Gravitational radiation from a charge in the field of an elliptically polarized wave

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We derive the electromagnetic and gravitational spectra radiated by a moving particle of mass m and charge e in the field of an elliptically polarized monochromatic electromagnetic plane wave. It is shown that these spectra are proportional to each other no matter what the charge velocity, and independent of the frequency of the radiation. The proportionality factor is $(4\pi G m^2/e^2)\gamma_*^2 \cot^2(\theta/2)$, where γ_* is the effective Lorentz factor of the particle and θ is the angle between the radiation wave vector and direction of propagation of the plane wave.

1. INTRODUCTION

The source of gravitational radiation emitted by a given system is its energy-momentum tensor. The energy-momentum tensor of a charged particle moving in an electromagnetic wave is composed of the local particle tensor $t_{\mu\nu}$ and a nonlocal energy-momentum tensor $\theta_{\mu\nu}$ due to the external field and self-field of the particle. The existence of a nonlocal source $\theta_{\mu\nu}$ makes gravitational radiation quite different from electromagnetic radiation, for which the source (a current density) is a local one. In the general theory of relativity, the energy-momentum tensor of a gravitational field (a pseudotensor) is not well-defined, but that does not prevent one from calculating the gravitational radiation emitted by gravitating systems. It would seem that a detailed comparison of gravitational radiation from nongravitating and gravitating systems ought to improve our understanding of the latter, and for that reason alone, gravitational radiation from electromagnetic systems should be of interest. In addition, however, there have been recent suggestions as to how the gravitational radiation from electromagnetic systems might be generated (and detected) here on earth.[1] Detailed study of the gravitational radiation from electromagnetic systems is also prerequisite to a comparison of the strength of the signals emitted by possible terrestrial and cosmic sources.[2]

In the same vein, a study of the gravitational radiation from electromagnetic systems has revealed the intriguing fact that the two types of spectra emitted by a charge moving in a linearly or circularly polarized monochromatic plane wave are proportional, and it has been conjectured that the proportionality continues to hold for a more general plane wave as well.[3] Confirmation of that suggestion would be highly significant, as it would provide a basis for understanding the proportionality of the gravitational and electromagnetic spectra emitted by an ultrarelativistic particle moving in an arbitrary field—in the rest frame of such a particle, any external field can be approximated by a plane wave.

In the present paper, we demonstrate that the two types of spectra radiated by a charge in the field of an elliptically polarized monochromatic plane wave are indeed proportional. In contrast to the situation for the special cases of linear and circular polarization, the charge does not follow a planar trajectory in the frame in which it is at rest on the average.

The spectrum of classical gravitational radiation is given by[4]

$$[d\varepsilon_{\mathbf{q}}]_{Grav}=8\pi G[T_{\alpha\beta}^{*}(q)T^{\alpha\beta}(q)-{}^{1}/_{2}|T_{\alpha}{}^{\alpha}(q)]d^3q/16\pi^3, \quad (1)$$

where $T_{\mu\nu}(q) = t_{\mu\nu}(q) + \theta_{\mu\nu}(q)$ is the Fourier transform of the system's conserved energy-momentum tensor.

The classical power spectrum of electromagnetic radiation from a charge is given by (see Ref. 5, §66; here we employ Heaviside-Lorentz units)

$$[d\varepsilon_{\mathbf{q}}]_{EM}=|j_\mu(q)|^2 d^3q/16\pi^3, \quad (2)$$

where the $j_\mu(q)$ are the Fourier components of the charge density, determined from the particle trajectory:

$$j_\mu(q)=e\int_{-\infty}^{\infty}\frac{d\tau\,\pi_\mu(\tau)}{m}\exp(-iqx(\tau)); \quad (3)$$

here $x_\mu(\tau)$ and $\pi_\mu(\tau)$ are the particle's position and momentum, and τ is the proper time.

We shall show that just as in the special cases examined in Ref. 3,

$$[d\varepsilon_{\mathbf{q}}]_{Grav}=\frac{4\pi G m^2\Gamma^2}{e^2}[d\varepsilon_{\mathbf{q}}]_{EM}. \quad (4)$$

The parameter Γ is independent of the frequency, but it depends on the direction of the radiation wave vector $\mathbf{q}$ and on the mean squared value of the plane-wave potential (see Eq. (11) below):

$$\Gamma=\gamma_*\operatorname{ctg}(\theta/2), \quad (5)$$

$$\gamma_*=\frac{m_*}{m}=\left[1+\frac{e^2(a_1{}^2+a_2{}^2)}{2m^2}\right]^{1/2}. \quad (6)$$

Here θ is the angle between $\mathbf{q}$ and the direction of plane-wave propagation, γ_* is the effective Lorentz factor, and m_*, is the charge's effective mass, which equals its mean kinetic energy in the system in which it is at rest on the average.

As noted in Ref. 6, Eqs. (1) and (2) imply that Γ^2 is an invariant characteristic of the system, and in Section 4 below, we provide an invariant expression for Γ^2 in the present case. The factor Γ characterizes the rate of conversion of virtual photons of the charge's self-field and photons of the external field into gravitons, and it contains information on the field that holds the particle in orbit.

2. ELECTROMAGNETIC RADIATION FROM A MOVING CHARGE IN THE FIELD OF AN ELLIPTICALLY POLARIZED ELECTROMAGNETIC PLANE WAVE

We describe the field of an electromagnetic plane wave in terms of the 4-potential $A_\mu = A_\mu(\varphi)$, which depends on

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the coordinates through the invariant quantity $\varphi = kx$, where the four-vector k_μ, the wave vector, has zero length: $k^2 = 0$. In the Lorentz gauge, the potentials of the field and wave vector satisfy $kA = 0$.

In order to find the Fourier components of the four-dimensional current vector (3), which enter into the expression for the electromagnetic spectrum, it is necessary to determine the trajectory of the particle in the force field. For a plane wave described by the potential $A_\mu(\varphi)$, with $\varphi = kx$, the solution of the equations of motion,

$$\frac{d\pi_\mu}{d\tau} = \frac{e}{m} F_{\mu\nu}\pi^\nu, \quad x_\mu(\tau) = \int \frac{\pi_\mu(\tau)}{m} d\tau, \tag{7}$$

is[5]

$$\pi_\mu(\tau) = p_\mu - eA_\mu + k_\mu\left(\frac{epA}{kp} - \frac{e^2A^2}{2kp}\right), \quad \varphi = \frac{kp\tau}{m} + \varphi_0, \tag{8}$$

where τ is the proper time. The four-vector p_μ is a constant of the motion that is equal to the momentum π_μ in the absence of a field. For a suitable choice of the origin of time, the constant φ_0 vanishes.

In considering motion in a periodic field, it is convenient to introduce the mean four-momentum or quasi-four-momentum

$$\bar{\pi}_\mu = p_\mu - \frac{e^2\overline{A^2}}{2kp} k_\mu, \tag{9}$$

whose square is $\bar{\pi}^2 = -(m^2 + e^2\overline{A}^2) \equiv -m_*^2$. The particle trajectory is thus given by

$$x_\mu(\varphi) = \frac{1}{k\bar{\pi}} \int_0^\varphi \left\{\bar{\pi}_\mu - eA_\mu + k_\mu\left[\frac{e\bar{\pi}A}{k\bar{\pi}} - \frac{e^2}{2k\bar{\pi}}(A^2 - \overline{A^2})\right]\right\} d\varphi. \tag{10}$$

Consider now the field of an elliptically polarized, monochromatic plane wave. The field potential $A_\mu(\varphi)$ takes the form

$$A_\mu = a_\mu^1 \cos(kx) + a_\mu^2 \sin(kx), \qquad a_\alpha{}^i a^{j\alpha} = (a^i)^2\delta_{ij}, \quad k_\alpha a^{i\alpha} = k^2 = 0, \quad i, j = 1, 2. \tag{11}$$

We choose the direction of wave propagation to be the 3-axis. Then $k_1 = k_2 = 0$, $k_3 = k^0 = \omega$, and in a gauge in which $A_0 = 0$ and $\mathbf{A}(\varphi)$ lies in the 1–2 plane, the potential is

$$A_1 = a_1 \cos(kx), \quad A_2 = a_2 \sin(kx). \tag{12}$$

In a coordinate system in which the charge is at rest, the trajectory may be described by a set of parametric equations:

$$x_1(\varphi) = \frac{\xi_1}{\omega}\sin\varphi, \quad x_2(\varphi) = -\frac{\xi_2}{\omega}\cos\varphi,$$
$$x_3(\varphi) = -\left(\frac{\xi_1^2 - \xi_2^2}{8\omega}\right)\sin 2\varphi, \tag{13}$$
$$x^0(\varphi) = -\left(\frac{\xi_1^2 - \xi_2^2}{8\omega}\right)\sin 2\varphi - \frac{\varphi}{\omega},$$

where

$$\xi_{1,2} = \frac{ea_{1,2}}{m_*}, \tag{14}$$

and

$$m_* = \left[m^2 + \frac{e^2(a_1^2 + a_2^2)}{2}\right]^{1/2}$$

is the charge's effective mass, which equals its mean kinetic energy in this frame.

We see then that the space curve traced out by the charge is a curvilinear trajectory whose projection on the 1–3 and 2–3 planes containing the wave vector $\mathbf{k}$ is a figure eight, while its projection on the transverse 1–2 plane is an ellipse.

The Fourier components of the current density $j_\mu(q)$ may be expressed in terms of a function of the form

$$\int_{-\infty}^{\infty} d\varphi \exp(-iqx(\tau))\{1, \sin\varphi, \cos\varphi, \cos^2\varphi\}. \tag{15}$$

In the rest frame,

$$-iqx(\tau) = i\left(\alpha\sin\varphi - \beta\cos\varphi - \gamma\sin 2\varphi - \frac{q^0}{\omega}\varphi\right), \tag{16}$$

where

$$\alpha = -\frac{\xi_1 q_1}{\omega}, \quad \beta = -\frac{\xi_2 q_2}{\omega}, \quad \gamma = \frac{(\xi_1^2 - \xi_2^2)}{8\omega} q_-, \tag{17}$$

$q^0 = |\mathbf{q}|$ is the frequency of the radiation, and $q_- = q^0 - q_3$. The integrals in (15) can be written in the form of sums

$$\sum_{s=-\infty}^{\infty} 2\pi\delta(s - q^0/\omega) A_n(s\alpha\beta\gamma), \quad \sum_{s=-\infty}^{\infty} 2\pi\delta(s - q^0/\omega) B_n(s\alpha\beta\gamma), \tag{18}$$

where the functions $A_n(s\alpha\beta\gamma)$ and $B_n(s\alpha\beta\gamma)$ are defined by

$$A_n(s\alpha\beta\gamma) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi \cos^n\varphi \exp[i(\alpha\sin\varphi - \beta\cos\varphi - \gamma\sin 2\varphi - s\varphi)],$$
$$B_n(s\alpha\beta\gamma) = \frac{1}{2\pi}\int_{-\pi}^{\pi} d\varphi \sin\varphi\cos^n\varphi \exp[i(\alpha\sin\varphi - \beta\cos\varphi - \gamma\sin 2\varphi - s\varphi)]. \tag{19}$$

Forming the invariant squared magnitude of the Fourier components of the current density, $|j_\mu(q)|^2$, we obtain the electromagnetic spectrum:

$$[d\varepsilon_q]_{EM} = t\sum_s 2\pi\delta(q^0 - s\omega)\frac{e^2}{\gamma_*^2} \times\{-(1+y^2)|A_0|^2 + x^2|A_1|^2 + y^2|B_0|^2 - (x^2 - y^2)\operatorname{Re} A_0A_2^*\}\frac{d^3q}{16\pi^3}. \tag{20}$$

Here

$$x = \frac{ea_1}{m}, \quad y = \frac{ea_2}{m}, \quad \gamma_* = \frac{m_*}{m} = [1 + {}^1/_2(x^2 + y^2)]^{1/2}, \tag{21}$$

and t denotes the radiation time, which results from the appearance of the double sum

$$\sum_s\sum_{s'} (2\pi)^2\delta(s - q^0/\omega)\delta(s' - q^0/\omega) \tag{22}$$

in calculating (20).

It is not difficult to show that in the special cases in which $a_1 = a_2$ or $a_2 = 0$, Eq. (20) reduces to the well-known results for the electromagnetic spectrum of a charge in the field of a circularly or linearly polarized wave, respectively, as given, for example in Ref. 7.

3. GRAVITATIONAL RADIATION FROM A MOVING CHARGE IN THE FIELD OF AN ELLIPTICALLY POLARIZED PLANE WAVE

To obtain an expression for the gravitational radiation spectrum of a charge in the present case, we need to determine the Fourier components of the conserved total energy-momentum tensor for the system, $T_{\mu\nu} = t_{\mu\nu} + \theta_{\mu\nu}$, which is composed of the energy-momentum tensor of a material object ($t_{\mu\nu}$) and that of the self and external electromagnetic fields ($\theta_{\mu\nu}$). The two latter tensors are the local and nonlocal sources of gravitational radiation, respectively.

The well known form for the energy-momentum tensor of the electromagnetic field is

$$\theta_{\mu\nu}=-F_{\mu\lambda}F^{\lambda}{}_{\nu}-{}^1/_4 g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}, \tag{23}$$

where the field-strength tensor $F_{\alpha\beta}$ is the sum of the self-field ($f_{\alpha\beta}$) and external field ($\varphi_{\alpha\beta}$) tensors:

$$F_{\alpha\beta}=\varphi_{\alpha\beta}+f_{\alpha\beta}. \tag{24}$$

We then find that the Fourier transform $\theta_{\mu\nu}(q)$ can be expressed in terms of the Fourier components of the tensors $\varphi_{\alpha\beta}$ and $f_{\alpha\beta}$:

$$\theta_{\mu\nu}(q)=-\int\frac{d^4k}{(2\pi)^4}[\varphi_{\mu\alpha}(k)f^{\alpha}{}_{\nu}(q-k)+\varphi_{\nu\alpha}(k)f^{\alpha}{}_{\mu}(q-k)+{}^1/_2 g_{\mu\nu}\varphi_{\alpha\beta}(k)f^{\alpha\beta}(q-k)], \tag{25}$$

where we have discarded the quadratic terms $\varphi\varphi$, which are not a source of gravitational radiation, and quadratic combinations ff, since we neglect the effect of the charge self-field on the charge itself.

In accordance with the definition (11) for the potential associated with an elliptically polarized plane electromagnetic wave, we write the field tensor $\varphi_{\alpha\beta}(x)$ in the form

$$\varphi_{\alpha\beta}(x)=-\varphi_{\alpha\beta}^{(1)}\sin(kx)+\varphi_{\alpha\beta}^{(2)}\cos(kx),$$
$$\varphi_{\alpha\beta}^{(i)}=k_\alpha a_\beta{}^i-k_\beta a_\alpha{}^i,\quad a_\alpha{}^i a^{j\alpha}=(a^i)^2\delta_{ij},\quad k_\alpha a^{i\alpha}=k^2=0. \tag{26}$$

The Fourier components of the field thereupon take the form

$$\varphi_{\alpha\beta}(q)=(2\pi)^4(i/2)[\Phi_{\alpha\beta}\delta(q-k)-\Phi_{\alpha\beta}{}^*\delta(q+k)],$$
$$\Phi_{\alpha\beta}=\varphi_{\alpha\beta}^{(1)}-i\varphi_{\alpha\beta}^{(2)}. \tag{27}$$

On the other hand, we may express the Fourier transform $f_{\alpha\beta}(q)$ of the charge's self-field in terms of the current components,

$$f_{\alpha\beta}(q)=\frac{i}{q^2}[q_\alpha j_\beta(q)-q_\beta j_\alpha(q)], \tag{28}$$

which can in turn be obtained with (3), making use of the trajectory (13):

$$j_1(q)=-e\xi_1\sum_s 2\pi\delta(q^0-s\omega)A_1(s\alpha\beta\gamma),$$
$$j_2(q)=-e\xi_2\sum_s 2\pi\delta(q^0-s\omega)B_0(s\alpha\beta\gamma), \tag{29}$$
$$j_3(q)=e\frac{\xi_1^2-\xi_2^2}{4}\sum_s 2\pi\delta(q^0-s\omega)[2A_2(s\alpha\beta\gamma)-A_0(s\alpha\beta\gamma)],$$
$$j^0(q)=e\sum_s 2\pi\delta(q^0-s\omega)\left[\left(1-\frac{\xi_1^2-\xi_2^2}{4}\right)\times A_0(s\alpha\beta\gamma)+\frac{\xi_1^2-\xi_2^2}{2}A_2(s\alpha\beta\gamma)\right].$$

The functions $A_n(s\alpha\beta\gamma)$ and $B_n(s\alpha\beta\gamma)$ have been defined in (19).

The delta function in the expression for $\varphi_{\alpha\beta}(q)$ makes it easy to carry out the integration in (25) and obtain the Fourier components $\theta_{\mu\nu}(q)$ as a function of the current-density components $j_\mu(q\pm k)$. As a result,

$$\theta_{11}(q)=-\theta_{22}(q)=\frac{a_1}{4}\left[j_1+j_1-\frac{q_1}{q_-}(j_-+j_-)\right]+\frac{ia_2}{4}\left[j_2-j_2-\frac{q_2}{q_-}(j_--j_-)\right],$$
$$\theta_{12}(q)=\frac{a_1}{4}\left[j_2+j_2-\frac{q_2}{q_-}(j_-+j_-)\right]+\frac{ia_2}{4}\left[-(j_1-j_1)+\frac{q_1}{q_-}(j_--j_-)\right],$$
$$\theta_{13}(q)=\theta_1{}^0(q)=\frac{1}{4}\frac{a_1}{q_-}[(q^0-k^0)j_3-(q_3-k_3)j^0+(q^0+k^0)j_3-(q_3+k_3)j^0]+i\frac{a_2}{4q_-}[q_1(j_2-j_2)-q_2(j_1-j_1)], \tag{30}$$
$$\theta_{23}(q)=\theta_2{}^0(q)=\frac{a_1}{4q_-}[q_1(j_2+j_2)-q_2(j_1+j_1)]+\frac{ia_2}{4q_-}[(q_3-k_3)j^0-(q_3+k_3)j^0-(q^0-k^0)j_3+(q^0+k^0)j_3],$$
$$\theta_{33}(q)=\theta_3{}^0(q)=\theta_{00}(q)=\frac{a_1}{4q_-}[q_1(j^0+j_3+j^0+j_3)-(q^0-k^0+q_3-k_3)j_1-(q^0+k^0+q_3+k_3)j_1]+\frac{ia_2}{4q_-}[-q_2(j^0+j_3-j^0-j_3)+(q^0-k^0+q_3-k_3)j_2-(q^0+k^0+q_3+k_3)j_2].$$

For the sake of brevity, we have not written out the arguments of the functions j_μ, but have instead used the convention that the first of each pair of terms with the same indices is a function of $q-k$, and the second is a function of $q+k$.

Notice that the components $j_\mu(q\pm k)$ are also given by (29), but with $A_n(s\alpha\beta\gamma)$ and $B_n(s\alpha\beta\gamma)$ replaced by $A_n(s\pm 1,\alpha\beta\gamma)$ and $B_n(s\pm 1,\alpha\beta\gamma)$.

The Fourier components of the energy-momentum tensor $t_{\mu\nu}$ of a point charge can be obtained from[4]

$$t_{\mu\nu}(q)=m\int_{-\infty}^{\infty}d\tau\,\dot{x}_\mu(\tau)\dot{x}_\nu(\tau)\exp(-iqx(\tau)), \tag{31}$$

and can be expressed in terms of $A_n(s\alpha\beta\gamma)$ and $B_n(s\alpha\beta\gamma)$. Then $t_{\mu\nu}(q)$ takes the form

$$t_{11}(q)=\xi_1{}^2 m.\sum_s 2\pi\delta(q^0-s\omega)A_2(s\alpha\beta\gamma),$$
$$t_{12}(q)=\xi_1\xi_2 m.\sum_s 2\pi\delta(q^0-s\omega)B_1(s\alpha\beta\gamma),$$

$$t_{13}(q)=-\frac{(\xi_1^2-\xi_2^2)}{4}\xi_1 m_* \sum_s 2\pi\delta(q^0-s\omega)(2A_3-A_1),$$

$$t_{23}(q)=-\frac{(\xi_1^2-\xi_2^2)}{4}\xi_2 m_* \sum_s 2\pi\delta(q^0-s\omega)(2B_2-B_0),$$

$$t_{33}(q)=\left(\frac{\xi_1^2-\xi_2^2}{4}\right)^2 m_* \sum_s 2\pi\delta(q^0-s\omega)(4A_4-4A_2+A_0), \tag{32}$$

$$t_1^0(q)=-\xi_1 m_* \sum_s 2\pi\delta(q^0-s\omega)\times\left[\left(\frac{\xi_1^2-\xi_2^2}{2}\right)A_3+\left(1-\frac{\xi_1^2-\xi_2^2}{4}\right)A_1\right],$$

$$t_{22}(q)=\xi_2^2 m_* \sum_s 2\pi\delta(q^0-s\omega)(A_0-A_2),$$

$$t_2^0(q)=-\xi_2 m_* \sum_s 2\pi\delta(q^0-s\omega)\times\left[-\left(\frac{\xi_1^2-\xi_2^2}{4}-1\right)B_0+\left(\frac{\xi_1^2-\xi_2^2}{2}\right)B_2\right],$$

$$t_3^0(q)=\frac{\xi_1^2-\xi_2^2}{4}m_* \sum_s 2\pi\delta(q^0-s\omega)\times\left[(\xi_1^2-\xi_2^2)A_4-(2-(\xi_1^2-\xi_2^2))A_2-\left(1-\frac{\xi_1^2-\xi_2^2}{4}\right)A_0\right],$$

$$t_{00}(q)=m_* \sum_s 2\pi\delta(q^0-s\omega)\left[\frac{(\xi_1^2-\xi_2^2)^2}{4}A_4+(\xi_1^2-\xi_2^2)\times\left(1-\frac{\xi_1^2-\xi_2^2}{4}\right)A_2+\left(1-\frac{\xi_1^2-\xi_2^2}{4}\right)^2 A_0\right].$$

We can now form the invariant product $T^*_{\mu\nu}T^{\mu\nu}-1/2|T^\mu_\mu|^2$, which yields the gravitational radiation spectrum. Since $\theta^\mu_\mu=0$, the spectrum consists of several terms:

$$T_{\mu\nu}{}^*T^{\mu\nu}-{}^1/_2|T_\mu{}^\mu|^2=\theta_{\mu\nu}{}^*\theta^{\mu\nu}+2\,\mathrm{Re}\,\theta_{\mu\nu}{}^*t^{\mu\nu}+t_{\mu\nu}{}^*t^{\mu\nu}-{}^1/_2|t_\mu{}^\mu|^2. \tag{33}$$

In doing the calculations, the following relations between $A_n(s\alpha\beta\gamma)$ and $B_n(s\alpha\beta\gamma)$ are required:

$$\begin{aligned}&A_n(s-1)+A_n(s+1)=2A_{n+1}(s),\\&A_n(s-1)-A_n(s+1)=2iB_n(s),\\&B_n(s-1)+B_n(s+1)=2B_{n+1}(s),\\&B_n(s-1)-B_n(s+1)=2i[A_n(s)-A_{n+2}(s)]\end{aligned} \tag{34}$$

and

$$\begin{aligned}&(s-2\gamma)A_0(s)-\alpha A_1(s)+4\gamma A_2(s)-\beta B_0(s)=0,\\&i[(s-2\gamma)A_{n+1}(s)-\alpha A_{n+2}(s)+4\gamma A_{n+3}(s)\\&-\beta B_{n+1}(s)]+(n+1)B_n(s)=0,\\&i[(s-2\gamma)B_n(s)-\alpha B_{n+1}(s)+4\gamma B_{n+2}(s)\\&-\beta(A_n(s)-A_{n+2}(s))]-(n+1)A_{n+1}(s)+nA_{n-1}(s)=0.\end{aligned} \tag{35}$$

The first of these equations can be derived directly from the definitions in (19); the remainder follow from the periodicity of the functions

$$F(\varphi)=\{1,\cos\varphi,\ \sin\varphi\}\times\exp[i(\alpha\sin\varphi-\beta\cos\varphi-\gamma\sin 2\varphi-s\varphi)], \tag{36}$$

for which

$$\int_{-\pi}^{\pi} dF(\varphi)=0.$$

According to Eq. (1), then, the spectrum of gravitational radiation emitted by a charge is given by

$$[d\varepsilon_q]_{Grav}=t\sum_s 2\pi\delta(q^0-s\omega)4\pi Gm^2\times\frac{q_\perp^2}{q_-^2}\{-(1+y^2)|A_0|^2+x^2|A_1|^2+y^2|B_0|^2-(x^2-y^2)\,\mathrm{Re}\,A_0A_2^*\}\frac{d^3q}{16\pi^3}. \tag{37}$$

Comparing this result with Eq. (20), we see that the gravitational spectrum differs from the electromagnetic spectrum by a proportionality factor $4\pi Gm^2\Gamma^2/e^2$, where Γ is given by Eq. (5).

Note that the system's total energy-momentum tensor $T_{\mu\nu}=\theta_{\mu\nu}+t_{\mu\nu}$, with $\theta_{\mu\nu}$ and $t_{\mu\nu}$ defined in (30) and (32), satisfies the conservation law

$$q^\mu T_{\mu\nu}(q)=0,\quad \nu=0,1,2,3. \tag{38}$$

In the special cases of linear and circular polarization, Eq. (37) goes into the results found in Refs. 6 and 8.

4. DISCUSSION AND CONCLUSION

For a charged particle in motion in the field of an arbitrarily polarized monochromatic electromagnetic plane wave, the gravitational and electromagnetic spectra are proportional to one another, no matter what the particle velocity. The proportionality results from the combined effect of local and nonlocal mechanisms for emitting gravitational radiation. The proportionality factor $4\pi Gm^2\Gamma^2/e^2$ is independent of the frequency of the radiation, but it does depend on the angle between the gravitational radiation wave vector and that of the plane wave, as well as the parameters of the latter. Since the electromagnetic spectrum is completely determined by the charge trajectory, all information about the nonlocal mechanism of gravitational radiation resides in the coefficient Γ^2, which in the present case is given by Eq. (5).

We may write (5) in the form

$$\Gamma=\frac{m_*}{m}\frac{q_\perp}{q_-}. \tag{39}$$

This equation does not depend on the wave polarization, and is invariant under rotations of the vector **q** about the 3-axis.

For circular polarization, the fact that this expression is independent of the azimuthal angle of **q** is related to the axial symmetry of the emission process. For linear polarization, all of the azimuthal asymmetry of the radiation is concentrated in the factor $|j_\mu(q)|^2$, while the factor Γ^2 remains symmetric.

Let us now derive the invariant expression for Γ^2. Inasmuch as (39) does not depend on the amplitude of the external field, we may use either of the constant tensors $\varphi^{(i)}_{\alpha\beta}$, $i=1,2$; see (26). Settling upon the first, we have

$$\Gamma^2=\frac{(\overset{(1)}{\varphi_{\alpha\beta}}\, q^\alpha\bar{\pi}^\beta)^2+(\overset{(1)*}{\varphi_{\alpha\beta}}\, q^\alpha\bar{\pi}^\beta)^2}{m^2(\varphi^{(1)}_{\alpha\beta}q^\beta)^2}, \tag{40}$$

where $\varphi^{(1)*}_{\alpha\beta}=\frac{1}{2}\varepsilon_{\alpha\beta\gamma\sigma}\varphi^{(1)\gamma\sigma}$ is the tensor dual to $\varphi^{(i)}_{\alpha\beta}$, and $\bar{\pi}_\beta$ is the quasi-four-momentum (9). Expanding (40) in a system in which $\bar{\pi}=0$, we obtain agreement with (39). Note also that $\varphi^{(1)*}_{\alpha\beta}$ differs from $\varphi^{(2)}_{\alpha\beta}$ solely by a constant factor.

In the ultrarelativistic limit, the radiation wave vector **q** makes a small angle of order γ^{-1} with the velocity vector of the charge, and the gravitational radiation is emitted almost tangent to the trajectory.

For the motion we are considering here, the charge's velocity vector makes an angle ϑ with the wave vector **k** of the plane wave that lies in the range

$$\operatorname{arctg}\left[\frac{4\xi_{max}}{|\xi_1^2-\xi_2^2|}\right]\leqslant\vartheta\leqslant\operatorname{arctg}\left[-\frac{4\xi_{min}}{|\xi_1^2-\xi_2^2|}\right]+\pi, \tag{41}$$

where $\xi_{\max}=\max[\xi_1,\xi_2]$ and $\xi_{\min}=\min[\xi_1,\xi_2]$. ξ_1 and ξ_2 have been defined in (14), and they depend on the amplitudes a_1 and a_2 of the plane wave.

In the ultrarelativistic limit, when $(x^2+y^2)/2\gg 1$ holds (see (21); in other words, when the effective velocity $v_*=[(\xi_1^2+\xi_2^2)/2]^{1/2}$ approaches unity), the range of effective emission angles is also given by (41), i.e., by ξ_1 and ξ_2 or a_1 and a_2. If the latter two values are equal—that is, if the wave is circularly polarized—then $\vartheta_{\max}$ and $\vartheta_{\min}$ reduce to $\pi/2$, since the charge will then circle in a plane perpendicular to **k**. If one amplitude is zero, then we are dealing with linear polarization, and the range of angles between the velocity vector and **k** in the ultrarelativistic limit is a maximum $(2\cot^{-1}2^{1/2}\leqslant\vartheta\leqslant\pi)$. This will also be the range of effective emission angles θ. For an arbitrary ratio between a_1 and a_2, the range of effective emission angles will lie somewhere between these limiting values.

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