Flux creep in superconductors with large defects

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A calculation is made of the rate of creep of a lattice of superconducting vortices under the influence of a transport current in the presence of pinning by large defects, which satisfy the Labusch criterion and also obey the theory of elasticity. Creep involves formation of kinks of vortex rows at the defect boundaries. The dependence of the activation energy of such a process on the current is logarithmic in the case of a film and it obeys a power law in the case of a bulk sample.

1. Many experiments have been made recently on the creep of a magnetic flux in superconductors (see, for example, Refs. 1–5). This is due to the fact that this thermally activated process is much stronger in high-temperature superconductors than in conventional materials. Moreover, apart from the Anderson phenomenological theory[6] and its generalization given in Ref. 7, the published theoretical investigations of the creep have been concerned mainly with fluctuation-type motion of a single vortex[8] and determination of the activation energy in the collective pinning case.[9,10]

We shall report a calculation of a fluctuation current-voltage characteristic of a superconducting sample containing large defects. Since in magnetic fields B, which are not too close to H_{c2}, the critical current due to small defects is low,[11] it follows that even a small number of large defects can alter significantly the critical current density j_c and the nature of the current-voltage characteristic. We shall consider the case of defects with a characteristic size (radius of curvature) R exceeding considerably the period $b=(2\Phi_0/B\sqrt{3})^{1/2}$, of a vortex lattice, satisfying the Labusch criterion of one-particle pinning[12] and also obeying the theory of elasticity. These defects can be particles of a new phase, grain boundaries, impurity clusters, dislocation pile-ups, or twins (twins are typical of yttrium-based high-temperature superconductors).

The critical current density j_c and the current-voltage characteristic, obtained using a current $j>j_c$ for a system of this kind, were found by Larkin and Ovchinnikov.[13] It follows from their paper that if $j<j_c$, then a vortex lattice which experiences a bulk Lorentz force $[\mathbf{j}\times\mathbf{B}]$ is maintained by the interaction between vortex layers and certain parts of the surface of a defect (contact areas); the dimensions of these parts and the pressure acting there can be calculated in the same way as in the Hertz contact problem.[19]

Our task was to determine the activation energy $E(j)$ for the creep of a vortex lattice across such contact regions. Consequently, the voltage V in the range $j<j_c$ is proportional to $\exp(-E/T)$ and the time dependence of the current in magnetic measurements is deduced from the expression $E(j(t))=T\ln\omega t$ (Ref. 7). We shall see that the characteristic scale is then small compared with the dimensions of the contact regions, so it should be sufficient to consider the interaction of a lattice with a plane infinite boundary of a defect. The boundaries should be regarded as parallel to the direction of close packing in the lattice, because for this orientation a critical pressure P_c (corresponding to the detachment of a lattice from a defect at $j=j_c$) is maximal and it is the parts of the boundaries oriented in this way that control the process. The creep involves formation of pairs of kinks of a vortex lattice at a boundary barrier and separation of these kinks under the influence of a pressure P created by a defect when a transport current is flowing. Formation of such pairs occurs at the center of a contact region where the pressure is maximal.

Equilibrium conditions for a vortex lattice are described by the following equation:[11–13]

$$(C_{11}-C_{66})\frac{\partial}{\partial\boldsymbol{\rho}}\left(\frac{\partial\mathbf{u}}{\partial\boldsymbol{\rho}}\right)+C_{66}\frac{\partial^2\mathbf{u}}{\partial\boldsymbol{\rho}^2}+C_{44}\frac{\partial^2\mathbf{u}}{\partial z^2}+[\mathbf{jB}]+\mathbf{f}_{pin}=0. \quad (1)$$

Here, $\mathbf{u}(\boldsymbol{\rho})$ is a two-dimensional displacement vector in a plane xy perpendicular to the field; $\mathbf{f}_{pin}$ is the pinning force; $(C_{11}-C_{66})$ is the bulk modulus; C_{66} is the shear modulus; C_{44} is the bending modulus (in fields not too close to H_{c1}, we have $C_{66}\ll C_{11}$). For calculating the activation energy of creep we need the expression for the energy of elastic distortions

$$F_{el}=\frac{1}{2}\int\left[(C_{11}-C_{66})\left(\frac{\partial\mathbf{u}}{\partial\boldsymbol{\rho}}\right)^2+C_{66}\left(\frac{\partial u_\alpha}{\partial\rho_\beta}\right)^2+C_{44}\left(\frac{\partial\mathbf{u}}{\partial z}\right)^2\right]dV. \quad (2)$$

2. We shall first consider a thin film of thickness d in a perpendicular field. The displacements caused by the pressure $P(x)$ acting on the boundary at $y=0$ [in this case we have $\mathbf{f}_{pin}=\{0,\ P(x)\delta(y)\}$] are described by the solutions of Eq. (1), which excludes the dependence on z:

$$u_x(x,y)=-\frac{1}{C_{66}}\int\frac{d^2k}{(2\pi)^2}\times\int\frac{P(x_1)k_xk_y\exp[ik_x(x-x_1)+ik_yy]}{(k_x^2+k_y^2)^2}dx_1,$$

$$u_y(x,y)=\frac{1}{C_{66}}\int\frac{d^2k}{(2\pi)^2}\times\int\frac{P(x_1)k_x^2\exp[ik_x(x-x_1)+ik_yy]}{(k_x^2+k_y^2)^2}dx_1. \quad (3)$$

In the course of the creep process a pair of vortex row kinks (Fig. 1) forms at a boundary and begins to move apart. In the case of a single kink located at $x=0$ and creating a pressure $P_1(x)$ (which is an odd function of x), we find from Eq. (3) at $y=0$ that

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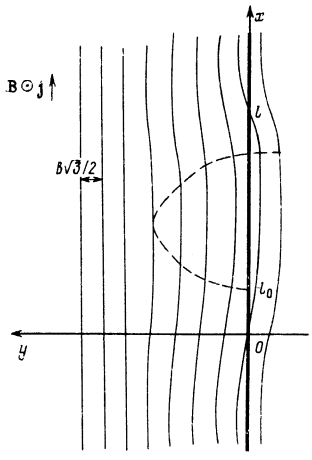

FIG. 1. Climb of vortex rows (continuous curves) across the boundary of a defect (x axis) when a current of density j flows in a film. The lattice shifts by a distance $b\sqrt{3}/2$ inside the region bounded by the dashed curve.

$$u_y(x,0) = -\frac{b\sqrt{3}}{4} - \frac{1}{4\pi C_{66}} \int P_1(x_1) \ln \left| 1 - \frac{x_1}{x} \right| dx_1. \tag{4}$$

Far from a kink the relative shift should be equal to the distance between the rows:

$$u_y(\pm\infty, 0) + \frac{b\sqrt{3}}{4} = \mp \frac{b\sqrt{3}}{4}. \tag{5}$$

The condition (5) is satisfied if in Eq. (4) we assume that

$$P_1(x) = -\left(\frac{2\sqrt{3}}{\pi}\right) C_{66} \frac{b}{x}. \tag{6}$$

The expression (6) for P_1 applies at distances x which are large compared with the size l_0 of the core of a kink, which is the region where the displacement differs considerably from the limiting values [Eq. (5)]. The size of this region is estimated by equating the characteristic (maximal) pressure for a defect $P_c \lesssim C_{66}$ to the elastic shear stress:

$$l_0 \sim bC_{66}/P_c. \tag{7}$$

Equations (6) and (7) are sufficient to find, with a logarithmic precision, the elastic energy F_{el} of a pair of kinks located at a distance $l \gg l_0$ from one another:

$$F_{el} = \frac{d}{2} C_{66} \int \left(\frac{\partial u_\alpha}{\partial \rho_\beta} \right)^2 d^2\rho = \frac{3}{\pi} C_{66} b^2 d \ln \frac{l}{l_0} \tag{8}$$

(the main contribution to the energy of a kink comes from the region $l_0 \ll \rho \ll l$). Since the force of attraction between kinks $\partial F_{el}/\partial l$ is equal to the force pushing them apart $Pdb\sqrt{3}/2$, the critical distance is

$$l^* = \frac{2\sqrt{3}}{\pi} C_{66} \frac{b}{P}, \tag{9}$$

and when it is exceeded the kinks begin to move apart continuously, so that in the final analysis the lattice shifts by a distance $b\sqrt{3}/2$.

The activation energy of the creep is

$$E = F_{el}(l^*) = \frac{3}{\pi} C_{66} b^2 d \ln \frac{P_c}{P}. \tag{10}$$

Bearing in mind the relationship

$$jB \approx n_l P^2 R / C_{66} \tag{11}$$

(n_l is the number of defects per unit area),[13] we find that the current-voltage characteristic of a film in the subcritical region is described by

$$V \propto (j/j_c)^{3\delta}, \quad \delta = \frac{C_{66} b^2 d}{2\pi T} \gg 1. \tag{12}$$

The parameter δ represents the degree of proximity of the film to the melting point of an Abrikosov lattice.[15] The dependence of the current on the voltage of the type described by Eq. (12) has been observed in recent resistive measurements.[16]

3. In the three-dimensional situation when the pinning boundary is infinite along the x axis (i.e., along the magnetic field), the climb of the lattice across the boundary involves formation of closed kink loops. In view of the large value of the bending modulus $C_{44} = B^2/4\pi = C_{11}$, compared with $C_{66} = \Phi_0 B/(8\pi\lambda)^2$ these loops are strongly elongated along the field. Their shape and, consequently, the activation energy of the process can in this case be found only to the nearest order of magnitude.

It is clear from Eq. (1) that the bending strain represented by the third term on the left-hand side of Eq. (1) becomes significant on deviation of the kink line from the z axis by an angle θ of the order of $\theta_0 \propto (C_{66}/C_{44})^{1/2}$. Using Eq. (2), we find that the energy per unit length of a kink as a function of the angle θ can be estimated from

$$\varepsilon \approx b^2 (C_{66} + C_{44}\theta^2), \tag{13}$$

which shows that the optimal dimensions of a kink loop across the field (l) and along the field (h) are related by

$$l/h \sim (C_{66}/C_{44})^{1/2}, \tag{14}$$

where the kink lines are oriented at an angle $\theta \sim \theta_0$ over a large part of the loop.

Since pushing across a boundary under the influence of a pressure P creates an energy gain of the order of PbS per one lattice period of a loop of area $S \sim hl$, it follows that a comparison of this quantity with the elastic energy of the loop distortion $\sim C_{66} b^2 h$, can yield—subject to Eq. (14)—the activation energy

$$E \approx b^3 \frac{C_{66}^{2/3} C_{44}^{1/3}}{P} \approx b^3 \frac{C_{66}^{2/3} C_{44}^{1/3}}{P_c} \left(\frac{j_c}{j} \right)^{1/3} \tag{15}$$

(we have allowed here for the fact that the relationship $j \propto P^3$ applies in the three-dimensional case—see Ref. 13).

If the current is sufficiently large, the loop width $l^* \sim bC_{66}/P$ can decrease to dimensions of the order of the field penetration depth λ, when the spatial dispersion of the moduli C_{11} and C_{44} becomes significant;[17] when the wave vectors obey $k \gg \lambda^{-1}$ these moduli decrease compared with the homogeneous case and the reduction is proportional to $(k\lambda)^{-2}$. In this case if $l \ll \lambda$, the quantity C_{44} in Eqs. (13)–(15) is understood to be

$$B^2 l^2 / 4\pi\lambda^2 \sim C_{66}^3 / P^2,$$

whereas the climb energy is given by

$$E \approx b^3 \frac{C_{66}^3}{P_c^2}\left(\frac{j_c}{j}\right)^{1/2}, \quad b \ll bC_{66}/P \ll \lambda. \tag{16}$$

4. In derivation of the expressions for the activation energy given by Eqs. (10), (15), and (16) it is assumed that the creep is independent at each defect. This is justified if the energy of the resultant compression, which is of the order of $C_{11}b^2d$ in the case of a film and of the order of $C_{11}b^2n_m^{-1/3}$ in the case of a bulk sample (n_m is the volume density of defects), is small compared with the energy gain associated with the shift of the lattice under the action of the Lorentz force ($jBbdn_f^{-1}$ and $jBbn_m^{-1}$, respectively). Therefore, at current densities $j<j_1$, where

$$\begin{aligned} j_1 &\sim B^{-1}n_f C_{11} b \text{ (film)}, \\ j_1 &\sim B^{-1}n_m^{2/3} C_{11} b \text{ (bulk)}, \end{aligned} \tag{17}$$

the fluctuations jumps occur simultaneously at $N>1$ defects. Since the elastic energy of compression is inversely proportional to N, and the work performed by the Lorentz force rises proportionally to N, the number of defects is governed by the relationship

$$N \propto (j_1/j)^{1/2}, \quad j \lesssim j_1. \tag{18}$$

The corresponding values of the activation energy increase by a factor of N if $j<j_1$.

The validity of the above expressions is limited, on the low current side, by the condition that the dimensions of the critical configurations l and h should be small compared with the dimensions of the contact regions. If in the case of the latter we use the expressions of Ref. 13, we find that these conditions are as follows:
for a film we have

$$\frac{j}{j_c} \gg \left(\frac{C_{66}}{P_c}\right)^2 \frac{b}{R} \ln^{-1}\frac{R}{b}, \tag{19}$$

whereas for a bulk sample, we obtain

$$j/j_c \gg C_{66}^{3/4} C_{44}^{9/4} P_c^{-3} (b/R)^{1/2}. \tag{20}$$

If we can show that these conditions agree with the requirement that the shift of the lattice at a single defect should exceed its period; therefore, the range of validity of our results is governed by the Labush criterion.

The results obtained, like the expressions from Refs. 8 and 10, indicate a steep rise of the creep activation energy when $j \ll j_c$ (in accordance with a power law or logarithmic). Therefore, the results differ considerably from the dependence $E(j) \propto (j_c - j)$ proposed in the Anderson model,[6] which is probably valid only if the current is close to the critical value.

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