which is treated as a perturbation.

The arbitrary potential has little effect for a large current $j \gg j_c$, and the vortices are driven at a velocity $v_0 = jB/c\Gamma$ (Ref. 13); the CVC is linear and the resistivity is $\rho = \rho_{\rm flow}$. The additions to vortex velocity associated with the arbitrary potential can be calculated from perturbation theory as a function of the velocity v_0 (or current j). If these additions are small compared to the velocity, i.e., $\delta v(v_0) \ll v_0$, for any (arbitrarily small) velocities, the CVC will always be linear and have a constant resistivity $\rho_{\rm flow}$ with zero pinning. However, the velocity additions grow as $v_0 \to 0$ for the flux lattice. Pinning becomes significant $\delta v(v_0) \sim v_0$ and $\delta v(v_c) = v_c$ determines the critical current $j_c = v_c \Gamma_c / B$. This corresponds to the onset of nonlinearity on the CVC, $\delta\rho(j_c) \sim \rho_{\rm flow}$. Obviously an arbitrary potential can no longer be analyzed by perturbation theory in the current range $j < j_c$, and this approach becomes inapplicable. The energy of interaction between a vortex and a random potential can be given as

$$U_{pin} = \sum_i \int V(\mathbf{r})\, p(\mathbf{r}_\perp - \mathbf{r}_{\perp i}(z,t)). \tag{2}$$

In this equation $V(\mathbf{r})$ is the "frozen" arbitrary potential with the correlator $\overline{V(\mathbf{r})V(\mathbf{r}')} = \delta(\mathbf{r} - \mathbf{r}')$; here the prime denotes averaging over the inhomogeneities; and $p(\mathbf{r}_\perp)$ describes the interaction of the vortex cores with the arbitrary potential, $p(\mathbf{r}_\perp) \to 0$ for $r_\perp > \xi$. Summation is carried out over all vortices, and the z axis lies in the direction of the magnetic field.

The exact form of the function $p(\mathbf{r}_\perp)$ depends on the microscopic nature of disorder.[29,30] In this case, when $V(\mathbf{r})$ represents the deviation from the average of the effective electron interaction constant, $p(\mathbf{r}_\perp)$ is proportional to the deviation from the average of the square of the absolute value of the order parameter near a vortex in the superconductor. The coordinate $\mathbf{r}_i$ of the ith vortex relative to "frozen" disorder can be given as

$$\mathbf{r}_i = \mathbf{r}_i(z,t) + \mathbf{v}t + \mathbf{u}_{pin,i}(z,t),$$

where $\mathbf{r}_i(z,t)$ is the position of the ith vortex unperturbed by the arbitrary potential (although independent of time due to thermal fluctuations) in the reference system traveling at a constant velocity $\mathbf{v}$ relative to "frozen" disorder, while $\mathbf{u}_{{\rm pin},i}$ is the small correction to the position of the $i^{\rm th}$ vortex attributable to the arbitrary potential. The constant velocity $v = v_0 + \delta v$, where $v_0 = \mathbf{j}\times\mathbf{B}/\Gamma c$ is the velocity of an unpinned liquid driven by the Lorentz force, while $\boldsymbol{\delta}\mathbf{v}$ is the correction to the average velocity attributable to the random potential.

We consider forces acting on the traveling vortices. A Lorentz force $F_L = \mathbf{j}\times\mathbf{B}/c$, a viscous friction force $\mathbf{F}_v = -\Gamma\mathbf{v}$ and a pinning force $\mathbf{f}_{\rm pin} = -\partial U_{\rm pin}/\partial \mathbf{r}_i$ as well as the internal interaction force with other vortices are all acting on the vortex system. The average vortex interaction force is equal to zero and hence (Newton's third law) the force equation can be written as

$$\langle \mathbf{F}_L + \mathbf{F}_v + \mathbf{f}_{pin} \rangle = 0. \tag{3}$$

Here $\langle \ldots \rangle$ denotes both thermal averaging and disorder averaging. Since we have defined the unperturbed velocity as $v_0 = F_L/\Gamma$ we obtain an equation for the velocity correction:

$$\langle \mathbf{f}_{pin} \rangle = + \left\langle \sum_i V(\mathbf{r})\, \nabla p(\mathbf{r}_\perp - \mathbf{r}_{\perp i} - \mathbf{v}t - \mathbf{u}_{pin,i}) \right\rangle. \tag{4}$$

We consider weak pinning when the vortex-defect interaction force is substantially weaker than the vortex-vortex interaction. In this case, u varies slowly from vortex to vortex and the index i can be dropped. Expanding $f_{\rm pin}$ in $u_{\rm pin}$, we have

$$\Gamma \boldsymbol{\delta}\mathbf{v} = -\left\langle \sum_i V(\mathbf{r})\, \nabla^2 p(\mathbf{r}_\perp - \mathbf{r}_{\perp i} - \mathbf{v}t)\, \mathbf{u}_{pin} \right\rangle. \tag{5}$$

The displacement $u_{\rm pin}$ associated with the pinning force can be written in a linear approximation as

$$\mathbf{u}_{pin}(\mathbf{r},t) = \int d\mathbf{r}'\, dt'\, G(\mathbf{r},\mathbf{r}';t,t')\, \mathbf{f}_{pin}(\mathbf{r}',t'), \tag{6}$$

where G is the response function of the vortex system. Substituting $\mathbf{u}_{\text{pin}}$ from Eq. (6) into Eq. (5) and averaging over the arbitrary potential $\overline{(V(\mathbf{r})V(\mathbf{r}')} = \gamma\delta(\mathbf{r}-\mathbf{r}'))$ we obtain

$$\Gamma\delta\mathbf{v} = -\gamma \int G(0,t'-t)dt' \sum_{i,j} \nabla^2 p[\mathbf{r}_\perp - \mathbf{r}_{\perp i}(t) - \mathbf{v}t] \times \nabla p[\mathbf{r}_\perp - \mathbf{r}_{\perp j}(t') - \mathbf{v}t], \tag{7}$$

where $G(0,t'-t) = G(\mathbf{r},\mathbf{r},t,t')$.

Going over to the Fourier components for $\mathbf{p}$ in the plane perpendicular to the z axis, we obtain

$$\Gamma\delta\mathbf{v} = \gamma \int G(0,t'-t) K_\perp^2 i\mathbf{K}_\perp' \, dt' \sum_{i,j} \exp\{i\mathbf{K}_\perp[\mathbf{r}_\perp - \mathbf{r}_{\perp i}(t) - \mathbf{v}t] + i\mathbf{K}_\perp'[\mathbf{r}_\perp - \mathbf{r}_{\perp j}(t') - \mathbf{v}t']\} \frac{d^2K_\perp \, d^2K_\perp'}{(2\pi)^2(2\pi)^2}. \tag{8}$$

Averaging over

$$\mathbf{r}_\perp \langle \exp[i(\mathbf{K}_\perp + \mathbf{K}_\perp')\mathbf{r}_\perp]\rangle = \frac{1}{V_L}\int dz \int d^2r \exp[i(\mathbf{K}_\perp + \mathbf{K}_\perp')\mathbf{r}_\perp'] = \frac{1}{V_L}(2\pi)^2 \delta(\mathbf{K}_\perp + \mathbf{K}_\perp') \int dz$$

(here V_L is the volume of the sample) we obtain

$$\frac{\delta v}{v} = \frac{\gamma}{\Gamma a^2} \int \frac{d^2K \, dt}{(2\pi)^2} K_v K_\perp^2 |p(\mathbf{K}_\perp)|^2 \times G(0,t) S(\mathbf{K}_\perp, t) \frac{\sin(K_v v t)}{v}. \tag{9}$$

Here a is the average distance between vortices; $B = \Phi_0/a^{2'}$, K_v is the component of the vector $\mathbf{K}$ along direction $\mathbf{v}$, while the structure factor $S(\mathbf{K}_\perp, t)$ is defined as

$$S(\mathbf{K}_\perp, t) = \frac{1}{L}\int dz \frac{1}{N}\sum_{i,j} \exp[i\mathbf{K}_\perp(\mathbf{r}_i(z,0) - \mathbf{r}_j(z,t))], \tag{10}$$

where N is the total number of vortices.

Equation (9) is the primary quantitative result of this paper. This equation relates the behavior of the vortex system in an arbitrary potential to the internal (i.e., pinning-independent) parameters: The Green's function $G(0,t)$ and the structure factor $S(\mathbf{K}_\perp,t)$.

4. PINNING OF THE FLUX LATTICE

Equation (9) is general in form and can be applied both to a vortex fluid and to a vortex lattice. The free energy of the vortex lattice as a function of vortex displacement relative to equilibrium in an elastic approximation takes the form[26,30]

$$F = \int d^3r \left[(C_{11} - C_{66}) \frac{(\nabla \mathbf{u})^2}{2} + \frac{1}{2} C_{66} (\nabla_\alpha u_\beta)^2 + \frac{1}{2} C_{44} \left(\frac{\partial \mathbf{u}}{\partial z}\right)^2 + V(\mathbf{r}) \right], \tag{11}$$

where C_{11}, C_{44}, C_{66} are the elastic compression, bending and shear moduli, respectively. Varying the energy (11) we obtain the equation of motion for the lattice:

$$\Gamma \frac{\partial \mathbf{u}}{\partial t} - C_{11} \operatorname{grad} \operatorname{div} \mathbf{u} - C_{66}(\nabla^2 - \operatorname{grad} \operatorname{div})\mathbf{u} - C_{44} \frac{\partial^2 \mathbf{u}}{\partial z^2} = \mathbf{f}. \tag{12}$$

Going over to the Fourier-components we find the Green's function $G(K,\omega)$:

$$G_{\alpha\beta}(K,\omega) = P_L \frac{1}{-i\Gamma\omega + C_{11}K_\perp^2 + C_{44}K_z^2} + P_T \frac{1}{-i\omega\Gamma + C_{66}K_\perp^2 + C_{44}K_z^2}, \tag{13}$$

where $P_L = K_\alpha K_\beta / K_\perp^2$ and $P_T = \delta_{\alpha\beta} - K_\alpha K_\beta / K_\perp^2$ are the two-dimensional ($\alpha,\beta = x,y$) longitudinal and transverse projection operators, respectively. In our fields $H_{c1} \ll H \ll H_{c2}$ the compression modulus C_{11} is far greater than the shear modulus C_{66} and the longitudinal part of the Green's function is small compared to the transverse part. In this section we therefore only consider transverse vibrations to the vortex lattice. The average amplitude of the thermal vibrations is[10,11,26]

$$u_{ph}^2 = \langle u_T^2 \rangle = T\Gamma \int \frac{d\omega \, d^3K}{(2\pi)^4[(\Gamma\omega)^2 + (C_{66}K_\perp^2 + C_{44}K_z^2)^2]} \approx \frac{4\pi^2 T\lambda^2 a}{\Phi_0^2} \left(\frac{M}{m}\right)^{1/2} \tag{14}$$

The integral in Eq. (14) is defined by large $K_\perp$ and breaks off at the boundary of the Brillouin zone $K_\perp \sim K_B \approx \pi/a$. The spatial dispersion of the bending modulus $C_{44}(K)$ becomes significant here. In the anisotropic case, the moduli C_{44} and C_{66} are equal to[10]

$$C_{44} = \frac{B^2}{4\pi} \left(\frac{1}{\lambda^2 K_z^2 + (M/m)\lambda^2 K_\perp^2 + 1} + \frac{\Phi_0}{4\pi B \lambda^2} \frac{m}{M} \right), \quad C_{66} = \frac{\Phi_0 B}{(8\pi\lambda)^2}. \tag{15}$$

The characteristic frequencies used in evaluating the integral (14) have values of the order of $\omega \sim (C_{66}K_\perp^2 + C_{44}K_\perp^2)/\Gamma$. The characteristic time of the short-wavelength lattice vibrations will therefore be

$$t_{ph} \sim \left(\frac{C_{66}K_B^2}{\Gamma}\right)^{-1} = \Gamma \frac{64\lambda^2}{\Phi_0^2} a^4. \tag{16}$$

We now find the Green's function $G(0,t)$ in Eq. (9):

$$G(0;t) = \int G(K,\omega) e^{-i\omega t} \frac{d\omega \, d^3K}{(2\pi)^4} = \frac{\theta(t)}{\Gamma} \int \exp\left[-\frac{C_{66}K_\perp^2 + C_{44}(K)K_z^2}{\Gamma} t\right] \frac{d^3K}{(2\pi)^3}. \tag{17}$$

The Green's function $G(0,t)$ will have a rather complicated time dependence due to the strong spatial dispersion of the bending modulus $C_{44}(K)$. At large times t the momenta K that substantially contribute to integral (17) are small, and the Green's function depends on time as

$$G(0,t) = \frac{\theta(t)}{4\pi^{1/2}\Gamma a^2 \lambda} \left(\frac{t_{ph}}{t}\right)^{1/2} \propto t^{-1/2}, \quad t \gg t_{ph} \frac{\lambda^2}{a^2} \frac{M}{m}. \tag{18}$$

For $t < t_{ph}(\lambda^2/a^2)(M/m)$ dispersion is significant in Eq. (17); we have $C_{44}(K) \propto 1/K^2$ and the Green's function is equal to

$$G(0,t)=\frac{\pi^{1/2}}{4}\frac{\theta(t)}{\Gamma a^3}\left(\frac{M}{m}\right)^{1/2}\left(\frac{t_{ph}}{t}\right)^2 \propto t^{-2},$$
$$t_{ph}<t<t_{ph}\frac{\lambda^2}{a^2}\frac{M}{m}. \tag{19}$$

The finite nature of the limits of integration over $K_\perp$ ($K_\perp < \pi/a$) becomes significant at very small times $t < t_{ph}$ in the integral (17) and hence the properties of the lattice are no longer important. The shear modulus C_{66} drops out, and the bending modulus C_{44} reduces to linear stretching of individual vortices [the second term in Eq. (15)], i.e., the response is determined by the behavior of individual vortices:

$$G(0,t)=\frac{\theta(t)}{2\Gamma a^3}\left(\frac{t_{ph}}{t}\right)^{1/2}\left(\frac{M}{m}\right)^{1/2} \propto t^{-1/2}, \quad t<t_{ph}. \tag{20}$$

We consider a vortex lattice at low temperatures. Then the structure factor $S(\mathbf{K}_\perp,t)$ in Eqs. (9), (10) consists of the sum of the delta-function peaks $\Sigma\delta(\mathbf{K}-\mathbf{K}_n)$ on the reciprocal lattice vectors $\mathbf{K}_n$. Integration over $\mathbf{K}_\perp$ in Eq. (9) is therefore replaced by summation over the reciprocal lattice. For large K, summation is cut off by the "form-factor" of an individual vortex $p(\mathbf{K}) \to 0$ for $K > 1/\xi$, i.e., $K_{max} \sim 1/\xi$.

If we expand the sine ($\sin(K_v vt) \approx K_v vt$) in Eq. (9) for small velocities v, the time integral

$$\frac{\delta v}{v} \propto \int G(0,t)\, t\, dt$$

for the Green's functions from (18)–(20) will diverge at large times. This divergence is cut off due to the multiplier $\sin(K_v vt)$ over times $t \sim 1/K_v v$ and hence the velocity correction $\delta v/v$ grows, when the velocity v tends to zero. Since $K_{max} \sim 1/\xi$, the characteristic times contributing to Eq. (9) will be $\sim \xi/v$. Due to the complicated time dependence of the Green's function $G(0,t)$ in (18)–(20), the dependence of the corrections $\delta v/v$ on velocity v will be different for different values of v: For

$$v < \frac{\xi a^2 M}{t_{ph}\lambda^2 m}$$

the Green's function is determined from Eq. (18) and $\delta v/v \propto v^{-1/2}$ for

$$\frac{\xi}{t_{ph}} > v > \frac{\xi}{t_{ph}}\frac{a^2 M}{\lambda^2 m}$$

the correction satisfies $\delta v/v \propto \ln(l/v)$ and for $v > \xi/t_{ph}$ the correction is determined by the behavior of individual vortices and $\delta v/v \propto v^{-1/2}$. Growth of the ratio $\delta v/v$ as $v \to 0$ suggests that we have a critical velocity (critical current) such that $\delta v(v_c)/v_c = 1$. This condition will hold in different regions with different dependences of $\delta v/v$ on v depending on the value of the arbitrary potential (the parameter γ). For strong disorder, the condition $\delta v(v_c) \approx v_c$ holds for $v_c > \xi/t_{ph}$ (i.e., for $j_c \gg j_0\xi^2/a^2$, where

$$j_0 = \frac{\Phi_0 c}{12\cdot 3^{1/2}\pi\lambda^2\xi}$$

is the depairing current); the pinning is single-particle pinning in this case and the critical current will be independent of the magnetic field. For weaker disorder, the significant times in Eq. (9) yield Green's function (19) and hence $\delta v/v \propto \ln(1/v)$ while the critical current is exponentially dependent on the magnetic field and the disorder force γ (Refs. 30, 26). For even weaker disorder, this regime is replaced and the critical current has a power law dependence on the magnetic field.[30,26]

For $T > 0$ the thermal fluctuations lead to the Debye-Waller factor $\exp(-K^2u_{ph}^2/2)$ in the structure factors $S(K, t)$ over times $t > t_{ph}$. As long as $u_{ph}^2 < \xi^2$ holds, this multiplier will not be signifcant, since integration in Eq. (9) is cut off at the mulitiplier $|p(K)|^2$ of the wave vector $K_{max} \sim 1/\xi$. As follows from Eq. (14), at temperatures above the depinning temperature

$$T_p \approx \frac{\Phi_0^2\xi^2}{4\pi^2\lambda^2 a}\left(\frac{m}{M}\right)^{1/2} \approx \frac{U_{pl}}{2}\frac{\xi^2}{a^2} \tag{21}$$

the mean square amplitude of thermal fluctuations u_{ph} exceeds the core radius ξ, at the same time that the Debye-Waller factor determines that the primary contribution to Eq. (9) derives from the range $K \sim 1/u_{ph}$. This means that u_{ph} is substituted for the effective core radius[26,27] ξ, which results in a strong temperature dependence of the critical current versus temperature relation. The characteristic times contributing to Eq. (9) for $\delta v(v_c) \approx v_c$ will be

$$t_{pin}=u_{ph}/v_c. \tag{22}$$

Substituting u_{ph} from Eq. (14) we can relate t_{pin} to t_{ph}:

$$t_{pin}\approx t_{ph}\frac{j_0}{j_c}\frac{\xi}{a}\left(\frac{T}{U_{pl}}\right)^{1/2}. \tag{23}$$

Equation (23) is provided for temperatures $T > T_p$ and hence for weak collective pinning [which means $j_c < j_0(\xi/a)^2$], we have $t_{pin} \gg t_{ph}$.

One important feature of the crystalline state is that the structure factor $S(K, t)$ is nonzero and does not depend on time for $t \gg t_{ph}$. Thermal fluctuations will therefore not change the fact nor the degree of the divergence of the additions to the velocity $\delta v/v$, and will only modify the coefficient. The condition $\delta v/v \approx 1$ is therefore satisfactory and a critical current will exist at any temperature. This means that arbitrarily weak disorder is significant to a flux lattice at any temperature, and a flux lattice in an arbitrary potential will be in the vortex glass state (although we once again wish to emphasize that if the barriers $U(j)$ associated with pinning remain limited for any current ($U(j) < U_0$), pinning will not be significant at temperatures $T > U_0$).

5. PINNING OF THE FLUX LIQUID

In the vortex-fluid phase the pinning-induced energy barriers to vortex motion are finite (see Sec. 2), i.e., in the limit of weak currents $j \to 0$ the vortex velocity v is proportional to j and has a finite linear conductivity σ. Here σ may be close to σ_{flow} (i.e., $\delta_\sigma = (\sigma - \sigma_{flow})/\sigma_{flow} \ll 1$ and disorder can be analyzed by perturbation theory) or $\sigma \gg \sigma_{flow}$ depending on the disorder force, and the TAFF regime results. In this section we obtain an estimate of δ_σ and establish the conditions for these two regimes to arise. We therefore anticipate that the relative correction $|\delta v/v|^{(0)}$ to the flux velocity is finite in the liquid state as $v \to 0$. When $|\delta v/v|^{(0)}$ is small, it coincides with the relative correction to the conductivity δ_σ.

First we show that the wave vector domain $K_\perp \sim K_0 \approx 2\pi/a$ makes the primary contribution to the integral (9) for $\delta v/v$ in the case of a vortex fluid.

Clearly, the domain $K_\perp \gg K_0 = 2\pi/a$ is not significant in the integral (9), since the structure factor of the liquid $s(\mathbf{K}_\perp, t)$ decays rapidly for $K_\perp > K_0$. A nontrivial contribution could arise from the domain of small $K_\perp \ll K_0$, as in the case of weak (compared to pinning) interaction between vortices. We, however, show that this does not occur. Expression (9) in the limit $v \to 0$ can be written as

$$\delta_\sigma = \left| \frac{\delta v}{v} \right|_{v\to 0} = \frac{\gamma}{\Gamma a^2} \int \frac{d^2 K_\perp}{(2\pi)^2} K_v^2 K_\perp^2 \int_0^\infty G(0,t) S(\mathbf{K}_\perp, t) t\, dt, \tag{24}$$

where the form factor $|p(K)|$ is replaced by unity, since it is essentially only dependent on K for $K \sim 1/\xi \gg K_0$. The condition $K_\perp \ll K_0$ makes possible a macroscopic determination of the structure factor:[31]

$$S(\mathbf{K}_\perp, \mathbf{K}_z, t) = \langle \delta n(\mathbf{K}, 0) \delta n(-\mathbf{K}, t) \rangle$$
$$= \frac{T K_\perp^2 n_v^2}{C_{11}(\mathbf{K}) K_\perp^2 + C_{44}(\mathbf{K}) K_z^2}$$
$$\times \exp\left[-\frac{t}{\Gamma} (C_{11}(\mathbf{K}) K_\perp^2 + C_{44}(\mathbf{K}) K_z^2) \right], \tag{25}$$

where $n_c = B/\Phi_0$ is the vortex density; the structure factor $S(K_\perp, t)$ entering into Eq. (24) is obtained by integration of Eq. (25) with respect to K_z [see definition (10)]. We derive an upper estimate of the contribution to integral (29) from small $K_\perp$, assuming $G(0,t) \to \text{const}$ for $t \to \infty$ [in fact, $G(0,t)$ decays with t following a power law]:

$$\delta_\sigma^{(1)} \propto \int dK_z \int d^2 K_\perp K_\perp^4 \frac{K_\perp^2 T}{[C_{11}(K) K_\perp^2 + C_{44}(K) K_z^2]^3}. \tag{26}$$

As is evident from expression (26) the integral over $K_\perp$ clearly converges; this domain makes a relatively small contribution.

The primary contribution to the integral (24) therefore derives from the domain $K_\perp \sim K_0$ where the static structure factor of the liquid $S(\mathbf{K}_\perp, t=0)$ has a peak corresponding to the short-range order maintained in the liquid. The most significant difference between the liquid and solid phase for our purposes lies in the fact that the structure factor $S(\mathbf{K}_\perp, t)$ in the liquid decays over time. For $K \sim K_0$ this decay is initiated at times $t \sim t_{\rm pl}$, when the atomic displacement nears the same order of magnitude as the interatomic distance, $u(t_{\rm pl}) \sim a$.

Here $t_{\rm pl}$ will be significant in a viscous liquid and will have an exponential temperature dependence:

$$t_{pl} \sim t_{ph} \exp(U_{pl}/T). \tag{27}$$

The mean square drift of the liquid particles over time $u^2(t) = 1/2 \langle (u(t) - u(0))^2 \rangle$ is related to the Green's function $G(0, t)$ through the fluctuation-dissipative theorem:

$$u^2(t) = \int \frac{d\omega}{2\pi} \langle |u_\omega|^2 \rangle (1 - \cos \omega t)$$
$$= \int \frac{d\omega}{2\pi} \frac{2T}{\omega} \operatorname{Im} G(0,\omega)(1 - \cos \omega t)$$
$$= T \int_0^t G(0,\tau)\, d\tau. \tag{28}$$

The fact that the Green's function $G(0,t)$ is a retarded function, $G(0,t) = 0$ for $t < 0$, was used in the last equality in Eq. (28).

In the small velocity limit ($v \to 0$), $t \sim t_{\rm pl}$, $K \sim K_0$, $u \sim a$ are significant in integral (24) and hence subject to Eq. (27)

$$\delta_\sigma \approx \frac{\gamma}{\Gamma} \frac{1}{T} K_0^6 t_{pl}. \tag{29}$$

This estimate derived from general considerations can be reinforced by results obtained from a model in which the dynamical structure factor of the liquid is expressed through a simultaneous structure factor by the equation

$$S(K,t) = S(K,0) \exp[-{}^1\!/_2 K^2 \langle [u(t) - u(0)]^2 \rangle]. \tag{30}$$

Relations (27) and (30) permit an evaluation of the time integral in Eq. (24) by integration by parts:

$$\delta_\sigma = \frac{\gamma}{\Gamma a_0^2} \frac{1}{T} \int \frac{d^2 K}{(2\pi)^2} \frac{K^2}{2} \int_0^\infty S(K,t)\, dt \approx \frac{\gamma}{\Gamma T} \frac{4\pi^3}{a_0^6} t_{pl}. \tag{31}$$

In evaluating integral (31) we assume that the integration leads to the multiplier $t_{\rm pl}$, while the result of integration with respect to K is similar to the result obtained for the structure factor of the crystal. It is assumed that the structure factor $S(K,t)$ can be represented as in Eq. (30) as $K \to 0$ as well in Eq. (22); this study also derived the convergence condition of the integral for δ_σ in the range of K. Indeed, the hydrodynamic approximation (25) is valid for small K and hence the domain of small values of K is indisputably insignificant.

Equation (31) was derived assuming $\Delta v \ll v$, which will hold at high temperatures when $t_{\rm pl}$ is not very large. In this range, pinning leads to a small conductivity correction and $\sigma \approx \sigma_{\rm fl}$. At low temperatures we have thermally-activated flux flow (TAFF). It is natural to suggest that the height of barriers defining such flow will be identical to the value in Eq. (28) for $t_{\rm pl}$. Hence, $\sigma_{\rm TAFF} \propto t_{\rm pl}$. Joining this expression with Eq. (31), we have the interpolation equation

$$\sigma = \sigma_{fl}(1 + \mathscr{A} t_{pl}/t_{ph}), \tag{32}$$

where the coefficient $\mathscr{A}$ can be represented as

$$\mathscr{A} \approx 10^4 \frac{\gamma}{T} \frac{\lambda^2 B}{\Phi_0^3} \approx 10^2 \left[\frac{j_c(0)}{j_0(0)} \right]^{3/2} \mathrm{Gi}^{-1/2} \frac{B}{H_{c2}} \frac{T_c}{T}, \tag{33}$$

$j_c(0)$ is the critical current in the single-vortex pinning range at low temperatures (see Refs. 26, 30); $j_0(0)$ is the depairing current for $T \to 0$; Gi is the Ginzburg number defining the width of the fluctuation range near T_c; $\mathrm{Gi} \approx 10^{-2}$ for YBaCuO compounds. Definition (16) of the characteristic phonon time $t_{\rm ph}$ was used in deriving Eqs. (32), (33).

The result (32) therefore shows that the following condition must hold to realize the TAFF regime in the weak current range

$$\mathscr{A} t_{pl}/t_{ph} \gg 1. \tag{34}$$

Below we see that Eq. (34) in fact coincides with the criterion $t_{\rm pl} \gg t_{\rm pin}$ deriving from the qualitative analysis in Sec. 2. If the condition (34) holds, the linear segment of the CVC with an exponentially small resistivity $\rho_{\rm TAFF} \propto t_{\rm pl}^{-1}$ will with increasing current be replaced by a nonlinear transition region and at strong currents $j > j_{\rm cr}$ will become linear with a resistivity $\rho_{\rm flow}$. The value of the crossover current $j_{\rm cr}$ can be estimated in the same manner as the critical current in the pinning theory of the flux lattice: We consider equation (9) in the domain $K_0 v t_{pl} \gg 1$ and estimate the value of v_{cr} for which $|\delta v|/v = 1$. The time integral in Eq. (9) converges over times $t \sim (K_0 v)^{-1} \ll t_{pl}$ and hence the time dependence of the structure factor $S(K, t)$ can be neglected, and we obtain

$$\frac{\delta v}{v} = \frac{\Upsilon}{\Gamma a^2} \int \frac{d^2K}{(2\pi)^2} K_v K^2 S(K, t=0) \frac{1}{v} \operatorname{Im} G(\omega = K_v v), \tag{35}$$

where $G(\omega)$ is the Fourier transform of the response function $G(0, t)$. To estimate Eq. (35) we need the explicit form of $g(\omega)$ which we obtain in the Maxwellian model of a highly viscous liquid,[32] i.e., we assume that the transition from elastic to viscous behavior of the vortex fluid can be described by replacing the shear modulus C_{66} by an interpolation expression of the type

$$\bar{C}_{66}(\omega) = C_{66}\left(1 + \frac{i}{\omega t_{pl}}\right)^{-1}. \tag{36}$$

Expression (36) reduces to the ordinary shear modulus C_{66} for $\omega t_{pl} \gg 1$; the inverse limit corresponds to a liquid of viscosity $\eta = t_{pl} C_{66}$. We then obtain

$$G(\omega) = \int \frac{d^2K_\perp \, dK_z}{(2\pi)^3} \frac{1}{-i\omega\Gamma + \bar{C}_{66}(\omega) K_\perp^2 + C_{44}(K) K_z^2}$$
$$= \frac{1}{4\pi} \frac{\Gamma^{1/2}}{C_{66}^{1/2}} \left\{ \frac{1}{[C_{44}(K_B) t_{ph}]^{1/2}} - \left[\frac{-i\omega}{C_{44}(0)}\right]^{1/2} \right\} \left(1 + \frac{i}{\omega t_{pl}}\right)^{1/2}. \tag{37}$$

A value $K_B \approx \pi/a$, i.e., the boundary of the Brillouin zone in the corresponding crystal, is used as the upper integration limit in integration over $K_\perp$ in (37). There are two competing contributions to $\operatorname{Im} G(\omega)$ in the frequency range $t_{pl}^{-1} \ll \omega \ll t_{ph}^{-1}$. One such contribution is the same as in the case of a flux lattice proportional to $\omega^{1/2}$; the second contribution reflects the breakdown of crystalline order over large time periods and is proportional to $(\omega t_{pl})^{-1}$. In the frequency range

$$t_{pl}^{-1} \ll \omega \ll (\lambda/a)^{2/3} t_{pl}^{-2/3} t_{ph}^{-1/3} \tag{38}$$

the second term is the principal term, which leads to a $\operatorname{Im} G(\omega)$ of the type

$$\operatorname{Im} G(\omega) = \frac{1}{8\pi} \left(\frac{\Gamma}{t_{ph}}\right)^{1/2} \frac{1}{C_{66} C_{44}^{1/2}(K_B)} \frac{1}{\omega t_{pl}}. \tag{39}$$

Expression (35) for velocities $v = a\omega/2\pi$ satisfying Eq. (38) therefore takes the form

$$\frac{\delta v}{v} = \frac{\Upsilon}{8\pi a^2 \Gamma} \left(\frac{\Gamma}{t_{ph}}\right)^{1/2} \frac{1}{C_{66} C_{44}^{1/2}(K_B)} \frac{1}{t_{pl} v^2}$$
$$\times \int \frac{d^2K}{(2\pi)^2} K^2 S(K, t=0)$$
$$\approx \frac{T}{T_M} \mathscr{A} \frac{1}{(K_0 v)^2} \frac{1}{t_{pl} t_{ph}}, \tag{40}$$

where T_M is the melting point of the flux lattice, $T/T_M \gg 1$.

The CVC of the flux lattice with weak pinning in the strong current domain therefore takes the form

$$\mathscr{E} = \rho_{flow} j (1 - j_{cr}^2/j^2), \tag{41}$$

where the characteristic transition current to the nonlinear regime j_{cr} is equal to (recalling that $v_{flow} = jB/c\Gamma$)

$$j_{cr} \approx 5 \left(\frac{t_{ph}}{t_{pl}}\right)^{1/2} c \frac{\Upsilon^{1/2}}{\Phi_0} \left(\frac{B}{\Phi_0}\right)^{5/4} \approx j_0 \left[\frac{j_c(0)}{j_0(0)}\right]^{3/4} \left(\frac{t_{ph}}{t_{pl}}\right)^{1/2} \left(\frac{B}{H_{c2}}\right)^{5/4}, \tag{42}$$

where j_0 is the depairing current; $j_c(0)$ and $j_0(0)$ are determined in Eq. (33). Note that j_{cr} grows with diminishing plastic time t_{pl}.

Note also that the CVC enters the TAFF linear regime with a conductivity σ determined from Eqs. (32), (33) at currents j significantly less than j_{cr}. The determination of the corresponding characteristic current j_T lies beyond the scope of this study, since it requires a detailed analysis of the flux flow mechanism in the TAFF domain.

Expression (41) in the TAFF domain will be valid as long as $j \gg j_{cr}$ holds, i.e., as long as the relative correction is small; the correction is always small in the flux flow region and is given by Eq. (41) for $j \gg j_{cr} (\mathscr{A} t_{pl}/t_{ph})^{1/2}$ or by Eq. (32) for small currents.

6. CONCLUSION

We have demonstrated that the concept of collective pinning by a highly viscous vortex fluid at weak defects can explain the temperature behavior of the linear resistivity $\rho(T)$ of high-T_c superconductors in a strong magnetic field that is characterized by a sharp transition from the flux flow regime for $T \gg T_k$ to the thermally activated flux flow (TAFF) regime with an activation decay of $\rho(T)$ with temperature for $T_g < T \ll T_k$. The T_k regime crossover temperature is given by

$$\mathscr{A} \frac{t_{pl}}{t_{ph}} \approx \mathscr{A} \exp\left[\frac{U_{pl}(T_k)}{T_k}\right] \sim 1, \tag{43}$$

where the coefficient $\mathscr{A}$ is determined in Eq. (33), while the plastic strain energy $U_{pl}(T)$ is estimated in Eq. (1). Condition (43) coincides with the condition $t_{pl} \approx t_{pin}$ obtained in section 2 on the basis of a qualitative argument, if the estimate of v_c from Eq. (40) is used to determine $t_{pin} \approx a/v_c$. The sharpness of the transition between the two regimes of $\rho(T)$ predicted by the inequality $\mathscr{A} \ll 1$ can be attributed to the weakness of the pinning process, i.e., the small value of the parameter γ characterizing the degree of disorder. The disorder force can be characterized by the ratio of the critical current to the depairing current $j_c(0)/j_0(0)$ measured at low temperatures and in weak fields. The ratio lies below 10^{-2} for YBaCuO compounds. The resistivity $\rho(T)$ in the TAFF domain takes the form

$$\rho(T) \approx \rho_{flow} \mathscr{A}^{-1} \exp\left[-\frac{U_{pl}(T)}{T}\right]. \tag{44}$$

Note that the preexponential multiplier in Eq. (44) far exceeds the value of ρ_{flow}, which was determined experimentally.[2,3] Expression (32) is an interpolation between the flux flow and the TAFF regimes. It is important to point out that the linear relation between $\sigma(T)$ and t_{pl} in the TAFF domain has not been proved: This is only one possibility that we believe is the most probable. Hence, the possibility for an experimental test of this relation is of special interest. Such a test could possibly make use of the fact that, as follows from Eqs. (32) and (42), the product σj_{cr}^2 does not contain t_{pl}, i.e., it has a relatively slow (nonactivation) temperature dependence:

$$\frac{\sigma}{\sigma_{flow}}\frac{j_{cr}^2}{j_0^2}\approx\frac{10^2}{\mathrm{Gi}^{1/2}}\left(\frac{j_c^{(0)}}{j_0^{(0)}}\right)^3\frac{T_c}{T}\left(\frac{B}{H_{c2}}\right)^{7/2}. \qquad (45)$$

In this case j_{cr} can be determined by measuring the CVE in the high current range. As follows from Eq. (41) a linear relation between the squared currents and voltages must be observed in this case:

$$\mathscr{E}^2(\gamma)=\rho_{flow}^2(j^2-2j_{cr}^2). \qquad (46)$$

Note that this CVC was a result of Eq. (39): $\mathrm{Im}G(\omega)\propto\omega^{-1}$, which was obtained using Maxwellian model (36) describing the shear resistance of a highly viscous liquid. If this is not a valid assumption, it may turn out that $\mathrm{Im}G(\omega)\propto\omega^{\alpha'-1}$ with $\alpha'>0$. In this case we obtain in place of Eq. (41)

$$\mathscr{E}=\rho_{flow}j\left[1-\left(\frac{j_{cr}}{j}\right)^{2-\alpha'}\right], \qquad (41')$$

while the combination $\sigma j_{cr}^{2-\alpha'}$ will be independent of t_{pl}. It is therefore possible in principle to use a measurement of the corrections to the linear CVC for $j\gg j_{cr}$ to test Maxwellian model (36) for a vortex fluid.

These results allow us to predict the variation of the temperature dependence of the resistivity $\rho(T)$ with increasing degree of disorder γ (from, for example, irradiating a sample[23,24]). In the flux flow region $T\gg T_k$, $\rho(T)\approx\rho_{\text{flow}}$ and is only weakly dependent on γ. In the TAFF domain the resistivity decays with increasing γ: $\rho\propto\gamma^{-1}$ [see Eq. (44)], where the activation exponent is independent of γ. The T_k regime crossover temperature grows logarithmically with increasing γ. At the same time the transition temperature T_g to the vortex glass state below which the linear resistivity vanishes, will always be close (to the degree that disorder is weak) to the melting point of the flux lattice T_M, and hence will be weakly dependent on the degree of disorder. The weak dependence of T_g on the magnitude of disorder is in agreement with the results of Ref. 23 which demonstrated that the transition line to a phase with irreversibility of the magnetic response remains unshifted upon irradiation of the crystal.

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