Bloch-line velocity limit in magnetic films

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(Submitted 1 March 1991)
Zh. Eksp. Teor. Fiz. **100**, 564–570 (August 1991)

We show that the peak velocity u_p with which vertical Bloch lines move in twisted domain walls of films with perpendicular anisotropy is given by $u_p \sim (sv_p)^{1/2}Q^{1/4}C^{-1/2}$, where s is the domain-wall (DW) flexural-oscillation velocity, v_p is the peak DW velocity, Q is the quality factor of the material, and $C \approx \pi/2$. The mechanism that limits the velocity of the Bloch-lines (BL) is their clustering into groups of five BL or more., The velocity limit of a BL-cluster velocity is discussed qualitatively.

1. A Bloch line (BL) is a magnetization-field vortex filament that can move along a domain wall (DW) under the influence of an external magnetic field directed along the DW or the gyroscopic force produced by the motion of the DW itself.[1] It is shown in Refs. 1–4 that the maximum BL velocity is

$$U_{\text{lim}} \approx s = \gamma (8\pi A)^{1/2}, \quad (1)$$

where γ is the gyromagnetic ratio and A is the exchange-rigidity constant of the material. The mechanism that limits the BL velocity in this case is dynamic transformation of an isolated BL into a cluster of five BL with no change of the initial topological charge. This BL velocity limit is similar in a certain sense to the Walker velocity limit[1] for a domain wall, and likewise takes no account of the influence of the demagnetization fields of the surface magnetic charges. Allowance for the latter is of fundamental importance for magnetic films with a perpendicular anisotropy, the behavior of BL in which has recently attracted much attention in connection with the development of a solid-state memory for super-dense information storage.

The domain walls in such films are "twisted" by the demagnetization fields, i.e., their structure varies in a direction perpendicular to the film plane. Equation (1) was derived for untwisted DW, which are formed in very thin films having uniaxial anisotropy perpendicular to the film plane, when the influence of the demagnetizing fields of the surface magnetic charges on the DW structure can be neglected, in films (slabs) with planar anisotropy, etc. The velocity limit of twisted DW is known[1] to be lower than the Walker velocity, owing to the generation and subsequent stability loss of horizontal Bloch lines.

2. Let us examine the effect of DW twisting on the BL velocity limit. A BL moving in a DW bends the latter by a gyroscopic force. It is obvious, as well as verified by computation,[5] that if the BL velocity is high enough HBL are generated on the bent section (see Fig. 1).

Let such a bent section be moved with velocity u along an immobile DW, say by an external field directed along the DW (parallel to the x axis).[1)] The DW velocity component normal to the DW wall at an arbitrary point A is then

$$u_n = uq'(1+(q')^2)^{-1/2}, \quad (2)$$

where $q(x)$ is the profile of the DW sag, and the prime denotes differentiation with respect to x.

It follows from the Slonczewski theory that for $u_n > v_p$ a horizontal BL is produced in the DW at the point

$$z = z_1 = h(1+e^2)^{-1}$$

(Fig. 1), where h is the film thickness.

As the velocity is increased, the horizontal BL moves from the surface (from the point $z = z_1$ into the interior of the sample all the way to a point $z = z_2 = he^2/(1+e^2)$ reached when the velocity becomes comparable with the so-called Slonczewski peak velocity.

$$v_p = 23{,}8\gamma A/hK_u^{1/2}, \quad (2a)$$

where K is the uniaxial-anisotropy constant.

The horizontal BL breaks at a velocity $u = u_p$ whch, according to Slonczewski, is in fact the maximum (peak) DW velocity. In this case, as seen from Fig. 2, a cluster of five BL is produced in our case, just as in bulk mechanism, defined by Eq. (1), which limits the BL velocity. The subsequent evolution of the BL and DW dynamics depends on the actual conditions.

In particular, the sag may slow down and the cluster can stabilize, or else new horizontal BL loops are generated and eventually break up, etc., i.e., BL lines accumulate and form a "stack." Similar processes are known in the dynamics of magnetic bubble domains.

Note that dissipative processes can alter somewhat the considered behavior of the BL near the velocity limit. The point is that dissipation makes the leading edge of the DW-sag wave accompanying the moving BL steeper than the trailing edge (Refs. 2, 4, 5, 7).[2)] The nucleation and breaking on the leading front will therefore occur earlier than on the trailing edge. If the dissipation is large enough one can expect the BL velocity to be limited by the dynamic transformation of the BL into a cluster of three rather than five BL.

3. Let us analyze in greater detail the conditions for

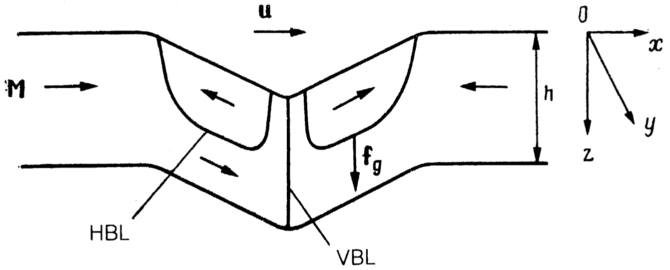

FIG. 1. Horizontal BL loops on the leading (a) and trailing (b) edges of a sag wave produced by a moving VBL. f_g—gyroscopic forces acting on the horizontal BL.

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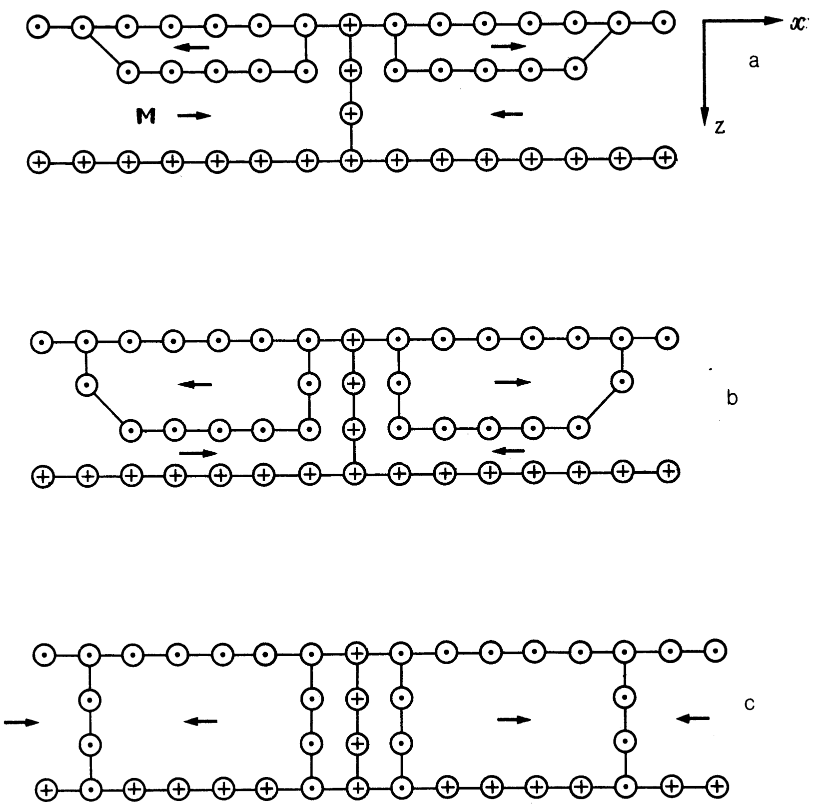

FIG. 2. Schematics of the horizontal BL break mechanism for a vertical BL moving in a DW (projection on the zx plane): a—nucleation of horizontal BL loops, (b)—"pre-breakthrough" situation, c—break through horizontal BL loops and formation of a cluster of five vertical BL.

dynamic equilibrium of a moving horizontal BL loop. This can be compared with the mechanical problem of equilibrium of an elastic filament acted upon by definite forces and moving over a curved surface. Note that such a mechanical system is nonholonomic, as is also the BL problem. This means, in particular, that a more natural approach to the derivation of the BL-equilibrium equations that takes automatic account of the nonholonomy of the constraints is a Newtonian approach based on the notion of forces, in contrast to the Lagrangian (energy-based) approach in which allowance for the constraints leads to complications. However, the Lagrangian formulation of this problem, being more physical for the situation considered, can be consistently applied here if a common procedure is used to derive the equations for the DW and the BL dynamics. The difficulties with the nonholonomy of the constraints are then eliminated, but at the expense of more cumbersome computations and more system degrees of freedom.

We project the components of the forces acting on the horizontal BL a) along the normal to the DW surface and b) on a plane tangent to the DW. The former (normal) forces are balanced by DW reaction forces. Since we are not interested in small distortions of the DW profile along the normal to the film, i.e., in its deviation from cylindrical (this is usually permissible in similar problems of the theory of DW in magnetic films), we disregard hereafter these (perpendicular) force components. The tangential force components acting on the horizontal BL can also be subdivided into forces along a tangent to the line itself, and forces acting along what is known as the geodesic vector or the tangential curvature. It is in fact the condition that these latter forces be in equilibrium which determines the sought horizontal BL profile and the conditions for its stability.

Let the DW surface be defined by the equations (in a parametric form)

$$x=x,\quad y=q(x),\quad z=z. \tag{3}$$

The vector normal to this surface is then

$$\mathbf{N}=(q'(1+(q')^2)^{-1/2},\ -(1+(q')^2)^{-1/2},0). \tag{4}$$

The horizontal BL equation can be defined as

$$x=x,\quad y=q(x),\quad z=z_L(x). \tag{5}$$

z_L is the position of the horizontal BL center). The unit vector tangent to this curve is

$$t=(\rho,\ q'\rho,\ z_L'\rho), \tag{5a}$$

where

$$\rho=(1+(q')^2+(z_L')^2)^{-1/2},$$

and the curvature vector of the curve is

$$r_L''=(\partial_s\rho,\ \partial_s q'\rho,\ \partial_s z_L'\rho)=\rho(\partial_x\rho,\ \partial_x q'\rho,\ \partial_x z_L'\rho). \tag{6}$$

The direction of the normal to the horizontal BL in the tangential plane is specified by the unit vector.

$$\mathbf{p}=[\mathbf{Nt}]. \tag{7}$$

The geodesic curvature of the horizontal BL is then

$$k_G=([\mathbf{Nt}]\,\mathbf{r}_L''). \tag{8}$$

A horizontal BL is acted upon in a moving DW by the following forces. a) The dynamic-reaction (gyroscopic) force[1]

$$\mathbf{T}=2M_s\gamma^{-1}\Phi_H[\mathbf{tu}], \tag{9}$$

where M_s is the magnetization, Φ_H is the increment of the azimuthal angle ψ at which the magnetization emerges from the DW plane on going through the HBL along its normal, and $\mathbf{u} = (u,0,0)$ is the horizontal BL velocity.

b) The pressure force on the horizontal BL, due to the non-uniform dependence of its linear energy density l_L on z:[1]

$$-\nabla l_L = (0,\ 0,\ -\partial l_L/\partial z). \qquad (10)$$

c) The surface tension force, also determined by the horizontal BL linear density.

d) The dissipative force $f_{\rm diss}$, which will not be specified.

Projecting on these forces along the horizontal BL geodesic curvature we obtain

$$l_L k_G - (\partial l_L/\partial z) p_z = 2M_s\gamma^{-1}\Phi_H([\mathbf{tu}]\,\mathbf{p}) + f_{diss} \qquad (11)$$

or

$$l_L k_G - (\partial l_L/\partial z)\cos\alpha = 2M_s\gamma^{-1}\Phi_H u q'(1+(q')^2)^{-1/2} + f_{diss}, \qquad (12)$$

where

$$\cos\alpha = [(1+(q')^2)/(1+(q')^2+(z_L)^2)]^{1/2},$$

α is the angle between the geodesic-curvature vector and the z axis. For $|q'| \ll 1$ we have

$$k_G = z_L''(1+(z_L')^2)^{-3/2}.$$

To solve (12) we must calculate $q(x)$, i.e., the DW profile defined by the equation

$$-\partial_x[\sigma_W q'(1+(q')^2)^{-1/2}] - \beta q - 2M_s\mu_W^{-1}uq' = 2M_s\gamma^{-1}u\psi'_1 \qquad (13)$$

where σ_W is the DW energy density, $-\beta q$ is the restoring force that ensures at $u = 0$ the stability of a planar DW to flexural perturbations (it can be produced, in particular by a nonuniform magnetic field, and then $\beta = 2M_S H'$, where $H' = dH_z/dy$ is the magnetic field gradient), and μ_W is the DW mobility. The term in the right-hand side is the gyroscopic force applied to the DW by the moving BL.

4. To determine the velocity limit it suffices to consider the nondissipative approximation. In addition we neglect q' compared with 1, which can be readily verified to be valid for all velocities up to the limit of u. The solution of (13) can then be written in the form[4]

$$q(x) = q_0\int_{-\infty}^{\infty} G(x-x')\psi'(x')\,dx', \qquad (14)$$
$$q_0 = -2M_s\gamma^{-1}u\sigma_W^{-1},$$

where

$$G(x) = (2b)^{-1}\exp(-b|x|), \qquad (15)$$

is the Green's function and $b = (\beta/\sigma_W)^{1/2}$. To calculate the profile using (15) we neglect the influence of the horizontal BL loop on the form of the profile. This is justified in part by the fact that the integral topological charge of the horizontal BL loop is zero, so that the DW deformation is cancelled out by the different sections of the horizontal BL loop. We take therefore into account only the DW sag due to the initial vertical Bloch line. Putting $\psi_1 = \Phi_V\delta(x)$ (Φ_V is the increase of the angle ψ after going through the vertical BL), which accords with the BL approximation and is fully justified for $b \ll 1$, and substituting this expression in (14), we obtain

$$q(x) = q_0\exp(-b|x|). \qquad 16)$$

Equation (12) with a right-hand side determined by (16) can be solved only numerically. To simplify the procedure we use a piecewise linear approximation for the $q(x)$ profile, namely

$$q(x) = \begin{cases} q_0(1-b|x|), & |x| \leqslant b^{-1}, \\ 0, & |x| \geqslant b^{-1}. \end{cases} \qquad (17)$$

In this case Eq. (12) on the actual interval $u_1 \leqslant u \leqslant u_p$, where the horizontal BL breaks when the velocity limit is reached, takes the simple form.

$$\gamma(2M_s\Phi_H)^{-1}\partial l_L/\partial z_L = \gamma^{-1}M_s\Phi_V\sigma_W u^2. \qquad (18)$$

This equation relates the position of the flat section of the horizontal BL loop with the vertical BL velocity u (Fig. 3). Strictly speaking, Eq. (18) can be used only in a definite velocity interval $u_1 \leqslant u \leqslant u_p$, in which the location of the horizontal BL loop is stable. It can be seen from Fig. 3 that as the vertical BL velocity u increases the vertex of the horizontal BL loop shifts towards the film surface and the horizontal BL becomes unstable at $z = z_2$. The corresponding value of the vertical BL velocity can be taken to be the peak vertical BL velocity, in full analogy with the determination of the peak velocity of a plane DW. The left-hand side of (18) is equal at $z = z_2$ to the Slonczewski peak velocity v_p (2a).

Equating the right-hand side of (18) to v_p, we get

$$CQ^{-1/2}u_p^2/s = v_p \qquad (19)$$

or

$$u_p = (sv_p)^{1/2}Q^{1/4}C^{-1/2}, \qquad (20)$$

where

$$Q = K_u/2\pi M_s^2,$$

and v_p is the Slonczewski peak velocity given by (2a) for a plane DW.[3)] For an isolated vertical BL the constant is $C = \pi/2$ and can reach a larger value ($3\pi/2$) when account is taken of the influence of the vertical BL loop on the DW sag. It is easy to verify by directly substituting (20) in (14) that $(q')^2 \ll 1$ for $u \leqslant u_p$.

We conclude the section by estimating the peak BL velocity for iron-garnet films. Putting $Q = 4$, $s = 500$ m/s and

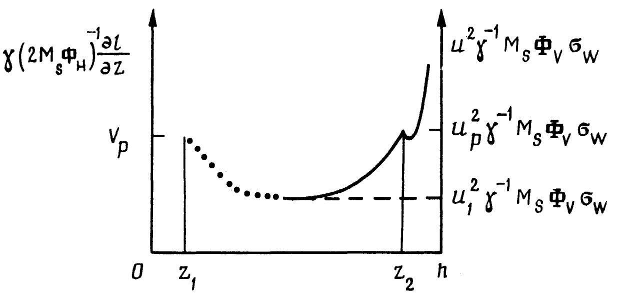

FIG. 3. Dependence of the form $\partial l_L/\partial z$ on z. The solid and dashed lines show the HBL corresponding respectively to stable and unstable equilibrium.[1]

$v_p = 10$ m/s we obtain $u_p = 100$ m/s, close enough to the experimental data of Ref. 3.

5. The equations for clusters consisting of a small number of BL can also be qualitatively generalized. For a cluster having N_L BL the value of Φ_V is increased N_L times. The right-hand side of (18) is also increased by N_L times and the peak velocity for a cluster decreases according to (20) by a factor $N_L^{1/2}$, i.e.,

$$(u_p)_{cl} = (sv_p)^{1/2} Q^{1/4} N_L^{-1/2} C^{-1/2} \tag{21}$$

This scaling is obviously valid so long as $|q'|$ remains much smaller than 1, i.e., for $N_L \ll (s/v_p) Q^{1/2}$.

It turns out in fact that Eq. (21) continues to hold also for $q' \gtrsim 1$. We demonstrate this by using Eq. (13) in the nondissipative approximation, and also by assuming, as in Sec. 4, that $\Delta_L \ll L^{-1}$, $\Delta_L = (A/2\pi M_s^2)^{1/2}$.

The shape of the DW is described then by the equation

$$\Delta_L^2 (1+(q')^2)^{-3/2} q'' - b^2 q + (\gamma 4\pi M_s)^{-1} (1+(q')^2)^{-1/2} q' u = 0 \tag{22}$$

with boundary conditions

$$q,\ q'(x=\pm\infty) = 0, \tag{23}$$

$$(1+(q')^2)^{-1/2} q' |_{x=x_L-0} - (1+(q')^2)^{-1/2} q' |_{x=x_L+0}$$
$$= (2M_s/\sigma\gamma)(\varphi'_{x=+\infty} - \varphi'_{x=-\infty})$$
$$\times u = Q^{-1/2} s^{-1} (\varphi_{x=+\infty} - \varphi_{x=-\infty}) u = N\pi u / sQ^{1/2}. \tag{24}$$

Equation (22) has a first integral

$$(1+(q')^2)^{-1/2} = 1 - (qb/\Delta_L)^2. \tag{25}$$

We have already used the boundary condition (23) here. Since the function $a(x)$ must be even in the absence of dissipation, it follows from (24) that

$$q'/(1+(q')^2)^{1/2} |_{x=x_L+0} = \pi N u / 2sQ^{1/2}. \tag{26}$$

Integrating Eq. (25) with allowance for (26) we obtain for $q(x)$ (if $\pi Nu/2sQ^{1/2} \leqslant 1$) an equation in the form

$$\frac{1}{2^{1/2}} \ln \frac{[2^{1/2} - (2-(qb/\Delta_L)^2)^{1/2}][2^{1/2} + (2-(q_0 b/\Delta_L)^2)^{1/2}]}{[2^{1/2} + (2-(qb/\Delta_L)^2)^{1/2}][2^{1/2} - (2-(q_0 b/\Delta_L)^2)^{1/2}]}$$
$$+ [2-(qb/\Delta_L)^2]^{1/2} - [2-(q_0 b/\Delta_L)^2]^{1/2} = -\frac{b}{\Delta_L} |x - x_L|, \tag{27}$$

where $q_0 = N\pi u \Delta_L / 2bsQ^{1/2}$ is the amplitude of the deflection of the DW at $q = q(x = x_L)$, i.e., at the center of the cluster. Together with (12) Eq. (2) determines the shape of the horizontal BL loop.

The maximum value of $q'(1+(q')^2)^{-1/2}$ is reached as $x \to x_L + 0$ and is equal to $\pi Nu/2sQ^{1/2}$ so that according to (2) and (26) the peak velocity of the vertical BL is determined also in this case by Eq. (20).

For parameter values $\pi Nu/2sQ^{1/2} > 1$ (i.e., for $|q'| \gg 1$) it can be concluded from the qualitative arguments advanced at the beginning of the article that the peak velocities of a vertical BL cluster and a DW should be close to one another.

It can also be assumed that there is no need to insert in the resultant Eq. (20) the peak velocity determined just by the Slonczewski equation. It seems sufficient to substitute the actually observed maximum DW velocity at which the self-similar motion of the DW is disturbed. Such approximation equations for v_p, which describe the experimental data well, are given, for example, in Refs. 7 and 8.

1) The results are easily extended to a more general situation, when the BL motion is caused by the gyroscopic force due to motion of the DW itself.

2) This conclusion is confirmed also by recent BL-dynamics computations based on the Landau–Lifshitz equations.[9]

3) To be exact, it must be noted that Eq. (12) and the corresponding equation (18.2) of Malozemoff and Slonczewski[1] have different forms of the $q(x)$ profile. But since $q' \ll 1$ holds we can determine u_p by using v_p values that are valid for a plane DW.

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Translated by J. G. Adashko