Interaction of current carriers with acoustic plasmons as a possible reason for anomalous relaxation in cuprate metal-oxide compounds

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It is shown that the experimentally observed anomalies of the kinetic, thermodynamic, and optical properties of cuprate metal-oxide compounds in the normal metallic state can be explained on the basis of a model of a two-band layered metal with degenerate "light" carriers in a wide band and nondegenerate "heavy" carriers in a narrow band, if quasiparticle relaxation occurs primarily via interaction with acoustic plasmons, whose spectrum in the tight-binding approximation is a periodic function of the quasimomentum and lies in the transmission range (outside the region of strong Landau damping) in the entire volume of the Brillouin zone.

1. INTRODUCTION

It is well known that high-T_c superconductors, discovered by Bednorz and Müller,[1] based on cuprate metal-oxide compounds (MOC) with a layered crystal structure[2–9] in the normal metallic state at temperatures T above the critical superconducting transition temperature $T_c \approx 30$–125 K exhibit a whole collection of anomalous kinetic, thermodynamic, optical, magnetic, and other properties. In particular, the following are typical a) nearly linear temperature dependence of the resistivity $\rho(T)$ and inverse Hall constant $R_H^{-1}(T)$ over a wide range of values of T;[10,11] b) supralinear increase of the electronic heat capacity $C(T)$ at high temperatures T;[12] c) violation of Korringa's law $\tau_s^{-1}(T) \propto T$ for the inverse relaxation time of the nuclear spin;[13] d) deviation of the high-frequency dependence $\sigma(\omega)$ from Drude's damping law in the infrared-reflection spectrum and the Raman scattering spectrum;[14,15] e) symmetric relative to the sign of the applied voltage V, linear dependence of the conductance $g_T(V)$ of tunneling contacts;[16] etc.

The analysis of experimental data performed in Ref. 17 showed that some features of the normal state of cuprate MOC can be explained on the basis of the assumption that current-carrier scattering or spin relaxation occurs via interaction with collective excitations of the charge or spin density, if the damping of these excitations is described by an imaginary part of the polarizability (electric susceptibility) of the following form ($\hbar = k_B = 1$):

$$\operatorname{Im}\Pi(\mathbf{q},\omega) \sim \begin{cases} -N(0)\omega/T, & |\omega| < T, \\ -N(0)\operatorname{sign}\omega, & |\omega| > T, \end{cases} \tag{1}$$

where $N(0)$ is the unrenormalized single-particle density of states at the Fermi level.

This construction of the dissipative part of the response of a Fermi system, on the one hand, implicitly presupposes the absence of complete degeneracy of the current carriers and spin (conduction electrons, holes) at $T \neq 0$ and, on the other hand, it leads to a logarithmic divergence of the renormalization factor $Z(\omega) = 1 - \partial \operatorname{Re}\Sigma(\omega)/\partial\omega$ as $\omega \to 0$ and $T \to 0$ [where $\Sigma(\omega)$ is the single-particle self-energy], i.e., it corresponds to an infinitely heavy effective mass of quasiparticles on the Fermi surface and a completely incoherent ground state of the Fermi system; in Ref. 17 such a Fermi system is termed marginal. In Ref. 17 no microscopic justification of the phenomenological dependence (1) is given, with the exception of an indication of the existence of a singularity in the amplitude of s-wave scattering in a $2D$ Fermi gas with an attractive interaction.[18] It should be noted, however, that the negative sign of the quantity $[\operatorname{Re}\varepsilon^{-1}(\mathbf{q},\omega) - 1]$, as implied by the Kramers-Kronig relations[19] for the inverse dielectric response function, does not at all signify that there exists near the Fermi surface an effective attractive interelectronic interaction, which is necessary, in particular, for Cooper pairing.

In this paper it is shown that all the paradoxes of a marginal Fermi liquid[17] vanish and the dissipative processes necessary for an adequate description of anomalous relaxation remain in the model of a two-component two-dimensional Fermi liquid with "light" and "heavy" charge carriers,[1)] if it is assumed that a) h-carriers are almost localized on lattice sites, so that for these carriers the tight-binding approximation is applicable, and b) at sufficiently high temperatures the h-carriers become nondegenerate, while the l-carriers remain degenerate.

This model corresponds, in particular, to a layered $2D$ metal with overlapping wide and narrow $2D$ bands (Fig. 1). In this case, as will be shown below, inelastic scattering of almost-free l-carriers by low-frequency collective excitations of the charge density of the h-carriers—acoustic plasmons (APs) with frequencies $\omega(\mathbf{q}) \approx q_\parallel u$ as $q_\parallel \to 0$, where $\mathbf{q}_\parallel$ is the longitudinal momentum of the APs in the plane of the two-dimensional layers and u is the velocity of "plasma sound"[2)]—makes the main contribution to the kinetic processes.

If the single-particle spectrum of h-carriers in a narrow $2D$ band of width W_h has the form

$$E_h(k_x,k_y) = \frac{W_h}{2}\left[1 - \frac{1}{2}(\cos k_x a + \cos k_y b)\right], \tag{2}$$

where a and b are lattice constants in the plane of the layers ($a \approx b$), then the acoustic-plasmon spectrum is a periodic function of the quasimomentum $\mathbf{q}_\parallel$ and lies in the region of transmission (outside the region of strong damping) in the entire volume of the Brillouin zone, if the following condition is satisfied:

$$W_h < \min\{e^2/\varepsilon_i a,\ \hbar^2/m_l^* a^2\}, \tag{3}$$

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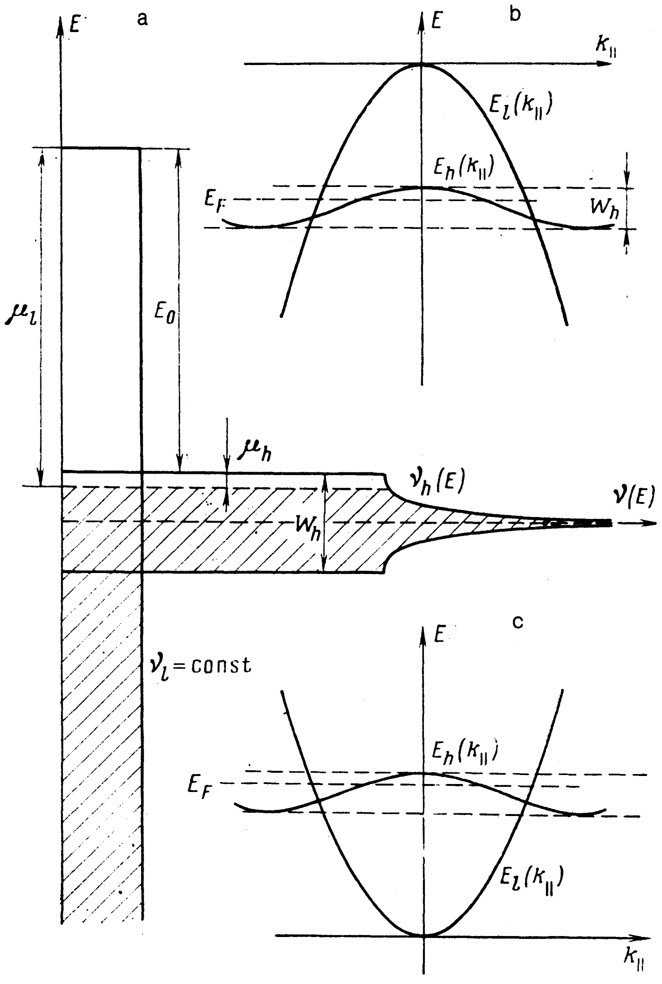

FIG. 1. The density of states (a) and the spectrum in overlapping wide and narrow $2D$ bands in the case of identical (b) and different (c) types of carriers.

where ε_i is the permittivity of the ionic lattice and m_l^* is the longitudinal effective mass of l-carriers in the wide $2D$ band.

This is attributable to the fact that Landau damping of nondegenerate h-carriers in a narrow band with the spectrum (2), as is shown in this paper, is not equal to zero in the limited energy range $|\omega| < W_h \sin(q_{\|} a/2)$, in contrast to $3D$ and $2D$ metals with wide bands and a quadratic spectrum of degenerate l- and h-carriers,[20–25] in which a wide region of quantum Landau damping by h-carriers delimits the acoustic-plasmon spectrum at large momenta, i.e., the spectrum has a cutoff point $q_{max} \approx \Omega_h / v_{Fh}$, where Ω_h and v_{Fh} are the plasma frequency and the Fermi velocity of h-carriers.[3)]

The imaginary part of the polarizability, calculated in the present paper, of nondegenerate h-carriers in a narrow $2D$ band with the spectrum (2) has the form

$$\operatorname{Im} \Pi_h(\mathbf{q}_{\|}, \omega, T) = -\operatorname{sh}\left(\frac{\omega}{2T}\right) \cdot \mathscr{F}(\mathbf{q}_{\|}, \omega, T), \qquad (4)$$

where the function $\mathscr{F}$ with $q_{\|} a \lesssim 1$ is a slowly varying quantity over a wide region of values of ω and T, so that for $|\omega| \lesssim T \lesssim W_h/4$ the expression (4) is actually identical to the phenomenological dependence (1) postulated in Ref. 17.

This makes it possible to explain the almost linear temperature dependence of the resistivity $\rho(T) \propto T$ (Refs. 10 and 11) by scattering of l-carriers by low-frequency collective excitations of the charge density of h-carriers (h-plasmons), while the weak temperature dependence of the relaxation rate of nuclear spins[13] can be explained by relaxation by excitations of the spin density (paramagnons). By taking into account the electron-electron scattering of nondegenerate h-carriers by degenerate l-carriers we can explain both the linear temperature dependence of the inverse Hall constant $R_H^{-1}(T) \sim T$ (Ref. 11) in the case of l- and h-carriers carrying charges (effective mass) of the same sign, and the inversion of the sign of R_H at high temperatures T for oppositely charged l- and h-carriers.

On the other hand, the interaction of degenerate l-carriers with acoustic plasmons gives rise to renormalization of the single-particle spectrum, which in turn results in a supralinear temperature dependence of the electronic heat capacity $C(T) \approx \gamma T + \beta T^3$ at high temperatures T, in qualitative agreement with the experiment of Ref. 12, while the frequency dependence of the high-frequency conductivity owing to relaxation of l-carriers by acoustic plasmons corresponds to the experimentally observed IR reflection spectra.[14,15]

It should be emphasized that the assumption that there exists a narrow band (peak in the density of states) near the Fermi level agrees with the results of numerical band calculations for layered-chain-type cuprate MOC of the type $YBa_2Cu_3O_7$ (Refs. 27–30) and is consistent with the generally accepted ideas of weak hybridization of the d and p orbitals of Cu^{2+} copper ions and O^{2-} oxygen ions (Refs. 31 and 32) and strong direct overlapping of the p orbitals of the nearest O^{2-} ions in the $2D$ CuO_2 layers.[33,34] In this case, d-holes strongly localized on Cu^{2+} ions can play the role of h-carriers while delocalized p holes of oxygen can play the role of l-carriers. We note that the model of a very narrow band (impurity level) was employed in Ref. 35 to describe the linear temperature dependences of $\rho(T)$ and $R_H^{-1}(T)$ in $YBa_2Cu_3O_{7-x}$, while in Ref. 36 the two-band model was used to explain the anomalous carrier-density dependence of R_H in $La_{2-x}Sr_xCuO_4$.

2. PLASMON SPECTRUM AND LANDAU DAMPING IN A LAYERED METAL WITH A NARROW BAND

We now study a layered metal (such as cuprate MOC), whose quasi-$2D$ electron spectrum contains, together with a partially filled wide $2D$ band of width $W_l \gtrsim 1$ eV with a quadratic spectrum of free l-carriers $E_l(\mathbf{k}_{\|}) = k_{\|}^2/2m_l^*$ and an almost constant density of states $\nu_l \approx m_l^*/2\pi$, near the Fermi level a narrow $2D$ band of width $W_h \ll W_l$ with the single-particle spectrum (2) and with a much higher density of states $\nu_h \gg \nu_l$, depending on the energy E (see Fig. 1):

$$\nu_h(E) = \frac{8}{\pi^2 a^2 W_h} K(\varkappa(E)), \quad \varkappa(E) = \frac{2}{W_h}[E(W_h - E)]^{1/2}, \qquad (5)$$

where $K(\varkappa)$ is a complete elliptic integral of the first kind, so that when the band is half filled pinning of the Fermi level occurs on account of the logarithmic singularity of the function $\nu_h(E)$ at the point $E = W_h/2$.

Neglecting the weak (on the order of the ratio ν_l/ν_h) hybridization of the spectra of the narrow and wide bands, we represent the dielectric response function of such a two-band metal in the form

$$\varepsilon(\mathbf{q},\omega)=\varepsilon_i(\mathbf{q},\omega)-V_c(\mathbf{q})[\Pi_l(\mathbf{q},\omega)+\Pi_h(\mathbf{q},\omega)], \quad (6)$$

where ε_i is the permittivity of the ion lattice taking into account the dispersion associated with optical phonons and interband transitions; V_c is the matrix element of the unscreened Coulomb interaction in a layered metal[37,38] with interlayer separation d:

$$V_C(\mathbf{q})=\frac{2\pi e^2}{q_\parallel}\frac{\mathrm{sh}(q_\parallel d)}{\mathrm{ch}(q_\parallel d)-\cos(q_z d)}, \quad (7)$$

and Π_l and Π_h are the polarization operators of the l- and h-carriers, respectively. If hops (tunneling) of electrons between layers and the fluting of the cylindrical Fermi surface associated with such hops are neglected, then Π_l and Π_h will depend only on the longitudinal quasimomentum $\mathbf{q}_\parallel(q_x, q_y)$ in the plane of the $2D$ layers.

In the region $|\omega|\lesssim E_{Fl}$ and $q_\parallel\lesssim 2k_{Fl}$, where $E_{Fl}=k_{Fl}^2/2m_l^*$ is the Fermi energy and $k_{Fl}=(2\pi N_l)^{1/2}$ is the Fermi momentum of the degenerate l-carriers in layers with l-carrier density N_l in the $2D$ layer (per unit area of the layer), we have

$$\mathrm{Re}\,\Pi_l(\mathbf{q}_\parallel,\omega)\approx -2\nu_l,$$
$$\mathrm{Im}\,\Pi_l(\mathbf{q}_\parallel,\omega)\approx -2\nu_l\pi^{1/2}\frac{\omega}{q_\parallel v_{Fl}}\theta(q_\parallel v_{Fl}-\omega), \quad (8)$$

where $v_{Fl}=k_{Fl}/m_l^*$ is the Fermi velocity of the l-carriers and $\theta(x)$ is the Heaviside function: $\theta(x)=1$ for $x>0$ and $\theta(x)=0$ for $x<0$.

In the limit $T\to 0$, when h-carriers are also degenerate, the chemical potentials (μ_l and μ_h) and the $2D$ densities (N_l and N_h) of the carriers in the overlapping $2D$ bands are related (see Fig. 1) by

$$\mu_l-\mu_h=E_0,\quad N_l+N_h=N_0, \quad (9)$$

where

$$\mu_l\equiv E_{Fl}=\frac{N_l}{2\nu_l},\quad \mu_h=\frac{N_h}{2\nu_h(\mu_h)}, \quad (10)$$

and the total $2D$ density N_0 is determined by the average volume carrier density $n_0\equiv 1/v_0$, where v_0 is the volume of a unit cell. Thus, for example, in a layered metal like $La_{2-x}Sr_x/CuO_4$ (Refs. 1–3) with one conducting layer per unit cell we have $N_0=n_0 d$, while in MOC like $YBa_2Cu_3O_{7-x}$,[4,5] $Bi_2Sr_2Ca_{n-1}Cu_nO_x$,[6,7] or $Tl_mBa_2Ca_{n-1}Cu_nO_x$,[8,9] containing several ($n\geqslant 2$) cuprate layers per cell, the relation between N_0 and n_0 is more complicated.

From Eqs. (5), (9), and (10) there follows a transcendental equation for the chemical potential of the h-carriers:

$$K\left\{2\left[\frac{\mu_h}{W_h}\left(1-\frac{\mu_h}{W_h}\right)\right]^{1/2}\right\}=\frac{\pi^2\nu_l W_h a^2}{8}\left[\frac{N_0-2\nu_l E_0}{2\nu_l\mu_h}-1\right], \quad (11)$$

whence for values of the parameters $W_h\approx 0.1$ eV, $E_0\approx 0.5$ eV, $a\approx 4$ Å, $d\approx 6$ Å and $m_l^*\approx m_0$ (where m_0 is the free-electron mass), so that $\nu_l\approx 2\cdot 10^{14}$ eV$^{-1}\cdot$cm^{-2} holds, for $n_0\approx 4\cdot 10^{21}$ cm^{-3} and $N_0\approx 2.4\cdot 10^{14}$ cm^{-2} we obtain the estimate

$$\mu_h\approx 2{,}5\cdot 10^{-2}W_h\approx 30\ \mathrm{K}.$$

For $T\gg\mu_h$, when the h-carriers become nondegenerate and are described by a Boltzmann distribution function

$$n_h(\mathbf{k}_\parallel,T)=\exp\left\{-\frac{E_h(\mathbf{k}_\parallel)-\mu_h(T)}{T}\right\}, \quad (12)$$

their $2D$ density N_h depends on T on account of carrier redistribution between bands and is connected with $\mu_h(T)$ by the relation ($a=b$)

$$N_h(T)\equiv 2\int\frac{d^2k_\parallel}{(2\pi)^2}n_h(\mathbf{k}_\parallel,T)$$
$$=\frac{2}{a^2}I_0^2\left(\frac{W_h}{4T}\right)\cdot\exp\left\{\frac{\mu_h(T)-W_h/2}{T}\right\}, \quad (13)$$

where $I_0(x)$ is a Bessel function of imaginary argument. In this case, with the help of Eq. (9) we obtain the following equation for N_h:

$$\ln[a^2N_h(T)]=\ln 2+2\ln I_0(W_h/4T)$$
$$-\frac{E_0+W_h/2}{T}+\frac{N_0-N_h(T)}{2\nu_l T}. \quad (14)$$

Figure 2a shows the dimensionless density $y\equiv a^2N_h$ (solid curves), determined from Eq. (14), and the ratio $4\mu_h/W_h$ (dashed curves) as a function of $x\equiv W_h/4T$ while Fig. 2b shows y as a function of x^{-1} for different values of the filling factor $y_0\equiv a^2(N_0-2\nu_l E_0)>0$, when the Fermi level intersects the narrow band ($E_{Fl}>E_0$). One can see that for $W_h\approx 0.1$ eV and $y_0=0.1$ the h-carriers become degenerate ($\mu_h>T$) when $T<W_h/40\approx 30$ K. On the other hand, for $y_0\geqslant 0$, on account of carrier redistribution between bands the h-carrier density increases almost linearly with T, while for $y_0<0$ ($E_{Fl}<E_0$) N_h depends exponentially on T:

$$N_h(T)\propto\exp\left(-\frac{E_0-\mu_l}{T}\right).$$

The calculation of the real part of Π_h performed in the Appendix makes it possible to find, on the basis of the dispersion equation $\mathrm{Re}\,\varepsilon(\mathbf{q},\omega)=0$ and taking into account Eqs. (7) and (8), the dispersion relation for long-wavelength acoustic plasmons ($q_\parallel a\ll 1$) in a layered crystal:

$$\omega(\mathbf{q})\approx q_\parallel\Omega_h\begin{cases}(q^2+4/a_l^*d)^{-1/2}, & qd\ll 1,\\ [d/2(q_\parallel+2/a_l^*)]^{1/2}, & q_\parallel d\gg 1,\end{cases} \quad (15)$$

where

$$\Omega_h=\left\{\frac{4e^2W_h}{\pi\varepsilon_\infty d}[E(\varkappa_h)-(1-\varkappa_h^2)K(\varkappa_h)]\right\}^{1/2} \quad (16)$$

in the case of degenerate h-carriers ($\varkappa_h\equiv\varkappa(\mu_h)$, $\mu_h>T$) and

$$\Omega_h(T)=\left[\frac{\pi e^2a^2W_hN_h(T)}{\varepsilon_\infty d}\frac{I_1(W_h/4T)}{I_0(W_h/4T)}\right]^{1/2} \quad (17)$$

in the case of nondegenerate h-carriers ($\mu_h<T$ or $\mu_h<0$). Here it is assumed that the frequency of the acoustic plasmons is much higher than all phonon frequencies, so that $\varepsilon_i=\varepsilon_\infty$, where ε_∞ is the optical permittivity of the lattice, and $a_l^*=\varepsilon_\infty/e^2m_l^*$. In the limit $q\to 0$ the intersection of the

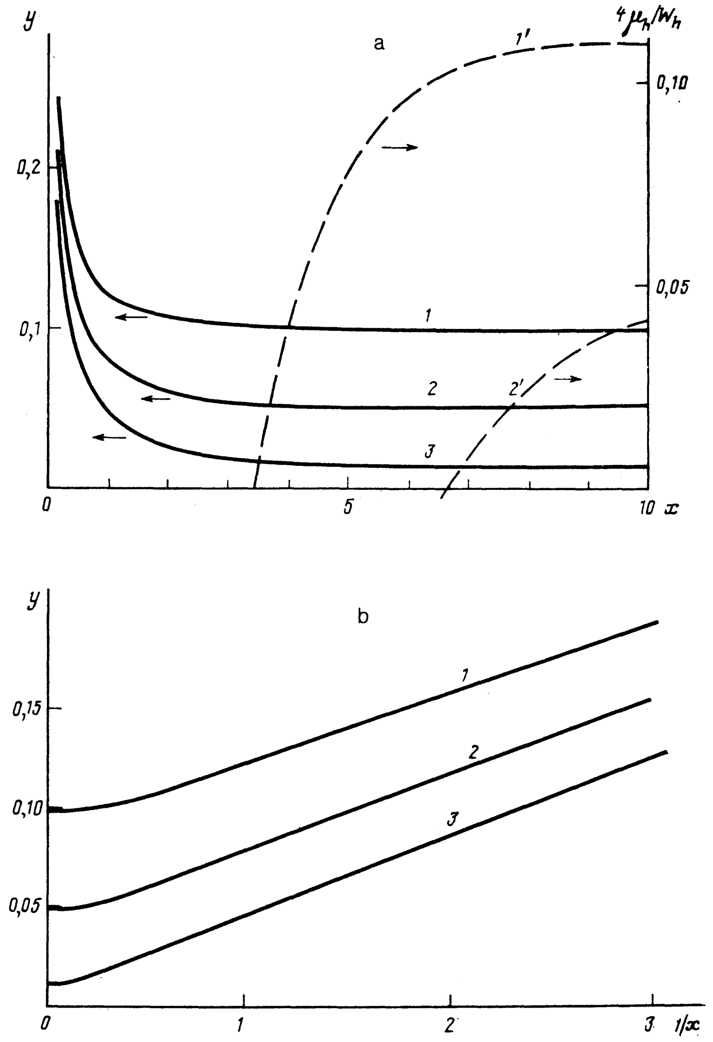

FIG. 2. The dimensionless $2D$ density $y \equiv a^2 N_h$ (solid curves) and the chemical potential $4\mu_h/W_h$ (dashed curves) of nondegenerate h-carriers in a narrow $2D$ band ($\mu_h < 0$ or $\mu_h > 0$, but $T > \mu_h$) as a function of the dimensionless parameter $x \equiv W_h/4T$ (a) and y as a function of x^{-1} (b) for different values of the band-filling parameter $y_0 \equiv a^2(N_0 - 2\nu_l E_0)$: $y_0 = 0.1$ (1), 0.05 (2), and 0.01 (3).

acoustic branch $\omega(\mathbf{q}) \approx 1/2 q_\parallel \Omega_h (a_l^* d)^{1/2}$ with the branches of optical phonons must be taken into account.

Figure 3a shows the dependence, obtained from the formula (16), of Ω_h on the parameter $\delta_h \equiv (1 - \varkappa_h^2)^{1/2}$, and Fig. 3b shows the dependence, obtained with the help of Eqs. (13), (14), and (17), of Ω_h on $x^{-1} \equiv 4T/W_h$. As we can see, on the one hand Ω_h increases rapidly as the narrow band is filled (i.e., as δ_h decreases and $\varkappa_h$ increases), which is as expected, while on the other hand, for $y_0 \geqslant 0.05$ the quantity Ω_h decreases as the temperature increases, in spite of the almost linear increase of the h-carrier density (see Fig. 2b). The latter fact is attributable to the effective increase of the effective mass of h-carriers in the narrow band as T increases, since the contributions of the h-carriers having effective masses of different signs (near the top and bottom of the band) partially cancel one another in collective oscillations.

As shown in the Appendix, the imaginary part of $\Pi_h(\mathbf{q}_\parallel, \omega, T)$ in the case of nondegenerate h-carriers in the narrow $2D$ band for one of the basis directions (for example, $q_\parallel = q_x, q_y = 0$) is equal to[4)]

$$\operatorname{Im} \Pi_h(q_x, \omega, T) = -\frac{N_h(T)\,\mathrm{sh}(\omega/2T)}{I_0(W_h/4T)\,[w^2(q_x) - \omega^2]^{1/2}} \times \mathrm{ch}\left\{\frac{W_h}{4T}\cos\left(\frac{q_x a}{2}\right)\cdot\left[1 - \frac{\omega^2}{w^2(q_x)}\right]^{1/2}\right\} \qquad (18)$$

in the region $|\omega| \leqslant w(q_x) \equiv (W_h/2)\sin(q_x a/2)$ and $\operatorname{Im} \Pi_h = 0$ for $|\omega| > w(q_x)$. Along the diagonal of the Brillouin zone ($q_x = q_y$) the width of the region of damping is doubled ($\operatorname{Im} \Pi_h \neq 0$ for $|\omega| \leqslant W_h \sin(q_\parallel a/2)$.

In the long-wavelength approximation ($q_x a \ll 1$) under the condition $W_h/4 \gg |\omega|$, T the expression (18) reduces to the expression derived in Ref. 39 for the imaginary part of the polarizability, describing quantum Landau damping by nondegenerate carriers in a wide band with a quadratic spectrum:

$$\operatorname{Im} \Pi_h(q_x, \omega, T) = -2N_h(T)\cdot\left(\frac{\pi m_h^*}{2T}\right)^{1/2} \frac{\mathrm{sh}(\hbar\omega/2T)}{\hbar q_x} \times \exp\left\{-\frac{\hbar^2 q_x^2}{8 m_h^* T} - \frac{m_h^* \omega^2}{2T q_x^2}\right\}, \qquad (19)$$

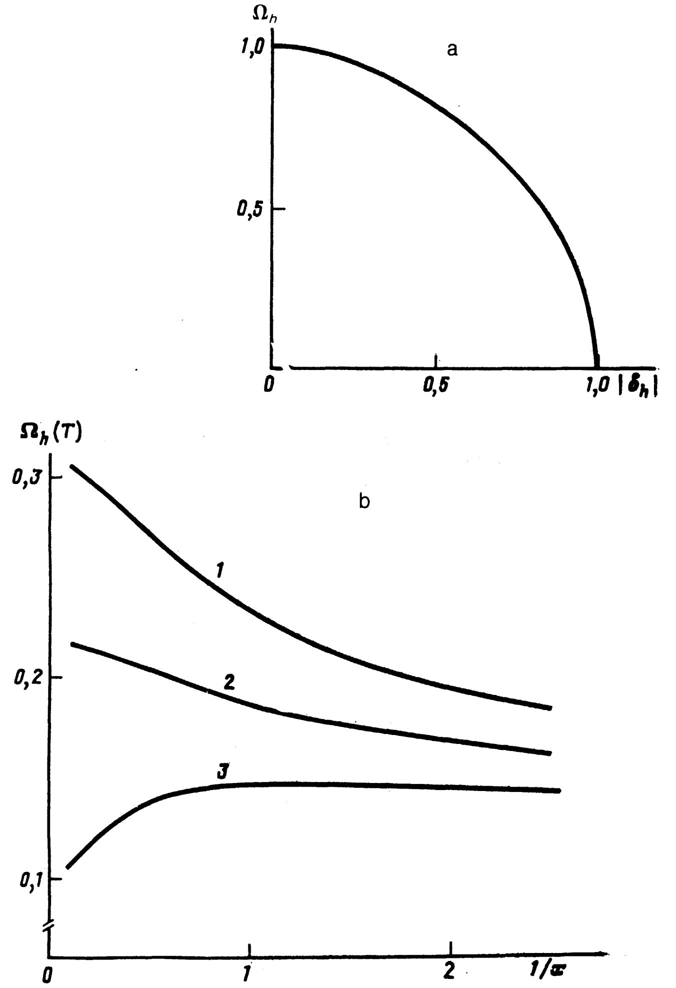

FIG. 3. The plasma frequency of h-carriers as a function of the degree of band filling $\delta_h \equiv 1 - 2\mu_h/W_h$ at $T = 0$ (a) and as a function of the temperature in the case of nondegenerate h-carriers (b) for different values of y_0:$y_0 = 0.10$ (1), 0.05 (2), and 0.01 (3).

where $m_h^* = 4\hbar^2/W_h a^2$ is the effective mass of the h-carriers near the bottom (top) of the band. In the classical limit ($\hbar \to 0$) the well-known expression for the Landau damping rate[40] in a Maxwellian plasma follows from Eq. (19):

$$\gamma = \frac{\pi^{1/2}}{2} \frac{\omega}{q_x^3 R_h^3(T)} \exp\left[-\frac{\omega^2}{q_x^2 \bar{v}_h^2(T)} \right], \tag{20}$$

where $R_h(T) = (T/4\pi e^2 N_h)^{1/2}$ is the Debye screening radius and $\bar{v}_h(T) = (2T/m_h^*)^{1/2}$ is the mean thermal velocity of the particles.[5)]

Since in the short-wavelength region near the edge of the Brillouin zone ($q_x = q_y = \pi/a$) in order of magnitude we have

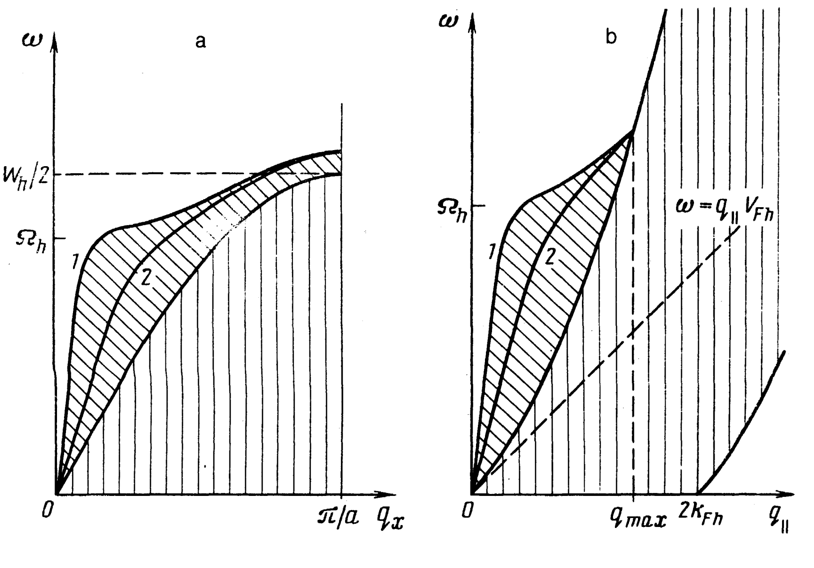

FIG. 4. The dispersion curves of acoustic plasmons in a layered crystal and the region of Landau damping (vertical hatching) in the tight-binding approximation for h-carriers in a narrow 2D band (a) and in the case of wide overlapping bands with a quadratic spectrum of l- and h-carriers (b). The oblique hatching designates regions where there is an effective attractive interelectronic interaction owing to exchange of virtual acoustic plasmons; in this region Re $\varepsilon(q, \omega) < 0$. The curves 1 and 2 correspond, respectively, to longitudinal ($q_z = 0$) and transverse ($q_z = \pi/d$) propagation of acoustic plasmons relative to the plane of the layers.

$\omega(\mathbf{q}) \approx (e^2 W_h / \varepsilon_\infty a)^{1/2}$

(see the Appendix), while the imaginary part Π_h is different from zero in the region

$$|\omega| \leqslant W_h \sin(q_\parallel a/2),$$

it can be shown with the help of Eqs. (15)–(17) that under the condition (3), the acoustic-plasmon branch lies above the region of strong Landau damping in the entire volume of the Brillouin zone, (Fig. 4a).

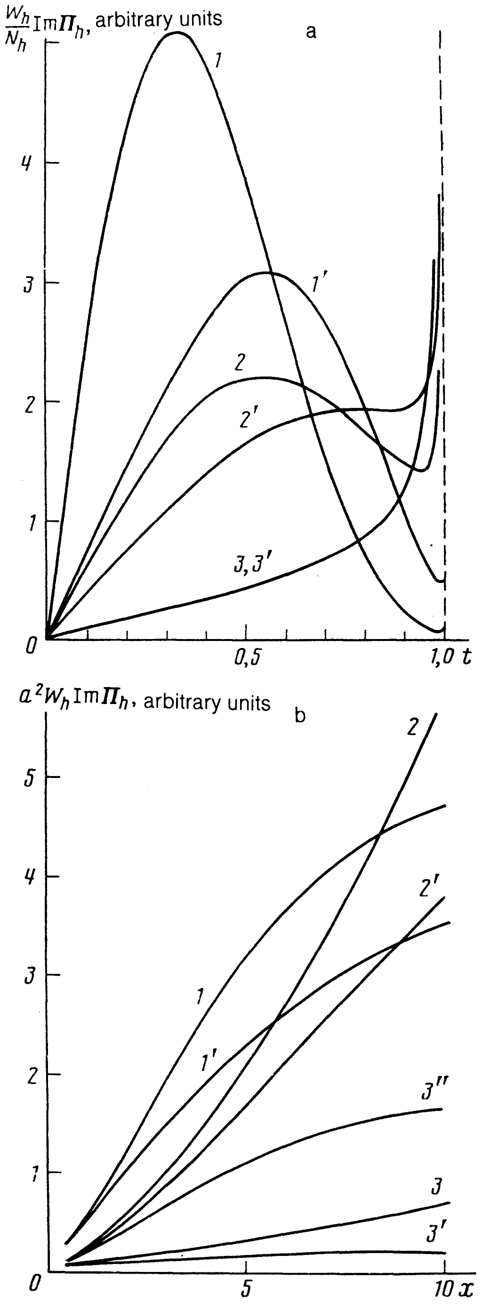

FIG. 5. a) Im $\Pi_h(q_x, \omega, T)$ as a function of $t = \omega/w(q_x)$ for different values of the parameters $q_x a$ and $x \equiv W_h/4T$: $x = 10$ (1, 1'), 4 (2, 2'), 1 (3, 3'); sin $(q_x a/2) = 0.1$ (curves 1–3) or 0.5 (curves 1'–3'). b) Im Π_h as a function of x for different values of $q_x a$, t, and y_0: 1, 1') $t = 0.5$, $y_0 = 0.1$; 2, 2') $t = 0.1$, $y_0 = 0.1$; 3, 3') $t = 0.1$, $y_0 = 0.01$; 3") $t = 0.1$, $y_0 = 0.05$; sin $(q_x a/2) = 0.1$ (curves 1–3) or 0.5 (curves 1'–3', 3").

Hence it follows that in the tight-binding approximation for h-carriers in a narrow $2D$ band the spectrum of the acoustic plasmons in the layered quasi-two-dimensional metal is a periodic function of the longitudinal quasimomentum $\mathbf{q}_\parallel$, analogous to the plasmon spectrum in a chain-type (quasi-$1D$) metal with a narrow $1D$ band[26] and in contrast to the spectrum of acoustic plasmons in quasi-isotropic $3D$ or $2D$ metals with wide overlapping bands and a quadratic spectrum of degenerate l- and h-carriers,[20–25] in which the region of transmission is bounded at large momenta by a wide region of quantum Landau damping (Fig. 4b). The periodicity of the acoustic-plasmon spectrum in the case of the narrow band is related to strong localization of h-carriers on lattice sites (for example, d holes on Cu^{2+} ions), so that the propagation of h-plasmons occurs in the system with an electron density periodically modulated in space.

It must be stressed that in the region

$$W_h \sin\left(\frac{q_\parallel a}{2}\right) < |\omega| < \omega(\mathbf{q})$$

for any $\mathbf{q}_\parallel$ the condition Re $\varepsilon(\mathbf{q}, \omega) < 0$ is satisfied (Fig. 4a), i.e., in the entire volume of the Brillouin zone there exists a retarded attractive interelectronic interaction $V_c(\mathbf{q})\mathrm{Re}\varepsilon^{-1}(\mathbf{q}, \omega) < 0$, which is governed by exchange of virtual acoustic plasmons and should give rise to much more effective Cooper pairing of degenerate l-carriers than in transition metals and doped semiconductors and semimetals with wide bands (valleys), for which the possibility of the "plasmon" mechanism of superconductivity was studied previously.[22–25,41–43]

Figure 5 shows Im $\Pi_h(q_x, \omega, T)$ as a function of $t \equiv \omega/w(q_x)$ and $x \equiv W_h/4T$ for different values of the parameter $q_x a$. One can see that over wide regions of values of ω and T Im Π_h depends almost linearly on ω and T^{-1}; this actually agrees with the phenomenological dependence Im $\Pi_h \propto \omega/T$ for $|\omega| < T$, postulated in Ref. 17. We note in this connection that all basic results concerning relaxation in the model of a marginal Fermi liquid[17] remain valid, if in Eq. (1) we set Im $\Pi = 0$ for sufficiently large $|\omega| > T$. The model of a narrow $2D$ band proposed in this paper corresponds precisely to this case—the absence of strong damping in the region $|\omega| > W_h$, where there remains only weak quantum Landau damping by degenerate l-carriers [see Eq. (8)] as well as Drude damping owing to elastic scattering of l-carriers by lattice defects and impurities.[6)]

Figure 6 shows the quantity $\mathscr{F} \equiv -$ Im $\Pi_h/\sinh(\omega/2T)$ as a function of ω and T taking into account the temperature dependence of the h-carrier density (see Fig. 2) for different degrees of filling of the narrow $2D$ band. One can see that under certain conditions (in particular, when $q_x a \lesssim 1$), neglecting the statistical factor $\sinh(\omega/2T)$ the frequency and temperature dependences of Im Π_h are comparatively weak in wide regions of values of ω and T; this also indicates a dependence close to Im $\Pi_h \propto \omega/T$ for $|\omega| < T$, as is required for a qualitatively correct description of relaxation processes in cuprate MOC. We call attention to the rapid (exponential) decrease of the function $\mathscr{F} = -$ Im $\Pi_h/\sinh(\omega/2T)$ at low temperatures $T \ll W_h/4$ (Fig. 6b). This decrease is associated with the exponential decrease of Landau damping in the region $|\omega| > q_x \bar{v}_h(T)$ (see Fig. 5a) in the classical limit. For the same reason, in

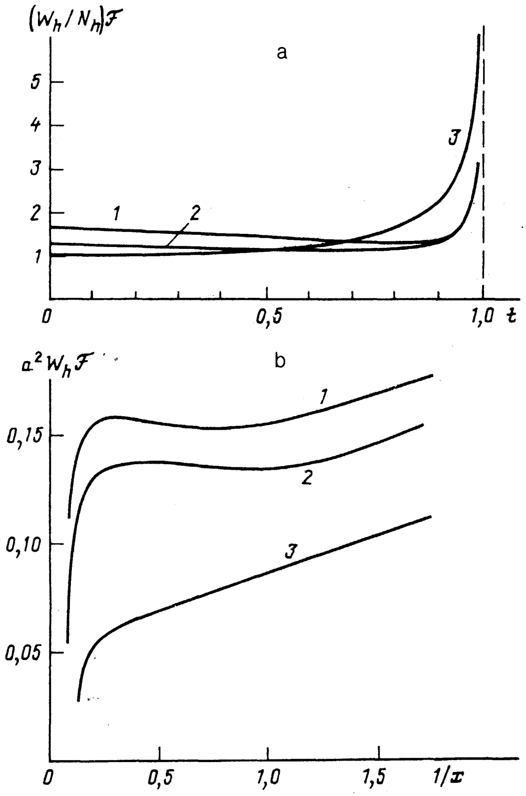

FIG. 6. a) $\mathscr{F} = -\operatorname{Im}\Pi_h(q_x, \omega, T)/\sinh(\omega/2T)$ as a function of $t=\omega/w(q_x)$ for different values of $x\equiv W_h/4T$ and q_xa: 1) $x=2$, $\sin(q_xa/2)=0.1$; 2) $x=2$, $\sin(q_xa/2)=0.5$; 3) $x=0.4$, $\sin(q_xa/2)=0.1$–0.5; b) $\mathscr{F}$ as a function of x^{-1} for $t=0.5$: 1) $y_0=0.1$, $\sin(q_xa/2)=0.2$; 2) $y_0=0.1$, $\sin(q_xa/2)=0.5$; 3) $y_0=0.05$, $\sin(q_xa/2)=0.6$.

the region $|\omega|<q_x\bar{v}_h(T)$ for $q_xa\ll 1$ the dependence $\operatorname{Im}\Pi_h \propto \omega/T$ changes to $\operatorname{Im}\Pi_h\propto\omega/T^{3/2}$ [see Eqs. (19) and (20)].

Thus, as will become evident in the subsequent analysis, in the tight-binding approximation Landau damping by nondegenerate h-carriers in a narrow $2D$ or $1D$ band could be responsible for the anomalous relaxational properties of cuprate metal-oxide compounds, if electron-electron scattering is the principal relaxation channel.

3. ELECTRON-PLASMON INTERACTION AND THE SELF-ENERGY OF l-CARRIERS

Optical measurements[14,15] and calculations[37] of the anisotropic plasma frequency of free current carriers (oxygen p holes) in single crystals of cuprate MOC indicate that the longitudinal effective mass is relatively small[7)] and band calculations[27,30] yield a quite large width for the $2D$ bands ($W_l>1$ eV). In this connection, for l-carriers with densities $n_0\gtrsim 10^{21}$ cm^{-3} (i.e., $N_0\gtrsim 6\cdot 10^{14}$ cm^{-2} and $k_{Fl}\gtrsim 2\cdot 10^7$ cm^{-1} for $d\gtrsim 6$ Å) the dimensionless density parameter in $2D$ layers is

$$r_s=\frac{2^{1/2}e^2}{\varepsilon_\infty v_{Fl}}\equiv\frac{2^{1/2}}{k_{Fl}a_l^*} \tag{21}$$

for $m_l^*\lesssim 2m_0$ and optical permittivity of the crystal $\varepsilon_\infty\gtrsim 4$ (Refs. 14 and 37) falls into in the range of typical values of the electron density of the metal ($r_s\lesssim 6$). Hence it follows that the properties of l-carriers can be described by means of the standard Fermi-liquid approach[19,44] taking into account the quasi-two-dimensionality of the electronic spectrum in layered metals with a cylindrical topology of the Fermi surface.

The screened Coulomb interaction between the l-carriers, taking into account retardation effects associated with the exchange of virtual plasmons, is determined by the equation[45,46]

$$\tilde{V}_C(\mathbf{q},\omega)=V_C(\mathbf{q})+V_C(q)\tilde{\Pi}(\mathbf{q},\omega)\tilde{V}_C(\mathbf{q},\omega)\equiv\frac{V_C(\mathbf{q})}{\varepsilon(\mathbf{q},\omega)}, \tag{22}$$

where $\tilde{\Pi}=\Pi_l+\Pi_h$ is the total electronic polarizability.[8)] In the frequency range $|\omega|>W_h$, where there is no strong Landau damping by h-carriers ($\operatorname{Im}\Pi_h=0$), for $q_\parallel d\gtrsim 1$ and $q_\parallel v_{Fl}\gtrsim|\omega|$, according to Eqs. (6)–(8) and (15), we obtain

$$\operatorname{Re}\varepsilon(\mathbf{q}_\parallel,\omega)\approx\varepsilon_\infty\left[1+\frac{2}{q_\parallel a_l^*}-\frac{\Omega_h^2}{\omega^2}q_\parallel d\right]. \tag{23}$$

As can be seen, $\operatorname{Re}\varepsilon(\mathbf{q},\omega)<0$ holds in the region $|\omega|<\Omega_h[q_\parallel d/(1+2/q_\parallel a_l^*)]^{1/2}$ (but $|\omega|>W_h\sin(q_\parallel a/2)$).

In the static limit ($\omega\to 0$), in the case of nondegenerate h-carriers we find

$$\varepsilon(\mathbf{q}_\parallel,0)\approx\varepsilon_0\left[1+\frac{2\varepsilon_\infty}{q_\parallel a_l^*\varepsilon_0}+\frac{d}{q_\parallel R_h^2(T)}\right], \tag{24}$$

where ε_0 is the static permittivity of the lattice, and $R_h(T)=[\varepsilon_0T/4\pi e^2N_h(T)]^{1/2}$ is the Debye radius of screening by nondegenerate h-carriers.

In order to describe the retarded interaction of degenerate l-carriers with acoustic plasmons we introduce the plasmon Green's function

$$D_{pl}(\mathbf{q},\omega)=V_C(\mathbf{q})[\varepsilon^{-1}(\mathbf{q},\omega)-\varepsilon^{-1}(\mathbf{q},\omega_{max})], \tag{25}$$

where $\omega_{max}\gg\Omega_h$, but $\omega_{max}\lesssim\min\{E_{Fl},\Omega_l\}$, since for energies in the range $|\omega|\gtrsim E_{Fl}$ rapid damping (decay) of quasiparticles occurs on account of the Fermi-liquid interaction,[44] while in the region $|\omega|\gtrsim\Omega_l$ (where Ω_l is the plasma frequency of l-carriers) strong quantum Landau damping by l-carriers occurs.

Since the Kramers-Kronig relation[19]

$$\frac{1}{\varepsilon(\mathbf{q},\omega)}=1+\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Im}\varepsilon^{-1}(\mathbf{q},\omega')}{\omega'-\omega-i\delta}d\omega'\ (\delta\to+0) \tag{26}$$

is valid for the response function $\varepsilon^{-1}(\mathbf{q},\omega)$ for any $\mathbf{q}$ and ω, the Green's function (25) satisfies the dispersion relation

$$D_{pl}(\mathbf{q},\omega)=-\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\operatorname{Im}D_{pl}(\mathbf{q},\omega')}{\omega-\omega'+i\delta}d\omega', \tag{27}$$

if in the region $\omega\sim\omega_{max}$ the damping is weak: $\operatorname{Im}\varepsilon^{-1}(\mathbf{q},\omega_{max})\to 0$.

The single-particle self-energy of l-carriers Σ_{pl}, associated with the electron-plasmon interaction, in a layered metal with a slightly fluted and isotropic (in the plane of the layers) cylindrical Fermi surface and with a quadratic spectrum in a wide $2D$ band for $T\neq 0$, taking into account Umklapp processes with respect to the transverse quasimomentum p_z, can be represented in the following form:

$$\Sigma_{pl}(\mathbf{p}_\parallel, \tilde{p}_z, i\omega_n) = \nu_l T \sum_{\omega_m} \int_0^{2\pi} \frac{d\varphi}{2\pi} \int_{-\pi}^{\pi} \frac{d\tilde{p}_z'}{2\pi} \int_{-E_{Fl}}^{\infty} d\xi_l$$

$$\times \sum_{k=-\infty}^{\infty} G_l(\mathbf{p}_\parallel', \tilde{p}_z' + 2\pi k, i\omega_m)$$

$$\times D_{pl}(\mathbf{p}_\parallel' - \mathbf{p}_\parallel, p_z' - p_z + 2\pi k, i\omega_m - i\omega_n), \tag{28}$$

where $\omega_n = (2n+1)\pi T$ are the discrete imaginary "frequencies" ($n = 0, \pm 1, \pm 2, ...$), φ is the angle between the momenta $\mathbf{p}_\parallel$ and $\mathbf{p}_\parallel'$ of the interacting electrons in the plane of the layers, G_l is the Green's function of the l-carriers, ξ_l is the energy of the l-carriers relative to the Fermi level, and $\tilde{p}_z \equiv p_z d$. Transforming to real continuous frequencies ω and separating from Eq. (28) the odd part as a function of ω

$$f_{pl}(\omega) = {}^1/_2[\Sigma_{pl}(\omega) - \Sigma_{pl}(-\omega)],$$

the pole part of G_l can be put into the form[9)]

$$G_l(\xi_l, \tilde{p}_z, \omega) = \frac{|\Psi_l(\tilde{p}_z)|^2}{\omega Z_{pl}(\omega) - \xi_l + i\gamma_{pl}(\omega)}, \tag{29}$$

where

$$Z_{pl}(\omega) = 1 - \frac{\mathrm{Re}\, f_{pl}(\omega)}{\omega}, \quad \gamma_{pl}(\omega) = -\mathrm{Im}\, f_{pl}(\omega), \tag{30}$$

and $\Psi_l(\tilde{p}_z)$ is the Fourier component of the transverse part of the wave function $\Psi_l(z)$ of the l-carriers, localized in 2D layers of thickness d_0. Note that in order to take into account the electron-phonon interaction the corresponding phonon function $f_{\text{ph}}(\omega)$, which contributes to the renormalization of the quasiparticle spectrum and to the damping of quasiparticles in the frequency range $|\omega| \lesssim \omega_D$ (where ω_D is the Debye frequency of the phonons), must be added to $f_{\text{pl}}(\omega)$. For $\Omega_h \gg \omega_D$, however, in the frequency range $|\omega| \gtrsim \Omega_h$ the effects associated with electron-phonon interaction can be neglected.

As a result we find from Eqs. (28) and (29), performing the averaging over the cylindrical Fermi surface of the l-carriers,

$$\mathrm{Re}\, f_{pl}(\omega) \equiv \omega[1 - Z_{pl}(\omega)]$$

$$= \frac{1}{2} \int_0^\infty d\omega' \Big\{ [\mathrm{Re}\, Q_{pl}(\omega' - \omega) - \mathrm{Re}\, Q_{pl}(\omega' + \omega)]$$

$$\times \mathrm{th}\left(\frac{\omega}{2T}\right) - [\Phi_{pl}(\omega' - \omega, T) - \Phi_{pl}(\omega' + \omega, T)] \Big\}, \tag{31}$$

$$\gamma_{pl}(\omega) = \frac{1}{2} \int_0^\infty d\omega' \Big\{ \mathrm{Im}\, Q_{pl}(\omega' - \omega) \cdot \Big[\mathrm{th}\left(\frac{\omega'}{2T}\right)$$

$$- \mathrm{cth}\left(\frac{\omega' - \omega}{2T}\right) \Big]$$

$$+ \mathrm{Im}\, Q_{pl}(\omega' + \omega) \cdot \left[\mathrm{th}\left(\frac{\omega'}{2T}\right) - \mathrm{cth}\left(\frac{\omega' + \omega}{2T}\right) \right] \Big\}, \tag{32}$$

where

$$Q_{pl}(\omega) = \frac{2}{\pi} \tilde{\nu}_l \int_0^{2k_{Fl}} \frac{dq_\parallel}{(4k_{Fl}^2 - q_\parallel^2)^{1/2}} \int_0^{2\pi} \frac{d\varphi}{2\pi} D_{pl}(q_\parallel, \varphi, \omega), \tag{33}$$

$$\Phi_{pl}(\omega, T) = -\int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \frac{\mathrm{Im}\, Q_{pl}(\omega')}{\omega - \omega'} \mathrm{cth}\left(\frac{\omega'}{2T}\right); \tag{34}$$

and $\tilde{\nu}_l$ is the renormalized (owing to Umklapp processes with respect to p_z) effective density of states, equal in order of magnitude to $\tilde{\nu}_l \approx \nu_l d/d_0$. It should be noted that, generally speaking, the kernel of the interaction $Q_{\text{pl}}(\omega)$, just like the polarization operators Π_l and Π_h, should contain a Coulomb vertex part $\Gamma_C(\mathbf{p}, \pi; \mathbf{p}', \omega')$, which describes the effects due to the local field[51] and satisfies, on the Fermi surface with zero transferred energies and momenta ($|\omega' - \omega| \to 0$, $|\mathbf{p}' - \mathbf{p}| \to 0$), the Ward-Pitaevskiĭ identity:[52,53]

$$\Gamma_C(k_{Fl}, 0; k_{Fl}, 0) \equiv 1 - \frac{\partial\, \mathrm{Re}\, f_{pl}(\omega)}{\partial \omega}\bigg|_{\omega=0} \equiv Z_{pl}(0). \tag{35}$$

Since, however, Γ_C decreases quite rapidly as $|\omega' - \omega|$ and $q \equiv |\mathbf{p}' - \mathbf{p}|$ increase and since on account of the square-root singularity large values of $q_\parallel \approx 2k_{\text{Fl}}$ make the main contribution to the integral (33), to a good approximation we can set $\Gamma_C = 1$. This gives an electron-plasmon interaction constant that is somewhat too low, while substituting into the integrand of the expression (33) and into $\tilde{\Pi} = \Pi_l + \Pi_h$ the value of Γ_C determined by the relation (35) gives values of $Q_{\text{pl}}(\omega)$ that are too high.

Near the Fermi surface ($\omega \to 0$), taking into account (7), for $q_\parallel d > 1$ we obtain from Eq. (31)

$$\mathrm{Re}\, f_{pl}(\omega) = 2\omega\, \mathrm{Re}\, Q_{pl}(0) = \omega(\mu_0 - \mu_\infty), \tag{36}$$

where

$$\mu_0 = 4e^2 \tilde{\nu}_l \int_0^{2k_{Fl}} \frac{dq_\parallel}{q_\parallel (4k_{Fl}^2 - q_\parallel^2)^{1/2}} \int_0^{2\pi} \frac{d\varphi}{2\pi} \varepsilon^{-1}(q_\parallel, \varphi, 0), \tag{37}$$

$$\mu_\infty = 4e^2 \tilde{\nu}_l \int_0^{2k_{Fl}} \frac{dq_\parallel}{q_\parallel (4k_{Fl}^2 - q_\parallel^2)^{1/2}} \int_0^{2\pi} \frac{d\varphi}{2\pi} \mathrm{Re}\, \varepsilon^{-1}(q_\parallel, \varphi, \omega_{max}). \tag{38}$$

The dimensionless positive quantity

$$\lambda_{pl} \equiv \mu_\infty - \mu_0 > 0 \tag{39}$$

is the electron-plasmon interaction constant, so that $Z_{\text{pl}}(0) = 1 + \lambda_{\text{pl}}$.[10)]

For frequencies in the range $|\omega| > W_h \sin k_{\text{Fl}} a$, where there is no strong Landau damping by h-carriers ($\mathrm{Im}\, \Pi_h = 0$), the real part of the kernel in Eq. (33) can be approximated by the simple expression

$$\mathrm{Re}\, Q_{pl}(\omega) \approx \lambda_{pl} \left[\frac{\tilde{\Omega}_h^2}{\omega^2 - \tilde{\Omega}_h^2} - \frac{\tilde{\Omega}_h^2}{\omega_{max}^2 - \tilde{\Omega}_h^2} \right], \tag{40}$$

where, according to Eq. (15), for $q_\parallel \approx 2k_{\text{Fl}} > d^{-1}$ we have

$$\tilde{\Omega}_h \approx \Omega_h \left(\frac{2k_{Fl} d}{1 + \alpha_\infty} \right)^{1/2}, \quad \alpha_\infty = \frac{e^2 m_l^*}{k_{Fl} \varepsilon_\infty} \equiv \frac{1}{k_{Fl} a_l^*}. \tag{41}$$

Thus, in contrast to the hypothetical marginal Fermi liquid,[17] in the case at hand we have a standard charged Fermi liquid with a well-defined single-particle spectrum [see Eq. (29)] and with a finite effective mass of quasiparticles on the Fermi surface. Taking into account the electron-plasmon and electron-phonon interactions this effective mass is determined by the expression

$$\tilde{m}_l^*=m_l^*(1+\lambda_{pl}+\lambda_{ph}).$$

4. ELECTRON-PLASMON INTERACTION AND ANOMALIES OF THE THERMODYNAMIC, KINETIC, AND OPTICAL PROPERTIES OF CUPRATE MOC

a) Electronic heat capacity

We assume that degenerate l-carriers in a wide $2D$ band make the main contribution to the temperature dependence of the electronic heat capacity of MOC.[11] To calculate their heat capacity $C_l(T)$ at constant volume V we determine the thermodynamic potential of the Fermi gas of quasiparticles with the spectrum $E_l(\xi)$, which is determined by the pole of the Green's function (29) in the limit $\gamma_{\rm pl}\to 0$ (compare with Ref. 54):

$$\Omega(T)=-BT\int_0^\infty d\xi \ln\left[1+\exp\left\{-\frac{E_l(\xi)}{T}\right\}\right]$$

$$=-B\int_0^\infty \frac{\omega-\operatorname{Re} f_{pl}(\omega)}{e^{\omega/T}+1}d\omega, \tag{42}$$

where $B=2\tilde{v}_l SN$, S is the area of one $2D$ layer, and N is the number of conducting layers of CuO_2 in the crystal (we have in mind an ideal single crystal with the volume $V=SNd$).

Neglecting the damping of quasiparticles ($\operatorname{Im} Q_{\rm pl}=0$), we use the approximate formula (40) to calculate $\operatorname{Re} f_{\rm pl}(\omega)$. At sufficiently low temperatures $T\ll\tilde{\Omega}_h$ we find from Eqs. (31) and (40)

$$\operatorname{Re} f_{pl}(\omega)=\frac{1}{2}\left[\int_{-\omega}^{\infty} d\omega' \operatorname{Re} Q_{pl}(\omega')-\int_{\omega}^{\infty} d\omega' \operatorname{Re} Q_{pl}(\omega')\right]$$

$$\approx -\lambda_{pl}\frac{\tilde{\Omega}_h}{2}\ln\left|\frac{\tilde{\Omega}_h+\omega}{\tilde{\Omega}_h-\omega}\right|. \tag{43}$$

The expression (43) satisfies the asymptotic relation (36), and the logarithmic singularity at the point $\omega=\tilde{\Omega}_h$ is suppressed by finite damping (see below).

Expanding the expression (43) in powers of the ratio $\omega/\tilde{\Omega}_h$ and substituting the series into Eq. (42), we obtain to within terms of order $(T/\tilde{\Omega}_h)^2$

$$\Omega(T)\approx-\frac{\pi^2}{12}BT^2\left[(1+\lambda_{pl})+\frac{\pi^2}{2}\lambda_{pl}T^2/\tilde{\Omega}_h^2\right]. \tag{44}$$

Hence we obtain the following expression for the entropy $S_l(T)$ and heat capacity $C_l(T)$ of l-carriers (for $\mu_l=$ const and $V=$ const):

$$S_l(T)\approx\frac{\pi^2}{6}BT[(1+\lambda_{pl})+\pi^2\lambda_{pl}T^2/\tilde{\Omega}_h^2], \tag{45}$$

$$C_l(T)\approx\frac{\pi^2}{6}BT[(1+\lambda_{pl})+3\pi^2\lambda_{pl}T^2/\tilde{\Omega}_h^2]. \tag{46}$$

As we can see, a supralinear temperature dependence of the electronic heat capacity ($\sim T^3$) should be observed at sufficiently high temperatures $T\gtrsim 0.1\tilde{\Omega}_h$; this agrees qualitatively with the experiment of Ref. 12.

b) Conductivity and resistivity

The conductivity of a two-band metal with wide and narrow bands is equal to

$$\sigma(T)=\sigma_l(T)+\sigma_h(T), \tag{47}$$

where σ_l and σ_h are the partial conductivities of l- and h-carriers.

Let us assume that the inelastic scattering of l-carriers by collective excitations of the charge density of h-carriers with characteristic time $\tau_l\equiv\gamma_{\rm pl}^{-1}(0)$, where $\gamma_{\rm pl}(0)$ is the damping rate of quasiparticles on the Fermi surface and is related with the Landau damping of l-plasmons by h-carriers, makes the main contribution to σ_l.

According to Eq. (32), we obtain

$$\gamma_{pl}(0)=\int_0^\infty d\omega \operatorname{Im} Q_{pl}(\omega)\left[\operatorname{th}\left(\frac{\omega}{2T}\right)-\operatorname{cth}\left(\frac{\omega}{2T}\right)\right]$$

$$\equiv-\int_0^\infty d\omega\frac{\operatorname{Im} Q_{pl}(\omega)}{\operatorname{sh}(\omega/2T)\operatorname{ch}(\omega/2T)}, \tag{48}$$

where, taking into account Eq. (15) and the square-root singularity in the integrand in Eq. (33) at the point $q=2k_{\rm Fl}$, to a good approximation we can set

$$\operatorname{Im} Q_{pl}(\omega)\approx\frac{e^2}{2v_{Fl}}\frac{\tilde{v}_l}{v_l}\int_0^{2\pi}\frac{d\varphi}{2\pi}\operatorname{Im}\varepsilon^{-1}(2k_{Fl},\varphi,\omega). \tag{49}$$

Taking into account the fact that $|\operatorname{Im}\Pi_h|\gg|\operatorname{Im}\Pi_l|$, and representing the quantity $\operatorname{Im}\varepsilon^{-1}\propto\operatorname{Im}\Pi_h$ averaged over φ, in accordance with Eq. (18), in the form of the product $\sinh(\omega/2T)\mathcal{F}(\omega, T)$, we can put Eq. (48) into the following form:

$$\gamma_{pl}(0)=T\int_0^\infty\frac{\tilde{\mathcal{F}}(x,T)}{\operatorname{ch} x}dx=TP(T), \tag{50}$$

where the function $P(T)$ is virtually independent of T, i.e., $P\approx$const, in a wide range of temperatures $T\gtrsim W_h/40$.[12] This is confirmed by the relatively weak ω and T dependence of the quantity $\operatorname{Im}\Pi_h/\sinh(\omega/2T)$ in wide intervals of ω and T (see Figs. 6a and b) and the almost linear temperature dependence of the quantity $|\operatorname{Im}\Pi_h(q_x,\omega,T)|/\sinh(\omega/2T)\cosh(\omega/2T)$, integrated over ω, for different values of the parameters $q_x a$ and $y_0\equiv a^2(N_0-2v_l E_0)$ shown in Fig. 7. It follows from Eq. (50) that the partial conductivity of degenerate l-carriers, which is governed by the scattering of these carriers by damping acoustic plasmons, is inversely proportional to T:

$$\sigma_l\equiv\frac{\Omega_l^2\tau_l}{4\pi}\propto\gamma_{pl}^{-1}(0)\propto T^{-1}. \tag{51}$$

The temperature dependence of the conductivity of nondegenerate h-carriers $\sigma_h\equiv\Omega_h^2\tau_h/4\pi$ is determined by the temperature dependence of the relaxation time τ_h and

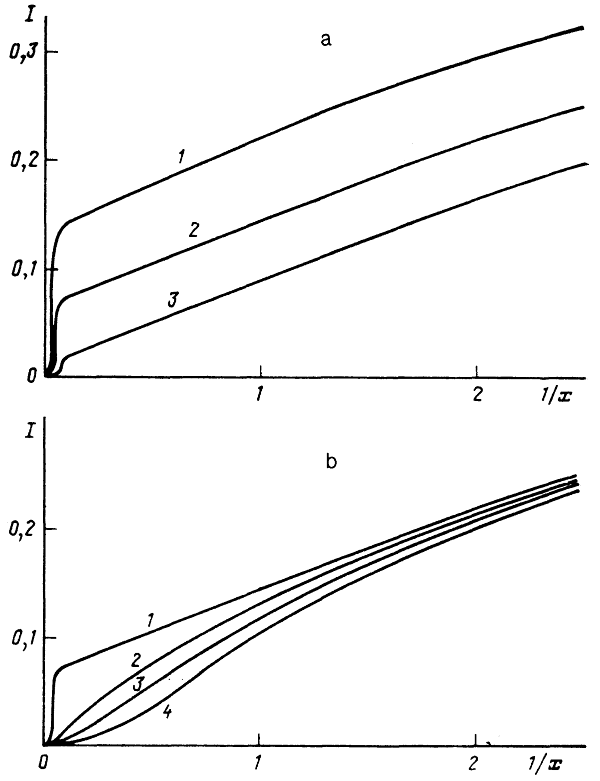

FIG. 7. The integral $I \equiv \int_0^\infty [|\mathrm{Im}\Pi_h(q_x, \omega, T)| d\omega]/[\sinh(\omega/2T)\cosh(\omega/2T)]$ as a function of T for different values of the parameters $q_x a$ and y_0: a) the curves 1–3 correspond to $y_0 = 0.1, 0.05$, and 0.01 with $\sin(q_x a/2) = 0.1$; b) the curves 1, 2, 3, and 4 correspond to $\sin(q_x a/2) = 0.1, 0.5, 0.7$, and 0.9 with $y_0 = 0.05$.

their plasma frequency Ω_h (see Fig. 3b).

The relaxation of h-carriers is determined primarily by their Coulomb scattering by l-carriers, since the electron-phonon interaction and the Drude scattering by defects and impurities are significantly suppressed for them (because of strong localization near lattice sites). In this case the relaxation time can be estimated from the formula $\tau_h \approx l_c/\bar{v}_h(T)$, where l_C is the temperature-independent Coulomb mean free path of a charged particle in the degenerate Fermi liquid of l-carriers, and $\bar{v}_h \propto T^{1/2}$ is the mean thermal velocity of nondegenerate h-carriers,[13] so that $\tau_h \propto T^{-1/2}$.

Figure 8 shows curves of Ω_h^2 versus T for different values of y_0 as well as the dependence of the quantity $(\Omega_h^2\tau_h)^{-1} \propto \sigma_h^{-1}$ on T, which to a good approximation is linear in the entire temperature interval studied, $0.025W_h < T < 0.6W_h$, corresponding for $W_h \approx 0.1$ eV to $30 < T < 700$ K. Hence it follows that in this interval we have $\sigma_h(T) \propto T^{-1}$.

Thus we obtain a nearly linear temperature dependence for the resistivity at temperatures $T > T_c$:

$$\rho(T) \equiv [\sigma_l(T) + \sigma_h(T)]^{-1} \approx \rho_0 + BT. \tag{52}$$

This agrees with the experimental data of Refs. 10 and 11 for most superconducting cuprate MOC in the normal (metallic) state.[14]

c) Nuclear-spin relaxation and magnetic susceptibility

In a normal paramagnetic metal the inverse nucelar-spin relaxation time caused by the creation of virtual excitations of the spin density (paramagnons) is determined, as is well known, by the expression

$$\tau_S^{-1}(T) \propto -\lim_{\omega\to 0}\left[\frac{T}{\omega}\int \frac{d^2q_\parallel}{(2\pi)^2}\,\mathrm{Im}\,\chi(\mathbf{q}_\parallel, \omega)\right]. \tag{53}$$

Here the imaginary part of the paramagnetic (spin) susceptibility χ has the same structure as the imaginary part of the electronic polarizability:

$$\mathrm{Im}\chi \propto \mathrm{Im}\,\Pi_h = -\mathrm{sh}(\omega/2T)\cdot\mathcal{F}(\mathbf{q}_\parallel, \omega, T),$$

where $\mathcal{F}$, according to Eq. (18), at $\omega = 0$ and in the limit $q_\parallel = q_x \to 0$ has the form

$$\mathcal{F}_0 \equiv \mathcal{F}(q_x, 0, T) = 2\frac{N_h(T)}{I_0(W_h/4T)}\,\frac{\mathrm{ch}(W_h/4T)}{W_h \sin(q_x a/2)}. \tag{54}$$

Figure 9 shows plots of the function (54) versus T, taking into account the temperature dependence of $N_h(T)$ for different degrees of filling of the narrow band (see Fig. 2b). It should be kept in mind that since in the limit $\omega \to 0$ $\mathcal{F}$ has a singularity at the point $q_\parallel = 0$, the region of small values of $q_\parallel a \ll 1$ makes the main contribution to the integral over $\mathbf{q}_\parallel$ in Eq. (53), so that the variables T and $\mathbf{q}_\parallel$ in Im χ separate for any directions, analogously to Eq. (54). For this reason the temperature dependence shown in Fig. 9 actually deter-

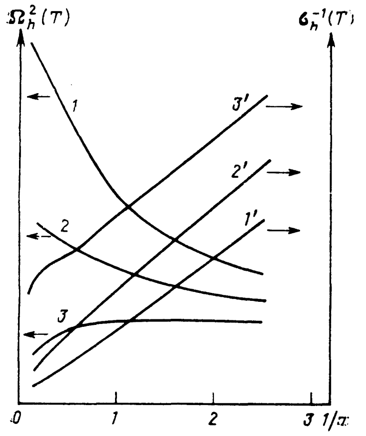

FIG. 8. Ω_h^2 as a function of T for different values of y_0 (curves 1–3) and $\sigma_h^{-1} \propto (\Omega_h^2\tau_h)^{-1}$ as a function of T (curves 1′–3′) for nondegenerate h-carriers in a narrow $2D$ band: $y_0 = 0.1$ (1, 1′), 0.05 (2, 2′), and 0.01 (3, 3′).

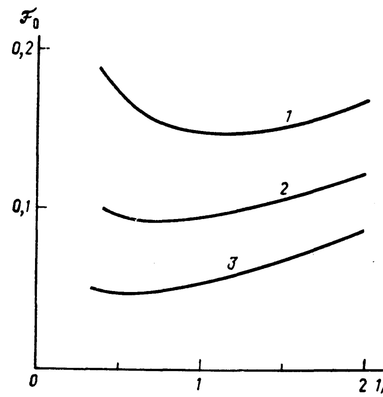

FIG. 9. The temperature dependence of the function $\mathcal{F}(q_x, 0, T)$ in the limit $q_x \to 0$, determining the temperature dependence of the inverse nuclear-spin relaxation time τ_S^{-1}: $y_0 = 0.1$ (1), 0.05 (2), and 0.01 (3).

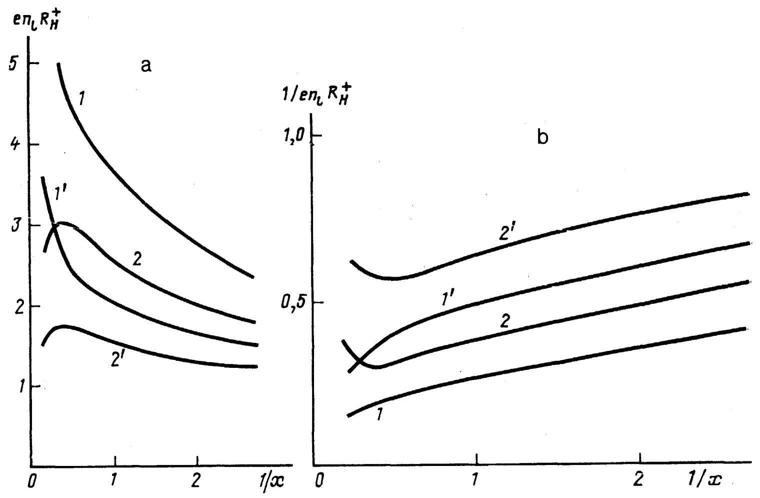

FIG. 10. The Hall constant R_H^+ (a) and $1/R_H^+$ (b) for l- and h-carriers of the same type for $y_0 = 0.05$ and $\sigma_l/\sigma_h = 4$ (curves 1 and 1') and for $y_0 = 0.1$ and $\sigma_l/\sigma_h = 2.5$ (curves 2 and 2').

mines the temperature dependence of τ_S^{-1}. One can see that this dependence differs significantly from Korringa's law $\tau_S^{-1} \propto T$ for normal metals; this agrees qualitatively with the experimental data for ^{63}Cu and ^{65}Cu nuclei in cuprate MOC.[13]

On the other hand, a nearly linear temperature dependence $\tau_S^{-1}(T)$ is observed for ^{17}O nuclei in $YBa_2Cu_3O_7$.[13] This could indicate that h-carriers (d-holes) and the spin-density excitations associated with them are localized near the copper ions Cu^{2+}, while the density of delocalized l-carriers (p-holes) predominates in the neighborhood of the oxygen ions O^{2-} thanks to the direct overlapping of p-orbitals in the CuO_2 cuprate layers.[33,34]

Note that the experimentally observed[55] weak temperature dependence of the magnetic susceptibility $\chi(T)$ in $YBa_2Cu_3O_{7-x}$ in the temperature interval $T = 100$–300 K can be explained by the contribution to $\chi(T)$ of the temperature-independent Pauli susceptibility χ_1 of degenerate l-carriers in the wide $2D$ band as well as by the weak temperature dependence of the paramagnetic susceptibility of nondegenerate h-carriers in the narrow $2D$ band $\chi_h(T) \propto N_h(T)/T$ thanks to the almost linear temperature dependence of N_h (see Fig. 2b).

d) Hall Effect

The Hall constant R_H of a two-band metal is expressed in terms of the partial conductivities (σ_l and σ_h) and the average number densities of the carriers in the bands (n_l and n_h) as follows:

$$R_H^{\pm} = \left(\frac{\sigma_l}{\sigma_l+\sigma_h}\right)^2 \frac{1}{en_l} \pm \left(\frac{\sigma_h}{\sigma_l+\sigma_h}\right)^2 \frac{1}{en_h}, \qquad (55)$$

where the plus and minus signs correspond, respectively, to l- and h-carriers with the same and different signs of the effective masses (charges), and the resulting sign of R_H is determined by the sign of the charge of those carriers (electrons or holes) that make the main contribution to the Hall effect.

Since, as shown above, σ_l and σ_h have virtually identical temperature dependences ($\sigma_{l,h} \propto T^{-1}$) and $n_l \approx$const, the temperature dependence of R_H^+ is determined by the temperature dependence of $n_h(T)$, which is shown in Fig. 2b.

Figure 10 shows, respectively, the temperature dependences of $R_H^+(T)$ and $1/R_H^+(T)$ in the case of l- and h-carriers of the same sign (p and d holes, see Fig. 1b). These dependences were obtained from the formula (55) using the curves in Fig. 2b for $y_0 = 0.05$ with $\sigma_l/\sigma_h = 4$ (curves 1 and 1', respectively) and for $y_0 = 0.1$ with $\sigma_l/\sigma_h = 2.5$ (curves 2 and 2') under the condition that the density of l-carriers corresponds to one l-carrier per unit cell. As we can see, for $W_h \approx 0.1$ eV the temperature dependences $1/R_h^+(T)$ are nearly linear in the temperature interval $100 < T < 700$ K; this is in good agreement with the experimental data.[10,56]

Note that for oppositely charged l- and h-charge carriers the sign of the Hall constant $R_H^-(T)$ can change, depending on the temperature.

Thus the two-band model, combined with electron-electron scattering of nondegenerate h-carriers by l-carriers and inelastic relaxation of degenerate l-carriers by acoustic plasmons, makes it possible to describe different temperature anomalies of the Hall effect in cuprate MOC.

We note that the anomalous carrier-density dependence of R_H is also explained on the basis of the two-band model with wide and narrow $2D$ bands which overlap near the Fermi level.[36]

e) Optical conductivity

Taking into account both elastic scattering of l-carriers by defects and impurities with characteristic time τ_D (Drude damping) and inelastic relaxation of quasiparticles by h-plasmons with the damping rate $\gamma_{pl}(\omega)$ owing to Landau damping by h-carriers, the ω-dependent high-frequency conductivity $\sigma(\omega)$, which determines the IR reflectance, has the following form:

$$\sigma(\omega) = \frac{e^2 n_l}{\tilde{m}_l^*(\omega)} \frac{\tilde{\gamma}_{pl}(\omega) + \tau_D^{-1}}{\omega^2 + [\tilde{\gamma}_{pl}(\omega) + \tau_D^{-1}]^2}, \qquad (56)$$

where, according to Eqs. (29) and (30), the renormalized mass and damping rate are equal to

$$\tilde{m}_l^*(\omega) = m_l^* Z_{pl}(\omega), \quad \tilde{\gamma}_{pl}(\omega) = \gamma_{pl}(\omega)/Z_{pl}(\omega). \qquad (57)$$

If the finite Landau damping of virtual h-plasmons in the region $\omega > W_h$ by l-carriers as well as the Drude damping owing to scattering of l-carriers by lattice defects and acoustic phonons are taken into account, then the real part of the electron-plasmon interaction kernel assumes the following form under the condition $\omega_{\max} \gg \tilde{\Omega}_h$ [compare with Eq. (40)]:

$$\operatorname{Re} Q_{pl}(\omega) \approx \lambda_{pl}\left[\frac{\tilde{\Omega}_h^2(\omega^2-\tilde{\Omega}_h^2)}{(\omega^2-\tilde{\Omega}_h^2)^2+4\gamma_h^2\omega^2} - O\left(\frac{\tilde{\Omega}_h^2}{\omega_{max}^2}\right)\right], \tag{58}$$

where γ_h is the damping rate of h-plasmons in the resonance region ($\omega \approx \tilde{\Omega}_h \gg \gamma_h$). In this case, instead of Eq. (43) we obtain, accurate to the leading-order (logarithmic) terms,

$$\operatorname{Re} f_{pl}(\omega) \approx -\frac{\lambda_{pl}}{4}\tilde{\Omega}_h \ln\left|\frac{(\tilde{\Omega}_h+\omega)^2+\gamma_h^2}{(\tilde{\Omega}_h-\omega)^2+\gamma_h^2}\right|. \tag{59}$$

Figure 11a shows the curves of $Z_{\text{pl}}(\omega) \equiv 1 - \operatorname{Re} f_{\text{pl}}(\omega)/\omega$ versus ω for different values of γ_h.

According to Eq. (32), at low frequencies $\gamma_{\text{pl}}(\omega)$ is a quadratic function:

$$\gamma_{pl}(\omega) \approx \gamma_{pl}(0) + \frac{\omega^2}{2}\gamma_{pl}''(0), \tag{60}$$

while at high frequencies $|\omega| \gg W_h$ terms with a small frequency difference $|\omega' - \omega| \lesssim W_h$ make the main contribution to $\gamma_{\text{pl}}(\omega)$, since in the region $|\omega' \pm \omega| > W_h$ there is no strong Landau damping by h-carriers. For this reason, to a good approximation in the high-frequency region we can set

$$\gamma_{pl}(\omega) \approx -\frac{1}{2}\int_{-\omega}^{W_h} d\omega' \operatorname{Im} Q_{pl}(\omega')\operatorname{cth}(\omega'/2T). \tag{61}$$

Comparing the expression (61) with Eq. (48), we can see that the integrand in the expression (61) contains the additional factor $\coth^2(\omega'/2T)$, which is always greater than unity and rapidly increases as ω' increases. For this reason, in spite of the factor 1/2, for finite ω the inequality $\gamma_{\text{pl}}(\omega) > \gamma_{\text{pl}}(0)$ is always satisfied, and as ω increases $\gamma_{\text{pl}}(\omega)$ approaches its limiting value $\gamma_\infty \equiv \gamma_{\text{pl}}(\omega)$.

Figure 11b shows γ_{pl} (dot-dashed curve) and $\tilde{\gamma}_{\text{pl}} \equiv \gamma_{\text{pl}}/Z_{\text{pl}}$ (solid and dotted curves) as a function of ω for different values of γ_h. Figure 11c shows the corresponding $\sigma(\omega)$ dependence. One can see that at plasma resonance $\omega = \tilde{\Omega}_h$, where the renormalization function $Z_{\text{pl}}(\omega)$ has a maximum (Fig. 11a), the frequency dependences $\tilde{\gamma}_{\text{pl}}(\omega)$ and $\sigma(\omega)$ have a minimum, which can be compared with the corresponding features in the experimental dependences.[14,15,57] The appearance of such a minimum at the frequency $\tilde{\Omega}_h$ of the h-plasmons owing to the electron-plasmon interaction is analogous to the Holstein effect[58,59] at phonon frequencies owing to the electron-phonon interaction.

When comparing theory with experiment it is necessary to take into account the fact that in the case when ionic bonding predominates in the crystal lattice of the MOC hybridization of acoustic h-plasmons with longitudinal (LO) and transverse (TO) optical phonons with frequencies ω_{LO} and ω_{TO} should occur (see Refs. 48 and 49). In this case, the characteristic frequency of the coupled phonon-plasmon oscillations, which determines the Holstein singularity in the dependence $\sigma(\omega)$, is equal to

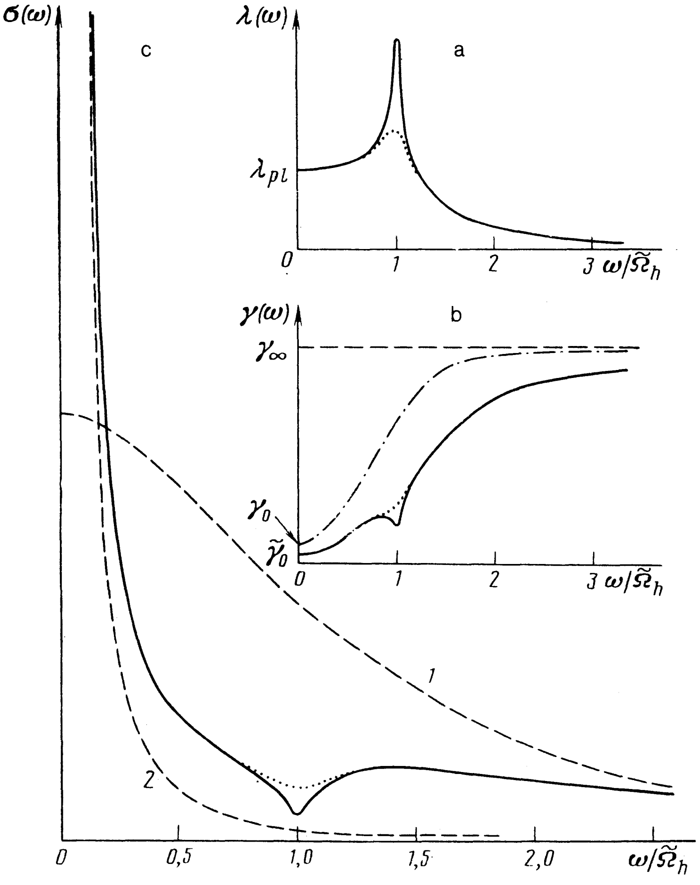

FIG. 11. a) $\lambda(\omega) \equiv Z_{\text{pl}} - 1$ as a function of ω for $\gamma_h/\tilde{\Omega}_h = 0.01$ (solid curve) and $\gamma_h/\tilde{\Omega}_h = 0.1$ (dotted curve). b) $\gamma_{\text{pl}}(\omega)$ as a function of ω (dot-dashed curve) and $\tilde{\gamma}_{\text{pl}}(\omega) \equiv \gamma_{\text{pl}}(\omega)/Z_{\text{pl}}(\omega)$ for $\gamma_h/\tilde{\Omega}_h = 0.01$ (solid curve) and $\gamma_h/\tilde{\Omega}_h = 0.1$ (dashed curve). The interpolation formula $\gamma_{\text{pl}}(\omega) = \gamma_0 + \gamma_1[1 - \exp(-\omega^2/\tilde{\Omega}_h^2)]$ was employed for γ_{pl}; $\gamma_\infty = \gamma_0 + \gamma_1$. c) $\sigma(\omega)$ as a function of ω in accordance with Eqs. (56) and (57) in the limit $\tau_D \to \infty$ for $\gamma_h/\tilde{\Omega}_h = 0.01$ (solid curve) and $\gamma_h/\tilde{\Omega}_h = 0.1$ (dotted curve). The dashed curves 1 and 2 are plots of the functions $\sigma(\omega)$ corresponding to Drude's law with relaxation time constants γ_∞^{-1} and $\tilde{\gamma}_0^{-1} \equiv \tilde{\gamma}_{\text{pl}}^{-1}(0)$, respectively.

$$\tilde{\Omega}\approx[\tilde{\Omega}_h^2+\omega_{LO}^2/(1+\alpha_\infty)]^{1/2}$$

for $\omega_{LO}\gg\omega_{TO}$ [compare with Eq. (41)].

5. CONCLUSIONS

Thus the simple model of a band spectrum containing only two overlapping $2D$ bands near the Fermi level—a wide band ($W_l\gtrsim1$ eV) and a narrow band ($W_h\lesssim0.1$ eV)—makes it possible to explain qualitatively a large number of anomalous physical properties of cuprate MOC in the normal state in terms of the interaction of charge (spin) carriers with low-frequency collective excitations of the charge (spin) density of nondegenerate h-carriers in the narrow band.

Of course, the real band structure of cuprate MOC is much richer and contains a large number of overlapping wide and narrow bands (see, for example, Refs. 27–30), and this must be taken into account when making a more detailed quantitative comparison of theory with experiment. In particular, the model of two bands of different width cannot explain the symmetric (relative to the sign of the applied voltage V) quasilinear dependence of the tunneling conductance $g_T(V)\approx g_0+g_1|V|$ (Ref. 16). This dependence can be obtained, for example, in a model of two close-lying identical narrow bands above and below the Fermi level superposed on a wide band.

Additional features should also arise in the carrier-density and temperature dependences of the Hall constant, the thermal-emf, and other characteristics. However, the main results obtained in this work and indicating the important role of low-frequency collective electronic excitations with a quasiacoustic spectrum in relaxation processes do not depend on the fine details of the band spectrum and apparently correctly reflect the physical nature of the electron-electron scattering and interaction in layered cuprate MOC at high temperatures.

In conclusion it should be noted that the interaction of acoustic plasmons with degenerate l-carriers in the wide band, as pointed out above, results in an effective attractive interelectronic interaction in the energy range $W_h<\omega<\tilde{\Omega}_h$ in the entire volume of the Brillouin zone. This attractive interaction should give rise to Cooper pairing of l-carriers and should increase T_c while at the same time suppressing the isotopic effect, as is observed in superconducting cuprate MOC.[60–62] The question of the role of the electron-plasmon interaction in the mechanism of high-temperature superconductivity is examined in Ref. 63.

We thank V. F. Gantmakher, V. B. Timofeev, L. I. Fisher, and G. M. Éliashberg for helpful discussions of a number of questions studied in this paper.

APPENDIX

In this section we calculate in the random-phase approximation the real and imaginary parts of the polarization operator of h-carriers in a narrow $2D$ band with the spectrum (2):

$$\Pi_h(\mathbf{q}_\parallel,\omega)=2\int\frac{d^2k_\parallel}{(2\pi)^2}\frac{f(E_h(\mathbf{k}_\parallel+\mathbf{q}_\parallel))-f(E_h(\mathbf{k}_\parallel))}{E_h(\mathbf{k}_\parallel)-E_h(\mathbf{k}_\parallel+\mathbf{q}_\parallel)+\omega-i\delta},$$

$$\delta\to+0, \tag{A1}$$

where $f(E)$ is the Fermi distribution function. In the case of degenerate h-carriers, in the limit $T\to0$ the expression for the polarization operator is found to be in many ways analogous to the expression for the polarization operator for electrons $\Pi_{1D}(q_\parallel,\omega)$ in a quasi-$1D$ (chain-type) metal with a narrow $1D$ band.[26] Thus, for example, along the diagonal of the Brillouin zone ($q_x=q_y$) the expression (A1) for the polarization operator of a quasi-$2D$ (layered) metal $\Pi_h\equiv\Pi_{2D}$ can be represented in the following form:

$$\Pi_{2D}(Q,\Omega)=\frac{1}{\pi a}\int_{-\tilde{k}_F}^{\tilde{k}_F}dv\tilde{\Pi}_{1D}(v,Q,\Omega). \tag{A2}$$

Here Π_{1D} differs from the expression obtained in Ref. 26 for Π_{1D} in that the parameters k_F and $2\sin Q$ are replaced by $\Delta(v)\equiv\cos^{-1}(\delta_h/\cos v)$ and $2\sin Q\cos v$, respectively, where $Q=(q_x+q_y)a/4$, $\delta_h=1-2\mu_h/W_h$, $\Omega=2\omega/W_h$ and $\tilde{k}_F=\arccos\delta_h$.

In order to calculate the real part of the polarization operator in the long-wavelength approximation we expand the integrand in Eq. (A2) in powers of Q through terms of fourth order, and performing the integration over v we find

$$\mathrm{Re}\,\Pi_h(q_\parallel,\omega)=\frac{W_hq_\parallel^2}{\pi^2\omega^2}P_0(\varkappa_h)\times\left\{1-\frac{q_\parallel^2a^2}{24}+\frac{W_h^2q_\parallel^2a^2[2P_1(\varkappa_h)+P_2(\varkappa_h)]}{4\omega^2P_0(\varkappa_h)}\right\}, \tag{A3}$$

where

$$P_0(\varkappa_h)=E(\varkappa_h)-(1-\varkappa_h^2)K(\varkappa_h),\quad \varkappa_h^2=\frac{4\mu_h}{W_h}\left(1-\frac{\mu_h}{W_h}\right), \tag{A4}$$

$$P_1(\varkappa_h)={}^2/_3(11\varkappa_h^2-10)E(\varkappa_h)+{}^4/_3(1-\varkappa_h^2)(5-3\varkappa_h^2)K(\varkappa_h), \tag{A5}$$

$$P_2(\varkappa_h)=4[(2-\varkappa_h^2)E(\varkappa_h)-2(1-\varkappa_h^2)K(\varkappa_h)]; \tag{A6}$$

$K(\varkappa_h)$ and $E(\varkappa_h)$ are complete elliptic integrals of the first and second kinds. Hence we obtain, taking into account Eq. (7), the following dispersion relation for the long-wavelength plasmons in a layered metal with a narrow $2D$-band ($q=(q_\parallel^2+q_z^2)^{1/2}$):

$$\omega(\mathbf{q})\approx\frac{q_\parallel}{q}\Omega_h\left\{1+\frac{q_\parallel^2d^2}{6}\left(1-\frac{q_\parallel^2}{2q^2}+\frac{q_z^4}{2q^2q_\parallel^2}\right)-\frac{q_\parallel^2a^2}{24}+\frac{W_h^2q^2a^2[2P_1(\varkappa_h)+P_2(\varkappa_h)]}{4\Omega_h^2P_0(\varkappa_h)}\right\}^{1/2},\quad \Omega_h=\left[\frac{4e^2W_h}{\pi\varepsilon_\infty d}P_0(\varkappa_h)\right]^{1/2}. \tag{A7}$$

When the contribution of the polarizability (8) of degenerate l-carriers in the wide $2D$ band to the permittivity is taken into account, we obtain instead of Eq. (A7) the spectrum of acoustic plasmons (15) with the plasma frequency (16).

On the other hand, by expanding Eq. (A2) near the corner of the Brillouin zone ($Q=\pi/2$) the plasmon spectrum in the short-wavelength limit ($q_\parallel\approx\pi/a$) can be obtained. In so doing it should be kept in mind that the expression (7) for the Coulomb matrix element is, generally speaking, not applicable in a layered crystal, since here the spatial localization of h-carriers near the lattice sites must be taken into account explicitly. In order to estimate the additional momentum dependence $V_c(\mathbf{q})$ associated with the periodic distribution of the electron density in the plane of the $2D$ layers, we employ the model potential

$$\tilde{V}_C(q_x, q_y) = \frac{\pi e^2 a}{[\sin^2(q_x a/2) + \sin^2(q_y a/2)]^{1/2}}, \quad (A8)$$

which for $q_\parallel a \ll \pi$ transforms into the Coulomb potential of a $2D$ system $V_{2D}(q_\parallel) = 2\pi e^2/q_\parallel$ (Refs. 37, 38) [see also the potential (7) for $q_\parallel d \gg 1$]. We note that in Ref. 26 the localization of the electron density on lattice sites was approximated by Gaussian functions.

The result is the following expression for the squared plasmon frequency near the corner of the Brillouin zone in the high-frequency approximation ($\omega^2 \gg W_h^2$; $\tilde{Q} \equiv \pi/2 - Q \ll \pi/2$):

$$\tilde{\omega}^2(\tilde{Q}) \approx \frac{2^{5/2} e^2 W_h \tilde{k}_F \sin \tilde{k}_F}{\pi \varepsilon_\infty a} \left(1 - \frac{\tilde{k}_F}{2} \tilde{Q}^2\right). \quad (A9)$$

Thus for $\tilde{k}_F \sim 1$ the frequency satisfies $\tilde{\omega}(0) > W_h$ for $W_h < e^2/\varepsilon_\infty a$.

In the case of nondegenerate h-carriers the polarization operator (A1), with Eqs. (2) and (12) taken into account, assumes the form

$$\Pi_h(q_x, q_y, \omega) = \exp\left(\frac{\mu_h - W_h/2}{T}\right) \cdot \int_{-\pi/a}^{\pi/a} \frac{dk_y}{\pi} \int_{-\pi/a}^{\pi/a} \frac{dk_\lambda}{\pi}$$

$$\times \frac{\operatorname{sh}[(W_h/4T) u(k_x, k_y, q_x, q_y)]}{\omega - (W_h/2) u(k_x, k_y, q_x, q_y) - i\delta}$$

$$\times \exp\left\{\frac{W_h}{4T}\left[\cos(k_x a)\cos\frac{q_x a}{2} + \cos(k_y a)\cos\frac{q_y a}{2}\right]\right\}, \quad (A10)$$

where

$$u(k_x, k_y, q_x, q_y) = \sin(k_x a)\sin\frac{q_x a}{2} + \sin(k_y a)\sin\frac{q_y a}{2}.$$

The real part of the polarization operator (A10) in the high-frequency approximation, to within terms of order $(W_h/\omega)^2$, is equal to

$$\operatorname{Re}\Pi_h(q_x, q_y, \omega) \approx \frac{N_h W_h}{\omega^2} \frac{I_1(W_h/4T)}{I_0(W_h/4T)} \left[\sin^2\left(\frac{q_x a}{2}\right) + \sin^2\left(\frac{q_y a}{2}\right)\right], \quad (A11)$$

where $I_n(x)$ is a Bessel function of imaginary argument of the first kind, whence Eq. (17) is obtained in the limit $q_\parallel \to 0$.

The imaginary part of the polarization operator, determined by the residues of the poles of the integrand in Eq. (A12), assumes the following form[15)] after the change of variables $\sin(k_y a) \equiv t$:

$$\operatorname{Im}\Pi_h(q_x, q_y, \omega) = -\frac{4\operatorname{sh}(\omega/2T)}{\pi^2 a^2} \exp\left(\frac{\mu_h - W_h/2}{T}\right) L(q_x, q_y, \omega), \quad (A12)$$

where

$$L(q_x, q_y, \omega) = \begin{cases} \int_{-1}^{1} dt F(t; q_x, q_y, \omega), & \frac{2|\omega|}{W_h} < \sin\left(\frac{q_x a}{2}\right) - \sin\left(\frac{q_y a}{2}\right), \\ \int_{\tilde{\Delta}(q_x, q_y, \omega)}^{1} dt F(t; q_x, q_y, \omega), & \sin\left(\frac{q_x a}{2}\right) - \sin\left(\frac{q_y a}{2}\right) < \frac{2|\omega|}{W_h} < \sin\left(\frac{q_x a}{2}\right) + \sin\left(\frac{q_y a}{2}\right), \\ 0, & \frac{2|\omega|}{W_h} > \sin\left(\frac{q_x a}{2}\right) + \sin\left(\frac{q_y a}{2}\right); \end{cases} \quad (A13)$$

$$\tilde{\Delta}(q_x, q_y, \omega) = \left[\frac{2|\omega|}{W_h} - \sin\left(\frac{q_x a}{2}\right)\right] \Big/ \sin\left(\frac{q_y a}{2}\right), \quad (A14)$$

$$F = (1-t^2)^{-1/2}\left[\sin^2\left(\frac{q_x a}{2}\right) - \left[\frac{2|\omega|}{W_h} - t\sin^2\left(\frac{q_y a}{2}\right)\right]^2\right]^{-1/2} \times \operatorname{ch}\left[\frac{W_h}{4T}\cos\left(\frac{q_y a}{2}\right)(1-t^2)^{1/2}\right] \operatorname{ch}\left(\frac{W_h}{4T}\cos\left(\frac{q_y a}{2}\right)\left\{1 - \left[\frac{2|\omega|}{W_h} - t\sin^2\left(\frac{q_y a}{2}\right)\right]^2 \Big/ \sin^2\left(\frac{q_x a}{2}\right)\right\}^{1/2}\right). \quad (A15)$$

For $q_y = 0$ and $q_\parallel = q_x$ Eq. (A12), substituting Eqs. (A13) and (A15), reduces to the expression (18). From Eqs. (A13), (A14), and (A15) it follows that when the direction of the wave vector $\mathbf{q}_\parallel$ changes the width of the region of Landau damping (in which $\operatorname{Im}\Pi_h \neq 0$) changes from $1/2 W_h \sin(q_\parallel a/2)$ at $q_y = 0$ to $W_h \sin(q_\parallel a/2^{3/2})$ along the diagonal of the Brillouin zone ($q_x = q_y = q_\parallel/2^{1/2}$).

We note that averaging over the directions of $\mathbf{q}_\parallel$ spreads the square-root singularity in the expression (18) at the point $|\omega| = \frac{1}{2} W_h \sin(q_x a/2)$ over ω into a peak of width $\Delta\omega \approx \frac{1}{2} W_h \sin(k_{Fl} a)$ at $q_\parallel \approx 2k_{Fl}$.

1) In what follows, for brevity, we shall denote them as l- and h-carriers, from the English words light and heavy.

2) The possibility of the existence of acoustic plasmons was studied previously in Refs. 20–23 for $3D$ multiband transition metals, semimetals, and multivalley semiconductors and in Ref. 23 for two-band layered metals like cuprate metal-oxide compounds.

3) In Ref. 23 it is incorrectly asserted that in $2D$ systems wide transmission windows occur in regions of quantum Landau damping at frequencies $|\omega| < q_\parallel v_F |q_\parallel/2k_F - 1|$, where k_F is the Fermi momentum. Such transmission windows exist only in $1D$ metals.[26]

4) This expression is valid both in the case of a narrow $1D$ band with nondegenerate carriers in conducting chains and for a narrow $3D$ band along the principal axes of a cubic crystal.

5) Here it is presumed that passage to the continuous limit, when $a \to 0$ but $\hbar/a = \text{const}$ and $m_h^* = \text{const}$, is made.

6) For almost localized h-carriers Drude damping should be largely suppressed, but the electron-electron scattering can be significant (see below).

7) The optical longitudinal mass in the plane of the layers satisfies $m_\parallel^* \sim m_0$, while the transverse mass (along the c axis, which is parallel to the z axis) satisfies $m_\perp^* > 10^2 m_\parallel^*$, since the ratio of the longitudinal and transverse plasma frequencies satisfies $\omega_{pl}^\parallel/\omega_{pl}^\perp > 10$. Thermodynamic, kinetic, and magnetic measurements give too high values of $m_\parallel^*$ on account of the renormalization of the spectrum of quasiparticles near the Fermi surface (see below).

8) On the basis of the "plasma" model of a metal[45,46] or in the polar model of an ionic crystal[47–48] the polarizability of the ionic lattice, i.e., exchange of virtual optical phonons, is also included in $\tilde{\Pi}$.

9) The even part of $\Sigma_{pl}(\omega)$ as well as the Coulomb self-energy Σ_C, which is determined by the screened matrix element of Coulomb repulsion $V_C(\mathbf{q})/\varepsilon(\mathbf{q}, \omega_{max})$, are included in the renormalization of the chemical potential μ_l and the effective mass m_l^* in the energy of the l-carriers ξ_l.

10) When the electron-phonon interaction is taken into account, the renormalization of the effective mass at the Fermi surface is determined by the renormalization factor $Z(0) = 1 + \lambda_{pl} + \lambda_{ph}$, where λ_{ph} is the electron-phonon interaction constant.

11) The heat capacity of the nondegenerate gas of h-carriers in a narrow $2D$ band $C_h(T) \sim k_B N_h(T)$, at least for $T \lesssim W_h/4$, i.e., $C_h \ll C_l$ for $N_h \ll N_l$.

12) For $T \ll W_h/4$, when the expression (19) can be used for $\operatorname{Im}\Pi_h$, we have $P(T) \propto T^{-1/2}$ and $\gamma_{pl}(0) \propto T^{1/2}$.

13) In a narrow $2D$ band with the spectrum (2) the exact dependence has the form $\bar{v}_h(T) \propto 4T \sinh(W_h/4T)/W_h I_0(W_h/4T)$; for $T \ll W_h/4$ we obtain $\bar{v}_h \propto T^{1/2}$.

14) If $\gamma_{pl}(0) \propto T^{1/2}$ and $\sigma_l \ll \sigma_h$, then $\rho(T) \propto T^{1/2}$. This is characteristic for intermetallic compounds of the form V_3Si and Nb_3Ge.[11,23]

15) A more detailed discussion of the calculation of $\operatorname{Re}\Pi_h$ and $\operatorname{Im}\Pi_h$ is given in Ref. 64.

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