Emission of acoustic waves and formation of heated jet as a fast source moves through a medium with a relativistic equation of state

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The field structure of the perturbations created in nuclear matter by a fast relativistic particle is studied. The equation of state of the medium is assumed to allow extremely high "sound" velocities $s \sim c$. The temperature distribution in the jet resulting from the thermal conductivity of the nuclear matter is calculated. Conditions for the phase matching of acoustic waves are found for the case in which the particle velocity V and also the "sound" velocity in the medium, s, are comparable to the velocity of light.

1. INTRODUCTION

Processes in which a large amount of energy is transferred to a nucleus have always attracted interest in research in nuclear physics. Such processes occur, in particular, in collisions of high-energy heavy ions, in which a large number of degrees of freedom are excited in the interacting subsystems. It might be suggested that the macroscopic properties of the nuclear subsystems are manifested particularly clearly here; e.g., motions of a hydrodynamic type, such as shock waves or jets,[1,2] etc., may arise in the nuclear matter.

Galitskiĭ *et al.*[1] have studied various aspects of the problem of the passage of a nonrelativistic particle through nuclear matter and the hydrodynamic motions which arise in the process. Glassgold *et al.*[2] have analyzed the validity of the hydrodynamic approximation for describing processes associated with the appearance of shock waves.

When the interaction energies of heavy ions are high, it is often necessary to consider relativistic effects. This comment applies in particular, to a possible interpretation of the experiments of Refs. 3 and 4, in which the fission of heavy nuclei by protons with a distinctly relativistic energy, on the order of 1–10 GeV, was studied (Fig. 1).

In the present paper, in contrast with Ref. 1, we focus on relativistic effects. In particular, we examine certain effects which arise from the passage of a fast particle ($V \sim c$) through matter whose equation of state allows perturbations to propagate through the medium at velocities comparable to the velocity of light. We find the conditions under which waves are excited. Two factors are taken into account in these conditions: the relativistic velocity of the particle itself and the relativistic nature of the equation of state of the matter. We discuss the effect of the thermal conductivity of the nuclear matter in the wake of the particle.

A brief outline of the paper is as follows. In Sec. 2 we derive the equations which we will need in the relativistic generalization. Section 3 is devoted to the structure of a source which models the interaction of the moving particle with the nuclear matter. In Sec. 4 we analyze in the acoustic approximation the field of perturbations created in the medium by the particle. The condition for phase matching in the emission of waves is discussed in Sec. 5 for the case in which the "sound" velocity in the medium and the velocity of the particle passing through it are comparable in magnitude to the velocity of light. Jet motion is analyzed in Sec. 6. In Sec. 7 we report numerical calculations of the temperature of the jet in which the thermal conductivity of the nuclear matter is taken into account.

2. DERIVATION OF HYDRODYNAMIC EQUATIONS WITH SOURCES

In the special theory of relativity, the equations of interest here can be derived from the energy and momentum conservation laws written in the form of a 4-divergence of the energy-momentum tensor of the system:[5,6]

$$\partial_k \hat{T}_i^{\,k} = \frac{\partial \hat{T}_i^{\,k}}{\partial x^k} = 0. \tag{1}$$

Here $x^0 = ct$, $x^a = \{x^1, x^2, x^3\}$, and c is the velocity of light. The Latin indices run over the values 0, 1, 2, 3; the Greek indices run over the values 1, 2, 3.

The tensor $T_i^k = T_i^k + \tau_i^k$, determined by its mixed components, includes, on the one hand, the constitutive tensor

$$T_i^{\,k} = \rho c^2 W u_i u^k - p\delta_i^{\,k},$$

which characterizes the ideal relativistic liquid and, on the other, the dissipative tensor τ_i^k, which describes both the dis-

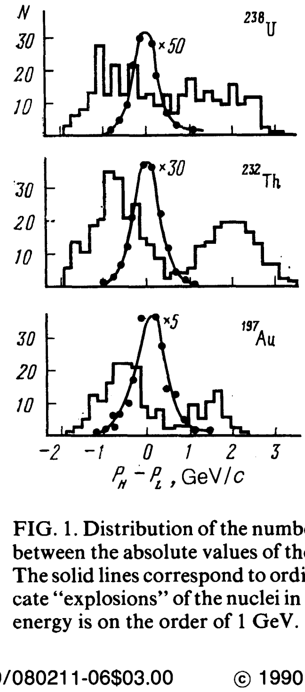

FIG. 1. Distribution of the number of events with respect to the difference between the absolute values of the transverse momenta of the fragments.[3] The solid lines correspond to ordinary fission events. The histograms indicate "explosions" of the nuclei in their collisions with protons. The proton energy is on the order of 1 GeV.

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sipative properties of the medium and the interaction between the medium and the particle moving through it. The quantity $c^2 W$ in this expression is the enthalpy per unit mass, ρ is the density, and p is the pressure. All these quantities are referred to the rest frame of an element of the medium. As usual, the 4-velocity is specified by its components:

$$u^i = dx^i/dl = \{\gamma,\ v^\alpha\gamma/c\}.$$

Here $\gamma^{-2} = (1 - \mathbf{v}^2/c^2)$, the interval dl is given by the expression $dl = c^2 dt^2 - d\mathbf{x}^2$, and $\mathbf{v}$ is the hydrodynamic velocity of the medium. It is useful to bear in mind that $u_i u^i = 1$ and $d/dl = u^i \partial/\partial x^i$. The specific enthalpy $c^2 W$ and the specific enthalpy σ are in turn determined by

$$c^2(W-1) = \mathscr{E} + p/\rho, \quad T d\sigma = c^2 dW - \rho^{-1} dp. \tag{2}$$

Here $\mathscr{E}$ is the internal energy per unit mass, and T is the temperature of the medium.

With these preliminary comments behind us, we turn now to Eqs. (1). Substituting the expression for $\hat{T}_i^k$ into (1), we find

$$c^2 W u_i \partial_k(\rho u^k) + u_i c^2 \rho u^k \partial_k W + c^2 \rho W u^k \partial_k u_i - \partial_i p + \partial_k \tau_i^k = 0. \tag{3}$$

projecting Eqs. (3) onto the 4-velocity u^i, we find

$$c^2 W \partial_k(\rho u^k) + c^2 \rho\, dW/dl - dp/dl + u^i \partial_k \tau_i^k = 0. \tag{4}$$

Using relations (2), we finally find the equations

$$c^2 W \partial_k(\rho u^k) + \rho T\, d\sigma/dl + u^i \partial_k \tau_i^k = 0. \tag{5}$$

To see the essence of the problem, we assume that no mutual conversions of particles occur in the medium. The following continuity condition must then hold:

$$\partial_k(\rho u^k) = 0. \tag{6}$$

Equation (5) therefore becomes

$$\rho T\, d\sigma/dl = -u^i \partial_k \tau_i^k. \tag{7}$$

This equation determines the heat evolution in the medium as a result of both the friction of the passing particle with the medium and dissipative processes in the medium itself.

Using the functional dependence $\sigma = \sigma(\rho, p)$, we can express the entropy production in terms of the derivatives of the field quantities ρ and p:

$$\rho T\, d\sigma/dl = -\beta s^2 W\, d\rho/dl + \beta\, dp/dl. \tag{8}$$

Here $\beta = \rho T(\partial\sigma/\partial p)_\rho$, and the sound velocity in the medium is $s^2 = W^{-1}(\partial p/\partial \rho)_\sigma$.

To derive a system of equations which generalize the Euler equation to the relativistic case, we introduce the tensor $P_i^k = \delta_i^k - u_i u^k$, which makes it possible to project Eqs. (1) onto the direction orthogonal to the 4-velocity u_i. Dotting (3) with this tensor, we find the equations which we need:

$$c^2 \rho W\, du_i/dl = P_i^k \partial_k p - P_i^k \partial_n \tau_k^n. \tag{9}$$

Here we have used $P_i^k du_k = du_i$. Using expressions (2) and (7), we can put Eqs. (9) in a slightly different form:

$$\rho c^2 d(W u_i)/dl = \partial_i p - \partial_k \tau_i^k. \tag{10}$$

3. STRUCTURE OF THE SOURCE

We now consider the model situation in which the role of the dissipative tensor reduces to just the contribution from the friction of the moving particle with the medium. It follows from Eqs. (6), (7), and (9) that it is sufficient to write out the explicit expression for the 4-vector $\hat{\tau}_i = \partial_k \tau_i^k$. To do this, we can use the approximation of Ref. 7. According to one of the results of that paper, the vector $\hat{\tau}_i$, which describes the effective friction in a two-component liquid, can be written in the form

$$\hat{\tau}_i = R n_1 n_2 (u_i^{(1)} - u_i^{(2)}). \tag{11}$$

The invariant coefficient R depends on the collision velocity and the average momentum transfer in the collision of the particles of the components; n_1 and n_2 are the invariant number densities of particles for each of the components; and $u_i^{(1)}$ and $u_i^{(2)}$ are the 4-velocities of respectively the first and second components. In the case of interest here, the second component must be understood as the incident particle. In the frame of reference moving with that particle, under the condition that the particle is at the origin, the density n_2 is then equal to a δ-function of $\mathbf{x}$: $n_2 = \delta^{(3)}(\mathbf{x})$.

Substituting (11) in Eqs. (7) and (10), we find

$$\rho T\, d\sigma/dl = R n_1 n_2 (u^{(1)i} u_i^{(2)} - 1), \tag{12}$$

$$\rho c^2 d(W u_i^{(1)})/dl = \partial_i p + R n_1 n_2 (u_i^{(2)} - u_i^{(1)}). \tag{13}$$

The last of these equations can be put in a form similar to that of (9):

$$\rho c^2 W\, du_i/dl = (\delta_i^k - u_i^{(1)} u^{(1)k}) \{\partial_k p + R n_1 n_2 (u_k^{(2)} - u_k^{(1)})\}. \tag{14}$$

4. ACOUSTIC FIELD IN THE COMOVING FRAME OF REFERENCE

Let us find the structure of the acoustic field in the frame of reference moving with the particle. In this case the partial derivative of the field quantities with respect to the time vanishes, so we have

$$d/dl = u^i \partial_i = (\mathbf{u}\nabla) = c^{-1}\gamma(\mathbf{v}\nabla). \tag{15}$$

Here $\mathbf{v}$ is the velocity of the medium in the comoving frame, and $\mathbf{u} = \mathbf{v}\gamma/c$.

To linearize Eqs. (6), (12) and (14), we set

$$p = p_0 + p_1, \quad \rho = \rho_0 + \rho_1, \quad \mathbf{u} = \mathbf{u}_0 + \mathbf{u}_1, \tag{16}$$

where $p_0, \rho_0, \mathbf{u}_0$ are the unperturbed values of the field quantities; and $p_1, \rho_1, \mathbf{u}_1$ are their variations which result from the interaction of the particle with the medium. Using (15) and (16), we can then put the continuity equation (6) in the form

$$\rho_0(\nabla\mathbf{u}_1) + (\mathbf{u}_0\nabla)\rho_1 = 0. \tag{17}$$

Correspondingly, Eq. (14) reduces in this approximation to

$$\rho_0 c^2 W_0 (\mathbf{u}_0\nabla)\mathbf{u}_1 = -(\nabla + \mathbf{u}_0(\mathbf{u}_0\nabla))p_1 - R n_0 \mathbf{u}_0 \gamma_0 \delta^{(3)}(\mathbf{x}). \tag{18}$$

Finally, using expression (8), we find

$$-\beta s^2 W_0 (\mathbf{u}_0\nabla)\rho_1 + \beta(\mathbf{u}_0\nabla)p_1 = R n_0 (\gamma_0 - 1)\delta^{(3)}(\mathbf{x}). \tag{19}$$

To solve Eqs. (17)–(19), we use spatial Fourier transforms, defined by the standard rule:

$$\varphi(\mathbf{x}) = \int d\mathbf{k}\,(2\pi)^{-3}\varphi \exp(i\mathbf{k}\mathbf{x}).$$

As a result we find a system of algebraic equations for the Fourier components of ρ, p, and $\mathbf{u}$:

$$\rho_0(\mathbf{ku})+(\mathbf{ku}_0)\rho=0,$$
$$p-s^2W_0\rho=-iRn_0(\gamma_0-1)/\beta(\mathbf{ku}_0),$$
$$\rho_0c^2W_0(\mathbf{u}_0\mathbf{k})\mathbf{u}+(\mathbf{k}+\mathbf{u}_0(\mathbf{u}_0\mathbf{k}))p=iRn_0\gamma_0\mathbf{u}_0.$$

Straightforward calculations lead to an equation in the variable p:

$$\{(\mathbf{u}_0\mathbf{k})^2(c^2/s^2-1)-\mathbf{k}^2\}p=-iA(\mathbf{u}_0\mathbf{k})\equiv-iRn_0\{\gamma_0+(\gamma_0-1)\beta^{-1}(c/s)^2\}(\mathbf{u}_0\mathbf{k}). \quad (20)$$

We separate the longitudinal and transverse components of the vector $\mathbf{k}=\{k_\parallel,\mathbf{k}_\perp\}$; here we have $(\mathbf{u}_0\mathbf{k}_\perp)=0$. We introduce $M=V/s$ (the Mach number) and $b^2=\gamma_0{}^2(M^2-1)$. Equation (20) then becomes

$$p=iAu_0k_\parallel(\mathbf{k}_\perp{}^2-b^2k_\parallel)^{-1}.$$

Integrating this expression over $\mathbf{k}_\perp$, making use of the causality principle, we find the following representation for $p_1(\mathbf{x})$ $(M>1)$:

$$p_1(\mathbf{x})=-(4\pi)^{-1}(Au_0)(\partial/\partial z)\left\{\int_0^\infty dk_\parallel N_0(bk_\parallel r)\cos(k_\parallel z)-\int_0^\infty dk_\parallel J_0(bk_\parallel r)\sin(k_\parallel z)\right\}. \quad (21)$$

After integration over $k_\parallel$, this expression becomes

$$p_1(\mathbf{x})=(2\pi)^{-1}Au_0(\partial/\partial z)\{\theta(z)\theta(z-br)(z^2-b^2r^2)^{-1/2}\}.$$

It follows from expression (21) that the field is localized in a conical region behind the source (Fig. 2), just as we would expect.

For the Fourier component of ρ we then have

$$\rho=p(W_0s^2)^{-1}+iRn_0(\gamma_0-1)/\beta s^2W_0(\mathbf{u}_0\mathbf{k}).$$

taking the inverse Fourier transforms of this expression, and circumventing the poles in the correct manner, we find

$$\rho_1(\mathbf{x})=p_1(\mathbf{x})/W_0s^2-[Rn_0(\gamma_0-1)/\beta s^2W_0u_0]\theta(z)\delta^{(2)}(\mathbf{r}). \quad (22)$$

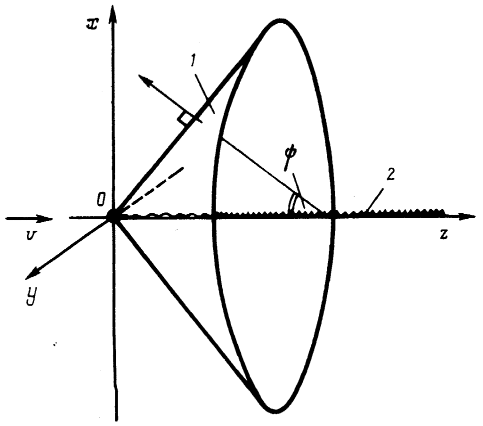

FIG. 2. Geometry of the problem. The analysis is carried out in the rest frame of the particle, which is moving at a velocity $-\mathbf{v}$. The particle is at the origin. Surface 1 is the Mach cone; the heated jet (2) forms at the axis.

Finally, we find expressions for the components of the velocity $\mathbf{u}_1=\{u_\parallel,\mathbf{u}_\perp\}$. For the longitudinal Fourier component of the velocity we have

$$u_\parallel=-[\gamma_0{}^2/\rho_0c^2W_0u_0]p+i[Rn_0\gamma_0/\rho_0c^2W_0k_\parallel].$$

In the coordinate representation we find the following expression for this component:

$$u_\parallel(\mathbf{x})=-[\gamma_0{}^2/\rho_0c^2W_0u_0]p_1(\mathbf{x})-[Rn_0\gamma_0/\rho_0c^2W_0]\theta(z)\delta^{(2)}(\mathbf{r}). \quad (23)$$

To determine the orthogonal component of the velocity, we introduce the triad of unit vectors of a cylindrical frame of reference $\mathbf{e}_z$, $\mathbf{e}_\varphi$, $\mathbf{e}_r$, where $\mathbf{e}_z$ runs along the z axis, and $\mathbf{e}_r$ and $\mathbf{e}_\varphi$ correspond to the polar radius r and the polar angle φ. By virtue of the axial symmetry of the problem, $\mathbf{u}_\perp$ must be parallel to $\mathbf{e}_r$; i.e., $\mathbf{u}_\perp=\mathbf{e}_ru_\perp(x)$.

For the $u_\perp$ Fourier component we have

$$(u_\perp)=-\frac{iA}{\rho_0c^2W_0}\frac{(\mathbf{ke}_r)}{k_\perp{}^2-b^2k_\parallel{}^2}$$

After taking inverse Fourier transforms, we find from this expression

$$\mathbf{u}_\perp(\mathbf{x})=\mathbf{e}_r(A/2\pi\rho_0c^2W_0)(b^2r/(z^2-b^2r^2)^{3/2})\theta(z)\theta(z-br).$$

Expressions (22) and (23) show that in addition to the familiar motion of the hydrodynamic type—Mach shock waves—there is an additional collective excitation: a jet motion.

We now consider the subsonic regime, with $b^2<0$. For convenience we use the notation $a^2=-b^2$. From expression (20) we find

$$p=iAu_0k_\parallel/(k_\perp{}^2+a^2k_\parallel{}^2).$$

Taking the inverse Fourier transforms of this expression, we find the following result for the pressure:

$$p_1(\mathbf{x})=-(Au_0/4\pi)z(z^2+a^2r^2)^{-3/2}.$$

Expression (22) for the density $\rho_1(\mathbf{x})$ remains valid. Differences arise because of a different variation of $p_1(\mathbf{x})$ along the coordinates. Nevertheless, again in the subsonic regime a jet forms behind the particle.

5. CONDITION FOR PHASE MATCHING IN CONNECTION WITH THE UNIFORM "SUPERSONIC" MOTION OF A PARTICLE IN THE MEDIUM

The direction in which the waves are radiated is determined by the locations of the poles in the integrands, for which we have (see also Fig. 2) $k_\parallel^2b^2-k_\perp^2=0$ and $\tan\psi=b$. It follows that

$$\cos\psi=(s/V)(1-V^2/c^2)^{1/2}/(1-s^2/c^2)^{1/2}. \quad (24)$$

In the nonrelativistic case, in which we have not only $V\ll c$ but also $s\ll c$, we find a known result: Radiation is possible for $\cos\psi=s/V<1$. This result is of course valid only for a particle in uniform motion.

6. EXISTENCE OF A HEATED JET

We turn now to the change in the temperature of the medium caused by the interaction with a particle. We first consider the case of an ideal Fermi liquid. For such a system, at a sufficiently low temperature ($T \ll \varepsilon_F$, where ε_F is the Fermi energy), the small change in the square of the temperature is a linear function of the small changes in the density and the pressure. Treating the pressure as a function of the square of the temperature and the density, and noting that under these conditions the isothermal compressibility is equal to the adiabatic compressibility,[8] we find $(\partial p/\partial \rho)_T = W_0 s^2$. For brevity we use the notation $\xi = (\partial p/\partial T^2)_\rho$. When these relations are taken into account, Eq. (19) becomes a closed equation for the temperature:

$$\beta\xi(\mathbf{u}_0\nabla)T^2=(\gamma_0-1)Rn_0\delta^{(3)}(\mathbf{x}).$$

A solution of this equation found by Fourier transforms is

$$T^2(\mathbf{x})=\frac{(\gamma_0-1)Rn_0}{\beta\xi u_0}\theta(z)\delta^{(2)}(\mathbf{r}). \qquad (25)$$

It follows from this result that heating occurs only in the jet. The temperature does not change in other spatial regions.

It is useful to compare expression (25) with the expression derived by Galitskiĭ *et al.*[1] in a nonrelativistic treatment. Using the equation of state of the medium which was used in Ref. 1 to calculate the coefficient $\beta\xi = \rho(\partial\mathscr{E}/\partial T^2)_\rho$ in (25), we find the following expression for the ratio of squared temperatures:

$$\left(\frac{T_{\mathrm{rel}}}{T_{\mathrm{nonrel}}}\right)^2=\frac{2c(\gamma_0-1)}{u_0v_0}.$$

We introduce $\mu = V/c$. We then have

$$(T_{\mathrm{rel}}/T_{\mathrm{nonrel}})^2=2[1-(1-\mu^2)^{1/2}]\mu^{-2}.$$

At small values $\mu \ll 1$, this ratio tends toward unity, as it should. At high velocities, $\mu \sim 1$, on the other hand, we have $(T_{\mathrm{rel}}/T_{\mathrm{nonrel}}) \sim \sqrt{2}$.

These relations show that the results calculated on the temperature in the jet in the nonrelativistic and relativistic approaches differ only negligibly.

7. TEMPERATURE DISTRIBUTION IN THE JET RESULTING FROM THERMAL CONDUCTIVITY; NUMERICAL ESTIMATES

To find numerical estimates of the temperature distribution, we need to consider the processes which cause the jet to spread out. In general, this is a rather involved problem. Galitskiĭ *et al.*[1] have shown that incorporating viscosity results in finite transverse dimensions of the jet only for the expression for the velocity of the liquid. To estimate the spreading of the jet in terms of temperature, we need to consider first the thermal conductivity of the medium. For this purpose we must add to the dissipative tensor terms which reflect the presence of heat fluxes, i.e., $Q^k u_i + Q_i u^k$. The 4-vector Q_i here is[6]

$$Q_k=c^{-1}KP_i^k(\partial_kT+T(du_k/dl)). \qquad (26)$$

In general, the thermal conductivity of the medium, K, is a function of the temperature and the density. For the case of interest here, in which the process takes place at a fairly low temperature, one can assume that K depends only weakly on the density.[9] For an infinite Fermi liquid, the thermal conductivity is inversely proportional to the temperature, i.e., $K \sim T^{-1}$, and the mean free path has a temperature dependence $L \sim T^{-2}$. For a finite Fermi system (at $T \ll \varepsilon_F$), the estimates of Ref. 10 lead to mean free paths considerably larger than the dimensions of the system. For this reason, the spatial boundedness of the region is taken into account, by assuming that the mean free path is equal in order of magnitude to the dimension of the system, d. A viscosity coefficient which agrees satisfactorily with semiphenomenological calculations for a nucleus was derived in this formulation of the problem in Ref. 10. In this case the thermal conductivity can be estimated from[9]

$$K \sim Cn\bar{v}L \sim Tnv_Fd,$$

where $C \sim T$ is the specific heat (per particle), and $\bar{v}$ is the mean velocity of the particles of the medium, which is on the order of v_F, the velocity at the Fermi surface. We can then write $K = \varkappa T$ for the thermal conductivity, where the coefficient $\varkappa$ does not depend on temperature. Substituting this expression into (26), we find

$$Q_i=(\varkappa/2c)P_i^k(\partial_kT^2+2T^2(du_k/dl)).$$

We can now find the expressions which we need to add to Eqs. (17)–(19) in order to incorporate the thermal conductivity. The dissipative tensor τ_i^k appears in the combination $u^i\tau_i^k$ in the equation for the entropy. To first order we then find

$$u^i\partial_k(Q^ku_i+Q_iu^k)=u^i\partial_k(\tau_i^k)_{\mathrm{therm}}\approx\partial_kQ^k.$$

To extend the Euler equation, we need to evaluate the expression $p_n^m\partial_k\tau_m^k$. Here we have $P_i^j\partial_k(\tau_j^k) \approx (dQ_i/dl)$. Substituting the resulting expressions into Eqs. (18) and (19), we find a system of linearized equations with the thermal conductivity:

$$-\beta W_0s^2(\mathbf{u}_0\nabla)\rho_1+\beta(\mathbf{u}_0\nabla)p_1=Rn_0\delta^{(3)}(\mathbf{x})(\gamma_0-1)+(\varkappa/2c)(\Delta+(\mathbf{u}_0\nabla)^2)T_1^2,$$

$$\rho c^2W_0(\mathbf{u}_0\nabla)\mathbf{u}_1+[\nabla+\mathbf{u}_0(\mathbf{u}_0\nabla)]p_1-Rn_0\delta^{(3)}(\mathbf{x})\gamma_0\mathbf{u}_0+(\varkappa/2c)(\mathbf{u}_0\nabla)[\nabla+\mathbf{u}_0(\mathbf{u}_0\nabla)]T_1^2.$$

There is no change in the continuity equation as a result.

Solving this system of equations, we find an equation for the square of the temperature:

$$\beta\xi(\mathbf{u}_0\nabla)T_1^2-(\varkappa/2c)(\Delta+(\mathbf{u}_0\nabla)^2)T_1^2=(\gamma_0-1)Rn_0\delta^{(3)}(\mathbf{x}). \qquad (27)$$

The Fourier component of the square of the temperature is then

$$(T^2)=(Rn_0(\gamma_0-1))/[(\varkappa/2c)(\mathbf{k}^2+(\mathbf{u}_0\mathbf{k})^2)+i\beta\xi(\mathbf{u}_0\mathbf{k})].$$

Inverting the Fourier transforms, we find an expression for the distribution of the square of the temperature:

$$T^2(\mathbf{x})=[Rn_0c(\gamma_0-1)/2\pi\varkappa](z^2+\gamma_0^2r^2)^{-1/2}\times\exp\left\{-\frac{k_0}{\gamma_0}[(z^2+\gamma_0^2r^2)^{1/2}-z]\right\}. \qquad (28)$$

Appearing as a parameter in this expression is the quantity k_0, which is determined by the values of $\varkappa$ and $\beta\xi$:

$$k_0=\beta\xi u_0c\varkappa^{-1}\gamma_0^{-1}.$$

Let us calculate these quantities.

The thermal conductivity K can be calculated from[11]

$$K=(4\pi^2\hbar^3p_F{}^3/3m_*{}^4T)\left[\int\frac{d\Omega}{4\pi}\frac{\omega(\theta,\varphi)}{\cos(^1/_2\theta)}(1-\cos\theta)\right]^{-1}.$$

Here m_* is the effective mass of the quasiparticle, and the function $\omega(\theta,\varphi)$ determines the probability for the scattering of quasiparticles through polar and azimuthal angles. The integration is over the solid angle Ω.

Evaluating the integral in brackets as in Ref. 10, we find

$$K=p_F{}^5(10\pi^5\hbar^3m_*T)^{-1}.$$

We define the mean free path by $L=1.33\hbar p_F^{-1}(\varepsilon_F/T)^2$, as in Ref. 12. Making use of the assumptions discussed above concerning the spatial boundedness of the system, and comparing K and L, we find a relationship between $\varkappa$ and L:

$$\varkappa=(2/7\pi^3)p_F{}^2\hbar^{-3}L.$$

If we set L equal to 7 fm (the nuclear size), for example, we find $\varkappa\approx 2\cdot 10^{14}$ ev$^{-1}\cdot$fm$^{-1}\cdot\sigma^{-1}$.

For the coefficient $\beta\xi$ we have, be definition,

$$\beta\xi=\frac{\rho}{2}(\partial\sigma/\partial T)_\rho,$$

Using the expression for the entropy of a Fermi liquid[8] (in the limit $T\to 0$),

$$\sigma=(\pi/3)^{2/3}m_*m^{-1/3}\rho^{-2/3}\hbar^{-2}T$$

we can write

$$\beta\xi=\frac{1}{2}\left(\frac{\pi}{3}\right)^{2/3}\frac{m_*}{m^{1/3}}\frac{\rho^{1/3}}{\hbar^2}=\frac{2^{1/3}}{6}\frac{m_*p_F}{\hbar^3}.$$

As a result we find

$$k_0\approx\left(\frac{7\cdot 2^{1/3}\pi^3}{12}\right)\frac{m_*}{Lp_F}\approx\frac{23v_0}{Lv_F}. \tag{29}$$

Finally, Eq. (27) contains the parameter R, which is as yet undetermined. Comparing the definition of R in (11) with the result of Ref. 13, we find $R=cmn_0^{-1}\tau^{-1}$, where $\tau=4\times 10^{-24}$ s. We show the final result in graphical form. Figure 3 shows the dependence of the temperature of the medium on the distance along the jet axis r for various values of z.

8. CONCLUSION

We have derived a theory for the excitation of collective motions in nuclear matter in the acoustic approximation, generalized to the relativistic case. In the model used here, it is assumed that the velocity at which excitations propagate through the medium can reach values comparable to the velocity of light.

We have derived expressions which describe the existence of Mach shock waves and a jet flow associated with the uniform supersonic motion of a relativistic particle.

For $V\sim c$ and $s\sim c$, the phase-matching condition leads to the result that the angle at which the waves are radiated is determined not only by the ratio s/V (the known relativistic limit) but also by the ratios V/c and s/c. Jet motion also arises behind a particle in the subsonic regime of the motion of the particle.

We have shown that in the jet formation process the temperature distribution within the jet is governed primarily by the thermal conductivity. We have calculated the values of parameters characterizing the effect. It has thus become possible to determine the temperature in the medium. This temperature reaches a level high enough that a phase transition occurs in the medium behind the particle[14] (Fig. 4), the nucleus "boils up," and an "explosion" of the nucleus is observed.

A few explanatory words are in order regarding the validity of the approximations which we have used. One might question whether the specific estimates of the kinetic coefficients of nuclear matter borrowed from the theory of a degenerate Fermi liquid can legitimately be applied to the phenomena which occur at temperatures on the order of 10–100 MeV. Using the Chapman-Enskog method for the relativistic Boltzmann equation, Galitskiĭ *et al.*[12] calculated the kinetic coefficients of nuclear matter for temperatures $T\sim$ 30–150 MeV. They examined both neutron matter and nuclear matter, with various isotopic compositions. They also studied the effects of an admixture of π mesons on the kinetic coefficients of neutron matter. A comparison of the corresponding quantities found in the present study with the results of Ref. 12 shows that (for example) the values of the thermal conductivity found by the quite different approaches differ by no more than a factor of 3 for temperatures on the order of 50–100 MeV. In view of the qualitative nature of our model, this agreement is completely satisfactory.

One might also question whether a purely hydrodynamic approach, which ignores the production of pions, Δ particles, etc., is valid. Previous analyses in several places

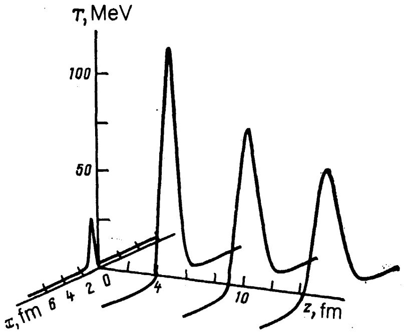

FIG. 3. Spatial distribution of the temperature in the medium for an energy $E\sim 1$ GeV of the impinging particle.

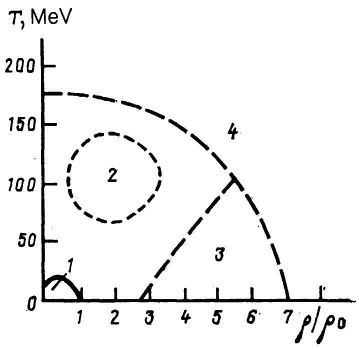

FIG. 4. Phase diagram of the nuclear matter.[14] 1—Liquid phase; 2—gas of nucleons; 3—condensed state; 4—quark-gluon plasma. The "boiling up" of the nucleus corresponds to a transition from region 1 to region 2.

(e.g., Refs. 15–17) have shown that the maximum concentration of pions and Δ particles resulting from collisions of heavy nuclei with energies on the order of 1 GeV per nucleon in the laboratory frame of reference is at most 15–30%. It is thus fairly safe to ignore these processes. The same factors lead to slight overestimates of the temperature of the medium in our estimates.

The arguments presented above thus lead to the conclusion that the model proposed in this paper is fairly successful in describing the process and that it yields satisfactory values of calculated quantities.

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