Two-level atom in a strong polyharmonic field

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The quasienergy spectrum of a two-level atom in a polyharmonic field is expressed in terms of the Floquet indices of a linear system of ordinary differential equations with periodic coefficients. An asymptotic technique is used to obtain an analytic description of the quasienergy spectrum in the case of a strong polyharmonic field. It is shown that "forbbidden bands" can arise for the quasilevel energies when the radiation incident on the atom is deeply modulated.

1. INTRODUCTION

The study of the dynamics of a two-level atom in an electromagnetic field is a classical problem of importance in spectroscopy and quantum optics.[1-11] A two-level atom in a field comprising a sum of two or more harmonics with frequencies close to that of the atomic transition is being diligently investigated theoretically and experimentally of late.[2-11] The present paper is devoted to an analysis of the case when the harmonics of the incident field are equidistant. This case has many physical applications for two reasons. First, it includes the particular case of a biharmonic field, when only two harmonics are present. No analytic description of the quasienergy spectrum existed up to now for this case. Second, such a structure is possessed by multimode-laser emission, where the modulation of the incident radiation can be made arbitrary by using appropriate optical elements.

The theoretical analysis of this physical system was restricted up to now either to simple situations, when a solution in terms of elementary function could be obtained[4,5] or a numerical procedure of constructing the solution was developed.[7,9,11] In the present paper are obtained explicit analytic expressions for the quasienergy spectrum of a two-level atom in a strong polyharmonic field under fairly general assumptions concerning the form of the field. We consider this problem here with the aid of the Schrödinger equation for the wave function of an atom. A more complete analysis of this physical situation is possible in the context of the density-matrix method, in which account is taken of the presence of pumping and damping. Our methods can apparently be used in this case, too. For damping that is small compared with the Rabi parameter, however, the location of the quasienergy spectrum can be sufficiently well determined also with the aid of the Schrödinger equation.

From the mathematical point of view, the problem considered here reduces to a calculation of the Floquet indices for a linear system of ordinary differential equations with periodic coefficients and with a large parameter. A procedure will be proposed here for an asymptotic calculation of the Floquet exponents and their asymptotic-expansion terms will be obtained up to $O(1)$.

2. FORMULATION OF PROBLEM AND DESCRIPTION OF MATHEMATICAL FORMALISM

We introduce the wave function of the atom in the standard form[1]

$$|\Psi(t)\rangle=[c_1(t)|1\rangle+c_2(t)|2\rangle]\exp[-i(E_1+E_2)t/2\hbar],$$

where E_m and $|m\rangle$ are the energy and wave function of the mth level, $m=1$ and 2, $\hbar$ is Planck's constant, and t is the time. The Schrödinger equation for a two-level atom in an electromagnetic field is given in terms of the amplitudes $c_m(t)$ by the system

$$\frac{dc_1}{dt}=i\omega c_1+\frac{i\mu}{\hbar}p(t)c_2,$$
$$\frac{dc_2}{dt}=-i\omega c_2+\frac{i\mu}{\hbar}p(t)c_1,$$

where μ is the dipole moment of the atom, $2\omega=(E_1-E_2)/\hbar$, and $p(t)$ is the external field. We consider a polyharmonic field

$$p(t)=\sum_{n=-\infty}^{\infty}A_n\cos[(\Omega+n\Delta)t+\psi_n],\qquad A_n=\operatorname{Re}A_n,\quad (1)$$

i.e., we assume the external field to be equal to a sum of equidistant harmonics. The frequencies Ω can, of course be shifted by $m\Delta$, where $m=\pm1,\pm2$, without changing the form of (1). We shall assume that Ω is chosen such that the harmonics with largest amplitudes have numbers of the order of unity. This condition will be formulated more accurately below. We assume here that $\Omega\approx2\omega\gg\Delta$. Using the substitution

$$c_1=a_1\exp(i\Omega t/2),\quad c_2=a_2\exp(-i\Omega t/2),$$

stretching the time scale $\tau=\Delta t$, and using the rotating-wave approximation, we obtain the system

$$\frac{da_1}{d\tau}=i\varkappa a_1+\frac{i\mu}{\hbar\Delta}R(\tau)a_2,$$
$$\frac{da_2}{d\tau}=-i\varkappa a_2+\frac{i\mu}{\hbar\Delta}\overline{R(\tau)}a_1,\quad (2)$$

$$\varkappa=(\omega-\Omega/2)/\Delta,\qquad R(\tau)=\sum_{n=-\infty}^{\infty}A_n\exp[i(n\tau+\psi_n)].$$

We impose on $R(\tau)$ the following conditions that refine the concept of a strong field and the choice of Ω:

$$\frac{\mu}{\hbar\Delta}R(\tau)=\rho q(\tau),\qquad \rho\gg1,\qquad q,q',q''=O(1),$$

with ρ having the meaning of the strong-field Rabi parameter. The system (2) takes then the form

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$$\frac{da_1}{d\tau}=i\varkappa a_1+i\rho q(\tau)a_2,$$

$$\frac{da_2}{d\tau}=-i\varkappa a_2+i\rho\overline{q(\tau)}a_1.$$

From this system we obtain an equation for the function $F(\tau)=a_2(\tau)(\overline{q(\tau)})^{-1/2}$:

$$F''+\left[\rho^2|q|^2+\varkappa^2+\frac{\bar q''}{2\bar q}-\frac{i\varkappa\bar q'}{\bar q}-\frac{3}{4}\frac{\bar q'^2}{\bar q^2}\right]F=0. \quad (3)$$

[it is the same, apart from notation, as Eq. (2.4) of Ref. 11]. It follows from the definition of the function $q(\tau)$ that Eq. (3) has coefficients periodic in τ. According to the Floquet theorem[12] its solution has the following structure:

$$F(\tau)=\alpha_1\exp(i\nu_1\tau)P_1(\tau)+\alpha_2\exp(i\nu_2\tau)P_2(\tau),$$

where ν_1 and ν_2 are constants called the Floquet exponents, α_1 and α_2 are constants determined by the initial data, and $P_1(\tau)$ and $P_2(\tau)$ are periodic functions. It follows from our equations that the functions $\alpha_{1,2}(\tau)$ have the same structure. We conclude thus that the quasienergy spectrum of our physical system is described by the relation

$$^1/_2(E_1+E_2\pm\Omega)+(\nu_{1,2}+m)\Delta,\ m=0,\ \pm1,\ \pm2,\ldots \quad (4)$$

Each of the two initial levels is split here into two infinite systems of quasilevels. An important role in practical application is played also by the Rabi spectrum of the physical system, i.e., the Fourier spectrum of the function $c_1\bar c_2-\bar c_1c_2$. It obviously consists of three sets:

$$\{N\Delta\pm\Omega\},\quad \{(\nu_1-\nu_2+N)\Delta\pm\Omega\},\quad \{(\nu_2-\nu_1+N)\Delta\pm\Omega\},$$
$$N=0,\ \pm1,\ \pm2,\ldots$$

As shown in Ref. 13, the location of the Rabi spectrum can be determined by experiment. The determination of the location of the quasienergy spectrum and of the Rabi spectrum of the system reduces to calculation of the Floquet indices of Eq. (3). This is in fact our aim and we shall use, recognizing the presence of the large parameter ρ, an asymptotic-calculation technique.[14,15]

Two possibilities exist for the parameter $\varkappa$ indicative of the detuning of the central frequency of the incident radiation from the transition frequency: $\varkappa=O(\rho)$ and $\varkappa^2=O(\rho)$. Next, different methods must be used to calculate $\nu_{1,2}$, depending on the presence or absence of zeros of the function $q(\tau)$ in the period. The function $q(\tau)$ describes the modulation of the radiation incident on the atom. We shall define the radiation as weakly modulated in the absence of zeros in the period and as deeply modulated in their presence. Altogether there are three situations which must be separately analyzed: 1) large detuning and weak modulation, $\varkappa^2=b^2\rho^2$, $|q(\tau)|>0$; 2) large detuning and deep modulation, $\varkappa^2=b^2\rho^2+a\rho$, $q(\tau)$ has zeros in the period; 3) small detuning, deep modulation $\varkappa^2=a\rho$, $q(\tau)$ has zeros in the period.

The procedure for calculating the Floquet indices for Eq. (3) reduces to the following. It is necessary to find in the vicinity of an arbitrary point τ_0 a pair of linearly independent solutions and "continue them analytically" to 2π. We obtain as a result in the vicinity of τ_0 a new pair of linearly independent solutions. The matrix relating this pair to the initial one is called the monodromy matrix. Its eigenvalues are equal to $\exp[2\pi i\nu_{1,2}]$. At different choices of both the initial point and the pairs of solutions in its vicinity, the corresponding monodromy matrices are similar to one another and their eigenvalues coincide. If the function $q(\tau)$ has no zeros in the period, $\nu(\tau)$ can be easily calculated by the WKB method (Sec. 3). If $q(\tau)$ has zeros in the period, the procedure becomes more complicated, since regular singular points arise in Eq. (3). To construct the monodromy matrix we use here the standard-equation method (Secs. 4 and 5). For large detuning we choose as standard the Whittaker equation with stretched scale, and in the case of small detuning a "hybrid" of the Whittaker equation with the parabolic-cylinder equation.

3. LARGE DETUNING AND WEAK MODULATION

Equation (3) takes for large detuning the form

$$F''+\left[\rho^2(|q|^2+b^2)+a\rho+\frac{\bar q''}{2\bar q}-\frac{i\varkappa\bar q'}{\bar q}-\frac{3}{4}\frac{\bar q'^2}{\bar q^2}\right]F=0. \quad (5)$$

Using the weak-modulation condition, we use the WKB method[14] to solve this equation and obtain

$$\nu_{1,2}=\pm\frac{1}{2\pi}\int_0^{2\pi}ds\left[\rho(|q|^2+b^2)^{1/2}+\frac{a-b\theta'(s)}{2(|q|^2+b^2)^{1/2}}\right]. \quad (6)$$

Here $\theta=\arg q$.

According to (4), each initial level splits into two quasienergy-level systems. For $2\nu_1=l$ with l an integer, however, these series coalesce into one. Thus, either one or two series of quasilevels can exist (for each initial level), depending on the values of the parameters and the form of the incident radiation. It follows from (6) that the location of the series is quite sensitive to the value of the Rabi parameter ρ: if it is relatively large ($\rho\gg1$) the changes of the harmonic can shift noticeably. A characteristic feature in this case is that for a monotonic increase (decrease) of ρ, i.e., of the radiation power, without change of the other parameters, the quasilevel series into which the initial level is split are shifted uniformly in opposite directions. Each quasilevel can have then an arbitrary value.

4. LARGE DETUNING AND DEEP MODULATION

In this situation Eq. (5) has regular singular points at $\tau=\tau_k$, $1\leqslant k\leqslant n$, where τ_k are the zeros of the function $q(\tau)$. We assume here that τ_k are simple zeros, i.e., $q'(\tau_k)\neq0$. We fix in some arbitrary manner the points τ_k':$0=\tau_1'<\tau_1<\tau_2'<\ldots<\tau_n<\tau_{n+1}'=2\pi$. In the vicinity of each point τ_k' we choose a pair of independent solutions of (5): $f(k,\tau)=(F_1(k,\tau),F_2(k,\tau))$. We assume that the choice of solutions is consistent, i.e., $f(1,\tau)=f(n+1,\tau)$. We consider the transition matrices $M(k)$ that describe the result of expanding the pair of solutions $f(k,\tau)$ in the basis of solutions $f(k+1,\tau)$. The monodromy matrix of Eq. (5), which correspond to the choice of $\tau_0=0$ as the initial point and $f(1,\tau)$ as the initial pair of solutions, is then

$$M=M(1)M(2)\ldots M(n).$$

It suffices thus to calculate the matrices $M(k)$. We construct the solutions of (5) in the vicinity of the singular point $\tau=\tau_k$ with the aid of a standard equation[15] which we choose, analyzing the chacter of the singularity in the vicinity of this point, to be the Whittaker equation with stretched scale:

$$\frac{d^2w}{dz^2}+\left[\rho^2-\frac{i\lambda\rho}{z}-\frac{3\zeta}{4z^2}\right]w=0. \tag{7}$$

We seek the solution of Eq. (5) in the vicinity of $\tau=\tau_k$ in the form

$$F(k,\tau)=[z'(k,\tau)]^{-1/2}w(z(k,\tau)), \tag{8}$$

where $w(z)$ is the solution of (7). Following Ref. 15, we can obtain a nonlinear equation for the function $z(k,\tau)$. Substituting in it the asymptotic series

$$z(k,\tau)=\sum_{m=0}^{\infty}z_m(k,\tau)\rho^{-m},$$

$$\lambda=\sum_{m=0}^{\infty}\lambda_m\rho^{-m},\qquad \zeta=\sum_{m=0}^{\infty}\zeta_m\rho^{-m},$$

we arrive at a recurrent system of equations for the functions $z_m(k,\tau)$. The parameters λ_m and ζ_m are chosen from the condition that there be no singularities in the equations for $z_m(k,\tau)$. To highest order we obtain

$$z_0(k,\tau)=\int_{\tau_k}^{\tau}ds\,(b^2+|q|^2)^{1/2},$$

$$z_1(k,\tau)=\frac{1}{2}\int_{\tau_k}^{\tau}ds\left[\frac{a}{(|q|^2+b^2)^{1/2}}-\frac{ib\bar q'}{\bar q(|q|^2+b^2)^{1/2}}+\frac{iz_0'}{z_0}\right], \tag{9}$$

$$\lambda_0=\zeta_0=1.$$

We calculate $\nu_{1,2}$ all the way to terms $O(1)$, so that it suffices to confine ourselves to the already constructed terms of the asymptotic expansions. At this accuracy, the standard equation (7) takes the form

$$\frac{d^2w}{dz^2}+\left[\rho^2-\frac{i\rho}{z}-\frac{3}{4z^2}\right]w=0. \tag{10}$$

We define a pair of solutions of (10) using their behavior as $z\to\pm\infty$:

$$W^\pm=\begin{pmatrix}w_1^\pm\\ w_2^\pm\end{pmatrix}\underset{z\to\pm\infty}{\sim}\begin{pmatrix}|z|^{-1/2}\exp(-i\rho z)\\ |z|^{1/2}\exp(i\rho z)\end{pmatrix}. \tag{11}$$

It is easy to show that

$$W^-=\begin{pmatrix}-i&0\\0&i\end{pmatrix}W^+. \tag{12}$$

Following (8), we introduce the solutions of (5) in the form $F^{\pm}_{1,2}(k,\tau)$, where we choose for $w(z)$ the functions $w^{\pm}_{1,2}(z)$. The transition matrix $M(k)$ is diagonal in the basis of the solutions $F^-_{1,2}(k,\tau)$ and by using (9), (11), and (12) we get

$$M(k)=\begin{pmatrix}\exp L(k)&0\\0&\exp[-L(k)]\end{pmatrix},$$

where

$$L(k)=-\frac{i\pi}{2}-\frac{1}{2}\ln\left|\frac{z_0(k,\tau)}{z_0(k+1,\tau)}\right|+i\int_{\tau_k}^{\tau_{k+1}}ds\left[-\rho(|q|^2+b^2)^{1/2}-\frac{a}{2(|q|^2+b^2)^{1/2}}\right]+\int_{\tau_k}^{\tau}ds\left[\frac{z_0'(k)}{2z_0(k)}-\frac{b\bar q'}{2\bar q(|q|^2+b^2)^{1/2}}\right]$$
$$+\int_{\tau}^{\tau_{k+1}}ds\left[\frac{z_0'(k+1)}{2z_0(k+1)}-\frac{b\bar q'}{2\bar q(|q|^2+b^2)^{1/2}}\right].$$

Summing over all the points τ_k and using the notation $\theta=\arg q$ and the periodicity of the function $q(\tau)$ we arrive at the final expression for the Floquet exponents in this case:

$$\nu_{1,2}=\pm\frac{1}{2\pi}\int_0^{2\pi}ds\left[\rho(|q|^2+b^2)^{1/2}+\frac{a-b\theta'(s)}{2(|q|^2+b^2)^{1/2}}\right].$$

It is assumed here that on going through a zero of the function the value of $\theta(s)$ changes jumpwise by π. With allowance for this stipulation, the last relation is essentially the same as (6). All the remarks made in the discussion of (6) are pertinent also in this case.

5. SMALL DETUNING, DEEP MODULATION

Equation (5) takes in this case the form

$$F''+\left[\rho^2|q|^2+a\rho+\frac{\bar q''}{2\bar q}-\frac{i(a\rho)^{1/2}\bar q'}{\bar q}-\frac{3}{4}\frac{\bar q'^2}{\bar q^2}\right]F=0. \tag{13}$$

Let τ_k, $M(k)$, and $1\leqslant k\leqslant n$ have the same meaning as before. We calculate the transition matrices $M(k)$ by the standard-equation method. We put

$$r_k=\frac{d}{d\tau}|q(\tau)|\,|_{\tau=\tau_k+0}>0$$

[as before, we assume the function $q(\tau)$ to have simple zeros]. A local analysis of the singularities shows that the standard equation should be chosen to be

$$\frac{d^2w}{dz^2}+\left\{\rho^2z^2+a(k)\rho-\frac{i[a(k)\rho]^{1/2}}{z}\lambda-\frac{3\zeta}{4z^2}\right\}w=0. \tag{14}$$

We construct the solution of (13) in the vicinity of $\tau=\tau_k$ in a form similar to (8). Writing down the equations for the higher-order terms $z(k,\tau)$, $a(k)$, $\lambda(k)$, $\zeta(k)$, of the asymptotic expansions we get

$$z_0^2(k,\tau)=2\int_{\tau_k}^{\tau}|q(s)|\,\mathrm{sgn}(s-\tau_k)\,ds,$$

$$2z_0(k,\tau)z_1(k,\tau)=\int_{\tau_k}^{\tau}\left[\frac{a}{z_0(k)z_0'(k)}-\frac{a_0(k)z_0'(k)}{z_0(k)}\right]ds,$$

$$a_0(k)=ar_k^{-1},\quad \lambda_0(k)=\zeta_0(k)=1. \tag{15}$$

Following the standard-equation method, we must calculate for Eq. (14) a monodromy matrix that connects a pair of solutions with a fixed behavior as $z\to\pm\infty$. It suffices here to determine this matrix with account taken of only the higher-order of the expansion of the coefficients of (14) in terms of the parameter ρ. We arrive thus at the problem of calculating the monodromy matrix for the equation

$$\frac{d^2w}{dz^2}+\left\{\rho^2z^2+a(k)\rho-\frac{i[a(k)\rho]^{1/2}}{z}-\frac{3}{4z^2}\right\}w=0. \quad (16)$$

Here and below $a(k) = ar_k^{-1}$. Generally speaking, Eq. (14) [and its particular case (16)] are "hybrids" of the parabolic cylinder equation and the Whittaker equation, and for arbitrary λ and ζ the explicit form of the monodromy matrix of (14) is as yet unknown. For Eq. (16), however, it can be calculated. We present here only the results, and relegate the details to the Appendix. Let $w_{1,2}^{\pm}(z)$ be solutions of (16) with fixed behavior as $z\to\pm\infty$:

$$w_1^{\pm}(z)\underset{z\to\pm\infty}{\sim}|z|^{-1/2}\exp[iR(z)],$$

$$w_2^{\pm}(z)\underset{z\to\pm\infty}{\sim}|z|^{-1/2}\exp[-iR(z)],$$

$$R(z)=\frac{\rho z^2}{2}+\frac{a(k)}{2}\ln(\rho^{1/2}|z|).$$

Then

$$\begin{pmatrix} w_1^- \\ w_2^- \end{pmatrix}=S\begin{pmatrix} w_1^+ \\ w_2^+ \end{pmatrix},$$

$$S=\begin{pmatrix} \sin\delta(k) & \cos\delta(k)\exp[i\varphi(k)] \\ \cos\delta(k)\exp[-i\varphi(k)] & -\sin\delta(k) \end{pmatrix}, \quad (17)$$

where

$$\delta(k)=\arcsin\frac{r_1^2(k)-r_2^2(k)}{r_1^2(k)+r_2^2(k)}, \quad (18)$$

$$r_1(k)=\frac{[a(k)]^{1/2}}{2|\Gamma(1-{}^1/_4ia(k))|},$$

$$r_2(k)=\left|\Gamma\left(\frac{1}{2}-\frac{ia(k)}{4}\right)\right|^{-1},$$

$$\varphi(k)=\frac{\pi}{4}+\arg\Gamma\left(\frac{1}{2}-\frac{ia(k)}{4}\right)+\arg\Gamma\left(1-\frac{ia(k)}{4}\right). \quad (19)$$

We introduce now pairs of solutions of (13) with fixed behavior at $\tau<\tau_k$ and $\tau>\tau_k$:

$$F_{1,2}^{\pm}(k,\tau)=[z'(k,\tau)]^{-1/2}w_{1,2}^{\pm}(z(k,\tau)). \quad (20)$$

Writing down their asymptotes at $\tau_k<\tau<\tau_{k+1}$, we obtain

$$F_2^-(k+1,\tau)=\exp[iN(k)]F_1^+(k,\tau),$$

$$F_1^-(k+1,\tau)=\exp[-iN(k)]F_2^+(k,\tau),$$

where

$$N(k)=\rho\int_{\tau_k}^{\tau_{k+1}}ds|q|+\frac{a(k)+a(k+1)}{4}\ln\rho+\frac{a(k)}{2}\ln|z_0(k,\tau)|$$

$$+\frac{a(k+1)}{2}\ln|z_0(k+1,\tau)|+\int_{\tau}^{\tau_{k+1}}ds\left[\frac{a}{|q|}+\frac{a(k+1)z_0'(k+1)}{z_0(k+1)}\right]$$

$$+\int_{\tau_k}^{\tau}ds\left[\frac{a}{|q|}-\frac{a(k)z_0'(k)}{z_0(k)}\right]. \quad (21)$$

Taking (17) and (20) into account we obtain a final expression for the transition matrices that connect the solution pairs $F_{1,2}^{-}(k,\tau)$, $F_{1,2}^{-}(k+1,\tau)$:

$$\boldsymbol{M}(k)\sim\begin{pmatrix} \cos\delta(k)\exp[i(N(k)+\varphi(k))] & \sin\delta(k)\exp[-iN(k)] \\ -\sin\delta(k)\exp[iN(k)] & \cos\delta(k)\exp[-i(N(k)+\varphi(k))] \end{pmatrix}.$$

It follows from our equations that det $M(k)\sim 1$ and consequently det $M\sim 1$. Thus, the equation for the eigenvalues of the monodromy matrix M is

$$\lambda^2-\lambda\,\mathrm{Tr}\,M+1=0,$$

whence

$$\nu_{1,2}=\pm\frac{1}{2\pi}\arccos\frac{\mathrm{Tr}\,\boldsymbol{M}}{2}. \quad (22)$$

The expressions obtained reduce the calculation of the Floquet exponents to calculation of the trace of a product of a finite number of matrices. The number of matrices, i.e., the number of zeros of the function $q(\tau)$ in the period, can be arbitrary and finite. We present here only final equations for $n=1$ and $n=2$.

If $n=1$ then $M=M(1)$ and

$$\nu_{1,2}=\pm\frac{1}{2\pi}\arccos\{\cos\delta(1)\cos[N(1)+\varphi(1)]\}.$$

If $n=2$ then $M=M(1)M(2)$ and

$$\nu_{1,2}=\pm\frac{1}{2\pi}\arccos\{\cos\delta(1)\cos\delta(2)$$

$$\times\cos[N(1)+N(2)+\varphi(1)+\varphi(2)]-$$

$$-\sin\delta(1)\sin\delta(2)\cos[N(1)-N(2)]\}.$$

Here $\delta(k)$, $\varphi(k)$, and $N(k)$ are given by (18), (19), and (21). Our equations point, as above, to explicit conditions for the quasilevel crossings. In this case however, the situation differs substantially from those considered above. It is connected with the behavior of the quasilevels when the parameter ρ is monotonically increased (decreased) at fixed values of the remaining parameters and of the form of the incident radiation. For example, let $n=1$ and $\delta(1)\neq\pi l$, with l an integer. For a monotonic variation of ρ the quasilevels will then vary but remain each in a limited interval. "Forbidden bands" appear in this case, i.e., intervals in which no quasilevel can land at any ρ. Quasilevel crossing is then impossible, since $2\nu_1$ cannot take on the integer values of m. A similar statement is valid also for $n=2$, $\delta(1)-\delta(2)\neq\pi l$, as well as for $n=3, 4, \ldots$

6. CONCLUSION

Let us sum up. An analytic description of the quasienergy spectrum (and of the Rabi spectrum) of a two-level atom was obtained in the case of a strong polyharmonic field under rather broad assumptions concerning the form of the field. Our equations yield in explict form the condition for the crossing of the quasilevels of this system. As noted in Ref. 11, resonances are observed in this case in the absorption spectrum of the system. (In Ref. 11, in which the formulation of the problem is similar to ours, the quasienergy spectrum is not described explicitly; it is reduced to the problem of calculating an infinite determinant whose values are obtained subsequently by numerical methods).

We have shown that at small detuning of the incident-radiation central frequency from the atomic-transition frequency and at deep modulation of the incident radiation there can appear in the quasienergy spectrum "forbidden bands" or intervals in which, at a fixed form of the incident radiation, no quasienergy levels can land at any radiation power. This phenomenon is similar to the corresponding effect in solid-state physics. No quasi-energy-level crossing is possible here.

We point out here a relation between our results and studies of the Stark effect for a two-level atom in a polyharmonic field, see for example Ref. 16. We have analyzed a situation in which the Rabi parameter is of the same order as (even larger than) the distance between the transition frequency and the central frequency of the incident radiation, $\varkappa^2 = b^2\rho^2 + a\rho$. Here, as noted in Ref. 11, the Stark phenomenon constitutes splitting of each level into two series of sublevels. Our equations make it possible to track the displacements of these quasilevels upon adiabatic variation of the incident-radiation power, i.e., they contain a complete description of the Stark effect in this case. A case that in a certain sense the opposite of ours was considered in Ref. 16, where the problem was treated under the assumption that the Rabi parameter is much smaller than the distance between the central frequency of the incident radiation and the transition frequency.

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APPENDIX

We describe here the method of calculating the monodromy matrix (17) for Eq. (16). It is based on the following fact. If $w(z)$ is the solution of (16) and $u(z)$ is defined as

$$u(z)=z^{1/2}w(z)+\frac{[z^{1/2}w(z)]'+i[a(k)\rho z]^{1/2}w(z)}{i\rho z}, \qquad (\mathrm{A1})$$

then $u(z)$ is a solution of the following parabolic-cylinder equation with variable scale:

$$u''+[\rho^2z^2+(a(k)-i)\rho]u=0. \qquad (\mathrm{A2})$$

Inverting (A1), we obtain

$$w(z)=\frac{iu'(z)+\{[a(k)\rho]^{1/2}+\rho z\}u}{z^{1/2}}.$$

The solutions of Eq. (16) and accordingly the monodromy matrices (17) can thus be constructed with the aid of solutions of Eqs. (A2). We derived (17) using standard equations,[13] but recognizing that the asymptotic solutions given in Ref. 13 for the parabolic-cylinder equation are valid only for real values of the parameter of the equation. We therefore constructed the monodromy matrix (17) using direct expressions for the solutions of the parabolic-cylinder equations in terms of confluent hypergeometric functions.

[1] S. Stenholm, *Foundations of Laser Spectroscopy* [Russ. transl.]. Mir, 1987, Ch. 2.
[2] S. G. Rautian and I. I. Sobel'man, Zh. Eksp. Teor. Fiz. **41**, 456 (1961) [Sov. Phys. JETP **14**, 334 (1962)].
[3] S. P. Apanasevich, Dokl. Akad. Nauk USSR **12**, 878 (1968).
[4] S. P. Goreslavskiĭ and V. P. Kraĭnov, Zh. Eksp. Teor. Fiz. **76**, 26 (1979) [Sov. Phys. JETP **49**, 13 (1979)].
[5] R. W. Boyd, M. G. Raumer, P. Narum *et al.*, Phys. Rev. A **24**, 411 (1981).
[6] A. M. Bonch-Bruevich, T. A. Vartanyan, and N. A. Chigir', Zh. Eksp. Teor. Fiz. **77**, 1899 (1979) [Sov. Phys. JETP **50**, 901 (1979).
[7] G. I. Toptygina and E. E. Fradkin, *ibid.* **82**, 429 (1982) [**55**, 246 (1982)].
[8] I. S. Zeĭlikovich, S. A. Pul'kin, L. S. Gaĭda, and V. N. Komar, *ibid.* **94**, No. 12, 76 (1988) [**67**, 2434 (1988)].
[9] Z. K. Yankauskas and G. I. Valyukenas, Opt. Spektrosk. **67**, 149 (1989) [Opt. Spectrosc. **67**, 84 (1989)].
[10] L. S. Gaida, I. S. Zelikovich, and S. A. Pul'kin, *ibid.* **64**, 695 (1988).
[11] G. I. Toptygina and E. E. Fradkin, Zh. Eksp. Teor. Fiz. **97**, 766 (1990) [Sov. Phys. JETP **70**, 428 (1990).
[12] J. C. Camparo and R. P. Fruenholz, Phys. Rev. A **38**, 6143 (1988).
[13] M. Abramovitz and I. A. Stegun, *Handbook of Mathematical Functions*, Dover, 1964, Ch. 13, 19, 20.
[14] M. V. Fedoryuk, *Asymptotic Methods for Linear Ordinary Differential Equations* [in Russian], Nauka, 1983.
[15] I. V. Komarov, L. I. Ponomarev, and S. Yu. Slavyanov, *Spheroidal and Coulomb Spheroidal Functions* [in Russian], Nauka, 1977, Ch. 1 and 5.
[16] N. F. Perel'man, Zh. Eksp. Teor. Fiz. **79**, 775 (1980) [Sov. Phys. JETP **52**, 395 (1980)].

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