Relaxed turbulent plasma state and anomalous transport processes

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Self-consistent kinetic and hydrodynamic equations are obtained which describe plasma dynamics under conditions such that magnetohydrodynamic turbulence is excited. The concept of the relaxed state of a turbulent plasma is introduced. It is shown that the magnetic field and current profiles in the relaxed state depend weakly on the form of the correlations of the magnetic field fluctuations. The resulting theory is in good agreement with experimental data taken from reversed-field pinches in the quasisteady discharge regime.

I. INTRODUCTION

The problem of anomalous transport in tokamaks is of fundamental importance in the controlled thermonuclear fusion program. It has been established that anomalous transport can be responsible for the transition to plasma turbulence. The difficulties in studying plasma turbulence arise, in part, because even when the amplitudes of the stochastic fields are quite small they can drastically change the macroscopic plasma state.[1]

It is significant that, despite the presence of turbulence, the plasma in a tokamak may be in a quiescent and, in a certain sense, stable state. For example, sharp artificially imposed density and temperature disturbances quickly die away in the steady state. It often appears as though the plasma has a preferred quiescent current profile to which it rapidly relaxes when perturbed.[2,3] Under certain specific conditions this quiescent plasma becomes unstable and a substantial amount of plasma is ejected, or relaxation oscillations, e.g., the so-called disruptive (sawtooth) oscillations,[4] are excited.

A similar situation is observed in other toroidal confinement systems, in particular the reversed-field pinch (RFP). In a tokamak the longitudinal B_z field (i.e., the field directed along the toroidal axis) produced by external currents in an order of magnitude larger than B_θ, the field produced by the current in the plasma. In the RFP B_z and B_θ are comparable. The B_z component decreases rapidly as a function of the distance r from the toroidal axis and vanishes at some point r_c inside the plasma. Beyond this point the sign of B_z is reversed, which is what gives the device its name.

Under RFP conditions the Kruskal–Shafranov[5] stability condition is not satisfied, in consequence of which MHD oscillations are excited. The amplitude of MHD waves in the steady state is

$$\frac{b}{B}\sim(1\div3)\cdot10^{-2},\quad B=(B_\theta^2+B_z^2)^{1/2}. \tag{1}$$

In a tokamak where the Kruskal–Shafranov condition holds, the ratio b/B is at least a factor of ten smaller, which makes it harder to measure. The main contribution to the anomalous transport here may come from electrostatic drift waves, and the MHD activity may be less important. But as the plasma pressure increases (more precisely, as $\beta=8\pi nT_e/B^2$ increases) the role of MHD disturbances grows.[6] Thus information about the level of turbulence in the plasma has a more precise character in an RFP. In what follows it is therefore more convenient for us to analyze this type of device, although the basic ideas and methods are applicable in the general case.

Taylor[7] first introduced the idea of the relaxed state of an RFP. He argued that, since the plasma pressure is small ($\beta\ll1$), the magnetic field in the system must be force-free ($\mathbf{J}\|\mathbf{B}$), i.e., it must satisfy the equations

$$\operatorname{curl}\mathbf{B}=\mu\mathbf{B},\quad \operatorname{div}\mathbf{B}=0. \tag{2}$$

Here the quantity μ in general depends on the magnetic surface, $\mu=\mu(r)$. Taylor's basic assumption has a phenomenological character: as a result of turbulent mixing, the system relaxes to a state with

$$\mu=\mu_0=\text{const} \tag{3}$$

due to conservation of the total magnetic field helicity.

The introduction of the idea of the relaxed state (3) proved to be extremely fruitful. Equations (2) and (3), together with the natural boundary conditions at the axis ($r=0$) and on the metal liner ($r=a$),

$$\left.\frac{\partial B}{\partial r}\right|_{r=0}=0,\quad B_r|_{r=a}=0, \tag{4}$$

and the conditions for conservation of the total current J and magnetic flux Φ,

$$J=2\pi\int_0^a J_\| h_z r\,dr, \tag{5}$$

$$\Phi=2\pi\int_0^d B_z r\,dr \tag{6}$$

completely determine the structure of the magnetic field and the current in the discharge, as well as their dependence on external parameters. In fact, it is easy to see that the solution of Eq. (2) in cylindrical geometry takes the form

$$B_z=B(0)J_0(\mu_0 r),\quad B_r=0,\qquad B_\theta=B(0)J_1(\mu_0 r), \tag{7}$$

where J_0 and J_1 are Bessel functions. The field B_z given by this solution is found to be not only in qualitative agreement, but in fair quantitative agreement with the experimental data (Fig. 1a).

On the other hand, the quantity μ itself, measured directly, turns out to vary over the cross section (Fig. 1b).[8] The structure as a function of the basic discharge parameters

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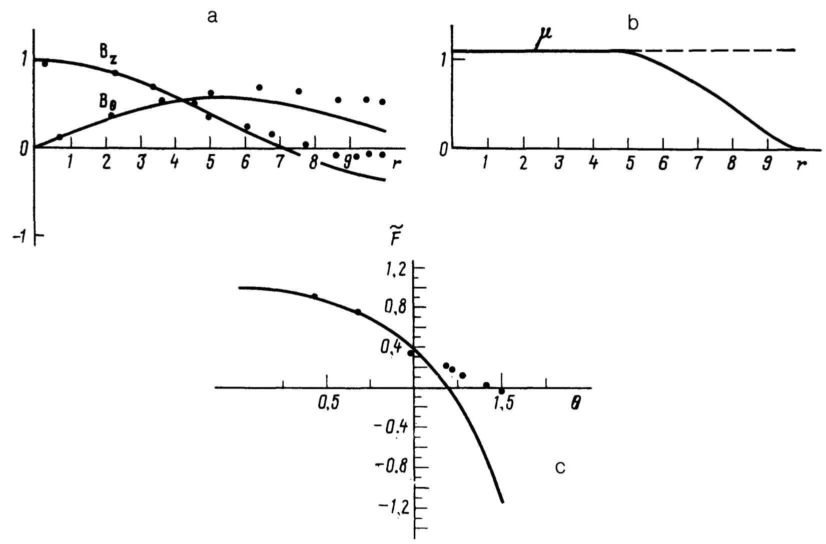

FIG. 1. (a) The functions $B_z(r)$ and $B_\theta(r)$ given by Eq. (7). The points represent experimental data. (b) Dependence of $\mu(r)$ found experimentally (solid curve) and from Eq. (7) (dashed curve). (c) $\tilde{F}$–θ plot obtained from Eq. (7). The points represent experimental data.

is observed to deviate significantly from the theoretical predictions (Fig. 1c). Just as importantly, the phenomenological theory, although it determines the current and magnetic field structure, says nothing about the transport processes, the density and temperature profiles, or the properties of the waves that are excited.

At the same time, experiment and numerical simulation convincingly demonstrate that the relaxed state is special: the plasma rapidly relaxes to it when the discharge is turned on or when pronounced disturbances are artificially introduced.[9,10] During relaxation the plasma wave energy usually grows; in the relaxed state it is quiescent and can be shown to assume a minimum value.[7]

Thus condition (3) is essentially violated in the relaxed state. What, then, is this state? The present paper is intended to study this question. We make no *ad hoc* phenomenological assumptions. The general equations are expanded in the amplitude of the stochastic oscillations (1) and in a small parameter related to the typical energy confinement time, which falls out automatically.

As will be seen, the relaxed state is a steady self-consistent state of the plasma, in which, thanks to the anomalous transport processes, the properties of the excited modes are adjusted to the structure of the current and the magnetic field, and also to the density and temperature profiles. It is very important that the main role here is played by the anomalous transverse spreading of current induced by the turbulence, which leads to the establishment of the current and magnetic field profiles in the discharge. That the specific form of the profile turns out to depend relatively weakly on the details of the turbulence is surprising.

In Sec. 2 we derive the general kinetic equation for electrons and ions under conditions such that MHD turbulence is excited in the plasma. In Sec. 3 we discuss the properties of the correlation integral which determines the wave–particle interaction in an RFP. In Sec. 4 we derive the full system of hydrodynamic equations describing an RFP plasma, including the effect of MHD turbulence on transport processes. In Sec. 5 we consider the resulting nonlinear quasistationary plasma turbulence state; we refer to it as relaxed turbulence. The current and magnetic field profiles and their dependence on the form of the correlation function are studied numerically in the relaxed state. The rapid convergence of the iterative solution of the nonlinear equations is demonstrated. In Sec. 6 we consider anomalous energy losses. In the concluding Sec. 7 we derive equations for MHD oscillations in a turbulent plasma. This completes the formulation of a closed self-consistent set of equations describing a relaxed turbulent plasma state.

2. THE KINETIC EQUATION

Let us consider a plasma in a magnetic field **B** consisting of regular and fluctuating components, $\mathbf{B}_0$ and **b**:

$$\mathbf{B}=\mathbf{B}_0+\mathbf{b}, \quad |\mathbf{b}|\ll|\mathbf{B}_0|. \tag{8}$$

The basic quantities characterizing the fluctuations are the correlation length L_c and the correlation time τ_c. In accord with the data, we will treat these as being large in comparison with the gyroradii and inverse gyrofrequencies of the ions and electrons.

Hence the kinetic equations describing the motion of electrons and ions in the field **B** can be written in the drift approximation as[11]

$$\frac{\partial f}{\partial t}+\mathbf{V}\frac{\partial f}{\partial \mathbf{r}}+\frac{\partial u}{\partial t}\frac{\partial f}{\partial u}+\frac{\partial \mu}{\partial t}\frac{\partial f}{\partial \mu}=St(f), \tag{9}$$

where $f=f(u,\mu,r,t)$ is the particle distribution function, $\mathbf{V}=u\mathbf{h}+\mathbf{V}_{dr}$ is the particle velocity, μ is the adiabatic invariant, $St(f)$ is the collision integral, and $\mathbf{V}_{dr}$ is the particle drift velocity in the electric and magnetic fields **E** and **B**:

$$\mathbf{V}_{dr}=\frac{c}{B^2}[\mathbf{EB}]+\frac{mc\tilde{\mu}}{2e}\left[\mathbf{h}\frac{\nabla B}{B}\right]$$
$$+\frac{\tilde{\mu}mc}{2e}(\mathbf{h}\,\mathrm{rot}\,\mathbf{h})\mathbf{h}+\frac{mcu^2}{eB}(\mathbf{h}(\mathbf{h}\nabla)\mathbf{h}),$$
$$\mathbf{h}=\frac{\mathbf{B}}{B}, \quad \tilde{\mu}=\frac{u_\perp^2}{B},$$
$$\mu=\frac{u_\perp^2}{B_0}, \quad \frac{d\tilde{\mu}}{dt}=0, \quad \frac{du}{dt}=\frac{e\mathbf{EV}}{mu}-\frac{\tilde{\mu}}{2u}\mathbf{V}\frac{\partial \mathbf{B}}{\partial r}. \tag{10}$$

Taking into account (8), we can write the distribution function f in Eq. (9) in the form

$$f=f_0+\delta f,\quad |\delta f|\ll|f_0|$$

and, following our previous work,[12,13] average (9) and (10) over an ensemble of random values of **b**. We find

$$\frac{\partial f_0}{\partial t}+\langle \mathbf{V}\rangle\frac{\partial f_0}{\partial \mathbf{r}}+\left\langle\frac{du}{dt}\right\rangle\frac{\partial f_0}{\partial u}+\left\langle\frac{d\mu}{dt}\right\rangle\frac{\partial f_0}{\partial \mu}=\langle St(f)\rangle-\langle\hat{H}\delta f\rangle. \tag{11}$$

Here the operator $\hat{H}$ has the form

$$\hat{H}=\delta\mathbf{V}\frac{\partial}{\partial\mathbf{r}}+\delta\left(\frac{d\mu}{dt}\right)\frac{\partial}{\partial\mu}+\delta\left(\frac{du}{dt}\right)\frac{\partial}{\partial u},$$

and the quantities $\delta\mathbf{V}$, $\delta(d\mu/dt)$, and $\delta(du/dt)$ are linear in the $\mathbf{b}/B$ corrections found from expanding Eqs. (10). Subtracting (11) from (9) we find an equation for the correction δf to within terms of order b^2/B_0^2:

$$\frac{\partial}{\partial t}\delta f+\langle\mathbf{V}\rangle\frac{\partial}{\partial\mathbf{r}}\delta f+u\frac{b_r}{B_0}\frac{\partial\delta f}{\partial r}+\left\langle\frac{du}{dt}\right\rangle\frac{\partial}{\partial u}\delta f$$
$$=-\hat{H}f_0+St(f)-\langle St(f)\rangle. \tag{12}$$

Equation (12), in addition to the linear terms, has one nonlinear term:

$$u\frac{b_r}{B_0}\frac{\partial}{\partial r}\delta f.$$

This nonlinear correction, as we will see shortly, becomes important close to resonant surfaces.

In the cases of interest the ion and electron mean free paths $l_{e,i}$ are considerably longer than the longitudinal (parallel to the magnetic field) correlation length $L_{\parallel c}$. Thus to lowest order we can neglect collisions in the right-hand side of Eq. (12). Hence solving Eq. (12) and using the result in (11), we finally get (from here on we omit the subscript 0 from average quantities)

$$\frac{\partial f}{\partial t}+\mathbf{V}\frac{\partial f}{\partial\mathbf{r}}+\left\langle\frac{du}{dt}\right\rangle\frac{\partial f}{\partial u}+\left\langle\frac{d\mu}{dt}\right\rangle\frac{\partial f}{\partial\mu}$$
$$=St(f)+\left\langle\hat{H}\int_0^{+\infty}d\tau\int_{-\infty}^{+\infty}G(\Gamma/\Gamma')\hat{H}'f'\,d\Gamma'\right\rangle. \tag{13}$$

Here $G(\Gamma/\Gamma')$ is the Green's function for Eq. (12), where Γ is the phase-space volume. Equation (13) is similar to an equation obtained in Refs. 12 and 13. Note, however, that in deriving it we have not assumed that the function f varies slowly over the correlation length L_c.

In order to simplify the rest of the analysis, we limit ourselves to cylindrical geometry, which corresponds to small-aspect-ratio tori. Then the Green's function takes the form

$$G(\Gamma/\Gamma')=\theta(\tau)\delta\left(\theta-\theta'-\frac{h_\theta}{r}u\tau\right)\delta(z-z'-h_z u\tau)$$
$$\times\delta\left(r-r'-\frac{b_{m,n}(r^*_{m,n})}{B}\Delta_{m,n}u\tau\right)\delta(\mu-\mu')\delta(u-u'). \tag{14}$$

Here decomposing the field $b_r(\tau,\theta,z)$ yields the Fourier harmonics $b_{m,n}$; $m,n=0,\pm1,\pm2,\ldots$ are the indices of the toroidal modes; r^*_{mn} $(=r^*_{m,n})$ is the radius of the resonant surface, defined by the condition

$$\frac{m}{r_{m,n}^*}B_\theta(r_{mn}^*)+\frac{n}{R}B_z(r_{mn}^*)=0,$$

where R is the major radius; and $\Delta_{m,n}$ is a number that equals unity for $r=r^*_{mn}$ and vanishes otherwise.

Assuming that the function f is independent of z and θ and that the magnetic field fluctuations b are homogeneous and isotropic in θ, z, and t, we finally get the following equation:

$$\frac{\partial f}{\partial t}+V_{dr}\frac{\partial f}{\partial r}+\left\langle\frac{du}{dt}\right\rangle\frac{\partial f}{\partial u}=I_{fl}(f)+St(f),$$
$$I_{fl}(f)=\frac{u}{B}\frac{1}{r}\frac{\partial}{\partial r}\{rK\}+\frac{\partial}{\partial u}\left\{\left(\frac{e}{m}\frac{E_r}{B}-\frac{\mu}{2}\frac{1}{B}\frac{\partial B}{\partial r}\right)K\right\}, \tag{15}$$
$$K=\frac{u}{B}\frac{F}{|u|}\frac{\partial f}{\partial r}+\frac{\partial f}{\partial u}\left[\left(\frac{e}{m}\frac{E_r}{B}-\frac{\mu}{2}\frac{1}{B}\frac{\partial B}{\partial r}\right)\frac{F}{|u|}\right],$$
$$F=\int_0^{+\infty}dL\langle b_r b_r'\rangle;\quad dL=u\,d\tau,$$
$$b_r'=b_r\left(\theta-\frac{h_\theta L}{r},\,z-h_z L,\,r-\frac{b_{mn}(r^*)}{B}\Delta_{mn}L\right).$$

In writing the correlation F in (15) we assumed $L_{\parallel c}<V_{T_{e,i}}\tau_c$. This relation is well satisfied in discharges at sufficiently high temperatures, in which case the correlation F is independent of the particle velocity u.

From (15) we see that the integral I_{fl} describing scattering of particles by fluctuations is a differential operator, in contrast to the integro-differential operator (13). This property of I_{fl} arises because the particles are magnetized; their gyroradii are small, so that the Green's function G away from the resonant surfaces is localized, $G\sim\delta(r-r')$.

In order to investigate the solutions of this kinetic equation, we have to know the form of the correlation integral F. This is discussed in the next section.

3. THE CORRELATION INTEGRAL

It follows from the kinetic equation (15) that in cylindrical geometry the plasma dynamics is described by a single correlation function $F(r)$, which is determined by the fluctuations b_r of the magnetic field. The approximation of cylindrical geometry is valid for tori of small aspect ratio, for which the minor radius a and the major radius R satisfy $a\ll R$. Then solutions that are periodic in z can be Fourier-expanded in θ and z:

$$b_r=\sum_{m,n}b_r^{m,n}(r)\exp(im\theta+inz/R). \tag{16}$$

From (16) we see that the period in z equals $2\pi R$. Substituting (16) in (15) after averaging in θ and z, we find the correlation $F(r)$:

$$F(r)=\sum_{m,n}\int_0^\infty b_r^{m,n}(r)b_r^{*m,n}\left(r-\frac{b_{mn}}{B}L\right)$$
$$\times\exp\left\{iL\left(\frac{m}{r}h_\theta+\frac{n}{R}h_z\right)\right\}dL. \tag{17}$$

Equation (17) shows it is essential to include the nonlinear term $b_{mn}L/B$, since otherwise the correlation function consists solely of a sum of δ-functions at the resonant frequencies:

$$F(r)=\pi\sum_{m,n}|b_r^{m,n}(r)|^2\delta\left(\frac{m}{r}h_\theta+\frac{n}{R}h_z\right). \qquad (18)$$

The actual relation (17) accounts for resonance broadening due to magnetic field fluctuations. Thus each resonance (m,n) broadens by an amount that scales with the amplitude b_{mn} of the magnetic field fluctuation of that mode.

Measurements of large-scale fluctuations in an RFP reveal that modes with $m=1$ and $n=5$–10, with a spectral width $\delta n\sim 5$, are strongly excited; modes with $m=0$ and $m=2$ are excited significantly less, and other modes are excited practically not at all.[14,15] When the fluctuation level b_{mn} is small, (17) and (18) show that the correlation F is localized in narrow regions close to the resonant surfaces. In this case the contribution of the fluctuations to transport processes is less essential. Let us now estimate the fluctuation amplitude at which resonances effectively overlap over the whole extent of the discharge. For this we assume that the scale on which $b_{mn}(r)$ varies is large compared with the width of a resonance. Assuming further that, like r, the quantity $r-b_{mn}L/B$ varies in the interval $[0,a]$, by integrating (17) we find

$$F(r)=\sum_{m,n}|b_r^{m,n}(r)|^2\frac{\sin\left\{\frac{a}{b_{mn}}(mB_\theta/r+nB_z/R)\right\}}{mh_\theta/r+nh_z/R}. \qquad (19)$$

From this we see that the characteristic width δr of a resonance is

$$\delta r_R=\frac{\pi b_{m,n}}{(d/dr_{mn})(mB_\theta+nr_{mn}B_z/R)}\sim\pi\frac{b_{mn}}{B}r_{mn}.$$

The separation between resonances is easily estimated using the relation

$$\delta r_{mn}=\left|\frac{B_r(r_{mn})}{R(d/dr_{mn})(B_\theta/r_{mn}+nB_z/R)}\right|\sim\frac{r_{mn}^2}{R}.$$

Hence the fluctuation amplitude for which this separation becomes comparable with the width of a resonance is

$$\frac{(|b_{mn}|^2)^{1/2}}{B}\geqslant\frac{r_{mn}}{\pi R}. \qquad (20)$$

For typical pinch parameters we have $r_{mn}\leqslant 0.3a$ (in the principal mode) and $a/R\sim 0.1$, from which we get $b/B\sim 10^{-2}$, which agrees with the observed fluctuation amplitude in an RFP.[14,15] Hence magnetic field fluctuations in RFP conditions can give rise to effective heat and particle transport throughout the whole discharge. Below we will assume that the condition (20) holds, i.e., we will suppose that the correlation $F(r)$ is a smooth function of r. The function $F(r)$ has several general properties that follow from the boundary conditions on the fluctuations b_r. In particular, since the discharge is enclosed in a conducting chamber, at $r=a$ we have $b_r=0$. Accordingly as $r\to a$ we find

$$F(r)_{r\to a}=\frac{1}{2!}\left(\frac{d^2F}{dr^2}\right)_{r=a}(a-r)^2. \qquad (21)$$

4. HYDRODYNAMIC EQUATIONS

Analysis of the kinetic equation (15) for real systems is extremely complicated. It is therefore natural to go over to a hydrodynamic description of the plasma. This is valid if the plasma variables do not change significantly over an electron collision time:

$$\frac{dT_e}{dt}<\nu_{ei}T_e,\quad \frac{du}{dt}<\nu_{ei}u,$$

where ν_{ei} is the electron collision frequency and T_e and u are the electron temperature and hydrodynamic velocity. Superthermal particles in a fluctuating magnetic field can also change the distribution functions considerably.[12,13] For this reason it is necessary in the present problem that the transverse electron temperature gradient dT_e/dr not be too large, or the hydrodynamic approximation will fail. In what follows we assume that these conditions are met.

Substituting the Maxwell distribution f_M in (15) and introducing the hydrodynamic velocity

$$V_\parallel=\frac{(m_eu_e+m_iu_i)}{(m_e+m_i)}$$

and the electric current $J=ne(v_i-v_e)$ parallel to the magnetic field, after taking moments we arrive at the following system of hydrodynamic equations for the plasma number density n, the average velocity $V_\parallel$, the current $J_\parallel$, and the electron and ion temperatures T_e and T_i:

$$\frac{\partial n}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rnV_r)=\frac{1}{r}\frac{\partial}{\partial r}r\left\{D_1\frac{\partial n}{\partial r}+D_2\frac{\partial T_e}{\partial r}+D_3\frac{\partial T_i}{\partial r}\right\}+Q_n,$$

$$\frac{\partial J_\parallel}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(rV_rJ_\parallel)=\frac{1}{r}\frac{\partial}{\partial r}(rR_1)+R_2-\nu_{ei}J_\parallel+\frac{e^2n}{m_e}E_\parallel,$$

$$\frac{\partial}{\partial t}(V_\parallel n)+\frac{1}{r}\frac{\partial}{\partial r}(rnV_rV_\parallel)=\frac{1}{r}\frac{\partial}{\partial r}(r\Pi_1)+\Pi_2,$$

$$\frac{3}{2}\frac{\partial}{\partial t}(nT_e)+\frac{3}{2}\frac{1}{r}\frac{\partial}{\partial r}(rV_rnT_e)=\frac{2}{r}\frac{\partial}{\partial r}r\left\{T_eJ_d+\frac{F}{\pi^{1/2}B^2}\left(\frac{2T_e}{m_e}\right)^{1/2}n\frac{\partial T_e}{\partial r}\right\}$$

$$+2eE_rJ_d+J_\parallel E_\parallel+\frac{1}{r}\frac{\partial}{\partial r}\left(r\varkappa_\perp^e\frac{\partial T_e}{\partial r}\right)+Q_e,$$

$$\frac{3}{2}\frac{\partial}{\partial t}(nT_i)+\frac{3}{2}\frac{1}{r}\frac{\partial}{\partial r}(rV_rnT_i)$$

$$=\frac{2}{r}\frac{\partial}{\partial r}\left\{T_iJ_d+\frac{F}{\pi^{1/2}B^2}\left(\frac{2T_e}{m_i}\right)^{1/2}n\frac{\partial T_i}{\partial r}\right\}$$

$$-2eE_rJ_d+\frac{1}{r}\frac{\partial}{\partial r}\left(r\varkappa_\perp^i\frac{\partial T_i}{\partial r}\right)+Q_i. \qquad (22)$$

In Eqs. (22) we have written

$$J_d=D_1\frac{\partial n}{\partial r}+D_2\frac{\partial T_e}{\partial r}+D_3\frac{\partial T_i}{\partial r}$$

for the particle diffusive flux; the coefficients D_1, D_2, and D_3 are defined by the relations

$$D_1=\frac{D_iD_e(T_i+T_e)}{D_eT_i+D_iT_e},\quad D_2=\frac{D_e^TD_i}{D_e\frac{T}{T_e}+D_i},\quad D_3=\frac{D_i^TD_e}{D_e+D_i\frac{T_e}{T_i}}$$

where

$$D_{i,e}=\frac{F}{\pi^{1/2}B^2}\left(\frac{2T_{ie}}{m_{i,e}}\right)^{1/2}+D_{i,e}^{\rm cl},$$

$$D_{i,e}^{T}=\frac{F}{\pi^{1/2}B^2}\frac{n}{2T_{i,e}}\left(\frac{2T_{i,e}}{m_{i,e}}\right)^{1/2}+D_{i,e}^{T^{\rm cl}}.$$

Here $D_{i,e}^{\rm cl}$ and $D_{i,e}^{T^{\rm cl}}$ are the classical particle and thermal diffusivities, respectively, and $\varkappa_\perp^{i,e}$ are the classical transverse ion and electron thermal conductivities. The expressions R_i and Π_i result from electron and ion scattering by fluctuations. This scattering gives rise to current and velocity diffusion:

$$R_1=\frac{F}{\pi^{1/2}B^2}\left\{2\left(\frac{2T_e}{m_e}\right)^{1/2}\frac{\partial J_\parallel}{\partial r}+\frac{2e}{m_e}E_r\left(\frac{m_e}{2T_e}\right)^{1/2}J_\parallel-\frac{1}{B}\frac{\partial B}{\partial r}\left(\frac{2T_e}{m_e}\right)^{1/2}J_\parallel\right\},$$

$$\Pi_1=\frac{F}{\pi^{1/2}B^2}\left\{2\frac{\partial}{\partial r}\left[n\left(\frac{2T_i}{m_i}\right)^{1/2}V_\parallel\right]-\frac{2e}{m_i}E_r,nV_\parallel\left(\frac{m_i}{2T_i}\right)^{1/2}-\frac{1}{B}\frac{\partial}{\partial r}B\left(\frac{2T_i}{m_i}\right)^{1/2}nV_\parallel\right\}.$$

A frictional force also arises as a result of the interaction between the charged particles and the fluctuations. it is described by the terms R_2 for the electrons and Π_2 for the ions:

$$R_2=\frac{F}{\pi^{1/2}B^2}\left\{\frac{2e}{m_e}E_r\frac{\partial}{\partial r}\left[\left(\frac{m_e}{2T_e}\right)^{1/2}J_\parallel\right]+\frac{1}{B}\frac{\partial B}{\partial r}\frac{\partial}{\partial r}\left[\left(\frac{2T_e}{m_e}\right)^{1/2}J_\parallel\right]+4\left(\frac{e}{m_e}E_r\right)^2\left(\frac{m_e}{2T_e}\right)^{3/2}J_\parallel+\frac{4e}{m_e}E_r\frac{\partial B}{\partial r}\frac{1}{B}\left(\frac{m_e}{2T_e}\right)^{1/2}J_\parallel+\frac{3}{2}\left(\frac{1}{B}\frac{\partial B}{\partial r}\right)^2\left(\frac{m_e}{2T_e}\right)^{-1/2}J_\parallel,\right.$$

$$\Pi_2=-\frac{F}{\pi^{1/2}B^2}\left\{\frac{2e}{m_i}E_r\frac{\partial}{\partial r}\left[nV_\parallel\left(\frac{m_i}{2T_i}\right)^{1/2}\right]-\frac{1}{B}\times\frac{\partial B}{\partial r}\frac{\partial}{\partial r}\left[\left(\frac{2T_i}{m_i}\right)^{1/2}nV_\parallel\right]-4\left(\frac{e}{m_i}E_r\right)^2nV_\parallel\left(\frac{m_i}{2T_i}\right)^{3/2}+4\frac{e}{m_i}E_r\frac{1}{B}\frac{\partial B}{\partial r}nV_\parallel\left(\frac{m_i}{2T_i}\right)^{1/2}-\frac{3}{2}nV_\parallel\left(\frac{2T_i}{m_i}\right)^{1/2}\left(\frac{1}{B}\frac{\partial B}{\partial r}\right)^2\right\}.$$

An electric field E_r appears in the system as a result of the difference in the electron and ion transverse diffusion coefficients. From the condition for quasineutrality ($\nabla\cdot\mathbf{J}=0$) we find

$$E_r=\frac{D_i^T\partial T_i/\partial r-D_e\partial T_e/\partial r+(D_i-D_e)\,\partial n/\partial r}{D_e/T_e+D_i/T_i}.$$

In Eqs. (22) V_r is the plasma drift velocity across the magnetic field, defined in Eq. (10). The equations describing heat and particle transport include external heat sources Q_e, Q_i and an ionization rate Q_n. Equations (22) must be supplemented by the Maxwell equations

$$\operatorname{div}\mathbf{B}=0,\quad \operatorname{rot}\mathbf{B}=\frac{4\pi}{c}\mathbf{J},$$

$$\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}=-\operatorname{rot}\mathbf{E},\quad \mathbf{E}=\frac{1}{c}[\mathbf{VB}]+E_\parallel\mathbf{h}. \tag{23}$$

The system of equations consisting of (22) and (23) describes the pinch dynamics in the presence of magnetic field fluctuations.

5. THE RELAXED STATE: CURRENT AND MAGNETIC FIELD PROFILES

In the preceding section we derived a system of hydrodynamic equations (22) and (23) describing the plasma dynamics in the presence of magnetic fluctuations. Under RFP conditions right after the voltage is switched on, transient processes occur, following which a quasisteady state (which we call relaxed) can be established, where the current and magnetic field change smoothly during the lifetime of the discharge. Under these conditions the process can be considered stationary and the total current and magnetic flux fixed [cf. Eqs. (5) and (6)]. Hence in the limit $\beta\ll1$ Eqs. (22) and (23) for the magnetic field B and the current $J_\parallel$ assume the following form:

$$\hat{L}J_\parallel=0,\quad \operatorname{rot}\mathbf{B}=\frac{4\pi}{c}J_\parallel\mathbf{h},\qquad \operatorname{div}\mathbf{B}=0, \tag{24}$$

where the differential operator $\hat{L}$ is defined by the following expression:

$$\hat{L}J_\parallel=\frac{1}{r}\frac{\partial}{\partial r}r\left\{\left[2\frac{\partial J_\parallel}{\partial r}\left(\frac{2T_e}{m_e}\right)^{1/2}+\frac{2e}{m_e}E_r\left(\frac{m_e}{2T_e}\right)^{1/2}J_\parallel-\frac{1}{B}\frac{\partial B}{\partial r}\left(\frac{2T_e}{m_e}\right)^{1/2}J_\parallel\right]\frac{F}{\pi^{1/2}B^2}\right\}+\frac{F}{\pi^{1/2}B^2}\left\{\frac{2e}{m_e}E_r\frac{\partial}{\partial r}\left[\left(\frac{m_e}{2T_e}\right)^{1/2}J_\parallel\right]+\frac{1}{B}\frac{\partial B}{\partial r}\frac{\partial}{\partial r}\left[\left(\frac{2T_e}{m_e}\right)^{1/2}J_\parallel\right]+4\left(\frac{e}{m_e}E_r\right)^2\left(\frac{m_e}{2T_e}\right)^{3/2}J_\parallel+\frac{4eE_r}{m_e}\frac{1}{B}\frac{\partial B}{\partial r}\left(\frac{m_e}{2T_e}\right)^{1/2}J_\parallel+\frac{3}{2}\left(\frac{1}{B}\frac{\partial B}{\partial r}\right)^2\left(\frac{2T_e}{m_e}\right)^{1/2}J_\parallel\right\}.$$

Physically the operator $\hat{L}$ is proportional to the resistivity; it describes the current diffusion due to the fluctuations. In $\hat{L}$ we have ignored a term which describes classical collisional resistivity, since at sufficiently high temperatures it is small compared with the resistivity due to the fluctuations. If the resistivity were purely classical (i.e., if $\hat{L}$ were a number), then Eqs. (14) would have only the trivial solution $J_\parallel=0$.

We append to Eqs. (24) the boundary conditions (4), (5), and (6), as well as the natural requirement that the current be nonsingular at $r=0$:

$$\left.\frac{dJ_\parallel}{dr}\right|_{r=0}=0. \tag{25}$$

We see that the quasistationary system of equations (24), together with the conditions (4), (5), (6), and (25), has nontrivial solutions which do indeed describe a relaxed state of magnetic field and current in a plasma that has undergone the transition to turbulence.

The numerical solution of the nonlinear equations (24) for a given correlation function $F(r)$ given by (19) is obtained by iteration. The iterations converge very rapidly in the region of parameter space where solutions exist: 1% accuracy is attained after three to five iterations. The functional form of $B_\theta(r)$ and $B_z(r)$ found numerically for the

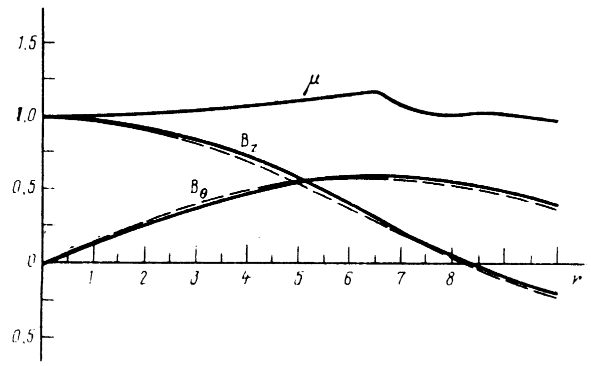

FIG. 2. $B_\theta(r)$, $B_z(R)$, and $\mu(r)$ obtained by solving Eqs. (24) with the correlation given by Eq. (19), where the modes $b_r^{m,n}(r)$ are specified as in Ref. 14. The results obtained using the solution (7) are shown for comparison (broken traces).

case n = const, T_e = const is shown in Fig. 2. Evidently this solution differs little from Eq. (7) found by Taylor for the same discharge parameters. But the quantity $\mu = J_\parallel/B$ is found to vary (Fig. 2).

An important property of the magnetic field profile described by Eqs. (24) is its weak dependence on the form of the correlation. This is because the function $d \ln F/dr$ appears in Eqs. (24) instead of the correlation itself. As a consequence of this, all the magnetic field profiles found for various choices of $F(r)$ are close to one another. Figure 3 shows two very different model correlation functions (curves a and b), and Fig. 4 shows the solutions of Eqs. (24) corresponding to these functions. It is clear that the curves in Fig. 4 are very similar even for such different correlations. Note that a uniquely defined correlation function $F(r)$ is associated with the solution (7) for n = const and T_e = const. In fact, taking $F(r)$ in the form

$$F(r)=\frac{C_0}{rB^{1/2}(dB/dr)}, \tag{26}$$

where C_0 is a normalization constant, we can easily convert Eqs. (24) into the linear equations (2) and (3), the solutions of which yield (7). Substituting (7) in (26), we find

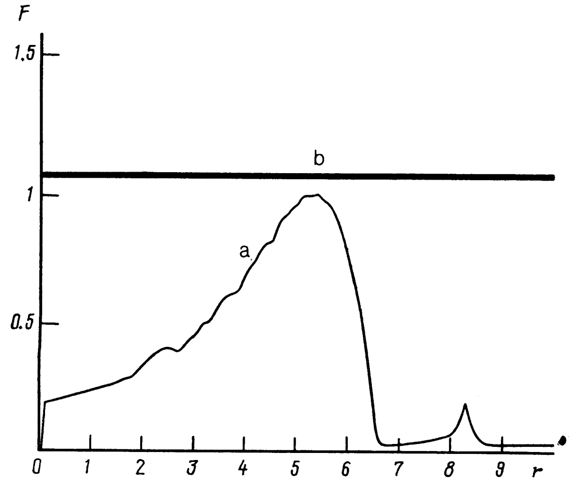

FIG. 3. (a) Correlation function evaluated according to Eq. (19) using data from Ref. 14. The straight line (b) represents the simplest possible correlation function.

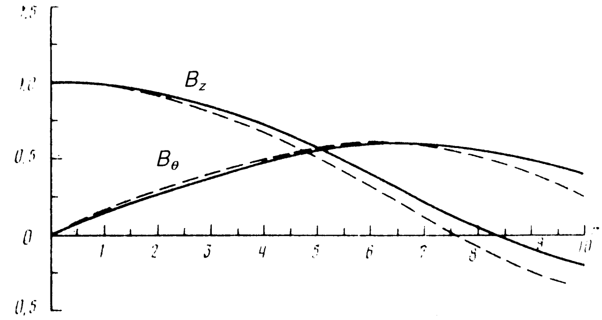

FIG. 4. $B_\theta(r)$ and $B_z(R)$ obtained by solving Eqs. (24) with the correlation functions shown in Fig. 3. The solid trace corresponds to Fig. 3a, the broken trace to Fig. 3b.

the exact correlation corresponding to Taylor's solution. Evidently it is singular as $r \to 0$: $F(r) \sim 1/r$. Strictly speaking, therefore, it is impossible to reach a state with such a correlation.

The behavior of the field structure as a function of the discharge parameters is usually determined[8,9] by a so-called $\widetilde{F}$–θ diagram, where

$$\widetilde{F}=B_z(a)/\overline{B}_z, \quad \theta=B_\theta(a)/\overline{B}_z$$

(here a is the pinch radius and $\overline{B}_z$ is the field averaged over the cross section). Hence the family of steady pinch states gives rise to the well defined $\widetilde{F}(\theta)$ curve shown in Fig. 5. Reversal occurs for $\theta \geqslant \theta_0 \approx 1.2$. On the other hand, the $\widetilde{F}(\theta)$ curve shown in Fig. 5 terminates at $\theta = \theta_{\max} \approx 1.5$. At large values of θ the iteration process no longer converges. This is taken to mean that the desired RFP solutions exist only for $\theta_0 \leqslant \theta \leqslant \theta_{\max}$.

We have examined the case of flat density and electron temperature profiles. Now we investigate how variable $T(r)$ and $n(r)$ affect the solutions, using for this purpose the realistic profiles

$$\frac{n}{n_0}=(1-(r/a)^2), \quad \frac{T_e}{T_0}=1-(r/a)^4.$$

Figure 6 shows that the forms of $T(r)$ and $n(r)$ have little effect on the shape of the magnetic field. The function $\mu(r) = J_\parallel(r)/B$ undergoes a more substantial modification. The form of the $\widetilde{F}$–θ diagram changes noticeably (Fig. 7). It

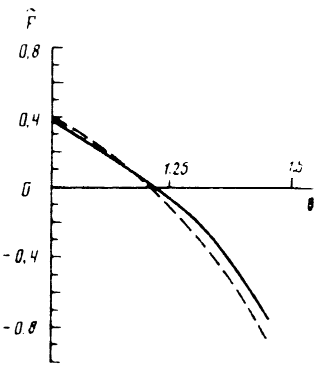

FIG. 5. $\widetilde{F}$–θ diagram describing quasistationary pinch states. The curve is calculated for n = const, T = const (solid trace). For comparison Taylor's solution is shown by a broken trace.

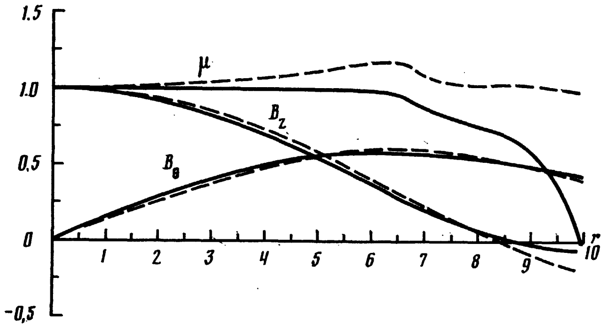

FIG. 6. $B_\theta(r)$, $B_z(R)$, and $\mu(r)$ obtained by solving Eqs. (24) with the correlation given by Eq. (19), for the cases $n = 1 - r^2/a^2$, $T = 1 - r^4/a^4$ (solid trace) and $n =$ const, $T =$ const (broken trace).

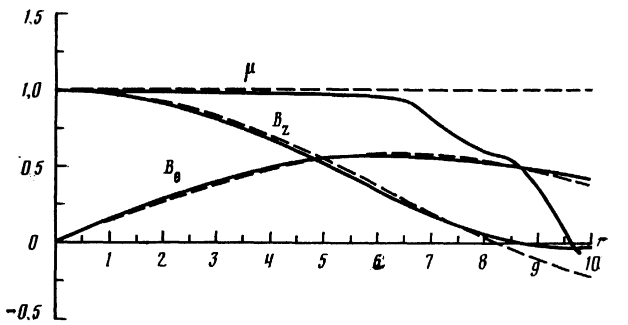

FIG. 8. $B_\theta(r)$, $B_z(R)$, and $\mu(r)$ for the case $n = 1 - r^2/a^2$, $T = 1 - r^4/a^4$, with $\lambda = \tau_E \nu_{ei} = 1$ (solid trace). The solution (7) is shown for comparison (broken trace).

follows from Figs. 6 and 7 that the results of calculations with realistic n and T profiles agree satisfactorily with the experimental data.[16]

In deriving Eqs. (24) we neglected the dissipation due to classical collisions in the equation for the current. Let us now examine how this affects the relaxed state. From Eqs. (22) and (24) we obtain

$$\hat{L}J_\parallel - \nu_{ei}J_\parallel = 0. \tag{27}$$

It follows from Eqs. (24) and (27) that collisional dissipation is negligible when we have $\lambda = \tau_E \nu_{ei} \ll 1$, where $\tau_e = a^2/D_E$ is the energy confinement time in the plasma. But it is noteworthy that these properties of the relaxed state are preserved even when the parameter $\lambda = \tau_E \nu_e$ is of order unity (Fig. 8). In this case the classical dissipation has its greatest effect at the edge of the discharge. Note that both classical transport processes and small changes in the correlation function exert a noticeable influence on the behavior of the solution near the edge of the discharge. For example, when the correlation function near the edge is raised to 0.1 (instead of 0.03), the function $\mu(r)$ vanishes at $r = a$ and no reversal occurs.

Thus the solution of Eqs. (24) provides a good description of the existence and properties of the relaxed state in an RFP, and also shows that the dependence on the form of the correlation function is weak.

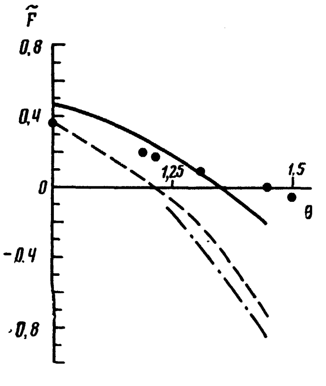

FIG. 7. $\tilde{F}$–θ diagram for the cases $n = 1 - r^2/a^2$, $T = 1 - r^4/a^4$ (solid trace) and $n =$ const, $T =$ const (broken trace), and for Taylor's solution (dot–dash trace). The points (●) are experimental results (Ref. 7).

6. ANOMALOUS THERMAL TRANSPORT

Now we discuss the possible forms of the electron temperature profile. To lowest order in $\tau_i^A/\tau_e^A = \varepsilon$, neglecting collisions and the additional heat sources Q_e, from (22) we get $T_e =$ const. To satisfy the boundary conditions in the same approximation, we should set $T_e = T_W$, where T_W is the wall temperature. But near $r = a$ conditions change sharply and the fluctuation amplitude drops, so that now we have to take into account electron collisions, the presence of impurities, radiative losses, etc. Consequently the shape of the $T_e(r)$ profile and the magnitude of $T_e(0)$ strongly depend on conditions near the wall. Consider an RFP discharge enclosed in a metal liner. In this case the main electron losses are associated with line radiation from iron and oxygen impurity atoms and with bremsstrahlung. The total loss rate $q(T_e)$ under these conditions was studied earlier.[17]

The thickness of the wall boundary layer including losses is determined by the parameter

$$\varepsilon_1 = \min\left\{\varepsilon, \frac{\nu_q a^2}{D_{\text{fl}}}\right\},$$

where ν_q is the effective rate at which particles lose energy due to radiation. Since $\varepsilon_1 \ll 1$ holds, we can assume that the layer is thin, i.e., the layer thickness Δx satisfies $\Delta x/a \ll 1$. Then under quasisteady conditions we find from (22), taking into account (15),

$$\frac{d}{dx}\left\{\left[\varkappa\left(\frac{T_0}{T_e}\right)^{1/2} + D_{\text{fl}}x^2\left(\frac{T_e}{T_0}\right)^{1/2}\right]\frac{dT_e}{dx}\right\} = \frac{J^2}{\sigma}T_e^{-3/2} - q(T_e). \tag{28}$$

Here x measures distance from the wall ($x = a - r$),

$$D_{\text{fl}} = \frac{2^{3/2}F}{\pi^{1/2}B^2a^2}\left(\frac{T_0}{m_e}\right)^{1/2}$$

is the anomalous thermal conductivity, and $\varkappa$ is the classical thermal conductivity.

Equation (28) must be supplemented with the boundary conditions

$$T_e|_{x=0} = T_W, \tag{29}$$

$$\left.\frac{dT_e}{dx}\right|_{x\to\infty} = 0.$$

Thus the temperature profile in the main part of the plasma is flat, with an abrupt change in the boundary layer given by Eqs. (28) and (29). Since ordinarily $T_0 \gg T_W$, to first order in T_W/T_0 we can take the wall temperature to be zero.

For convenience in what follows we introduce the functions

$$y=\left(\frac{T_e}{T_0}\right)^{5/2}, \quad A=\frac{3J_\parallel^2}{2\sigma T_0^{5/2}D_\parallel},$$

$$q_1(y)=\frac{3}{2}\frac{q(T_e)}{T_0 D_{\phi л}}, \quad \delta=\frac{\varkappa}{D_\parallel}.$$

Then Eq. (28) takes the form

$$\frac{d}{dx}\left\{(\delta y^{-1/5}+x^2)\frac{dy}{dx}\right\}=\frac{A}{y}-q_1(y). \tag{30}$$

It is convenient to express the right-hand side of (30) as the derivative of an effective "potential" $U(y)$:

$$\frac{dU}{dy}=\frac{A}{y}-q_1(y).$$

Analysis of Eq. (30) shows that the boundary condition (29) can be satisfied in the limit $x\to\infty$ only if there is a point where $dU/dy=0$. It is this point which determines the temperature T_0 in the interior of the discharge. Then the equation for T_0 takes the form

$$A=q_1(1). \tag{31}$$

By expanding (30) in the neighborhood of $y=1$ we can determine the form of the temperature drop at the wall. Setting $y=1+y_1$ we get

$$\frac{d}{dx}\left\{(\delta+x^2)\frac{dy_1}{dx}\right\}=U''(1)y_1. \tag{32}$$

From the definition of the potential U it is clear that $U''\propto\varepsilon_1^{-1}$. Accordingly, by using $(U'')^{1/2}\ll 1$ we can solve Eq. (32) in the WKB approximation. Following the usual procedure, we find

$$y=1-\left(\frac{\delta}{\delta+x^2}\right)^{1/4}\exp\left\{-|U''|^{1/2}\sinh^{-1}\left(\frac{x}{\delta^{1/2}}\right)\right\}. \tag{33}$$

In getting (33) we used the boundary conditions (29) at $x=0$. From the relation (33) we obtain the value of the thermal flux q_T reaching the wall:

$$q_T=\left[\varkappa+D_\parallel x^2\left(\frac{T_e}{T_0}\right)^{5/2}\right]\frac{dT_e}{dx}\bigg|_{x=0}=\frac{2}{3}T_0 D_\parallel(\delta|U''(1)|)^{1/2}. \tag{34}$$

From (34) we can determine the effective thermal transport coefficient:

$$D_{eff}=\frac{q_T a}{T_0-T_W}\approx\frac{2}{3}D_\parallel a(\delta|U''(1)|)^{1/2}.$$

The energy confinement time corresponding to electron thermal transport is

$$\tau_E=\frac{a^2}{4D_{eff}}=\frac{3}{2}\frac{a}{D_\parallel(\delta|U''(1)|)^{1/2}}.$$

Note that ohmic heating under the present conditions cannot give rise to very large temperatures T_0, since by virtue of (31) and the results of Ref. 16, T_0 is restricted to values <1–3 keV.

If additional sources heat the plasma strongly, then the temperature profile and energy confinement time change. To within terms of order $V_{T_i}/V_{T_e}\ll 1$ Eq. (22) takes the form

$$\frac{1}{r}\frac{d}{dr}r\left\{\frac{2^{5/2}Fn}{3\pi^{1/2}B^2 m_e^{1/2}}\frac{d}{dr}(T_e^{3/2})\right\}+Q_e=0. \tag{35}$$

For simplicity we have ignored ohmic heating. The solution of Eq. (35) at $r=a$ when $T_e(r)$ is well behaved at $r=0$ and Eqs. (29) hold can be written in the form

$$T_e(r)=\left\{\int_r^a\frac{3(m_e\pi)^{1/2}B^2(r_2)}{2^{5/2}F(r_2)n(r_2)r_2}dr_2\int_0^{r_2}Q_e(r_1)r_1dr_1\right\}. \tag{36}$$

The heat flux to the wall is then

$$q_T=\frac{1}{a}\int_0^a Q(r_1)r_1dr_1,$$

and the effective thermal transport coefficient D_{eff}, as before, is given by

$$D_{eff}=\frac{q_T a}{T_0-T_W}=\frac{\int_0^a Q(r_1)r_1dr_1}{T_e(0)}.$$

Taking F, B, and n to be constant, we have the rough estimate

$$T_e=\left(\frac{B^2Q_e}{4Fn}\right)^{2/3}(a^2-r^2)^{2/3},$$

$$D_{eff}=4\left(\frac{Fn}{4m_e^{1/2}B^2}\right)^{2/3}(Qa^2)^{1/3}, \quad \tau_E=\frac{a^2}{D_{eff}}.$$

The energy confinement time τ_E is proportional to $b^{-1/2}$. This dependence implies that the energy diffusion coefficient scales as $D_E\sim T_e^{1/2}$.

7. THE SELF-CONSISTENT TURBULENT APPROXIMATION

We have examined the average current, magnetic field, and electron temperature profiles in a discharge. The ion temperature and plasma density can be investigated completely analogously. The equations for the average quantities, however, will contain the correlation function of the fluctuations,

$$F(r)=\int_0^{+\infty}d\tau\langle b_r b_r'\rangle,$$

which is assumed to be given. To close the system of equations we have to be able to evaluate $F(r)$.

For this purpose, recalling Eq. (1), we must consider small (but nonlinear) oscillations in the system. Subtracting the average quantities (24) from (22), we obtain the following equations for the fluctuations u and b:

$$\frac{\partial\mathbf{u}}{\partial t}+(\mathbf{u}\nabla)\mathbf{u}=\frac{1}{4\pi m_i n}\{[\mathbf{B}\,\mathrm{rot}\,\mathbf{b}]+[\mathbf{b}\,\mathrm{rot}\,\mathbf{B}]+[\mathbf{b}\,\mathrm{rot}\,\mathbf{b}]\}$$
$$+\frac{\mathbf{B}}{Bn}\left\{\frac{1}{r}\frac{\partial}{\partial r}(r\Pi_1)+\Pi_2\right\},$$

$$\frac{\partial\mathbf{b}}{\partial t}=\mathrm{rot}[\mathbf{u},\mathbf{B}+\mathbf{b}]+\mathrm{rot}\left\{\mathbf{h}\frac{m_e}{4\pi e^2 n}\mathcal{L}\left(\frac{\mathbf{B}}{B}\mathrm{rot}\,\mathbf{b}\right)\right\}$$
$$+\eta_m\Delta\mathbf{b}, \quad \mathrm{div}\,\mathbf{b}=0. \tag{37}$$

Here $B(r)$ is the average magnetic field defined by (24). The operators Π_1 and Π_2 were defined following (22), and $\hat{L}$ was defined following (24).

We emphasize that Eq. (37) includes terms both linear and nonlinear in the fluctuation amplitude. It is also important that the oscillations take place in a turbulent medium, which changes not only the form of the average profiles but also helps damp the oscillations by transporting the average field produced by the action of the fluctuations. This means that the fluctuations affect the properties of the eigenmodes, their growth rates, and the way they saturate. The equations must be augmented by the relation (19) for the correlation function $F(r)$. Equations (19), (22), and (37) constitute a closed self-consistent system describing the relaxed turbulent state of the plasma.

This system is, of course, highly nonlinear. But, as our calculations have shown, the magnetic configuration in an RFP depends only weakly on the form of the correlation function $F(r)$. This gives us grounds to believe that a complete solution exists. An algorithm for finding it may be constructed as follows: Let an initial magnetic configuration $B_\theta^0(r)$, $B_z^0(r)$ be given, with the corresponding $\theta = 2\pi a J/\Phi$. Then using (37) we find the unstable modes and simultaneously calculate their saturation levels, determined by both nonlinear processes and dissipation due to scattering off of fluctuations. Note that Eqs. (37) are close to the classical MHD system, so their solution must be similar to that found by Schnack *et al.*[10] Then, using (19), we calculate $F^{(0)}(r)$. Plugging $F^{(0)}(r)$ into Eq. (22) we solve for $B_\theta^{(1)}(r)$ and $B_z^{(1)}(r)$, which are then used as the magnetic fields in the next iteration step. The iteration process continues until it converges.

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