Intense electromagnetic-pulse self-focusing accompanied by plasma compression

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The case in which nonlinear beam refraction is caused primarily by the relativistic oscillation of the electron mass in an rf field is analyzed. The plasma density increases in time-varying self-focusing of short pulses with large transverse dimensions in the running-focus region.

Self-focusing of laser light in an isotropic plasma is usually accompanied by a decrease in the particle density.[1-3] It is shown below that there are self-focusing situations in which the particle density increases in regions where the radiation energy is localized.

Let us consider the self-focusing of short pulses of electromagnetic radiation as a result of a slightly relativistic nonlinearity.[4] This process can be explained as follows: In the slightly relativistic case, and with small perturbations of the electron density, the refractive index for a circularly polarized pump wave can be written in the form

$$N=N_0+\delta N, \tag{1}$$

where $N_0 = (1-\omega_{pe}^2/\omega_0^2)^{1/2}$ is the linear part of the refractive index, and the nonlinear increment δN is given by

$$\delta N=\frac{1}{2}\frac{\omega_{pe}^2}{\omega_0^2}\frac{1}{N_0}\left(\frac{\delta m_e}{m_e}-\frac{\delta n_e}{n_0}\right). \tag{2}$$

Here ω_{pe} is the electron plasma frequency, ω_0 is the frequency of the rf pump wave, and δn_e is the perturbation of the electron density (this perturbation is small in comparison with the unperturbed density: $\delta n_e \ll n_0$). In the slightly relativistic case, the relative increase in the electron mass is

$$\frac{\delta m_e}{m_e}=\frac{1}{2}\frac{v_{0s}}{c^2}\ll 1$$

(v_{0s} is the electron velocity in the rf field of the pump wave, m_e is the rest mass of the electron, and c is the velocity of light).

If self-focusing is to occur, δN must be positive.

In the nonrelativistic case, in which the change in the electron mass can be completely ignored ($\delta m_e = 0$), the necessary condition for self-focusing is $\delta n_e < 0$. It follows from (2) that self-focusing in an unmagnetized collisionless plasma may be accompanied by compression of the plasma; i.e., δn_e may be greater than zero. This case is possible if the electromagnetic waves have relativistic intensity, with

$$\frac{\delta m_e}{m_e}>\frac{\delta n_e}{n_0}, \tag{3}$$

i.e., if the relative increase in the electron mass outweighs the relative density perturbation.

During the steady-state propagation of electromagnetic pulses, this type of self-focusing does not occur.[5] It turns out that in the case of a time-varying self-focusing of spatially wide and short pulses the plasma density can increase near a running focus.

1. Let us assume that a circularly polarized rf electromagnetic wave is propagating along the z axis in a collisionless plasma:

$$\mathbf{E}={}^1/_2(\mathbf{e}_x+i\mathbf{e}_y)E(z,\ r,\ t)e^{i\omega_0 t-ik_0 z}+\text{c.c.}, \tag{4}$$

where $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors along the corresponding axes, and $r = (x^2+y^2)^{1/2}$ is the transverse coordinate. We assume that the plasma is transparent: $\omega_0^2 \gg \omega_{pe}^2$.

For such frequencies, the contribution to the dynamics of the rf field from the low-frequency magnetic field which is generated can be ignored in comparison with the relativistic effect.[6] In describing the space-time dependence of the complex amplitude $E(z, r, t)$, we assume that the ponderomotive and relativistic nonlinearities are small and take them into account in the first nonvanishing approximation.

To study the time-varying self-focusing of electromagnetic beams we need to solve an equation for the amplitude E along with an equation for δn_e. The analytic solutions have many shortcomings, even for a cubic nonlinearity.[7-10] Let us attempt to study the effect in which we are interested here in the simplest (aberrationless) approximation, ignoring the inverse effect of the perturbation of the electron density on the amplitude of the rf field:

$$\frac{\delta n_e}{n_0}\ll\frac{1}{2}I,\quad I=\frac{e^2|E|^2}{m_e^2\omega_0^2c^2}\ll 1. \tag{5}$$

We introduce the variables

$$\tau=t-z/v_g,\quad \xi=z,\quad v_g=\partial\omega_0/\partial k_0\approx c. \tag{6}$$

Assuming that the field amplitude in (4) varies slowly, ignoring terms on the order of ω_{pe}^2/ω_0^2, and using (5), we find the following equation for the amplitude E:

$$2ik_0\frac{\partial E}{\partial\xi}+\Delta_\perp E+\frac{1}{2}\frac{\omega_{pe}^2}{c^2}IE=0. \tag{7}$$

Time enters this equation through the parameter τ.

To pursue the analysis we choose a solution of Eq. (7) in the geometric-optics approximation ($k_0 a \gg 1$, where a is the beam width) with a parabolic intensity profile and a Gaussian time profile. We assume that the longitudinal part of the field in (4), which stems from the nonlinear term in (7), makes a small contribution, on the order of $(k_0 a)^{-1}$, and can be ignored. If the beam intensity is far higher than the critical intensity, a solution of Eq. (7) can be written in the form[7]

$$I=I_0 f^{-2}(1-r^2/4a^2f^2)\exp(-\tau^2/2\tau_0^2), \tag{8}$$

$$f^2=1-\frac{\xi^2}{R^2}\exp\left(-\frac{\tau^2}{2\tau_0^2}\right), \tag{9}$$

$$R=2\left(\frac{2}{I_0}\right)^{1/4}\frac{k_0 c}{\omega_{pe}}a,\qquad r\leqslant 2af, \tag{10}$$

where f is a dimensionless beam width which satisfies the boundary conditions

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$$f(\xi=\theta,\tau)=1, \quad \left.\frac{\partial f}{\partial \xi}\right|_{\xi=0}=0. \tag{11}$$

The quantity τ_0 characterizes the pulse length. The self-focusing length, determined by the condition $f=0$, varies in time, following the changes in the pulse. At the focus, $f=0$, Eq. (7) becomes of course meaningless, since we find $I\to\infty$ at this point. We choose accordingly the difference $1-\xi^2/R^2$ to be so small that inequality (5) is not violated.

2. Starting from the hydrodynamic equation and taking into account the average rf-pressure force, we can construct an equation for the perturbations of the electron and ion densities, δn_e and δn_i respectively. Introducing the new variables in (6), and ignoring the weak dependence on the coordinate ξ, we find the following equations for δn_e and δn_i:

$$\left\{\left(1+\omega_{pe}^{-2}\frac{\partial^2}{\partial\tau^2}\right)\frac{\partial^2}{\partial\tau^2}-c_s^2\Delta_\perp\left(1+\omega_{pe}^{-2}\frac{\partial^2}{\partial\tau^2}\right)\right\}\frac{\delta n_e}{n_0}$$
$$=\frac{1}{2}\frac{c^2}{v_g^2}\frac{m_e}{m_i}\left(1+\omega_{pi}^{-2}\frac{\partial^2}{\partial\tau^2}\right)\left(\frac{\partial^2}{\partial\tau^2}+v_g^2\Delta_\perp\right)I(\xi,r,\tau), \tag{12}$$

$$\left(1+\omega_{pi}^{-2}\frac{\partial^2}{\partial\tau^2}\right)\frac{\delta n_i}{n_0}=\frac{\delta n_e}{n_0}. \tag{13}$$

Let us assume that the time scale and the length scale (in the transverse dimension) of the localization of the rf field are on the order of τ_0 and a respectively [cf. (8)]. We can then assume

$$I(\xi,r,\tau)\equiv I(\xi,r/a,\tau/\tau_0). \tag{14}$$

For the calculations below it is convenient to introduce the dimensionless variables

$$\rho=r/a, \quad \tau'=\tau/\tau_0, \tag{15}$$

in which the derivatives in (12) can be assumed to be of equal order of magnitude.

We consider short pulses, with

$$v_g^2\tau_0^2/a^2\ll 1. \tag{16}$$

We assume

$$\omega_{pi}^2\tau_0^2\gtrsim 1 \quad (\omega_{pe}^2\tau_0^2\gg 1). \tag{17}$$

It follows immediately from (12) that the electron density perturbation δn_e is positive.

Using the distribution of the rf field in (8), assuming condition (16), and using (12) we find the following expression for the value of δn_e on the axis (at $r=0$):

$$\left.\frac{\delta n_e}{n_0}\right|_{r=0}=\frac{1}{2}\frac{c^2}{v_g^2}\frac{m_e}{m_i}\left\{1-\frac{1}{\omega_{pi}^2\tau_0^2}\left(1-\frac{\xi^2}{R^2}e^{-\tau^2/2\tau_0^2}\right)\right.$$
$$\left.\times\left[1+\frac{\tau^2}{\tau_0^2}-\frac{2\tau^2}{\tau_0^2}\left(1-\frac{\xi^2}{R^2}e^{-\tau^2/2\tau_0^2}\right)^{-1}\right]\right\}I|_{r=0}. \tag{18}$$

Consequently, under the condition

$$\omega_{pi}^2\tau_0^2>\left(1-\frac{\xi^2}{R^2}\right)^{-1} \tag{19}$$

the perturbation of the electron density will be positive: $\delta n_e>0$. For the perturbation of the ion density we find the following expression from (12) and (13)

$$\frac{\delta n_i}{n_0}=\frac{1}{2}\frac{c^2}{v_g^2}\frac{m_e}{m_i}I>0. \tag{20}$$

The ion density thus also increases in the field localization region, and its space-time evolution follows that of the pulse intensity.

3. Let us consider the case in which the pulse length satisfies the condition

$$\omega_{pi}^2\tau_0^2\gg 1. \tag{21}$$

We then find from (13) the following equation for the perturbation of the electron density:

$$\left(\frac{\partial^2}{\partial\tau^2}-c_s\Delta_\perp\right)\frac{\delta n_e}{n_0}=\frac{1}{2}\frac{c^2}{v_g^2}\frac{m_e}{m_i}\left(\frac{\partial^2}{\partial\tau^2}+v_g^2\Delta_\perp\right)I. \tag{22}$$

Under the conditions considered below, the second term on the left side of this equation is small in comparison with the first. We will nevertheless retain it, to avoid the appearance of terms proportional to τ and in order to find a correct description of the z profile of the density. Specifically, the density perturbation must vanish in the limit $z\to\infty$.

From Eq. (13) we find the following expression for the perturbation of the ion density:

$$\delta n_i\approx\delta n_e. \tag{23}$$

A solution of Eq. (22) which vanishes as $t\to-\infty$ can be written in the form

$$\frac{\delta n_e}{n_0}=\frac{1}{2}\frac{m_e}{m_i}\frac{c^2}{v_g^2}\left\{I+\frac{v_g^2}{c^2}\frac{c_s}{2\pi}\Delta_\perp\int_{-\infty}^{\tau}dt'\right.$$
$$\left.\times\int dr'\frac{\theta(c_s(\tau-t')-|r-r'|)}{[c_s^2(\tau-t')^2-|r-r'|^2]^{1/2}}I(\xi,r',t')\right\}, \tag{24}$$

where

$$\theta(x)=\begin{cases}1, & x>0,\\ 0, & x\leqslant 0\end{cases}$$

is the unit step function. Analysis of this expression shows that the electron density perturbation is localized if the intensity I is localized. Specifically, the step function $\theta(x)$ in the integrand means that the perturbation δn_e is localized in the transverse direction outside the pulse localization region, at $r\lessdot c_s\tau$. To study the z profile of the perturbation δn_e, it is convenient to use an equation for the Green's function corresponding to Eq. (22). It can be shown directly on the basis of (25) that behind the pulse ($z<v_g t$), and far from it, the relation

$$c_s(v_g t-z)\gg v_g a \tag{25}$$

holds, and the density perturbation falls off as $(c_s\tau/a)^{-3}$. To pursue the analysis we rewrite expression (24) in the form

$$\frac{\delta n_e}{n_0}=\frac{1}{2}\frac{m_e}{m_i}\frac{c^2}{v_g^2}$$
$$\times\left\{I+\frac{v_g^2}{2\pi}\Delta_\perp\int_0^{2\pi}d\varphi\int_0^{\infty}dS\int_0^{S}dx\,I(r+\mathbf{i}c_s(S^2-x^2)^{1/2},\tau-S)\right\}, \tag{26}$$

where $\mathbf{i}$ is a unit vector $[\mathbf{i}\equiv(\cos\varphi,\sin\varphi,0)]$. It follows from (26) that there is no density perturbation ahead of the pulse: $|z-v_g t|\gg v_g\tau_0$, $z>v_g t$. The leading edge of the perturbation reproduces the profile of the intensity I. Using (14), and switching to dimensionless variables (15) in the integrand, we find that the second negative term in (26) may be smaller than the first under the condition

$$v_g^2\tau_0^2<a^2. \tag{27}$$

The meaning here is that the particle density increases, i.e., $\delta n_e>0$, in the pulse localization region.

The general conclusions drawn above can be illustrated by substituting expression (8) into (24). The integration over the transverse coordinate is carried out to the boundary of the pulse, at $r=2af$ [see (10)]. To simplify the analysis we assume

$$(1-\xi^2/R^2)^{1/2}\gg c_s\tau/a. \tag{28}$$

In the region

$$-\infty<\tau/\tau_0<(1-\xi^2/R^2)^{1/2}a/c_s\tau_0 \tag{29}$$

we find the following expression for the z profile of the perturbation (at $r=0$):

$$\frac{\delta n_e}{n_0}\bigg|_{r=0}=\frac{1}{2}\frac{m_e}{m_i}\frac{c^2}{v_g^2}\left\{\left(1-\frac{v_g^2\tau_0^2}{a^2}\right)I|_{r=0}-\frac{v_g^2\tau_0^2}{a^2}I_0\frac{\tau}{2\tau_0}\frac{R^2}{\xi^2}\int_{-\infty}^{\tau}\frac{dt'}{\operatorname{sh}^2(t'^2+\alpha)}\right\}, \tag{30}$$

where $\alpha=\ln(R/\xi)$.

At the maximum of the pulse ($\tau=0$) we find the following expression for the density perturbation:

$$\frac{\delta n_e}{n_0}\bigg|_{\tau=0}=\frac{1}{2}\frac{m_e}{m_i}\frac{c^2}{v_g^2}\frac{I_0}{1-\xi^2/R^2}\left(1-\frac{v_g^2\tau_0^2}{a^2}\right). \tag{31}$$

Under condition (27) we thus find $\delta n_e|_{r=0}>0$.

Near the leading edge, $|z-v_gt|\gg v_g\tau_0$, $z>v_gt$ the density perturbation has the same profile as the pulse:

$$\frac{\delta n_e}{n_0}\bigg|_{r=0}=\frac{1}{2}\frac{m_e}{m_i}\frac{c^2}{v_g^2}I_0e^{-\tau^2/2\tau_0^2}>0. \tag{32}$$

Behind the pulse, outside the region in which this pulse is localized, and also outside region (29),

$$|v_gt-z|\gg v_g\tau_0a/c_s\tau_0,\quad z<v_gt\,,$$

the density perturbation falls off in a power-law fashion:

$$\frac{\delta n_e}{n_0}\bigg|_{r=0}=(2\pi)^{1/2}\frac{m_e}{m_i}\frac{c^2}{v_g^2}I_0\frac{v_g}{c_s}\frac{v_g\tau_0}{a}\left(\frac{a}{c_s\tau}\right)^3 \tag{33}$$

[cf. (25)].

The parameter values which the plasma and the electromagnetic pulse would have to assume to satisfy conditions (16), (17), (21), and (27) are easily attainable with existing experimental apparatus. For electron densities in the interval $n_0\approx10^{18}$–10^{19} cm^{-3}, for a laser pulse length $\tau\approx10^{12}$ s, and for a pulse width $a\approx10^{-1}$ cm, these conditions hold, and an increase in the particle density can be expected in the region in which a self-focusing pulse is localized.

4. We can draw the following physical picture of the plasma-compression effect described above.

The fast time dependence of the pulse amplitude [see (8)] has the result that the ponderomotive force "plows" plasma particles along the signal propagation direction, as was shown in Ref. 11. As was also shown there, this plowing of particles is possible only in the "supersonic" regime if there is a relativistic nonlinearity. This condition is analogous to the requirement that the pulse length τ_0 (discussed above) be small.

Using (6) and (8), we easily see that the ratio of the transverse and longitudinal components of the ponderomotive force is equal to a parameter which plays an important role in the plasma compression according to the results of this paper:

$$\frac{\nabla_rI}{\nabla_zI}\approx\frac{v_g\tau_0}{a}. \tag{34}$$

[We ignored the weak dependence on the coordinate ξ in deriving (34).] For wide beams, for which this parameter is small, the rate at which the particles are plowed in the longitudinal direction will be greater than the rate at which the particles are scattered in the transverse direction, so the particle density near the focus will increase.

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