Nonlinear theory of relaxation of unstable magnetization precession in ^{3}He-A

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A theory of relaxation of the magnetization in the superfluid A-phase of liquid ^{3}He under conditions of the pulsed NMR method is proposed. The relaxation is due to the development of instability of spatially homogeneous precession. The localized initial perturbations create moving fronts on which the transition of the precessing magnetization from the unstable initial state to the equilibrium state occurs. The main dissipating mechanism is spin diffusion in the region of the front. In the limiting cases of strong and weak diffusion the velocity of the fronts is found analytically, and in the intermediate region—numerically.

1. INTRODUCTION

Spatially homogeneous precession of magnetization in the superfluid A-phase of helium-3 is unstable.[1] This circumstance is important for the interpretation of pulsed NMR experiments in ^{3}He-A. According to experiments,[2,3] at not too small initial magnetization tipping angles, it is just this instability that determines the duration of the induction signal. The linear theory[4] allows one to describe only the initial stage of the development of the instability. The solution of the nonlinear problem in general form is probably impossible. In the case of simpler systems it is clear, however, that when the initial perturbations are localized, the development of the instability can proceed as in combustion, i.e., a front propagates from the initial perturbation, in front of which the medium is in an unstable stationary state, and behind which in the equilibrium state. Dissipation of excess energy takes place on the front. Kolmogorov, Petrovskiĭ, and Piskunov[5] showed that such a solution is possible in the nonlinear diffusion theory. They also found the speed of propagation of the front.

Recently the question of the transition of unstable systems to equilibrium was considered for a wider class of equations by Kamenskiĭ and Manakov,[6] who proposed a procedure which allows one to determine whether the indicated regime exists for some given concrete problem, and to find the speed of propagation of the front on the basis of an analysis of the dependence of the increment of growth of the perturbations on the wave vector **k**. In the present article the indicated procedure is applied to the description of a possible path of development of the instability of the precession of the magnetization in ^{3}He-A in the nonlinear stage. A brief exposition of the results obtained for the case of strong spin diffusion has already been published.[7]

Motion of the magnetization in ^{3}He-A takes place under the action of two moments—the Zeeman moment created by the magnetic field $\mathbf{H}_0$ and the moment of the dipole forces $\mathbf{N}_d$. The ratio of the dipole moment to the Zeeman moment is characterized by the parameter $(\Omega/\omega_L)^2$, where Ω is the frequency of longitudinal oscillations and ω_L is the Larmor frequency for the field $\mathbf{H}_0$. The frequency Ω depends on the temperature and the pressure and, since its maximum value corresponds to the field $H_{\max} \approx 25$ Oe, the dipole contribution for fields H_0 several times greater than $H_{\max}$ can be considered as a correction. In the first approximation in this correction the Leggett equations for the A-phase have a two-parameter family of periodic solutions which describe a spatially homogeneous precession of the magnetization at a frequency that differs from the Larmor frequency by the amount $\sim(\Omega/\omega_L)^2$. The parameters are the angle β between the magnetization and the field $\mathbf{H}_0$ and the phase α of the precession. Motion of the spin part of the order parameter—the vector **d**—for these solutions is uniquely determined by the motion of the magnetization. A linear analysis of the evolution of .he small spatially homogeneous perturbations of the precession shows that there is an unstable mode for which the small harmonic perturbations of the angles α and β, as well as the parameters which describe the motion of **d**, grow exponentially with time. The dependence of the growth rate Γ on the perturbation wave vector **k** is described by the following formula:

$$\Gamma(\mathbf{k}) = \left\{ \frac{C_{kk}}{4\omega_L{}^2} \left[\frac{3}{4} \Omega^2 \sin^2\beta - C_{kk} \right] \times \frac{\Omega^2(3-\cos\beta)(1+\cos\beta)+C_{kk}}{\Omega^2(1+\cos\beta)^2+C_{kk}} \right\}^{1/2} - D_{kk}. \tag{1}$$

Here we have introduced the notation $C_{kk} = C_{\xi\eta}k_\xi k_\eta$ and $D_{kk} = D_{\xi\eta}k_\xi k_\eta$; $C_{\xi\eta}$ and $D_{\xi\eta}$ are respectively the spin velocity tensor and the spin diffusion tensor. Since the problem has a preferred direction **l**, the spin velocity tensor of the waves is determined by its two principal values:

$$C_{\xi\eta} = C_\perp{}^2(\delta_{\xi\eta} - l_\xi l_\eta) + C_\parallel{}^2 l_\xi l_\eta.$$

We shall assume that the spin diffusion tensor has an analogous structure with principal values $D_\perp$ and $D_\parallel$. The velocities $C_\perp$ and $C_\parallel$ vanish as $T \to T_c$ like $(1 - T/T_c)^{1/2}$ and $C_\perp^2 \to 2C_\parallel^2$, so that for the same magnitude of **k** the perturbations with wave number perpendicular to **l** grow faster.

We will discuss the question of real perturbation sources in Sec. 6, but for now we will assume that these perturbations depend only on the one coordinate z reckoned in the direction antiparallel to $\mathbf{H}_0$ and perpendicular to **l**, and that they are localized with respect to this coordinate. In this case the problem becomes one-dimensional and we may apply the approach of Ref. 6, according to which we make the substitution

$$C_{kk} = C_\perp{}^2 k^2 = C^2 k^2, \quad D_{kk} = D_\perp{}^2 k^2 = D^2 k^2$$

for the increment in expression (1). Below we show that under the given assumptions about the form of the initial

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perturbations in ^{3}He-A relaxation of the precession of the magnetization can take place by way of propagation of a front, and we find the speed of the front and estimate its width for various values of the parameters entering into the problem. This can be done by using only the form of the increment $\Gamma(k)$. The nonlinear system of equations of motion is required only for the analysis of processes in the region of the front. This system is derived for the corrections in the Appendix; however, we do not solve it here. To describe magnetic relaxation in ^{3}He-A and interpret the corresponding experiments, it is enough to know the speed of the front and to show that the width of the front is small in comparison with the characteristic dimensions of the cell.

2. PROCEDURE FOR FINDING THE SPEED OF THE FRONT

Let $\eta_0(z)$ be the initial perturbation, e.g., of the angle β, localized near the origin. Then for $t>0$ and as long as $\eta(z, t)$ is sufficiently small so that the problem can be considered to be linear the evolution of η is determined by the Fourier method:

$$\eta(z,t)=\sum_\nu\int A_\nu(k)\exp[i(kz-\omega_\nu t)]dk. \quad (2)$$

The summation here is carried out over all of the various branches of the spectrum $\omega_\nu(k)$ of the system of equations, linearized about the initial state of the system of equations, and $A_\nu(k)$ is the Fourier transform of the projection of $\eta_0(z)$ on the corresponding normal coordinate. We will treat the perturbations from the point of view of an observer moving in the direction of the z axis with velocity V. For this purpose it is convenient to introduce in place of z the variable $\zeta=z-Vt$. We then have for the contribution of the νth mode

$$\eta_\nu(\zeta,t)=\int A_\nu(k)\exp[ik\zeta+h_\nu(k)t]dk, \quad (3)$$

where $h_\nu(k)=i[kV-\omega_\nu(k)]$. Then, following the procedure in Ref. 6 (cf. also Refs. 8 and 9), we find the asymptotic form of $\eta(z, t)$ at large t and z, but finite ζ. Restrictions on t and ζ will be formulated later. For what follows only the unstable branches of $\omega_\nu(k)$ will be significant; the contributions of the other modes decay near the origin. The indicated asymptotic form can be found by the method of steepest descent, and is determined by the saddle points k_s of the arguments of the exponentials $h(k)$ as functions of complex k. The form of the integration contour in the complex k plane is determined by the concrete $h(k)$ dependence. The significant saddle points make the following contribution to the asymptotic form:

$$\eta_\nu(\zeta, t)\approx A_\nu(k_s)\exp[ik_s\zeta+i\delta+h(k_s)t][2\pi/h''(k_s)t]^{1/2}. \quad (4)$$

The additional phase shift δ is determined by the direction of the integration contour at the saddle point. The nature of the variation of the perturbation with time depends on the sign of the real part of $h(k_s)$. The perturbation grows if Re $h(k_s)>0$, and decays if Re $h(k_s)<0$. The locations of the saddle points and the signs of Re $h(k_s)$ depend on the velocity V as a parameter. If there exists $V=V_c$ such that for $V>V_c$ for all of the significant saddle points Re $h(k_s)<0$, then for an observer moving with a velocity greater than V_c *small initial* perturbations will remain small and their asymptotic form will be described by formula (4). The minimum value of V_c, i.e., the one for which any of the values of Re $h(k_s)$ changes sign, should be taken as the speed of propagation of the front since an observer moving with a smaller velocity will see perturbations growing in time and a transition to the nonlinear regime.

For sufficiently large times formula (4) can be used to describe the form of the perturbation a distance ζ ahead of the front. To do this the variation of the phase $\zeta * \delta k$ over the region $\delta k\simeq 1/[h''(k_s)t]^{1/2}$ of influence of the saddle must be small, i.e., the condition $\zeta^2\ll h''(k_s)t$ must be fulfilled. It is natural to define the width of the front l_{fr} such that the perturbations can be assumed to be small for $\zeta\gg l_{fr}$. The condition so obtained may therefore be thought of as a restriction on the time after which the front is formed:

$$t\gg l_{fr}^2/h''(k_s). \quad (5)$$

Below in the consideration of concrete cases we will adduce estimates both of the width of the front and of its formation time.

For the calculations it is convenient to transform to dimensionless quantities:

$$-i\omega_\pm(k)=\Gamma_\pm(k)=\frac{3\Omega^2}{4\omega_L}\gamma_\pm(q)\sin^2\beta,$$

$$k^2=\frac{3\Omega^2}{4C^2}q^2\sin^2\beta,\quad \Lambda=\frac{2D\omega_L}{C^2}.$$

The subscripts "$\pm$" denote the two branches of the root in formula (1). For $\gamma_\pm(q)$ we then have

$$\gamma_\pm(q)=\pm\left[q^2(1-q^2)\frac{(3-\cos\beta)+3(1-\cos\beta)q^2}{(1+\cos\beta)+3(1-\cos\beta)q^2}\right]^{1/2}-\Lambda q^2. \quad (6)$$

The growth rate $h(k)$ should also be reduced to dimensionless form:

$$h(k)=\frac{3\Omega^2\sin^2\beta}{8\omega_L}\tilde{h}(q),$$

where $\tilde{h}(q)=iqw+\gamma(q)$ and we have introduced the dimensionless velocity w according to the relation

$$V=\frac{3^{1/2}\Omega}{4\omega_L}Cw\sin\beta.$$

The saddle points are found from the equation

$$d\tilde{h}(q)/dq=0. \quad (7)$$

An analytic solution of Eq. (7) with growth (6) with subsequent analysis of the sign of Re $h(k_s)$ turns out to be possible only at the limiting values of the diffusion coefficient $\Lambda\gg 1$ and $\Lambda=0$. Just these cases will be considered in the following two sections. In the intermediate region the solution is found numerically.

3. THE CASE OF STRONG DIFFUSION

From formula (6) it is clear that with growth of Λ the interval of wave vectors for which the perturbations grow with time narrows down and for $\Lambda\gg 1$ the development of the instability is determined by the region of small q. If we exclude from consideration the region of angles β close to π (this case will be considered separately), then for $q\ll 1$

$$\gamma_\pm(q)=\pm\left[\frac{3-\cos\beta}{1+\cos\beta}\right]^{1/2} q-\Lambda q^2. \tag{8}$$

Let us first consider the branch $\gamma_+(q)$. Then

$$\tilde h=iqw+q\varphi(\beta)-\Lambda q^2,\quad \varphi(\beta)=\left(\frac{3-\cos\beta}{1+\cos\beta}\right)^{1/2}.$$

The function $\tilde h$ has a saddle point

$$q_{s+}=[iw+\varphi(\beta)]/2\Lambda.$$

The initial integration contour, passing along the real axis, must be shifted upward parallel to itself so that it passes through the point q_{s+}. Then the time dependence of the corresponding contribution to the asymptotic limit will be determined mainly by the exponential

$$\exp[(3\Omega^2/8\omega_L)\tilde h_+(q_s)t],$$

where

$$\tilde h_+(q_s)=(1/4\Lambda)[(\varphi^2-w^2)+2iw\varphi].$$

The sign of the real part of $\tilde h$ changes at $w=\varphi(\beta)$; this value of w is to be taken as the speed of propagation of the front w_c. Transforming to dimensioned quantities, we obtain for it the following expression:

$$V_c=C\frac{\Omega}{4\omega_L}\sin\beta\left(3\frac{3-\cos\beta}{1+\cos\beta}\right)^{1/2}. \tag{9}$$

Next setting $z=V_ct+\zeta$ and using the standard procedure of the method of steepest descent, we find the asymptotic form of the perturbation ahead of the front in a system moving with velocity V_c:

$$\eta\approx A(k_s)\left(\frac{\pi}{Dt}\right)^{1/2}\exp\left\{\frac{3^{1/2}\Omega}{8\Lambda C}\varphi(\beta)[iV_ct+(i-1)\zeta]\sin\beta\right\}. \tag{10}$$

For the branch $\gamma_-(q)$ the saddle point is

$$q_{s-}=[iw-\varphi(\beta)]/2\Lambda$$

and the integration contour must be made to pass through it just as for q_{s+}—parallel to the real axis. The speed of the front is found to be the same as for γ_+, but the asymptotic form of the perturbation ahead of the front is given by the complex conjugate of expression (10). Thus, the front propagates with velocity given by formula (8), and the perturbation ahead of the front decays at a length of the order of

$$\lambda\sim\Lambda C/\Omega\sim\Lambda l_d.$$

Here we have introduced the dipole length $l_d=C/\Omega\approx10^{-3}$ cm. An estimate of the width of the front can be obtained from energetic considerations. From formula (10) we conclude that the characteristic scale of the inhomogeneity in the front region is the length λ. At the front width l_{fr} the dissipation rate per unit area of the front is $\approx l_{fr}D\omega_L^2/\lambda^2$, while on the other hand the same quantity should be of the order of $V_c\omega_L^2$. Comparing both expressions, we obtain the estimate

$$l_{fr}\sim\lambda^2V_c/D\sim\Lambda C/\Omega\sim\Lambda l_d\sim\lambda.$$

Using next condition (5), we arrive at an estimate for the formation time of the front:

$$t\gg\Lambda\omega_L/\Omega^2,$$

for typical experimental conditions this time is small in comparison with the total relaxation time.

Let us now consider the region of angles close to π. The first term in formula (6) has a singularity as $q\to0$ and $\beta\to\pi$.

The result of taking the limit depends on the order in which it is carried out. We set $\beta=\pi-\psi$ and keep the main terms in formula (6) as ψ and $q\to0$. As a result we obtain

$$\gamma\approx[8q^2/(\psi^2+12q^2)]^{1/2}-\Lambda q^2.$$

If $\psi^2\gg12q^2$ for the values of q that are significant in the problem, then, arguing as before, we obtain for the speed of the front expression (9) expanded about $\beta=\pi$. In this case the region $q\propto1/\Lambda^{1/2}$ is significant since the saddle point is located a distance $\propto1/\Lambda\psi$ from the origin and has a region of influence $\propto1/\Lambda$.$^{1/2}$ This condition limits the applicability of formula (9) to the region $\psi\gg1/\Lambda^{1/2}$.

In the opposite case, $12q^2\gg\psi^2$, we have

$$\gamma\approx(2/3)^{1/2}-\Lambda q^2, \tag{11}$$

the saddle point in this limit is equal to $q_s=iw/2\Lambda$, and the front speed corresponding to it is equal to $w=(32/3)^{1/4}\Lambda^{1/2}$, which in dimensioned units corresponds to

$$V_c=(3/8)^{1/4}(\Omega/\omega_L)\Lambda^{1/2}C\sin\beta. \tag{12}$$

The region of applicability of this expression for the front speed is $\psi\ll1/\Lambda^{1/2}$. Note that in this limit the increment (11) has the same form as in the problem considered by Kolmogorov *et al.*,[5] and the answer, of course, coincides with that obtained there.

4. THE CASE OF WEAK DIFFUSION

The presence of dissipative terms in the equations of motion is important for the establishment of the considered regime of development of the instability since it is dissipation that carries the solution to the equilibrium state in the process of motion of the front and determines the width and shape of the front. However, the front propagation speed found by the procedure described in Section 2 remains finite also as $\Lambda\to0$. Let us first consider this limit. To simplify the calculations further, we will assume that the angle β is small. Then, according to formula (6), $\gamma=\pm q(1-q^2)^{1/2}$ and we must find the stationary points of the function $\tilde h_\pm=iqw+\gamma_\pm(q)$, i.e., of the equation

$$w=(2q^2-1)(q^2-1)^{-1/2}. \tag{13}$$

For $w>2^{3/2}$ Eq. (13) has two real roots, and for both of them $q^2>1$ and the function $\tilde h$ has purely imaginary values at the stationary points. This means that for $w>2^{3/2}$ the perturbations will not grow at large times. For $w<2^{3/2}$ the roots of Eq. (13) become complex, which leads to exponential growth, i.e., in this case $w=2^{3/2}$ is the speed of the front, or in dimensioned units

$$V_c=(3/2)^{1/2}(\Omega/\omega_L)C\sin\beta. \tag{14}$$

At this speed the two stationary points coalesce and a stationary point of higher order appears: $q_0^2=3/2$. This circumstance leads to the result that the perturbation ahead of the front decays only according to a power law: as $(z - V_c t)^{-1/4}$. In such a situation there is hardly any sense in speaking of the existence of a front; however, addition of even a small diffusion makes the decay exponential, and the change in the speed of the front turns out to be small.

To consider this question in more detail, let us expand $\tilde{h}_-(q)$ in the vicinity of the point $q_0 = (3/2)^{1/2}$, setting $q = q_0 + p$ and $w = w_0 - u$:

$$\tilde{h}_- \approx i[3^{3/2}/2 - u(3/2)^{1/2}] - ipu - 2^{3/2} i p^3 - 6^{1/2}\Lambda p - {}^3/_2\Lambda.$$

The values of $p = p_s$ at which the function $\tilde{h}$ is stationary are given by the expression

$$p_s^2 = \frac{2iq_0\Lambda - u}{3\cdot 2^{3/2}}.$$

The addition u to the front speed is found from the condition that the real part of $\tilde{h}_-$ vanish. Analysis of the equation

$$\mathrm{Re}\left(2^{5/2} i p_s^3 - \frac{3}{2}\Lambda\right) = 0$$

shows that $u \gg \Lambda$ and to leading order in Λ

$$u = {}^1/_4 ({}^9/_2)^{5/6} \Lambda^{2/3},$$

i.e., the addition to the front speed is small as long as $\Lambda^{2/3}$ is small.

To find the asymptotic form of the perturbation ahead of the front, as we set $z = V_c t + \zeta$ in Sec. 3 and use the method of steepest descent. The decay of the perturbation ahead of the front is then described by the factor

$$\exp\left(-\frac{3^{5/6}}{4\cdot 2^{1/6}} \frac{\Omega}{C} \Lambda^{1/3} \zeta \sin\beta\right),$$

i.e., the characteristic length over which the decay takes place is proportional to $l_d/\Lambda^{1/3} \sin\beta$. According to criterion (5) the formation time of the front in this case is $\sim \omega_L / \Lambda\Omega^2 \sin\beta$.

Formula (14) for the front speed is applicable only for small angles β; however, at such angles, as was shown by Sonin,[10] the instability of the precession is suppressed by the spatially homogeneous relaxation by the Leggett-Takagi mechanism. For applications it is possible to obtain a satisfactory approximate formula for the front speed at small Λ and arbitrary angles β. Toward this end it is necessary to represent the radicand in formula (6) in the form

$$q^2(1-q^2)\left\{1 + \frac{2}{3[\chi(\beta)+q^2]}\right\}, \quad \chi(\beta) = \frac{1+\cos\beta}{3(1-\cos\beta)}$$

and expand the radical in formula (6) with respect to the second term in the braces in the above expression. As a result we have

$$\tilde{h}(q) = iqw - iq(1-q^2)^{1/2}\left[1 + \frac{1}{3(\chi+q^2)}\right],$$

which gives after differentiation

$$w = \frac{2q^2-1}{(q^2-1)^{1/2}} + \frac{1}{3(q^2-1)^{3/2}(\chi+q^2)}[(2\chi+1)q^2 - \chi].$$

Regarding the second term on the right-hand side as a correction, we obtain after the first iteration

$$w = 2^{1/2}\left[1 + \frac{(1-\cos\beta)(13-5\cos\beta)}{(11-7\cos\beta)^2}\right].$$

This formula gives a good approximation for the speed even at $\beta = \pi$, when the error should be maximal. The angle-dependent correction does not exceed 10%.

5. INTERMEDIATE VALUES OF Λ

The experiments that have been conducted on the relaxation of ^{3}He-A used magnetic fields of ~ 100 Oe. For such fields and for temperatures not too close to T_c we have $\Lambda \approx 0.2$–0.3, but as $T \to T_c$ the parameter $\Lambda \to \infty$ owing to the vanishing of the spin wave velocity, making the region $\Lambda \sim 1$ of greatest interest from the point of view of applications. For such Λ the front speed was found numerically using the procedure described in Sec. 2. Figure 1 shows the results of the calculations. As $\Lambda \to 0$ and $\Lambda \to \infty$ the numerical values of the front speed approach the corresponding limits found using the formulas in Secs. 3 and 4. The variation of the ratios of the limits at the left and right infinities as one goes from small angles to angles close to π is connected with the aforementioned singularity of the increment as $\beta \to \pi$.

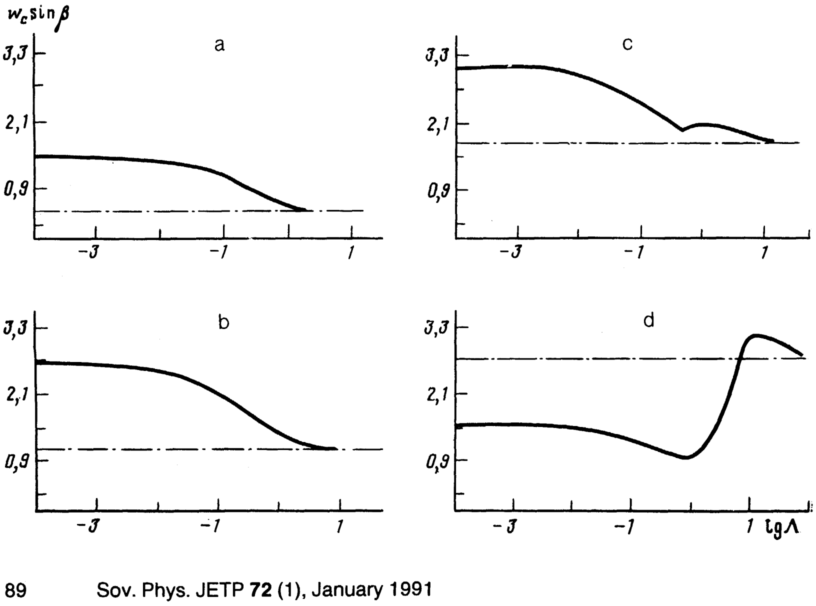

FIG. 1. Dependence of the dimensionless propagation speed of the front $w \sin\beta = 4\cdot 3^{-1/2}(\omega_L/\Omega)(V/C)$ on the logarithm of the dimensionless spin diffusion coefficient $\Lambda = 2D\omega_L/C^2$ for four values of the initial tipping angle of the magnetization β: a) 30°, b) 60°, c) 90°, d) 150°.

Figure 2 shows the dependence of $w_c \sin\beta$ on β at different Λ. This dependence is not very strong and is determined mainly by the factor $\sin\beta$. The dependence of the front speed w on Λ is also not very strong. The order of magnitude of the front speed and its temperature dependence are determined mainly by the scaling factor $\Omega C/\omega_L$. At moderate magnetic fields the entire existence region of the A-phase is located near T_c and it can be assumed that $\Omega C \propto 1 - T/T_c$. For rough estimates one can assume that the coefficient in this dependence does not depend on the pressure, then

$$\frac{\Omega C}{\omega_L}\left[\frac{\text{cm}}{\text{s}}\right] \approx 10^5 \frac{1-T/T_c}{H[\text{Oe}]},$$

which for the typical values $H \sim 100$ Oe and $T \approx 0.9T_c$ gives the speed $V \sim 1$ m/s. To obtain more exact values of the front speed one must use the dependences shown in the figures.

6. INITIAL PERTURBATIONS AND RELAXATION RATE

To apply the results it is necessary to refine the assumption made in Section 1 that the initial perturbations are localized. This assumption was used in applying the method of steepest descent, in which it was assumed that the Fourier transform of the initial perturbations $A(k)$ is a slowly varying function in the vicinity of the saddle points. For $\Lambda \sim 1$ the saddle points are located at $k \lesssim l_d^{-1}$ and the initial perturbations can be assumed to be localized if they are characterized by a scale less than $l_d \sim 10^{-3}$ cm. Deviations of the initial conditions from homogeneity at such scales can arise in the bulk of the ^{3}He as well as at the walls.

Bun'kov, Dmitriev, and Mukharskiĭ[3] have experimentally investigated the influence of the walls on the development of the instability of precession in ^{3}He-A and found that the introduction of additional walls oriented perpendicular to $\mathbf{H}_0$ accelerates the relaxation, but has practically no effect if oriented parallel to $\mathbf{H}_0$. They connected this result with the presence near the walls perpendicular to the field of an inhomogeneous texture of the vector $\mathbf{l}$ that characterizes the orientation of the orbital part of the order parameter. The boundary conditions require that $\mathbf{l}$ be oriented along the normal to the wall. In the bulk of the liquid in equilibrium we have $\mathbf{l} \perp \mathbf{H}_0$. Both of these conditions can be simultaneously satisfied only for walls parallel to $\mathbf{H}_0$. Near walls perpendicular to $\mathbf{H}_0$ or making a finite angle with the direction of the field there appears a transitional layer with thickness of the order of l_d. The spin precession frequency in ^{3}He-A depends

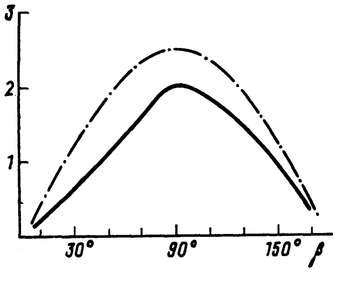

FIG. 2. Dependence of the dimensionless propagation speed of the front on the initial tipping angle of the magnetization for $\Lambda = 1$ (solid curve) and $\Lambda = 0.1$ (dot-dashed curve).

on the mutual orientation of the vectors $\mathbf{l}$ and $\mathbf{H}_0$, for which reason the local precession frequency near walls not parallel to $\mathbf{H}_0$ differs from the spin precession frequency in the bulk. As a result, after turning off the tipping pulse there arises a state whose spatial homogeneity is violated near such walls. If the investigated helium volume is bounded only by walls parallel to $\mathbf{H}_0$ (side walls) and perpendicular to $\mathbf{H}_0$ (base walls), then each of the base walls serves as a source from which a front propagates after the tipping pulse is turned off.

In this case the time dependence of the total (integrated over the investigated volume) longitudinal component of the magnetization should be linear, and the total relaxation time τ—if the formation time of the fronts is not taken into account—should be proportional to the distance L between the bases and equal to $L/2V_c$. The dependence of τ on the magnetic field, on the temperature, and on the initial tipping angle is a result of the dependence of the front speed on the indicated quantities. As has been shown, it is determined mainly by the factor $\omega_L/\Omega \propto H/(1 - T/T_c)\cdot \sin\beta$. Dependences have been experimentally observed[2,3,11] that are close to those indicated. The observed relaxation time of ~ 1 ms coincides to within an order of magnitude with the estimate for fields $H \sim 100$ Oe and cells with characteristic size ~ 1 cm. However, the cell geometry in these experiments was far from ideal for the realization of the simple relaxation regime considered here.

A serious limitation of the applicability of the above-described relaxation picture is the assumption of the absence in the investigated helium volume of significant initial perturbations. Such perturbations may be solitons or domain walls separating the regions in which $\mathbf{l}\|\mathbf{d}$ from those in which $\mathbf{l}\|-\mathbf{d}$. These walls also have thickness $\approx l_d$. The domain walls are difficult to prepare or annihilate in a controllable way, and only a small number of them is needed to initiate instability in the volume. To realize pure conditions it is necessary, in the preparation of the initial state, to avoid procedures in which solitons appear. It is well known, for example, that they arise during the relaxation of the magnetization after it has been tipped by an angle close to 180°. Solitons can also form when the A-phase is obtained by reheating from the B-phase since this is a first-order transition.

In the absence of solitons direct observation of the propagation of fronts is possible with the help of a few detector coils in analogy with the way this was done in ^{3}He-B.[12] However, it is important to emphasize the fundamental difference in the mechanism and character of the relaxation in both phases. In ^{3}He-B the relaxation proceeds quasistatically, and when the dissipative mechanisms is turned off the emerging two-domain structure is formed infinitely slowly. In ^{3}He-A the homogeneous precession decays even in the absence of dissipation, after a time of the order of the inverse growth rate of the perturbations, and the formation of the fronts takes place as a consequence of the existence of dissipation.

The type of magnetization relaxation considered here is new also in comparison with types known for other magnetic materials. Comparison with the available experimental data does not allow one to definitely conclude that relaxation in ^{3}He-A takes place by formation and propagation of fronts, wherefore it would be helpful if experiments would be devised having as their aim the direct observation of fronts and the measurement of their speed of propagation.

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APPENDIX

The motion of the order parameter—the vector **d**—is parametrized by the Euler angles α, β, and γ according to the definition

$$\mathbf{d}(t)=R(\alpha,\beta,\gamma)\mathbf{d}_0=R_z(\alpha)R_y(\beta)R_z(\gamma)\mathbf{d}_0.$$

Here $\mathbf{d}_0$ is the initial orientation of the order parameter. For definiteness, we will assume that it coincides with the y axis. $R_z(\alpha)$ is the matrix of the rotation by the angle α about the z axis, etc. The angles α, β, and γ are canonically conjugate with the spin projections S_z, S_ζ, and S_β, respectively, on the $\hat{z}$ axis, the $\hat{\zeta}=R(\alpha,\beta,\gamma)\hat{z}$ axis, and the direction $\hat{\zeta}\times\hat{z}$. The complete system of equations of motion consists of six equations for the indicated variables; however, for $(\Omega/\omega_L)^2\ll 1$ it can be reduced to a system of four equations for the variables α,β, $\Phi=\alpha+\gamma$, and S_ζ. We will assume that the variables depend only on the one spatial coordinate z, and we will denote differentiation with respect to this coordinate by a prime. We will choose the units to be such that the magnetic susceptibility of ^{3}He-A and the gyromagnetic ratio for the ^{3}He nuclei are equal to unity. Then the equations of motion take the form

$$\frac{\partial\alpha}{\partial t}+\omega_L=-\frac{1}{S_\zeta\sin\beta}\left[\frac{\delta U}{\delta\beta}-D\omega_L(2\alpha'\beta'\cos\beta+\alpha''\sin\beta)\right],$$

$$\frac{\partial\beta}{\partial t}=\frac{1}{\omega_L\sin\beta}\frac{\delta U}{\delta\alpha}-\frac{D}{\sin\beta}[(\cos\beta)''+(\beta')^2+(\alpha')^2\sin^2\beta],$$

$$\frac{\partial\Phi}{\partial t}=S_\zeta-\omega_L-\frac{1}{S_\zeta(1+\cos\beta)}\left[\frac{\delta U}{\delta\beta}-D\omega_L(2\alpha'\beta'\cos\beta+\alpha''\sin\beta)\right],$$

$$\frac{\partial S_\zeta}{\partial t}=-\frac{\delta U}{\delta\Phi}-D\omega_L[(\alpha')^2\sin^2\beta+(\beta')^2].$$

To abbreviate the equations we have used the notation

$$\frac{\delta U}{\delta\beta}=\frac{\partial(U+G)}{\partial\beta}-\frac{\partial}{\partial z}\left(\frac{\partial G}{\partial\beta'}\right)$$

etc., and the dipole energy and gradient energy, which enter here, have the following forms, respectively:

$$U=\frac{\Omega^2}{8}[\cos^2\beta+{}^1/_2(1+\cos\beta)^2\cos2\Phi],$$

$$G=\frac{1}{4}C^2[(1-\cos\beta)(3-\cos\beta)(\alpha')^2+(\beta')^2+2(\Phi')^2-4(1-\cos\beta)\alpha'\Phi'].$$

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