# Gradient catastrophe of a loaded supersonic magnetized plasma flow and shock-wave formation

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We show that stationary one-dimensional supersonic magnetized plasma flow with heavy ion loading is possible only for Mach numbers larger than 2. Under such conditions mass loading leads to a smoothing of the flow perturbations with time whereas for Mach numbers less than 2 the loading of the flow facilitates the steepening of the plasma flow perturbations and the formation of a standing shock wave in that flow. In the case of plasma flow motions which are exactly at right angles to the magnetic field the dispersion caused by the heavy ions cannot stop the steepening of the plasma flow and this indicates the unavoidable inclusion of smaller scales at the front of the shock wave, i.e., the appearance of a proton compression discontinuity.

## 1. INTRODUCTION

The interaction of a magnetized plasma flow with a neutral gas manifests itself first of all in the form of the effect of the loading of that flow with ions which are formed in the flow thanks to the ionization of the atoms of the neutral gas both by the external ionizing radiation and by the magnetized plasma flow itself. The best known example of this kind is the interaction of the solar wind (a magnetized plasma flow from the solar corona into interplanetary space) with comets. The new ions in the flow of the solar wind are formed here mainly by the photoionization of the atoms in the cometary gas evaporated from the nucleus of the comet by the ultraviolet solar radiation.[1] The ionization of a neutral gas by a magnetized plasma flow at flow velocities above the critical $V_{\rm cr} = (2e\varphi_I/m)^{1/2}$ ($\varphi_I$ and $m$ are the ionization potential and the mass of the atoms in the neutral gas) was predicted by H. Alfvén and became well known as the critical ionization velocity phenomenon (see the review in Ref. 2).

Although the consideration of the dynamics of the flow of a plasma loaded by new ions is given in the present paper in a very general form, it is first and foremost aimed at elucidating the mechanism for forming a shock wave near a comet. The fact is that the outgoing shock waves in the supersonic solar wind that flows around the planetary magnetospheres and ionospheres are formed in exactly the same way as those in front of blunt bodies in a supersonic gas flow. The position of the front of these shock waves can easily be calculated using the ordinary hydrodynamic equations for a given shape of the body or of the planetary magnetospheres and ionospheres.

A feature of the interaction of the solar wind with comets is that the gravitational field of the fragmentary cometary nucleus (altogether about a few tens of kilometers from edge to edge) is unable to retain the gaseous atmosphere formed when the matter evaporates from the surface of the nucleus under the action of the solar radiation. As a result atmospheric atoms and molecules fly away from the nucleus with a velocity of about 1 km/s before they are ionized by the ultraviolet solar radiation and penetrate millions of kilometers into the interplanetary space. An ion formed in the solar wind flow is immediately captured by this flow, increases in mass and decelerates. Biermann *et al.*[1] were the first to note that the interaction of the solar wind with a comet starts already at distances of a few million kilometers from its nucleus. Moreover, Biermann *et al.*[1] reached the conclusion that such a loading of the solar wind by cometary ions is the cause of the formation of shock waves far in front of the comet, where the effect of fragmentary nucleus of the comet or its ionosphere can be completely ignored.

This conclusion was reached in the framework of a simple stationary one-dimensional hydrodynamic kind of model. The analysis showed that the deceleration of the supersonic solar wind flow proceeds until the local Mach number $M$ is decreased to 1, which corresponds to an increase of the mass flux to a well defined critical value. A further loading of the solar wind would lead to acceleration of the flow. As a solution for this paradox Biermann *et al.*[1] assumed that a frontal shock wave is formed prior to Mach numbers $M > 1$. A numerical simulation confirms this conclusion[3–5] and gives a value of the local Mach number for which a shock wave is formed. For comets with a large gas production the local Mach number of the shock wave, found by numerical calculations, is independent of the parameters of the solar wind and of the cometary atmosphere and is equal to $M = 2$. The observations by space instruments can, unfortunately, not give sufficiently exact information about the Mach number of the cometary shock wave; at least $M = 2$ does not contradict the observations.

From this point of view the understanding of the physical mechanism of the cometary shock wave is particularly important. The first part of this paper is devoted to this problem. It was recently shown[6] that the linear analysis of the stability of the stationary solution of Ref. 1 reveals oscillation instability of magnetosonic type in the region of solutions with a local Mach number less than 2. Here we show, by means of an analysis of motions using characteristics, that this instability is the consequence of the development of a more dramatic effect than the simple instability of plasma oscillations, namely, a gradient catastrophe. It is just by means of the gradient catastrophe that a shock wave is formed in the flow of a loaded plasma; its fine structure is considered in the second part of the paper.

## 2. GRADIENT CATASTROPHE IN A LOADED MAGNETIZED-PLASMA FLOW

The Mach number and the position of the cometary shock wave are, clearly, determined by the physical nature of

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the shock wave. Whereas in the case of a supersonic flow interacting with a solid obstacle the shock wave is completely determined by the boundary conditions of the problem—the conditions in the unperturbed solar wind and the conditions on the obstacle, in the case with a comet the obstacle is "fragmentary" and therefore the boundary condition on the obstacle loses its meaning. Strictly speaking, one can change the distribution of the neutral matter arbitrarily in the region where the plasma flow becomes subsonic and, if in that case the distribution of neutral matter in the supersonic region remains unchanged, this can in no way affect the conditions in the supersonic region, and hence the shock wave. The cometary nucleus can affect the supersonic region of the solar wind flow and the shock wave only mediated through changes of the parameters of the neutral cometary atmosphere. One may thus assume that in the case of a comet the information about the shock wave parameters does not come from the "solid obstacle" as in the case of a piston but is produced in the solar wind flow itself in the loading process. We take this statement as a working hypothesis in the further analysis of the loading of the plasma flow.

From the mathematical point of view the present problem is reduced to the determination of the transition layer in the solution of a differential equation with a small coefficient of the leading derivative (the dispersion term in the wave equation in the case of a collisionless shock wave or the dissipative term in the Burgers equation). In the stationary variant the solution of the problem with a piston is equivalent to the solution of the boundary problem with boundary conditions determined from physical considerations as in the above mentioned example of a circumplanetary shock wave. These boundary conditions ultimately determine all the parameters of the shock wave.

From this point of view the nature of the cometary shock wave is not so clear. It is impossible to formulate the problem in terms of a boundary-value problem; essentially the second boundary itself is absent because the obstacle is friable and extended in space. However, there remains the possibility of formulating it in terms of the initial problem that mathematically corresponds to our working hypothesis that the cometary shock wave is completely determined by the prehistory itself of the loading of the solar wind flow.

Strictly speaking, Biermann *et al.*[1] were the first to solve the initial problem for the loaded solar wind in the vicinity of the comet; they found a stationary-one-dimensional solution for the magnetohydrodynamic solar wind with homogeneous conditions far from the comet. This result was widely used in what follows, with small modifications, for an analytical study of the interaction of the comet with the solar wind.[7,8] The modifications usually reduce to taking into account the isotropization of the distribution of the new ions of cometary provenance in the case when these ions have a finite velocity component along the magnetic field; this leads to the excitation of strong Alfvén turbulence. The adiabatic index of the plasma then changes from $\gamma = 2$ in the case when there is no isotropization (the new cometary ions have a distribution in the shape of a ring in velocity space) to $\gamma = \frac{5}{3}$ in the case of total isotropization. In the $\gamma = 2$ case this result was studied with regard to stability in Ref. 6 in which it was shown that in the linear approximation the local dispersion equation gives a positive local growth rate for magnetosonic oscillations in the flow region where the local Mach number is less than 2.

For a further analysis of a loaded plasma flow we consider a nonstationary one-dimensional magnetohydrodynamic system describing such a flow:[1]

$$\rho_t+u\rho_x+\rho u_x=\nu m,\quad B_t+uB_x+Bu_x=0,$$

$$u_t+uu_x+\frac{1}{\rho}\left[P+\frac{B^2}{8\pi}\right]_x=-\frac{\nu m u}{\rho},\tag{1}$$

$$u_t+uP_x+\gamma P u_x=(\gamma-1)\frac{\nu m u^2}{2};$$

here $\rho$ is the mass density, $u$ the mass velocity, $P$ the thermal plasma pressure, $B$ the magnetic field of the solar wind, $m$ the mass of a cometary ion loading the flow, and $\nu$ the rate at which they are formed. This system is exactly the same as that used by Biermann *et al.*,[1] except that we are taking the temporal dependence into consideration, and it has thus the stationary solutions found in that paper.

In the $\gamma = 2$ case we can disregard the equation for the freezing-in of the magnetic field in the plasma (the second equation of this system). Indeed, if we multiply that equation by $B/4\pi$ and equate the result to the equation of the plasma pressure (the last equation) it becomes clear that the magnetic field can be eliminated from the system if we use instead of the thermal pressure the total pressure, taking into account the magnetic field pressure: $P + B^2/8\pi \rightarrow P$. Taking this remark into account we shall thus consider the contracted system:

$$\rho_t+u\rho_x+\rho u_x=\nu m,\qquad u_t+uu_x+\frac{1}{\rho}P_x=-\frac{\nu m u}{\rho},$$
$$u_t+uP_x+\gamma P u_x=(\gamma-1)\frac{\nu m u^2}{2},\tag{2}$$

in which we understand by $P$ the total plasma pressure.

We rewrite the system (2) in characteristic form. To do this we multiply the equation of motion (the second) by $\pm (2P/\rho)^{1/2}$ and add the result to the pressure equation (the third one). As a result we get two equations describing the propagation of perturbations along the sound characteristics $C_\pm$:

$$\hat{D}_\pm P\pm\rho c\hat{D}_\pm u=\frac{\nu m u}{2}(u\mp 2c);\tag{3a,b}$$

we have here introduced the notation $c = (2P/\rho)^{1/2}$ for the sound speed and the operators $\hat{D}_\pm = \partial/\partial t + (c \pm u)\partial/\partial x$ for differentiation along the sound characteristics $C_\pm$.

Carrying out a similar procedure for the equation of continuity (the first one), using the factor $2P/\rho$, we get an equation for the propagation of entropy perturbations along the contact (or entropy) characteristic $C_0$ with the appropriate operator $\hat{D}_0 = \partial/\partial t + u\partial/\partial x$.

$$\hat{D}_0 P-c^2\hat{D}_0\rho=\frac{\nu m}{2}(u^2-2c^2).\tag{3c}$$

It is clear from the set (2) that it is convenient, for the analysis of the features of a loaded plasma flow, to introduce[9] the following combinations of the spatial derivatives of the flow parameters:

$$R=u_x+c_x+\frac{1}{2\rho}c\rho_x,\quad L=u_x-c_x-\frac{1}{2\rho}c\rho_x,\quad M=c_x-\frac{1}{2\rho}c\rho_x.\tag{4}$$

Let us get the transport equations for these quantities. To do this we apply the differentiation operator $\hat{D}_x = \partial/\partial x$ to each of the equations of the set (3), using the commutation rules

$$\hat{D}_x\hat{D}_0=\hat{D}_0\hat{D}_x+\hat{D}_x(u)\hat{D}_x,\quad \hat{D}_x\hat{D}_\pm=\hat{D}_\pm\hat{D}_x+\hat{D}_x(u\pm c)\hat{D}_x,$$

that follow directly from the definitions of these operators. One easily obtains the required transport equations in the following form:

$$\hat{D}_+R+\frac{3}{4}R^2+\frac{1}{4}RL+\frac{M}{4}(R-L)+\Delta=\hat{D}_x\left[\frac{\nu mu}{2\rho c}(u-2c)\right],$$

$$\hat{D}_-L+\frac{3}{4}L^2+\frac{1}{4}LR+\frac{M}{4}(R-L)-\Delta=-\hat{D}_x\left[\frac{\nu mu}{2\rho c}(u+2c)\right],$$

$$\hat{D}_0M+\frac{3}{4}M(R+L)=\hat{D}_x\left[\frac{\nu m}{2\rho c}(u^2-2c^2)\right], \tag{5}$$

where for the sake of simplicity we have introduced the notation

$$\Delta=\frac{1}{8}\frac{\nu m}{\rho}\left[(2M+R-L)+\frac{u^2-c^2}{c^2}(2M-R+L)\right].$$

We note moreover that the following useful relations for the quantities (4) follow from the system (3):

$$R=-\hat{D}_-\ln[(c^2\rho)^{1/4}]+\frac{\nu mu^2}{2\rho c^2},\qquad L=-\hat{D}_+\ln[(c^2\rho)^{1/4}]+\frac{\nu mu^2}{2\rho c^2},$$

$$M=\pm\hat{D}_\pm\ln[(c^2\rho)^{1/4}]\mp\frac{\nu m}{2\rho}\frac{u^2-2c^2}{c^2}. \tag{6}$$

If there is no loading ($\nu=0$), the system (5) for isentropic flow ($c^2/\rho=$ const and $M=0$) can be solved analytically.[9] We consider, for instance, the first of the equations of the system (5):

$$\hat{D}_+R+{}^3/_4R^2+{}^1/_4RL=0. \tag{7}$$

It is well known that this nonlinear Riccati equation has solutions that become infinite on a finite time interval. It can be reduced to linear by the substitution $R=1/z$:

$$\hat{D}_+z-{}^3/_4-{}^1/_4zL=0. \tag{8}$$

Using the second of Eqs. (6) we get

$$\hat{D}_+[z(c^2\rho)^{1/4}]={}^3/_4(c^2\rho)^{1/4}. \tag{9}$$

We note that it is necessary to integrate this equation along the characteristic $C_+$. Denoting all quantities at the time $t=0$ by an index "0", one can write the solution of (9) in the form

$$z(c^2\rho)^{1/4}=z_0(c_0{}^2\rho_0)^{1/4}+\int_{0(C_+)}^{t}\frac{3}{4}(c^2\rho)^{1/4}dt, \tag{10}$$

or, returning to the variable $R$, we get the following solution:

$$R=R_0\left(\frac{c^2\rho}{c_0{}^2\rho_0}\right)^{1/4}\left[1+\frac{3}{4}R_0\int_{0(C_+)}^{t}\left(\frac{c^2\rho}{c_0{}^2\rho_0{}^2}\right)^{1/4}dt\right]^{-1}. \tag{11}$$

A similar expression is valid for the quantity $L$ if one carries out the integration in (11) along the characteristic $C_-$. Hence it is clear that if $R_0\geqslant0$ at any time $t\geqslant0$ we shall have $R\geqslant0$, and the plasma flow remains continuous. However, if at any point of the flow $R_0\leqslant0$, the quantity $R$ will remain less than zero along the characteristic $C_+$ at all times as long as the denominator in Eq. (11) does not vanish. At this point of the plasma flow a gradient catastrophe occurs. The moment $t_k$ at which the gradient catastrophe appears is determined by the equation

$$\int_{0(C_+)}^{t_k}\frac{3}{4}\left(\frac{c^2\rho}{c_0{}^2\rho_0{}^2}\right)^{1/4}dt=-\frac{1}{R_0}\quad(R_0<0). \tag{12}$$

For times $t\geqslant t_k$ a discontinuous motion of the plasma flow is impossible and only dispersion effects, neglected in the system (2), can in principle stop the wave breaking process.

Returning to the case of the loading of a plasma flow by new ions, we note that there exists a stationary solution of the system (2) (Ref. 1) but only the fact of its existence itself is now important for us. One can see from Eq. (3a) that for the stationary solution of the system (2) the quantity $R$

$$(u+c)\left[(u+c)_x+\frac{1}{2}c(\ln\rho)_x\right]=(u+c)R=\frac{\nu mu}{2\rho c}(u-2c) \tag{13a}$$

changes sign in the stationary-flow point of where the local Mach number $M=2$. In the region of the stationary solution where the Mach number $M>2$ the quantity $R$ is positive and it follows from the solution (11) that any sufficiently small perturbation of the quantity $R$ relative to the stationary value smoothes out with time. On the contrary, in the region of a flow with a local Mach number less than 2, where the quantity $R$ is negative in the stationary solution, any infinitesimally small perturbation inevitably leads to the appearance of a gradient catastrophe and, what is more, the stationary solution assists the evolution of the catastrophe. It was shown in Ref. 6 that the boundary of the instability of the region for small sound perturbations $\omega-ku=kc$ is the same in the linear approximation as the Mach number $M=2$.

The evolution of the quantity $L$, the spatial derivative of the second Riemann invariant in gas dynamics, is determined by the supersonic nature of the flow motion. The fact is that its value for the stationary solution remains less than zero for all Mach numbers $M>1$:

$$(u-c)\left[(u-c)_x-\frac{1}{2}c(\ln\rho)_x\right]=(u-c)L=-\frac{\nu mu}{2\rho c}(u+2c), \tag{13b}$$

and has a singularity in the $M=1$ point of the flow. A linear analysis shows[6] that a sound perturbation $\omega-ku=kc$ turns out to be stable for larger Mach numbers $M>1.74$, owing to loading effects, but for smaller Mach numbers even as infinitesimally small perturbation grows and breaks, and this gradient catastrophe takes place necessarily ahead of the sound point, since the perturbation propagates along the characteristic $C_-$, approaching the sound point $u=c$ after an infinitely long time.

We can thus reach the conclusion that the stationary solution found by Biermann *et al.*[1] has a nonremovable singularity at the point where the local Mach number is $M=1.74$, and becomes inapplicable in the region of smaller Mach numbers. The flow region with Mach numbers $1.74<M<2$ also turns out to be unstable. In principle a shock wave can exist in that region, but the region before its front, up to a local Mach number $M = 2$, is in that case a generator of sound waves which are carried away together with the flow into the region behind the front and are additionally amplified on the front of the shock wave.[10] The case of a shock wave with an intermediate Mach number $1.74 < M < 2$ thus turns out to be a dynamic one and only shock waves with Mach numbers $M \geqslant 2$ guarantee a completely stationary structure of a flow with loading.

It is well known that the steepening of the front of a flow in a collisionless plasma is due to the dispersion of the magnetosonic waves in the plasma and to formation of a shock wave front with a characteristic dispersion scale of the order of the ion cyclotron radius.[11,12] The analysis given here enables us therefore to state that in the region where the local Mach number of the solar wind drops to $M = 2$ there must be formed the front of a circumcometary shock wave.

We note, finally, that the analysis given is independent of the actual nature of the plasma loading. The ionization rate $\nu$ can not only vary in space because of the inhomogeneous neutral gas distribution, as in the case of a comet when the ionization of the neutral gas is mainly caused by the photoionization by the solar ultraviolet radiation, but can also have an arbitrary dependence on the plasma flow parameters, for instance, on the electron density or temperature in the case of ionization by electron impact. Taking these dependences into account can only affect the existence of stationary solutions of the kind found in Ref. 1. However, even in the case when there are no stationary solutions, the main conclusion about that the flow has a singularity at a point with a local Mach number $M = 2$ remains in force. It will remain true as long as the magnetohydrodynamic description of the plasma flow is justified, i.e., as long as the characteristic time and space scales of the loading process are small as compared to the gyroperiod and the gyroradius of the plasma particles, respectively.

## 3. STRUCTURE OF THE FRONT OF THE CIRCUMCOMETARY SHOCK WAVE

In the general case the structure of the circumcometary shock wave is described by the mass, momentum, and energy flow conservation equations, taking into account thermal conductivity and wave dispersion in the plasma:[10]

$$\rho u = j = \text{const}, \tag{14}$$

$$p + ju - a_D{}^2 \frac{ju_1}{B} \frac{d^2B}{dx^2} = P_1 + ju_1, \tag{15}$$

$$\frac{\gamma P u}{\gamma - 1} + \frac{1}{2} ju^2 - \varkappa \frac{dT}{dx} = \frac{\gamma P_1 u_1}{\gamma - 1} + \frac{1}{2} ju_1{}^2, \tag{16}$$

where the index 1 indicates the plasma parameters before the shock wave front, $a_D$ is the dispersion parameter, and $\kappa$ the thermal-conductivity coefficient. The dispersion of a magnetosonic wave was in the case considered above of adiabatic plasma flow strictly transverse to the magnetic field calculated in Ref. 13 by the method of expanding the Vlasov equation in the reciprocal of the gyrofrequency of the particles. The expansion method used in this paper is practically linear in the products of the derivatives of the parameters of the flow, whereas with respect to the magnitudes of the plasma flow parameters themselves the expansion is applicable also in the nonlinear case, i.e., in the case when there are changes in the flow parameters over distances longer than the gyroradius of the plasma particles. In the case of a dominant contribution of the ions to the pressure the expression for the dispersion parameter has the form[13]

$$a_D{}^2 = \frac{P}{4\rho\Omega^2}\left[1 + \frac{3P}{\rho u^2}\left(\frac{2\Pi\rho}{mP^2} - 3\right)\right]. \tag{17}$$

Here $\Omega$ is the ion gyrofrequency and $\Pi$ the second moment of the ion distribution function in the magnetic moment of the particles $\mu = v_\perp^2/2B$. We note that for a Maxwell distribution the ratio $\lambda = 2\Pi\rho/mP^2$ equals 4 whereas for a ring distribution with respect to transverse velocities $\lambda = 2$. In the first case for small amplitude waves $\rho u^2 \approx 2P + B^2/4\pi$, we can obtain in the linear approximation[14] an appropriate limiting expression which shows that the dispersion of the magnetosonic waves is negative so that the magnetosonic solitons are compression solitons.

The equation for the magnetic-field profile in such a soliton can easily be obtained from Eq. (15) using instead of Eq. (16) the conservation of the energy flow in the adiabatic approximation and the condition that the magnetic field be frozen-in. As a result Eq. (15) reduces to the Sagdeev equation:[11]

$$a_D{}^2 \frac{d^2B}{dx^2} = -\frac{\partial V(B)}{\partial B}, \tag{18}$$

where

$$V(B) = \frac{1}{2}(B - B_1)^2\left[\frac{(B + B_1)^2}{16\pi\rho_1 u_1{}^2} - 1\right].$$

The introduction of a small magnetic viscosity into Eq. (15) turns the soliton solution of Eq. (8) into a shock wave with extended oscillatory drift.[11]

Returning to the circumcometary shock wave we must note that under the conditions of a solar-wind adiabatic plasma flow strictly transverse to the magnetic field the cometary ions which give the main contribution to the plasma pressure are distributed in velocity space in a ring of finite thickness.[7] Calculating the second moment II of the distribution function of the cometary ions with respect to the magnetic moment $M = 2$ in the region immediately in front of the shock wave we find that the parameter $\lambda = 2.12$. The dispersion length $a_D$ which is positive in front of the cometary shock wave with Mach number $M = 2$ can thus become zero inside its front. The latter occurs at the point where the local Mach number drops to $M_{cr} = \sqrt{1.32}$ [see Eq. (17)]. It is clear that at this point the dispersion caused by the heavy cometary ions cannot contain the steepening of the velocity profile of the plasma and there occurs thus a condensation discontinuity ("subshock") in the electron-proton plasma with a thickness much smaller than the ion dispersion length. This conclusion is in agreement with the numerical simulation of the formation of the shock wave in the circumcometary region.[15]

## 4. CONCLUSION

In the present paper we have considered the dynamics of the loading of a supersonic plasma flow by heavy ions formed in the flow, in particular, the case of plasma flow strictly transverse to the magnetic field when in the adiabatic approximation one can obtain a set of quasihydrodynamic equations to describe the plasma. As a result we showed that the stationary loading of a supersonic magnetized plasma flow is possible only for Mach numbers $M \geqslant 2$. In the region of local Mach numbers $1 \leqslant M \leqslant 1.74$ a gradient catastrophe of the plasma flow becomes inevitable and a small perturbation of the flow shows wave breaking. Although a shock wave can have an intermediate Mach number $1.74 \leqslant M \leqslant 2$, in this case it turns out that the region ahead the front up to $M = 2$ is nonstationary and sound waves are generated in it at a characteristic generation rate of the order of the ionization time and are carried away by the flow into a region behind the front of the shock wave. Such a picture is impossible for purely stationary flow but may follow-up the dynamics of the changes in the flow parameters of the solar wind.

Although the larger part of the front of the shock wave is controlled by the ion dispersion, and therefore its thickness turns out to be of the order of the cyclotron radius of the heavy ions, inside the front there appears a condensation discontinuity ("subshock") with a considerably smaller characteristic size.

In the more general case the hydrodynamic description of the plasma[8] is based upon the theory of Alfvén turbulence generated by the heavy ions produced in the flow and, in turn, guaranteeing an efficient exchange between the different degrees of freedom of the motions of these ions. It turns out that in the framework of these equations the loading of a supersonic plasma flow by heavy ions leads to a gradient catastrophe in the flow region with Mach numbers $M \leqslant 3$. The change in the numerical value is connected with the change in the adiabatic exponent which, thanks to the efficient exchange between the degrees of freedom, equals here $\gamma = \frac{5}{3}$. Moreover, in the case of flow at an angle to the magnetic field ($\neq 90°$) the ion thermal conductivity along the magnetic field controls the structure of the shock wave which is formed. However, here too a thin condensation discontinuity occurs under well defined conditions inside the front (for details see Ref. 16) similar to the isothermal discontinuity in ordinary gas dynamics.[10]


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