Anistropy of diamagnetism induced by fluctuations in superconductors with nontrivial pairing

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(Submitted 16 May 1990)
Zh. Eksp.Teor. Fiz. **98**, 1476–1479 (October 1990)

The fluctuation diamagnetism in a tetragonal normal metal exhibits anisotropic behavior near the superconducting transition with nontrivial pairing, even when the magnetic field lies in the basal plane. This anisotropy is not present if the superconducting state is described by a single complex order parameter; only the nonlinear response is anisotropic. The linear diamagnetic susceptibility is isotropic because for all degenerate superconducting phases, the fluctuations contributing to diamagnetism are the same above the critical temperature T_c.

Recently much attention has been paid to the properties of anistropic superconductors with nontrivial pairing. This nontrivial pairing occurs for certain types of heavy-fermion superconductors[1–3] and in several organic superconductors (Bechgaard salts).[4,5] The *d*-type pairing is not ruled out for high-temperature superconductors. If the symmetry breaking during superconducting transition takes place according to one of the multidimensional representations of the discrete symmetry groups of the normal phase, the anistropy of the upper critical field can have a very specific character.[6,7] For example, in tetragonal superconductors, for which the order parameter is transformed according to the two-dimensional representation of the D_{4h} group, anisotropy of the upper critical field is possible when the field is oriented in basal plane of the crystalline structure.[6] Observation of this kind of anisotropy would help in identifying superconductivity of this type, because in other cases (corresponding to the one-dimensional representation of D_{4h}) the upper critical field in a tetragonal superconductor is isotropic in the plane and orthogonal to the 4th-order symmetry axis.

The question arises whether this kind of anisotropic behavior in the basal plane of a tetragonal metal occurs not only for the superconducting state ($T< T_c$), but for the normal phase ($T> T_c$) as well due to fluctuations. In the present paper the anisotropy of the fluctuational diamagnetism in the normal phase of a tetragonal metal near T_c is considered. Since the anisotropy in question is connected with the specific character of the symmetry-breaking, it is natural to expect it to be weaker above T_c than below T_c. In fact contributions from all the degenerate superconducting phases are present in the normal metal. However, below the critical temperature the symmetry is broken, so only one phase is present. The immediate consequence of this, found in what follows, is the isotropy of the linear fluctuational diamagnetic response in the basal plane of the tetragonal metal for $T> T_c$. At the same time we show that the nonlinear contribution to the fluctuational diamagnetic response is anisotropic. In particular, the magnetic field lying in the basal plane and the field-induced magnetic moment generally have a nonzero angle betwen them, if the nonlinear contributions are taken into account. This takes place only for tetragonal metals for which the superconductivity below T_c is due to an order parameter with two complex components. Nonlinear effects in fluctuational diamagnetism have been subjected to extensive experimental studies.[8] This is why the results listed below make possible a new technique for the experimental identification of this type of superconductivity.

The corresponding Ginzburg-Landau functional in the Gaussian approximation has the following form:

$$\Delta F(\boldsymbol{\eta})=a(\boldsymbol{\eta}\boldsymbol{\eta}^*)+\frac{1}{2m_1'}(|\partial_x\eta_x|^2+|\partial_y\eta_y|^2)$$
$$+\frac{1}{2m_1''}(|\partial_x\eta_y|^2+|\partial_y\eta_x|^2)$$
$$+\frac{1}{2m_2}(|\partial_z\eta_x|^2+|\partial_z\mu_y|^2)+\frac{1}{4m_3'}[(\partial_x\eta_y)(\partial_y^*\eta_x^*)$$
$$+(\partial_x^*\eta_y^*)(\partial_y\eta_x)]$$
$$+\frac{1}{4m_3''}[(\partial_y\eta_y)(\partial_x^*\eta_x^*)+(\partial_y^*\eta_y^*)(\partial_x\eta_x)] \quad (1)$$

with $a=\alpha(T-T_c)$, $\boldsymbol{\eta}=(\eta_x,\eta_y)$, $\partial_k\boldsymbol{\eta}=\partial\boldsymbol{\eta}/\partial x_k-(2ie/c)A_k\boldsymbol{\eta}$. The *z*-direction is along the tetragonal axis of the crystal and the *x* and *y* coordinates are directed along other crystal axes. For simplicity we take $m_1'=m_1''=m_1$, $m_3'=m_3''=m_3$ (the equality $m_3'=m_3''$ is valid up to terms of order T_c^2/T_F^2). In that case the choice of the variables η_+ and the new coordinates $x_\pm=(y\pm x)/\sqrt{2}$ [Analogously $A_\pm=(A_y\pm A_x)/\sqrt{2}$]. The Gaussian fluctuations of the quantities η_+, η_- are found to be independent:

$$\Delta F(\boldsymbol{\eta})=\Delta F_+(\eta_+)+\Delta F_-(\eta_-),$$

where

$$\Delta F_+(\eta_+)=a|\eta_+|^2+\frac{1}{2}\left(\frac{1}{m_1}+\frac{1}{2m_3}\right)|\partial_+\eta_+|^2$$
$$+\frac{1}{2}\left(\frac{1}{m_1}-\frac{1}{2m_3}\right)|\partial_-\eta_+|^2+\frac{1}{2m_2}|\partial_z\eta_+|^2, \quad (2)$$

and $\partial_\pm=\partial/\partial x_\pm-(2ie/c)A_\pm$. The expression for $\Delta F_-(\eta_-)$ is obtained from Eq. (2) by replacing the quantities η_+ and $\pm\frac{1}{2}m_3$ by η_- and $\mp\frac{1}{2}m_3$, respectively. To be stable, a homogeneous superconducting state must satisfy $|2m_3|>m_1$, $m_{1,2}>0$.

The effective functional (2) assumes the usual form corresponding to an isotropic superconductor, after the transformations

$$x_\pm'=\left[m_2\left(\frac{1}{m_1}\pm\frac{1}{2m_3}\right)\right]^{-1/2}x_\pm,$$
$$A_\pm'=\left[m_2\left(\frac{1}{m_1}\pm\frac{1}{2m_3}\right)\right]^{1/2}A_\pm. \quad (3)$$

 0038-5646/90/100825-02$03.00

This enables us to write down the expression for the contribution of F_+ to the free energy using the well known result for isotropic superconductors (see, e.g., Ref. 9).

$$F_+=-V'\frac{|e|TB_+}{2\pi^2c}\sum_{n=0}^{\infty}\int_{-\infty}^{\infty}dp\ln\left\{\pi T\left[\left(n+\frac{1}{2}\right)\frac{|e|}{m'c}B_+ +\frac{p^2}{4m'}+a'\right]^{-1}\right\}, \quad (4)$$

where

$$V'=\frac{2m_1|m_3|}{m_2(4m_3^2-m_1^2)^{1/2}}V,\quad a'=a\frac{m_2(4m_3^2-m_1^2)^{1/2}}{2m_1|m_3|},\quad (m')^{-1}=\frac{(4m_3^2-m_1^2)^{1/2}}{m_1|m_3|}, \quad (5)$$

$$B_\pm=\left(\frac{m_2}{m_1}B^2\mp\frac{m_2}{m_3}B_xB_y\right)^{1/2} \quad (6)$$

and the magnetic field is assumed to lie in the basal plane. The expression for F_- is obtained from (4) by substituting B_+ into B_-.

The fluctuation contribution $F=F_+ + F_-$ to the free energy due to the magnetic field component in the xy plane depends in general on the direction of the field in this plane. However, the anisotropy of F is found to be much weaker than that of F_+ and F_- separately and drops out entirely to second order in the field. Taking into account the fourth-order terms one has

$$F=\frac{1}{2}|\chi|B^2\left[1-\left(1+\frac{m_1^2}{4m_3^2}\sin^2 2\theta\right)\frac{B^2}{B_0^2}\right], \quad (7)$$

where

$$\chi=-V\frac{2^{1/2}e^2T_c|m_3|}{3\pi c^2[am_2(4m_3^2-m_1^2)]^{1/2}},\quad B_0^2=\frac{80m_1m_2}{7}\left(\frac{ca}{e}\right)^2 \quad (8)$$

and θ is the angle through which the magnetic field is inclined toward the axis. Note that $B_0\approx 1.69H_{c2}^{\perp}(\varphi=0)$ where $H_{c2}^{\perp}(\varphi=0)$ is the upper critical field near T_c for direction parallel to the x and y axes.

Using Eq. (4) for F_+ and the analogous expression for F_- one can obtain, following Ref. 10, the general expression for the field-induced magnetic moment $\mathbf{M}$, in which nonlinear dependence of this quantity on the field is taken into account. We cite here only the result for relatively weak fields $B\ll B_0$ in which the lowest nonlinear corrections are accounted for:[2]

$$M_i=\chi_{ij}(\mathbf{B})B_j,\quad \chi_{xx}(\mathbf{B})=\chi_{yy}(\mathbf{B})=\chi(1-2B^2/B_0^2), \quad (9)$$

$$\chi_{xy}(\mathbf{B})=\chi_{yx}(\mathbf{B})=-\chi\frac{m_1^2B^2}{2m_3^2B_0^2}\sin 2\theta. \quad (10)$$

The angle θ_M between the magnetic moment and the x direction is related to the inclination θ of the magnetic field by

$$\tan\theta_M=\left(1-\frac{m_1^2B^2}{m_3^2B_0^2}\cos 2\theta\right)\tan\theta. \quad (11)$$

The magnitude of the magnetic moment is thus described by the expression,

$$M=|\chi|B\left[1-\frac{2B^2}{B_0^2}\left(1+\frac{m_1^2}{4m_3^2}\sin^2 2\theta\right)\right]. \quad (12)$$

Thus we conclude that in tetragonal superconductors with nontrivial pairing the anistoropy of the fluctuational diamagnetism emerges even in the case when the applied field lies in the basal plane. This anisotropy is described by expressions (10)–(12).

[1)]The isotropy of the linear diamagnetic response in the basal plane follows just from the fact that the second-rank tensor χ_{ij} in the linear approximation goes over to δ_{ij} $(i,j=x,y)$ in the linear approximation.

[2)]Nonlinear corrections of the same form arise also from fourth-order gradient terms. These nonlocal terms make a contribution to the diamagnetic response which is smaller than the one we have considered by a factor of order $(H_{c2}(T)/H_{c2}(0))^2$. Close to T_c the quantity $(H_{c2}(T)/H_{c2}(0))^2$ is negligible. The quantity $H_{c2}(T)$ is associated with a temperature $T<T_c$ for which the quantity $|T-T_c|$ is the same as that considered here for $T>T_c$.

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Translated by P. B. Lerner