Metastable states and magnetic resonance of the domain structure in a ferromagnet

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The existence of metastable states of a plane-parallel domain structure with a varying number of domain walls N in uniaxial ferromagnetic plates is demonstrated theoretically and experimentally. Under quasistatic remagnetization or demagnetization the sample is in a metastable state with $N < N_0$, where N_0 is the ground state. The energy barriers associated with nucleation inhibit the $N \to N+1$ processes, while the magnetostatic barriers inhibit the $N \to N-1$ processes. The $N \to N+1$ transitions may occur under the influence of radio-frequency fields. A theory of domain structures with a finite N is developed to explain the nature and properties of the metastable states. The theory also predicts asymmetry of the magnetization curve and the existence of an average magnetization for $H = 0$ for odd N.

INTRODUCTION

The domain structure in a ferromagnet arises in order to diminish (or completely cancel that portion of magnetostatic energy caused by the jump of the normal component of the magnetization vector $\mathbf{M}(\mathbf{x})$ along the edges of the sample. The total dipole moment of the sample may be equal to zero when a domain structure is present (most commonly, two oppositely magnetized domains). However the multipole part of the magnetostatic energy F_m remains nonzero in this case and is dependent on the number of domains N in the sample. As N grows the energy F_m decreases although the magnetic inhomogeneity energy F_γ associated with the domain wall structure will rise. The equilibrium number of domains N_0 is determined from the minimum sum of these two energies.[1)]

Therefore the ground state of the domain structure for $H = 0$ is determined by satisfying the following two conditions:

$$\int_V \mathbf{M}(\mathbf{x})\, d^3x = 0, \tag{1}$$

$$N = N_0. \tag{2}$$

The metastable states associated with the violation of condition (1) are well known in actual crystals. If the sample is magnetized to saturation and the magnetic field then drops to zero, the so-called residual magnetization, as a rule, is conserved. This is either due to the inhibited formation of domain walls or their inhibited displacement in actual crystals. Permanent magnets, magnetic memories, etc., operate in such metastable states.

Condition (1) can be satisfied if the specimen is exposed to alternating fields of diminishing amplitude. Such "shocking" of the magnetic system is conventionally used for demagnetization of ferromagnetic materials and it is ordinarily assumed that it results in satisfaction of both conditions, (1) and (2).

In the present study we demonstrate that this is not always valid and investigate the metastable states associated solely with the violation of condition (2) while condition (1) holds (precisely or approximately). The experiments carried out on $FeBO_3$ weak ferromagnet plates have demonstrated that there exists, in addition to the ground state of the domain structure corresponding to $N = N_0$, an entire series of metastable states with $N < N_0$. These states are resistant to the effect of alternating fields and transitions between them (transitions of the form $N \to N+1$) will occur only when certain critical frequencies and fields are achieved.

The present paper discusses the nature and properties of such metastable states and considers the linear resonance of the domain walls in the metastable states.

1. EXPERIMENT

The experiment was carried out on rectangular $FeBO_3$ single crystal plates of thickness $\approx 5 \cdot 10^{-3}$ cm with transverse dimensions of $\approx 10^{-1}$ cm. $FeBO_3$ is a weak easy-plane ferromagnet ($M = 9.2$ Gauss).[1] A regular through plane-parallel domain structure (Fig. 1) was generated by applying uniform elastic stresses which produce moderate uniaxial magnetic anisotropy in the easy plane. Forced oscillations of the domain walls relative to their equilibrium position were generated by a radio-frequency magnetic field H along the direction of magnetization of the domains; the equilibrium positions were observed by a magnetooptic technique described previously.[2] The frequency of field H was tunable over a broad range, from tens of Hz to 100 MHz.

We discovered that a specific number N of domain walls, dependent on the sample aspect ratio, arose in the sample in its initial state. (For example for the sample with $a = 1.5$ mm, $b = 50\ \mu$m, $c = 1$ mm, which we shall henceforth refer to as sample No. 1, we found $N_{min} = 3$.) Ordinary behavior of the domain structure was observed for any frequency of field H less than a certain critical frequency ω_m (for sample No. 1, we found $\nu_m = \omega_m/2\pi = 4$ MHz): The amplitude of the forced oscillations of the domain wall η grew to maximum values corresponding to total remagnetization of the sample for $H \geqslant H_s$ (H_s is the saturation field) as the amplitude of the radio-frequency field H increased; the demagnetized state with $N = N_{min}$ domain walls was reestablished in the sample when the field amplitude decreased.

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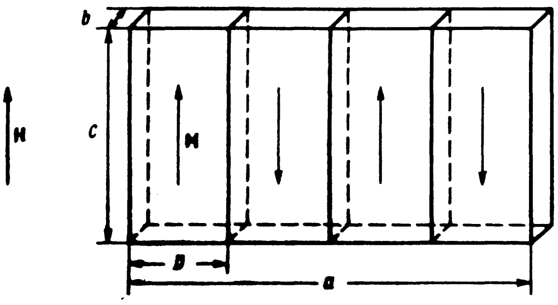

FIG. 1. Structure of the plane-parallel domains.

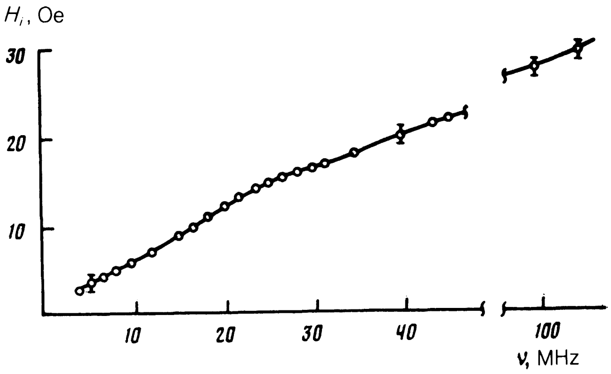

FIG. 3. Minimum of field amplitudes required for the $N \to N+1$ transition.

If a field of frequency exceeding ω_m was applied, an irregular transition to a state with a large number of domains occurred as a certain critical field amplitude H_m ($H_m < H_s$) was reached; this state was conserved after the field H was removed. By tuning from 4 to 28 MHz for sample No. 1 it was possible to obtain all values of N from 3 to 13. A further increase in frequency (to 100 MHz and higher) had no effect on N.

The basic trend associated with this effect is shown in Fig. 2, which shows the number of domain walls that appeared in this sample when a magnetic field was applied at a given frequency. The horizontal line at $N = 3$ from 0 to 4 MHz indicates that no demagnetized state aside from $N = 3$ can be obtained by applying and then removing a field of any amplitude. The horizontal segment at the $N = 4$ level indicates that for frequencies corresponding to this range there will always be an $H < H_s$ such that the system goes from the $N = 3$ state to the $N = 4$ state. This then proceeds up through the segment with $N = 13$ which begins at 28 MHz and remains unchanged for all higher frequencies we studied. Figure 3 shows a different aspect of this effect: The minimum amplitude H_i of the rf field required to obtain the value of N shown in Fig. 2 for each frequency value. It is clear that H_i grows with frequency while remaining below H_s.

The experiment was carried out on an entire series of samples with different ratios of the dimensions a, b and c. Different values of N_{min} and N_0, as well as various critical frequencies and fields corresponded to different samples although the behavior of the transition from the N to the $N + 1$, etc., states shown in Figs. 2 and 3 were common to all samples.

This effect therefore made it possible to establish in each sample a predetermined specific number of domain walls N from the specific set N_{min}, $N_{min} + 1$, ..., N_0 characteristic of each sample. This was used to learn how physical characteristics such as the magnetic susceptibility and the magnetic resonance frequency of the domain walls ω_a depend on N. The static magnetic susceptibility remained nearly independent of N, while the dependence of the squared frequency $\nu_a = \omega a/2\pi$ on N is given in Fig. 4.

We can conclude from all these experimental results that an entire set (up to 10 or more) of metastable states of the domain structure with a varying number of domain walls corresponds to the demagnetized state in the test samples. These metastable states corresponding to the relative energy minima are separated by rather high potential barriers, which permits observations of magnetic resonance in each such state.

In the following sections we discuss the physical nature of these metastable states.

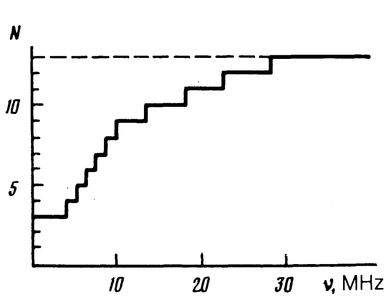

FIG. 2. The minimum number of domain walls N plotted as a function of frequency ν.

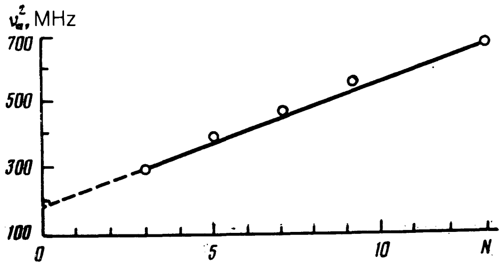

FIG. 4. The squared resonant frequency of the domain walls ν_a plotted as a function of N.

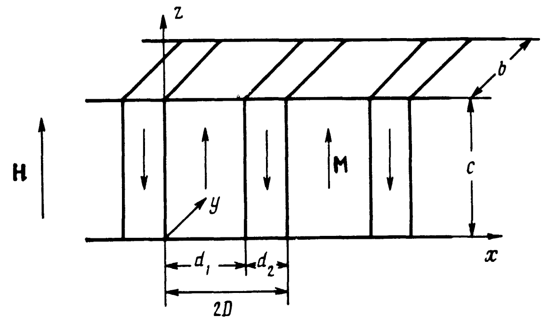

FIG. 5. Domain structure periodic in x.

2. EXPANSION AND COMPRESSION OF THE DOMAIN STRUCTURE PERIOD UPON REMAGNETIZATION

The idea of central concern to us among results of the numerous theoretical studies devoted to plane-parallel domain structures is the concept independently confirmed by Kooy and Enz[3] and Ignatchenko, Degtyarev and Zakharov.[4] These studies demonstrated that a magnetic field **H** not only alters the ratio of the domain volumes magnetized along and across **H** but also alters the period of the domain structure $2D$. As H grows the period will also grow and as H decreases, the period decreases. We recall in this section the primary results of these studies and then develop these ideas further.

For unequal neighboring domains the magnetostatic energy depends on both the domain structure period $2D$ and the ratio $q = d_2/D$ (see Fig. 5). In this case the term associated with the dipole part of the energy of the entire sample (and hence dependent solely on q) can always be distinguished; however the remaining multipole part of the energy F_m depends on both D and on q and cannot be represented as a sum of terms, each of which depends on only one of these variables.

The total energy density accounting for the domain wall energy and the magnetic field energy takes the form

$$F(D,\ q;\ H)={}^1/_2N_zM^2(1-q)^2+F_m(D,\ q)-MH(1-q)+\gamma/D, \tag{3}$$

where N_z is the demagnetization factor of the sample in the direction of the applied field **H** and γ is the surface energy density of the domain walls.

The equilibrium values of q and D for the given H are determined by two equations:

$$\frac{\partial F}{\partial D}=0,\quad \frac{\partial F}{\partial q}=0. \tag{4}$$

These equations are coupled equations by virtue of the interaction term $F_m(D,q)$ and changes in the magnetic field will not only cause a change in q but also in D, i.e., will produce an expansion or compression effect of the domain structure period. Ordinarily only numerical techniques can be used to analyze this effect given the mathematical complexity of the term $F_m(D,q)$. We note that this term completely determines the structure of the first equation in Eqs. (4), i.e., the dependence of D on q. In the second equation in (4), which describes the dependence of q on H (the magnetization curve), the term $F_m(D,q)$ ordinarily yields only minor deviations from the linear law corresponding to neglect of this term:

$$1-q\approx H/N_zM. \tag{5}$$

Both studies[3,4] examined a model of a sample infinite in x, which made it possible to carry out a Fourier expansion of the magnetostatic potential and magnetization in this coordinate. The sample in Ref. 3 was assumed to be unbounded along the y axis as well. A more general case was examined in Ref. 4 which accounted for the finite size of the sample in this direction. In this case the expression for the multipole energy density takes the form

$$F_m(D,q)=\frac{32M^2}{\pi^2bc}\sum_{n=1}^{\infty}\frac{1-\cos(\pi nq)}{n^2}\int_0^{\infty}\frac{\sin^2(bk/2)(1-e^{-cp})}{pk^2}\,dk, \tag{6}$$

where $p^2 = k^2 + (\pi n/D)^2$. This expression describes a rather broad class of physical situations. For $c \ll b$ it can be applied to a plate with an easy axis perpendicular to the surface, while for $c \gg b$ it can be applied to a plate with an easy axis in its plane. The exponential in the numerator of (6) can be neglected for $D \ll \pi c$ although even in this case the integral is not taken in general form. We consider certain limiting cases.

For the case $D \ll c \ll b$ (plate with the easy axis perpendicular to the plane) and passing to the limit $b\to\infty$ in (6) (after substitution of the integration variable $v = bk/2$) or as is done in Ref. 3, considering such a limiting configuration from the outset, we obtain

$$F_m=\frac{8M^2}{\pi^2c}Df_1(q),\quad f_1(q)=\sum_{n=1}^{\infty}\frac{1-\cos[\pi nq]}{n^3}. \tag{7}$$

In this case we obtain for the domain width the following dependence of D on q from the first equation in (4):

$$D=\left(\frac{\pi^2c\gamma}{8M^2f_1(q)}\right)^{1/2}. \tag{8}$$

For $H = 0$ we have $q = 1$ and this reduces to the familiar expression for the equilibrium domain width in a plate with the easy axis perpendicular to the surface, first derived by Kittel.[5]

For a thin film with the easy axis lying in the plane ($b \ll D \ll c$) we obtain from (6)

$$F_m=\frac{8M^2b}{\pi^2c}\sum_{n=1}^{\infty}\frac{1-\cos(\pi nq)}{n^2}\ln\frac{\nu D}{nb}, \tag{9}$$

where ν is a certain constant. In this case we obtain from the first equation in (4)

$$D=\frac{\pi^2 c\gamma}{8M^2 b f_2(q)}, \quad f_2(q)=\sum_{n=1}^{\infty}\frac{1-\cos(\pi n q)}{n^2}=\left(\frac{\pi}{2}\right)^2 q(2-q). \tag{10}$$

The dependence of D on both the domain wall energy γ and other parameters is substantially different from Eq. (8). For $q=1$ Eq. (10) reduces to the expression for an equilibrium domain width in a thin film, first proposed in the study by Ignatchenko and Zakharov.[6]

The experimental case of interest to us here does not satisfy either of these limiting cases. The easy axis lies in the plane of the plate, but the ratio D/b lies in the intermediate range of parameters ($1\lesssim D/b\lesssim 10$). A numerical calculation was carried out for this case, which demonstrated that (6) can be roughly approximated by the expression

$$F_m\approx\frac{2M^2(bD)^{1/2}}{3c}f_3(q), \quad f_3(q)\approx\left(\frac{\pi}{2}\right)^2 q(2-q). \tag{11}$$

From the equation of system (4) we have

$$D\approx\left[\frac{3\gamma c}{M^2 b^{1/2} f_3(q)}\right]^{2/3}. \tag{12}$$

Thus although in different situations the dependence of D on q (and, consequently, on H as well) is determined by different expressions, from the qualitative viewpoint it has the same universal character first identified in Refs. 3, 4: Expansion of the domain structure period is observed as the field increases and compression of this period is observed as the field decreases. The same difficulty that occurs for an infinite expanding Universe occurs here for a sample that is infinite along the x axis: The velocities of the domain walls grow without limit at large distances from the coordinate origin.

Large changes in D correspond to a change in the number of domain walls N in the sample for a sample bounded along x. If there is nothing to inhibit the formation or destruction of the domain walls, remagnetization of the bounded crystal will appear as follows. At values of H exceeding the saturation field H_s we have $q=0$ and $N=0$. A reduction in H to values below H_s will produce one (or two) domain walls and a nonzero $q<1$. A further reduction in H will produce new domain walls whose number reaches the maximum value N_0 for $H=0$; here $q=1$. As the negative value of H grows, the number of domain walls N will again drop to $N=0$. However in reality the creation (or destruction) of each domain wall is associated with overcoming the potential barrier.

3. THE DOMAIN STRUCTURE WITH A FINITE N

We consider a sample that is finite in all three directions with plane-parallel domain walls at random points with coordinates $x_1, \ldots, x_N$; here x_0 and x_{N+1} denote the coordinates of the left and right ends of the sample. For definiteness we assume positive magnetization of the far left domain.

In calculating the magnetostatic energy we use the general expression obtained in Ref. 4 for the case of magnetization $M(x,y)$ having any configuration in the xy plane and uniform in z in section c:

$$F=\frac{(2\pi)^3}{abc}\iint_{-\infty}^{+\infty}|M(k_x,k_y)|^2\frac{(1-e^{-c\varkappa})}{\varkappa}dk_x dk_y, \tag{13}$$

where $\varkappa^2=k_x^2+k_y^2$,

$$M(k_x,k_y)=\frac{1}{(2\pi)^2}\iint_{-\infty}^{+\infty}M(x,y)\exp[-i(xk_x+yk_y)]dx\,dy.$$

For this structure we derived an exact analytic expression for the magnetostatic energy as "lattice" sums:[7]

$$\begin{aligned}F(x_1,x_2,\ldots,x_N)&=-\frac{8M^2}{ac}\Big\{\sum_{n=1}^{N}(-1)^n f(x_n-x_0)+\frac{1}{2}(-1)^{N+1}f(a)\\&+\sum_{n=1}^{N}(-1)^{n+N+1}f(x_{N+1}-x_n)+2\\&\times\sum_{\substack{n\ m=1\\(n>m)}}^{N}\sum(-1)^{n+m}f(x_n-x_m)\Big\},\end{aligned} \tag{14}$$

where

$$\begin{aligned}f(x)&=\frac{x^2}{2}\operatorname{arsh}\frac{b}{x}+\frac{1}{2}(c^2-x^2)\\&\times\operatorname{arsh}\frac{b}{(x^2+c^2)^{1/2}}+\frac{bx}{2}\operatorname{arsh}\frac{x}{b}\\&-\frac{c^2x}{2b}\operatorname{arsh}\frac{x}{c}+\frac{x}{2b}(c^2-b^2)\operatorname{arsh}\frac{x}{(c^2+b^2)^{1/2}}\\&-\frac{1}{6b}[(x^2+b^2)^{3/2}-|x|^3+(x^2+c^2)^{3/2}-(x^2+b^2+c^2)^{3/2}\\&+(b^2+c^2)^{3/2}-b^3+2c^3]\\&+\frac{c^2}{2b}[(b^2+c^2)^{1/2}+(x^2+c^2)^{1/2}-(x^2+c^2+b^2)^{1/2}]\\&-\frac{c^2}{2}\operatorname{arsh}\frac{b}{c}+cx\operatorname{arctg}\frac{bx}{c(x^2+b^2+c^2)^{1/2}}.\end{aligned} \tag{15}$$

Therefore in the lattice representation the energy density (14) breaks into sums of the interaction potentials between the n^{th} and m^{th} domain walls:

$$\frac{16M^2}{ac}f(x_n-x_m),$$

between a domain wall and the left and right edges of the sample, respectively

$$\frac{8M^2}{ac}f(x_n-x_0), \quad \frac{8M^2}{ac}f(x_{N+1}-x_n),$$

and the interaction potential between the sample edges,

$$\frac{4M^2}{ac}f(x_{N+1}-x_0)=\frac{4M^2}{ac}f(a).$$

The signs of these potentials are such that the interaction between nearest neighbors is attractive, while the interaction with the domain wall of the second coordination "sphere" is repulsive, the interaction with the third wall is again attractive, etc. Therefore if an even number of domain walls (or an odd number of domains) is placed between two domain walls, they will attract, while if an odd number of domain walls is used they will repel; here the edges of the sample are formally assumed to be domain walls. Henceforth we will also require the value of the magnetostatic fields in the domains. For the present plane-parallel domains with random positions of the domain walls we have

$$H_z=M\Big[g(x_0-x,y,z)+2\times\sum_{n=1}^{N}(-1)^n g(x_n-x,y,z)+(-1)^{N+1}g(x_{N+1}-x,y,z)\Big] \tag{16}$$

where

$$\begin{aligned}g(x_n-x,y,z)&=\operatorname{arctg}\frac{(x_n-x)(b/2-y)}{(c/2-z)[(x_n-x)^2+(b/2-y)^2+(c/2-z)^2]^{1/2}}\\&+\operatorname{arctg}\frac{(x_n-x)(b/2+y)}{(c/2-z)[(x_n-x)^2+(b/2+y)^2+(c/2-z)^2]^{1/2}}\\&+\operatorname{arctg}\frac{(x_n-x)(b/2-y)}{(c/2+z)[(x_n-x)^2+(b/2-y)^2+(c/2+z)^2]^{1/2}}\\&+\operatorname{arctg}\frac{(x_n-x)(b/2+y)}{(c/2+z)[(x_n-x)^2+(b/2+y)^2+(c/2+z)^2]^{1/2}}.\end{aligned}$$

Formally the situation is such that each domain wall generates an oscillating magnetic field obeying an arc tangent law with the center at the wall position; the edges of this sample also produce fields, although these fields are of half the magnitude.

The problem of equilibrium domain structure is posed substantially differently in a sample with a finite number of domain walls compared to the model with a domain structure that is periodic in x. For a periodic structure $D=0$ corresponds to the magnetostatic energy minimum, and a finite value of D is obtained only when the surface energy of the domain walls is taken into account. In finite samples there exists a domain wall configuration for each N corresponding to the minimum magnetostatic energy independent of the boundary energy. Hence

1) For each fixed value of N there is a domain wall configuration (values of the coordinates $x_1, \ldots, x_N$) that minimizes the magnetostatic energy (14) and represents a solution of a system of N equations

$$\frac{\partial F}{\partial x_1}=0,\ldots,\frac{\partial F}{\partial x_N}=0; \tag{17}$$

2) in comparing the sums of the magnetostatic and boundary energies of each minimizing configuration it is necessary to find the configuration corresponding to the ground state;

3) by examining the behavior of the system in a magnetic field (or under other actions) it is necessary to determine the ranges within which a change in the domain wall configuration will occur without changing the number of walls and where (and how) the phase transitions will occur when the number of domain walls N changes.

In order to simplify the mathematical expressions the analysis below will be carried out for the limiting case of a sample that is unbounded along y. Since the finiteness along x is conserved in this case, all qualitative features of systems attributable to the finiteness of N are also preserved.

Expression (14) in this case remains unchanged while the function $f(x)$ is substantially simplified:

$$f(x)={}^1/_4x^2\ln(1+c^2/x^2)-{}^1/_4c^2\ln(1+x^2/c^2)+cx\operatorname{arctg}(x/c). \tag{18}$$

Expression (16) for the z-projection of the magnetostatic field as a function of x and z takes the form

$$\begin{aligned}H_z(x,z)=2M\Big[&\operatorname{arctg}\frac{x_0-x}{c/2+z}+\operatorname{arctg}\frac{x_0-x}{c/2-z}\\&+2\sum_{n=1}^{N}(-1)^n\Big(\operatorname{arctg}\frac{x_n-x}{c/2+z}+\operatorname{arctg}\frac{x_n-x}{c/2-z}\Big)\\&+(-1)^{N+1}\Big(\operatorname{arctg}\frac{x_{N+1}-x}{c/2+z}+\operatorname{arctg}\frac{x_{N-1}-x}{c/2-z}\Big)\Big].\end{aligned} \tag{19}$$

It is assumed in all these expressions that the coordinate origin is at the center of the sample [specifically, $x_0=-a/2, x_{N+1}=a/2$].

We consider the equilibrium domain structure, the magnetization curve and the magnetostatic field structure for different values of N.

a) $N=1$. Figure 6 gives the magnetostatic field energy F plotted as a function of the position of the domain wall x_1 together with the dependence of the magnetostatic field H_z on x when the equilibrium position of the domain wall is $x_1=0$. It is clear that the field H_z may reach its peak value both at the sample edge and in the center of the domain, depending on the ratio of dimensions of the sample.

b) $N=2$. In principle this is a two-particle problem and the state of the system is described by the two-dimensional energy surface $F=F(x_1,x_2)$ shown in Fig. 7. However from symmetry considerations we have $x_2=-x_1$, both for $H=0$ and for nonzero values of H. Hence the problem reduces to a one-particle problem and is described solely by the OA cross section of the energy surface $F(x_1,x_2)$ shown in Fig. 8, a (here we use the rotation $D=x_2=-x_1$). Figure 8, b, shows the magnetization curve for this case.

The most immediately evident characteristic of Fig. 8 is that the minimizing domain wall configuration does not correspond to the demagnetized state of the sample. The difference between D_0 and $0.25a$ when $m=0$ holds is slight, but is fundamental. This effect arises because magnetostatic ener-

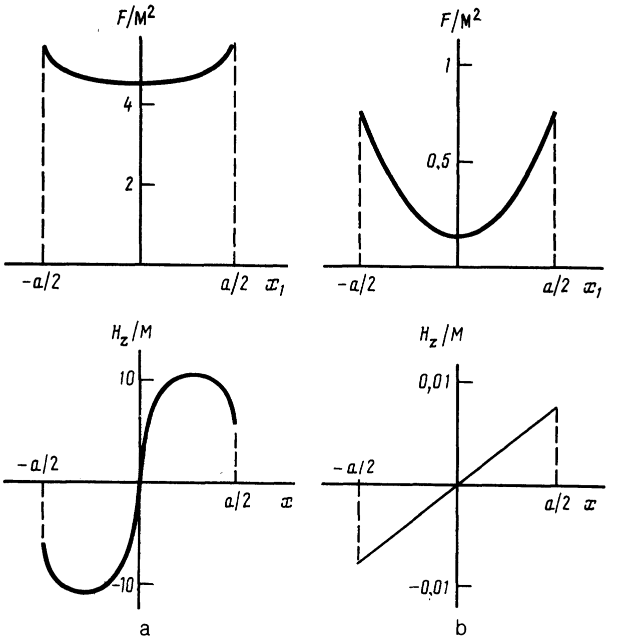

FIG. 6. The energy at domain wall position x_1 and the magnetostatic field (for $z = 0$, $x_1 = 0$) plotted as a function of the coordinate x for $N = 1$ for different dimensional ratios of the sample: $c/a = 0.1$ (a); $c/a = 10$ (b); $b = \infty$.

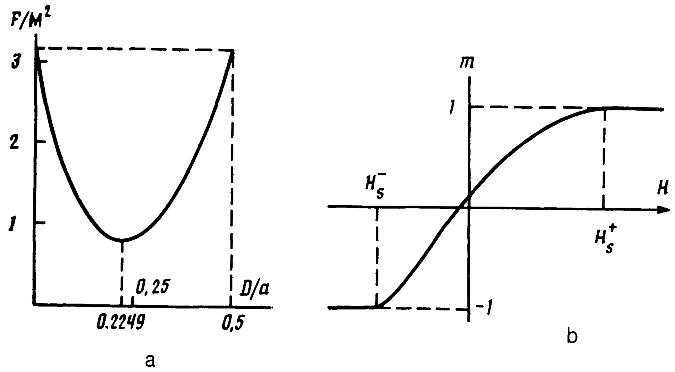

FIG. 8. Energy plotted as a function of $D = x_2 = -x_1$ (a) and the asymmetrical magnetization curves $m = \langle M_z \rangle / M$ (b) for $N = 2$ for $a = c$, $b = \infty$.

gy minimization corresponds to minimization of the sum of the dipole and multipole energy parts, and this sum is not always minimized when the condition for the vanishing of the magnetic moment of the system holds exactly. The second unusual effect—the asymmetry of the remagnetization curve—is due to the difference of the magnetization paths: In a positive field the domain walls converge and are annihilated, while in a negative field they are repelled to the edges of the sample. Both of these effects grow with a/c.

In the planes $x = \pm D_0$ corresponding to the equilibrium positions of the domain walls only the magnetostatic field values averaged over the sample thickness z vanishes. Figure 9 illustrates the local fields in these planes and shows $H_z(z)$ along the domain wall in precise equilibrium (b) and with a wall slightly shifted to one (a) and the other (c) side of equilibrium. It is clear that in equilibrium there are a number of alternating sections where the field may be nonzero and have different signs. This means that the domain wall is under bending forces. The plane domain wall model corresponds to a rather high surface tension energy of the domain wall compared to the magnetostatic energy; the domain wall is located at the site where the magnetic field pressure is equalized over its different sections. When the excess surface tension energy condition is violated, the domain wall will be bent along z in accordance with the magnetic field picture shown in Fig. 9b.

c) $N = 3$. The three-particle problem reduces to a one-particle problem based on symmetry considerations only for

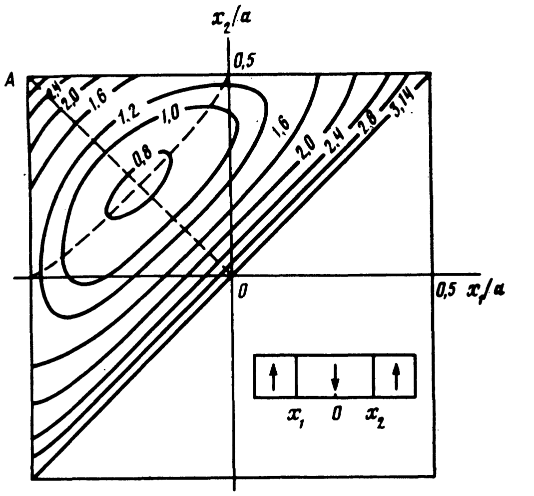

FIG. 7. $F(x_1, x_2)/M^2$ energy relief for $N = 2$ for $a = c$, $b = \infty$.

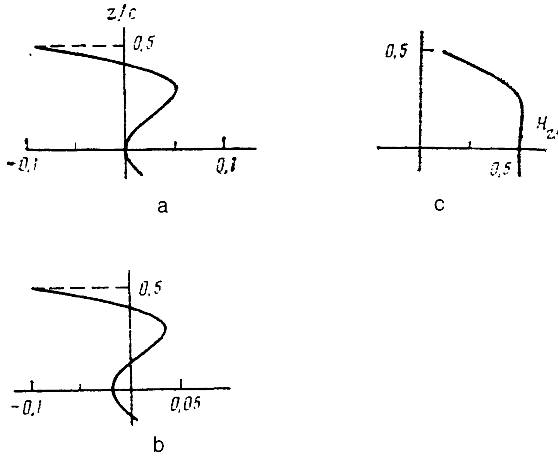

FIG. 9. Field distribution in the plane of the domain wall for $N = 2$ for the equilibrium position of the domain walls $D/a = 0.2249$ (b) and for those shifted from equilibrium: $D/a = 0.2258$ (a), $D/a = 0.25$ (c), for $a = c$, $b = \infty$.

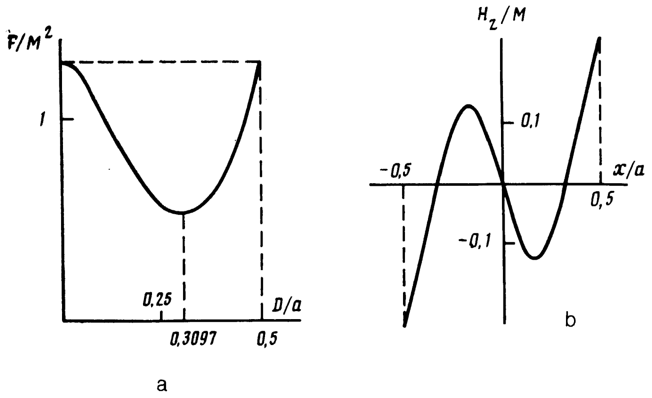

FIG. 10. Energy plotted as a function of $D = x_3 = -x_1$ for $x_2 = 0$ (a) and the magnetostatic field for $z = 0$ as a function of the coordinate x (b) for $N = 3$ for $a = c$, $b = \infty$.

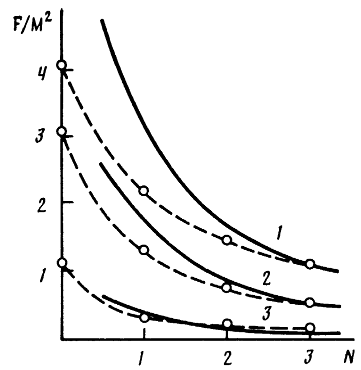

FIG. 11. Energy plotted against N for samples with a finite number of domains (the circles connected by dashed lines) and for the periodic domain structure model with $N = a/D$ (solid lines) and for different c/a: 1—0.5; 2—1; 3—5.

$H = 0$, when $x_2 = 0$, $x_3 = -x_1 = D$; application of a magnetic field breaks the symmetry. The dependence of F on D for $H = 0$ is shown in Fig. 10a. This case is interesting because the zero-value condition of the total magnetic moments of the sample holds for any D and hence the minimum on the $F(D)$ curve is due entirely to the multipole part of the magnetostatic energy.

Figure 10b, shows the distribution of the z-component of the magnetostatic field in the $z = 0$ plane. The highest field value is achieved for $a \geqslant c$ along the edges of the sample, while the highest value is achieved within the domains for $a > c$.

d) $N = 4$. The energy relief of the problem was also investigated for $N = 4$ in Ref. 7; this problem reduces to a two-particle problem both for $H = 0$ and when there is a nonvanishing external magnetic field, for $x_1 = -x_4$, $x_2 = -x_3$. As in the case $N = 2$ the situation is characterized by an asymmetrical hysteresis loop and moderate residual magnetization in an equilibrium demagnetized state, although these effects are much less clearly expressed than for $N = 2$.

Based on these examples we can draw certain general conclusions regarding systems with a finite number of plane-parallel domain walls. The domain structure configuration for $H = 0$ is such that the widths of the domains along the edges $x = \pm a/2$ of the sample are approximately half the widths of the other domains. This rule, which holds for both even and odd values of N, will be more accurately satisfied the larger the values of N. The most fundamental difference between such systems with a finite N and a periodic domain structure is the resulting magnetization for H_0 and the asymmetry of the hysteresis loop for states with an even value of N. However these effects are significant only for small N and decrease as a function of N.

Figure 11 shows the magnetostatic energy plotted as a function of the number of domains for equilibrium configurations with different a/c ratios (circles). Here the solid curves represent the $F(N)$ relation for the same samples calculated by the Kittel formula[5] obtained by assuming a domain structure periodic in x with a domain width $D = a/N$ [Eq. (7) for $q = 1$]. It is clear that the discrepancy between the exact and approximate values of F becomes insignificant for $N > 3$–4 and the equations from Section 2 can be used in samples that are finite in x. Here, however, the potential barriers between states with different N must be taken into account in the finite samples.

4. BARRIERS BETWEEN STATES WITH DIFFERENT *N* AND THE REMAGNETIZATION PROCESS

We approximate the actual domain structure of a finite sample by using expressions for a domain structure that is periodic in x. In this case the series in Eq. (19) can be summed exactly and the magnetostatic field $H_2(x,z)$ as $b \to \infty$ takes the form

$$H_z = -4\pi M_z - 4M \times \Big\{ \operatorname{arctg}\Big[\operatorname{th}\Big(\frac{\pi}{2D}\Big(\frac{c}{2}+z\Big)\Big)\operatorname{ctg}\Big(\frac{\pi}{4}\Big(q-\frac{2x}{D}\Big)\Big)\Big] + \operatorname{arctg}\Big[\operatorname{th}\Big(\frac{\pi}{2D}\Big(\frac{c}{2}+z\Big)\Big)\operatorname{ctg}\Big(\frac{\pi}{4}\Big(q+\frac{2x}{D}\Big)\Big)\Big] + \operatorname{arctg}\Big[\operatorname{th}\Big(\frac{\pi}{2D}\Big(\frac{c}{2}-z\Big)\Big)\operatorname{ctg}\Big(\frac{\pi}{4}\Big(q-\frac{2x}{D}\Big)\Big)\Big] + \operatorname{arctg}\Big[\operatorname{th}\Big(\frac{\pi}{2D}\Big(\frac{c}{2}-z\Big)\Big)\operatorname{ctg}\Big(\frac{\pi}{4}\Big(q+\frac{2x}{D}\Big)\Big)\Big]\Big\}. \tag{20}$$

We then have for the magnetostatic fields at the domain center

$$H_m^+ = -4\pi M + 16M \operatorname{arctg}\Big(\operatorname{th}\frac{\pi c}{4D}\operatorname{tg}\frac{\pi q}{4}\Big),$$
$$H_m^- = 4\pi M - 16M \operatorname{arctg}\Big(\operatorname{th}\frac{\pi c}{4D}\operatorname{ctg}\frac{\pi q}{4}\Big). \tag{20a}$$

The indices $\pm$ correspond to the fields in positively and negatively magnetized domains.

When an external magnetic field is applied, the magnetostatic fields change due to the change in q and D. As the positive values H rise, the magnetostatic fields in the posi-

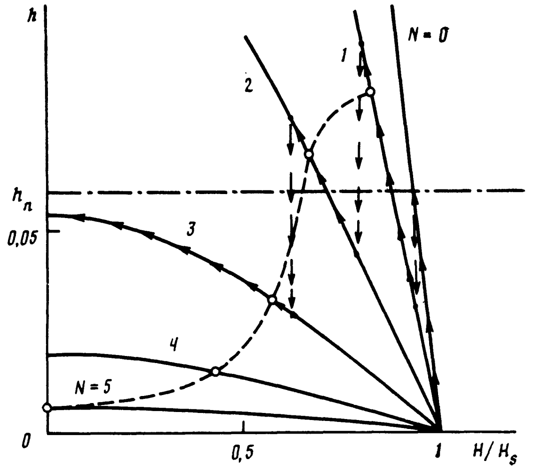

FIG. 12. The total magnetic field at the domain center $h = |H + H_m|/H_s$ ($H_s = 4\pi M$) plotted as a function of the external static field H for different N ($c/a = 2/3$, $b = \infty$). The circles connected by the dashed line correspond to the $N(H)$ relation determined by Eq. (21). The arrows indicate the phase trajectory of the change in $N(H)$ with decreasing magnetic field ($h_n = H_n/H_s$).

tively-oriented domains, whose width grows, decay, while in the negatively-magnetized domains these fields grow. However, the total magnetic field $H + H_m^{\pm}$ decreases with increasing H in each domain and then vanishes at $H = H_s$. The dependence of the total field on H calculated from Eqs. (20a) is shown in Fig. 12 for a number of values $N = a/D$. The internal fields in the domains would change in the same manner if N were conserved while q changed.

However, the change in the period of the domain structure[3,4] causes the change in q to be accompanied by a change in the number of domain walls in the sample. Corresponding Eqs. (8) and (12) can easily be rewritten for finite samples. For domains perpendicular to the plate,

$$N \approx N_0 f_1^{1/2}(q)/f_1^{1/2}(1), \tag{21}$$

for domains in the plane of the plate corresponding to our experimental situation,

$$N \approx N_0 f_3^{3/4}(q)/f_3^{3/4}(1) \approx N_0 q(2-q), \tag{22}$$

where N_0 is the equilibrium value of N for $q = 1$ defined for situations (21) and (22) by different expressions.

Figure 12 provides a sample curve corresponding to Eq. (21) for $N_0 = 5$ (the circles represent its intersection with the curves of the internal field as a function of H for $N =$ const). With no barrier between states with different N an irregular change in N would occur upon remagnetization of the finite sample in fields with values between the corresponding points (these values can be estimated more exactly by comparing the energies of the N^{th} and $(N + 1)^{\text{th}}$ states). Here the same critical fields at which N changes will correspond to a rise and decay in the magnetic field.

However each new domain, i.e., one domain (if the domain was formed along the $x = \pm a/2$ edge of the sample) or two domains immediately (if the new domain was formed within the old domain) of the domain walls will form only if the z-component of the internal field is opposite the magnetization and exceeds the nucleation field H_n:

$$|H + H_m(N,q)| \geqslant H_n\,. \tag{23}$$

In an ideal crystal H_n is determined by the sum of the magnetic anisotropy field and the effective magnetic field related to the formation of the new domain wall; it depends on the shape and critical dimensions of the nucleus. In an actual crystal H_n is largely determined by defects and inhomogeneities. Hence we will treat H_n as a parameter of the theory under development, treating H_n as a certain constant of the sample which in a first approximation is independent of N; the latter is valid for all nucleation models if the dimensions of the seed are much less than the domain dimensions.

The horizontal dot and dash line in Fig. 12 corresponds to $H_n = 0.06H_s$, provided as an example. We consider how remagnetization occurs in this situation. As the external magnetic field decreases from large positive values the first domain wall will not form when $H = H_s$. The field will decrease even further so that the curve corresponding to $N = 0$ intersects the internal field value H_n and the first nucleus is formed. Fields corresponding to transitions from the state $N = 1$ to the state $N = 2$ and then to the state $N = 3$ also shift, although these transitions occur because the transition points in this example lie above $H = H_n$. However, the transition from the state $N = 3$ to the state $N = 4$ is no longer possible since it requires $H < H_n$. Hence $N_{min} = 3$ domains will remain in the sample up through $H = 0$, i.e., the system will be in a metastable state.

Three domains will also be preserved as the field goes from zero to the point of intersection of the curve $N = 3$ with curve (21). This would then be followed by a reduction in the number of domains in accordance with Eq. (21), since the energy barriers associated with nucleation examined above are one-way barriers and do not inhibit the $N \to N - 1$ transitions. However these transitions will be inhibited by purely magnetostatic barriers (Fig. 10, *a*). Indeed, adding the external field interaction energy to the magnetostatic energy (14)

$$F_H = \frac{MH}{a}\left[x_0 + 2\sum_{n=1}^{N}(-1)^n x_n + (-1)^{N+1} x_{N+1}\right] \tag{24}$$

and solving Eq. (17) we obtain the critical values of the external fields H which are independent of the surface energy density of the domain walls γ at which the "excess" domain walls are driven to the edges of the sample (or the two walls forming the "excess" domain collapse).

We can show that these quantities exceed the magnetic field values for which the energies of the N^{th} and $(N - 1)^{\text{th}}$ states given in Fig. 12 become equal. Therefore the $N \to N - 1$ transition at the point where the energies coincide will not occur with a growing magnetic field: the field must grow further to achieve critical values.

Therefore for $H = 0$ there may exist, in addition to the equilibrium state $N = N_0$, an entire series of metastable states:

$$N_0 - 1,\ N_0 - 2, \ldots,\ N_{min}.$$

However, like N_0 all these states remain inaccessible when quasistatic remagnetization occurs: For $H = 0$ the system will always be in the state $N = N_{min}$ which is given by condition (23). [We have 0.5 Oe $\lesssim H_n \lesssim 1$ Oe for sample

No. 1 from (23) and (16) if we assume that condition (23) is satisfied at the domain center for nucleation of the new domain.]

Such a situation can be modeled by adding a certain periodic potential to total energy (3); for example, a simple potential of the type

$$F_\varepsilon=\varepsilon \sin^2[\pi(N-N_0)]. \qquad (25)$$

Indeed minimizing the total energy $F(N)$ with respect to N we obtain for this model an entire series of metastable states between $N=N_{min}$ and $N=N_0$ [there is no minimum on the $F(N)$ curve for $N<N_{min}$].

A comparison with experiment makes it possible to estimate the parameter ε which is found to be highly dependent on the dimensional ratio of the sample a/c. As a/c goes from 0.5 to 4 the value of ε decreases from 0.8 γ/a to 0.4 γ/a.

The periodic potential model (25) is simple and useful for a qualitative analysis of remagnetization if it is remembered that this model is both conditional and approximate. Therefore it does not reflect the way the size of the potential barriers changes as the magnetic field rises and falls since it is determined by different physical mechanisms, etc.

5. RESONANCE OF THE DOMAIN STRUCTURE IN METASTABLE STATES

We can easily show that the magnetic resonance frequency of the antiphase oscillations of the domain walls of periodic structure is determined by the expression

$$\omega_a^2=\frac{D}{m}\frac{\partial^2F(D,\eta)}{\partial\eta^2}\bigg|_{\eta=0}=\frac{4}{Dm}\frac{\partial^2F(D,q)}{\partial q^2}, \qquad (26)$$

where m is the surface density of the domain wall mass; η are small deviations of the domain walls from equilibrium for the given H corresponding to specific values of D and q.

For the case $b\to\infty$ Liberts[8] has derived an expression for ω_a^2 that precisely accounts for the multipole part of magnetostatic energy (7) with a random q:

$$\omega_a^2=\frac{16M^2}{cm}\ln\frac{\operatorname{ch}(\pi c/D)-\cos(\pi q)}{2\sin^2(\pi q/2)}. \qquad (27)$$

We have obtained an expression for ω_a^2, precisely accounting for the magnetostatic energy of the general type (6):

$$\omega_a^2=\frac{4M^2}{m}\left[\frac{N_z}{D}+\frac{8}{c}\psi(D,q)\right],$$

where

$$N_z=8\left[\operatorname{arctg}\frac{b}{c}-\frac{c}{4b}\ln\left(1+\frac{b^2}{c^2}\right)+\frac{b}{4c}\ln\left(1+\frac{c^2}{b^2}\right)\right],$$

$$\psi(D,q)=-\frac{cN_z}{8D}+\sum_{n=-\infty}^{+\infty}\left\{\operatorname{arsh}\frac{b}{D|q+2n|}-\operatorname{arsh}\frac{b}{(D^2(q+2n)^2+c^2)^{1/2}}+\frac{1}{b}[(D^2(q+2n)^2+b^2+c^2)^{1/2}-(D^2(q+2n)^2+b^2)^{1/2}-(D^2(q+2n)^2+c^2)^{1/2}+D|q+2n|]\right\}.$$

Expression (28) becomes (27) as $b\to\infty$. Both of these expressions have a minimum in q at $q=1$ and grow without limit as $q\to0$. Both expressions for sufficiently large values of N approach an asymptotic form

$$\omega_a^2=-\omega_m^2+4N_zM^2N/am, \qquad (29)$$

where the second term of the sum corresponds to the squared resonance frequency, neglecting the multipole terms in the magnetostatic energy. The role of the latter with large N reduces to formation of a negative addition to the squared frequency. The natural frequencies of the oscillations of the domain walls in the other limiting case of small N were investigated in Ref. 7.

The linear resonance of the antiphase oscillations of the domain walls was observed experimentally in all metastable states. The dependence of the squared frequency on N (see Fig. 4) is approximately described by the expression

$$\omega_a^2=\omega_1^2+\omega_2^2N, \qquad (30)$$

where ω_1 and ω_2 are parameters independent of N. The second term of the sum here corresponds to the second term of the theoretical equation (29); the first terms in (29) and (30) have both different values and signs. Therefore the experimentally observed shift of the squared frequency, which is dependent on N, has a nonmagnetostatic origin.

The possibility of experimental observation of magnetic resonance with various values of N indicates the relative stability of the metastable states: they do not break down under a radio-frequency field if its amplitude is less than a certain frequency-dependent critical value.

CONCLUSION

The possibility of metastable states with a differing number of domain walls N was discovered experimentally in a weak $FeBO_3$ ferromagnet. Analysis revealed that the states are rather resistant to the effect of both quasistatic and radio frequency fields if the frequencies and amplitudes of the latter are less than certain critical values that are different for states with different N. Applying radio frequency fields with frequencies and amplitudes exceeding the critical values causes transitions between the metastable states which serve to increase the number N.

A theoretical explanation for the existence and the properties of such metastable states is offered. As initially demonstrated in Refs. 3, 4 the remagnetization process will be accompanied not only by a change in the volume relation of the domains magnetized along and across the fields, but also by changes in the period of the domain structure. For the finite samples this corresponds to an increase in the number of domain walls N as the field H decreases and a reduction in N with increasing H. However, the creation of each new domain wall involves overcoming an energy barrier: The internal field (the sum of the magnetostatic and external fields) in the domain directed opposite the magnetization will exceed the nucleation field H_n. The internal field magnitude decays with increasing number N (with diminishing H) and may drop below H_n until the system reaches an equilibrium domain number N_0. In this case the $N\to N+1$ processes no longer occur as H decreases further and through $H=0$ the system will remain in the metastable state $N=N_{min}<N_0$. There is a whole set of metastable states between N_{min} and N_0 that remain inaccessible upon quasistatic remagnetization (as in the case $N=N_0$).

Energy barriers also inhibit $N\to N-1$ processes with increasing magnetic field, although these energy barriers are of a different physical nature: extending the domain walls outside the sample or the convergence of such walls, which is required for annihilation, serve to increase the magnetostatic energy.

The existence of metastable states with different values of N has made it possible for the first time to experimentally investigate the dependence of the resonance frequency of the domain walls on N in the same sample. This relation corresponds to the theoretical relation. There is, however, an additional N-dependent contribution to the frequency whose origin requires further analysis.

The theoretical analysis of magnetostatic energy and the remagnetization processes of samples with few domain walls, which was carried out here in order to identify the nature of the metastable states, allowed us to obtain a number of unusual additional effects characteristic of this situation. These include asymmetry of the magnetization curve, a nonzero average magnetization for an odd number of domains in both the metastable states and in the state corresponding to the global energy minimum, and the requirement that the width of domains along the edge of the sample for $H = 0$ for sufficiently large values of N must be approximately half the width of the remaining domains.

[1)] When the domains form that shut off the magnetic flux and cause F_m to vanish, N_0 is determined by the minimum of the sum of F_γ and the energy associated with the existence of the closure domains. This case is not examined here.

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