Temperature of Λ atoms during two-frequency cooling

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The temperature dependence is derived for a mechanism of two-frequency cooling for Λ atoms having nearly zero velocities. The result agrees qualitatively with the temperature dependence observed experimentally. The lowest temperature which can be attained by two-frequency cooling is calculated.

Several cooling mechanisms which have recently been discussed in the literature might be capable in principle of cooling atoms to a temperature well below the one-photon limit $T_0 = \hbar\gamma/2k_B : 10^{-4}$ K (Refs. 1–3). For example, cooling schemes based on the interaction of three-level V atoms and Λ atoms with standing light waves were studied by Dalibard and Cohen–Tannoudju[1] and Chang *et al.*[2] Minogin *et al.*[3] have studied a two-frequency mechanism for cooling Λ atoms which is based on coherent trapping of population.

The appearance of these studies stimulated some very interesting experiments,[4] in which the temperatures of the cooled atoms were found to be lower than T_0 by a factor of nearly 6. An important part of these experiments was a study of the temperature dependence of the cold atoms near zero velocity associated with changes in the frequency detuning of the light waves and in the power of the laser light. The temperature dependence seen experimentally is sharply different from the corresponding prediction based on the well-studied model of a two-level atom.[5]

A derivation of the corresponding temperature dependence for the cooling mechanisms proposed in Refs. 1–3 and a comparison of the results with experimental data should naturally provide evidence in favor of one or another of the atomic cooling models for describing the experiments of Ref. 4.

In the present paper we wish to point out that it is possible to derive a temperature dependence which agrees qualitatively with that observed in the experiments by Lett *et al.*,[4] by using a simple model of the interaction of a three-level Λ atom with the field of oppositely directed waves (this is a two-frequency cooling mechanism). The resulting expression for the temperature is distinguished in a favorable way from the corresponding expression of Ref. 1 in that in its very derivation we see the limitations on the laser power and the frequency detunings which rule out the attainment of arbitrarily low temperatures for the cold atoms as the laser power is reduced or as the frequency detuning is increased.

To solve this problem, we specify that the optical field with which the Λ atom interacts consists of two oppositely directed plane waves with frequencies ω_m ($m = 1, 2$):

$$\mathbf{E}=E_1\mathbf{e}_1\cos(\omega_1 t+k_1 z)+E_2\mathbf{e}_2\cos(\omega_2 t-k_2 z), \quad (1)$$

where $\mathbf{e}_m$ are unit polarization vectors, $|\mathbf{k}_m| = \omega_m/c$ are the wave vectors, E_m are the amplitudes of the light waves, and z is the spatial coordinate.

We assume that the upper level $|3\rangle$ in the Λ atom decays to the lower levels $|1\rangle$ and $|2\rangle$ with respective probabilities γ_1 and γ_2 ($\gamma_1 \neq \gamma_2$) (Fig. 1). The transitions $|1\rangle-|3\rangle$ and $|2\rangle-|3\rangle$ are electric dipole transitions with a dipole matrix element $d_{3,m}$, and the transition $|1\rangle-|2\rangle$ is dipole-forbidden. We also assume that the relaxation of the coherence between levels $|1\rangle$ and $|2\rangle$, i.e., γ_3, is nonzero.[6] For all cases of practical importance it is smaller than the widths of the optical transitions: $\gamma_3 \ll \gamma_{1,2}$. Under the condition that the frequency detunings are equal,

$$\Omega_1=\Omega_2=\Omega_0, \quad (2)$$

where

$$\Omega_1=\omega_1-\omega_{31}, \quad \Omega_2=\omega_2-\omega_{32},$$

we then find an expression for the radiation-pressure force acting on the Λ atom and an expression for the components of the momentum diffusion tensor.[5] For this purpose we solve an equation for the density matrix elements ρ_{ij} of the Λ atom in the steady state, and we find the population ρ_{33} of the upper level $|3\rangle$:

$$\rho_{33}=g^2 a\mathscr{L}, \quad (3)$$

where

$$a=4(kv)^2+2g^2\gamma_3/\gamma,$$
$$\mathscr{L}=[4(kv)^4+8\Omega_0 b(kv)^3+4(\Omega_0^2+\gamma^2+g^2)(kv)^2 -4g^2\Omega_0 kv+2g^2((\gamma_3/\gamma)\Omega_0^2+2g^2)]^{-1},$$
$$b=(\gamma_1-\gamma_2)/\gamma, \quad \gamma=\gamma_1+\gamma_2.$$

Here $g = d_m E_m/2\hbar$ is the Rabi frequency, which is the same for both transitions of the Λ atom,[1)] v is the projection of the atomic velocity onto the z axis, $k \approx k_m$ ($m = 1, 2$), and we are assuming $\gamma_3/\gamma \ll 1$. In the case of a three-level Λ atom, the steady-state level population ρ_{33} unambiguously deter-

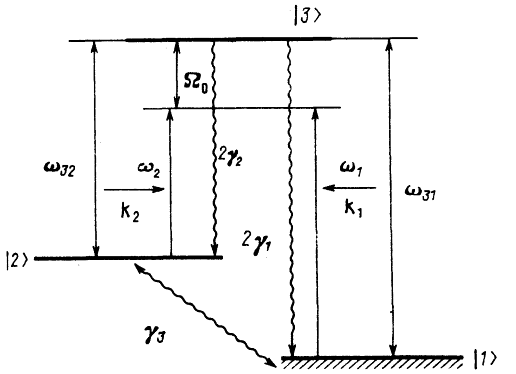

FIG. 1. Interaction of a Λ atom with the field of oppositely directed waves as in (1).

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mines, at times $t \gg \gamma^{-1}$, both the radiation-pressure force F_z,

$$F_z = 2\hbar k \gamma b \rho_{33}, \tag{4}$$

and the components of the momentum diffusion tensor,

$$2\mathscr{D}_{xx} = 2\mathscr{D}_{yy} = \mathscr{D}_{zz} = 2(\hbar k)^2 \gamma \rho_{33}, \tag{5}$$

where small nonadiabatic corrections for the statistics of the reradiated photons are being ignored.[5] In the derivation of (3)–(5) it was also assumed that the following condition holds on the intensities of the light waves:

$$2g^2/\gamma^2 \gg \gamma_3/\gamma. \tag{6}$$

According to Ref. 6, this inequality, along with (2), determines the necessary condition for the occurrence of coherent population trapping.

Again, we wish to stress that condition (6) is of fundamental importance, since it specifies the light intensities at which coherent trapping is even possible. If condition (6) does not hold, coherent trapping does not occur in the Λ system, and the two-frequency cooling mechanism which we are discussing here correspondingly does not operate.

According to Ref. 5, expressions (4) and (5) for the radiation-pressure force and for the velocity diffusion completely determine the behavior of the Λ atom in field (1). Correspondingly, the temperature of the atoms near zero velocity ($v \approx 0$) can be estimated by the method of Ref. 5:

$$T = \mathscr{D}_{zz}/k_B \beta, \tag{7}$$

where k_B is the Boltzmann constant, $\mathscr{D}_{zz}$ is the z component of the velocity-diffusion tensor (5), calculated for zero velocity ($v \approx 0$), and β is a dynamic friction coefficient. This coefficient arises from an expansion of the force (4) near a velocity $v \approx 0$:

$$F_z = F_z|_{v=0} + \frac{dF_z}{dv}\bigg|_{v=0} v$$

$$= F_z^0 - \frac{\hbar k 4g^2 |\Omega_0| b^2 \gamma_3 (kv)}{(\Omega_0^2 \gamma_3/\gamma + 2g^2)^2} = F_z^0 - \beta v. \tag{8}$$

According to (7) and (8), the temperature of the cold atoms near $v \approx 0$ is then

$$T = T_0 \frac{\Omega_0^2 \gamma_3/\gamma + 2g^2}{b^2 |\Omega_0| \gamma}, \tag{9}$$

where, as before, $T_0 = \hbar\gamma/2k_B$.

Expression (9) gives the temperature of the Λ atoms interacting with field (1) near zero velocity as a function of both the frequency detuning and the laser power. It can be seen in particular from (9) that the temperature of the atoms can become arbitrarily high as the detuning Ω_0 decreases. This situation corresponds to a vanishing value of the dynamic friction coefficient β in (8) under the condition $|\Omega_0| = 0$. In other words, only diffusive heating of the Λ atoms occurs in this case, and the force in (4) does not cool atoms whose velocities are close to zero.[6] Cooling of the atoms can occur only at detunings $\Omega_0 < 0$ in the case $b > 0$ or $\Omega_0 > 0$ in the case $b < 0$.

If the detuning $|\Omega_0|$ is large, the atomic temperature in (8) increases linearly, as it would for a two-level atom.[5] At the same time, there is a fairly broad range of $|\Omega_0|$ in which the temperatures in (8) are well below the single-photon limit T_0.

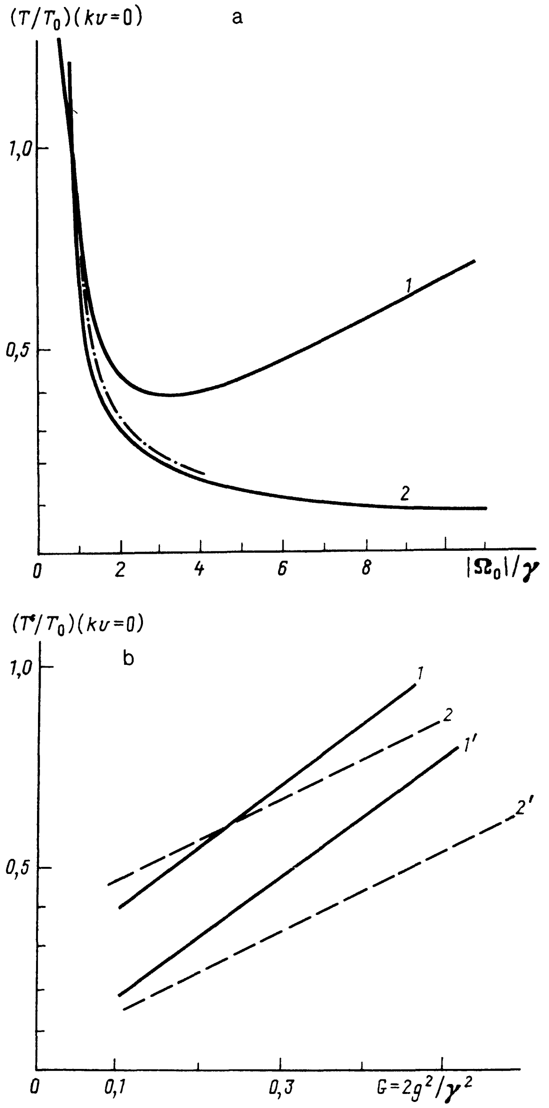

FIG. 2. Temperature of the cooled atoms near zero velocity according to (9). a: As the magnitude of the frequency detuning of the light waves, $|\Omega_0|$, is varied at $G = 2g^2/\gamma^2 = 0{,}2$, and (1) $\gamma_3/\gamma = 10^{-2}$ and (2) 10^{-3}. Dot-dashed line—Extrapolation of the data from Ref. 4. b: As the power of the laser light is varied. 1,1′—$|\Omega_0| = 4\gamma$ and (1) $\gamma_3/\gamma = 10^{-2}$ and (1′) 10^{-3}; 2,2′—$|\Omega_0| = 6\gamma$ (2) and $\gamma_3/\gamma = 10^{-2}$ and (2′) 10^{-3}.

Figure 2a shows the temperature of the Λ atoms according to (9) as a function of the frequency detuning $|\Omega_0|$. The dot-dashed line is an extrapolation of the experimental data of Lett *et al.*[4] We see a surprisingly good qualitative agreement between the behavior of the temperature of the cold atoms as described by (9) as a function of the detuning, on the one hand, and the experimental results, on the other. In addition, as we have already mentioned, we would expect an increase in the temperature of the atoms with increasing $|\Omega_0|$ on the basis of (9).

For the minimum value of the temperature in (9) we typically find

$$T_{min} = T_0 \frac{4g}{\gamma b^2}\left(\frac{\gamma_3}{2\gamma}\right)^{1/2}. \tag{10}$$

This value is reached at a detuning

$$\tilde{\Omega}_0=(2g^2\gamma/\gamma_3)^{1/2}. \qquad (11)$$

With the parameter values $\gamma = 10$ MHz and $b = 0.4$, for example, which correspond to curve *2* in Fig. 2a, we find $T_{min} = 2\cdot 10^{-2}\ T_0 \approx 4\cdot 10^{-6}$ K from (10), and from (11) we find a frequency detuning $\tilde{\Omega}_0 \approx 14\gamma$. In this case expression (10) predicts a temperature slightly higher than in the case of an exact resonance ($|\Omega_0| = 0$) [compare expression (10) above with expression (3) in Ref. 6].

How does the temperature (9) of the cold atoms near zero velocity depend on the laser power? In the experiments by Lett *et al.*, the temperature was observed to fall off linearly with decreasing laser power. The slope of the line was determined primarily by the frequency detuning of the light waves. Again in the case (9), the temperature is a linear function of the power, and the slope of the line is determined by the frequency detuning and by the relaxation γ_3 (Fig. 2b). It is important that condition (6) on the intensities of the light waves hold at all times, since otherwise the two-frequency cooling mechanism would not operate. For sodium atoms with a saturation intensity $I_{sat} = 10$ mW/cm^2, for example, condition (6) means that the mechanism of two-frequency cooling would be observed at intensities $I \gg I_{sat}\gamma_3/\gamma \gg 10^{-2}$ mW/cm^2 with $\gamma_3/\gamma = 10^{-3}$. A coherence relaxation γ_3 might stem from a variety of physical factors. The most important point here, however, is that the light waves have a spectrum of finite width.[7] In such a case, the effects described above will always occur.

Again, we wish to stress that we are not claiming that we have offered an exhaustive description of such a complex experimental situation as that in Ref. 4. Nevertheless, a qualitative comparison of the results on the temperature dependence, on the one hand, and the simplicity and clarity of the mechanism of two-frequency cooling based on a coherent trapping of population, on the other, can serve as arguments in favor of the operation of this mechanism in this situation and in other situations of this sort.

[1)] Since we have $\gamma_1 \neq \gamma_2$, the matrix elements of the dipole transitions $|m\rangle - |3\rangle$ are not equal to each other. However, all that is necessary here is that the Rabi frequencies of the exciting waves be equal.

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