Dynamics of solitary acoustoelectromagnetic waves in crystals having nonlinear electrostriction

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The interaction and nonlinear dynamics of acoustic and electromagnetic fields in crystals with quadratic electrostriction are considered. An exact solution of the dynamic equations for the wave amplitude is obtained. This solution describes the evolution of self-compressing coupled acoustoelectromagnetic pulses, which are nonlinear solitary waves evolving from an acoustic initializer. The interaction of such waves is manifest in their quasisoliton behavior. In a dissipative medium, nonlinear pulses can be formed when two thresholds are exceeded, one connected with the initial energy of the acoustic initializer pulse and the other with the input amplitude of the pump wave.

1. INTRODUCTION

Wave propagation in media with cubic nonlinearity can cause, depending on the input-signal power, various nonstationary excitations of both soliton and nonsoliton origin.[1–4] The overall picture, however, becomes much more complicated if waves of different nature (electromagnetic and acoustic waves[5]) are coupled in such a medium.

The complexity of such processes usually makes it necessary to restrict their description either to a stationary interaction regime or to self-similar solutions of the traveling-wave $[f(x-vt)]$ type.[6] This, however leaves unclear the physical picture and the conditions under which such waves are formed, as well as the dynamics of the transition from the linear regime to the nonlinear regime as a result of the onset of various instabilities. Allowance for wave dissipation only adds to these difficulties.[7]

From the formal point of view the main difficulty lies in the absence of explicit solutions of the nonlinear dynamic equations with initial or boundary conditions corresponding to the physical meaning of the problem. Progress is made possible in this field at present by advances in the mathematical formalism of the inverse scattering matrix problem[9] and some other methods.[3,4] These have made possible a complete study of the formation of three-wave soliton envelopes in quadratically nonlinear media[9] and, in particular for parallel acoustoelectromagnetic interaction.[10,11] Interactions of distinct types of waves in cubically nonlinear media have been much less studied.[7,12]

It is shown in the present paper that acoustoelectromagnetic interactions can produce solitary wave envelopes in anisotropic crystals with nonlinear electrostriction. These waves become enhanced as they evolve and are strongly compressed, and their speed is of the order of that of sound. An exact solution of the nonlinear dynamic equations has revealed a number of distinct features of such a process due to the thresholdlike decay of the initializing pulse and to the attenuation of sound.

The main difference between this system and the much-studied case of quadratically nonlinear medium is that the former is nonlinear even in an approximation with a constant pump-wave amplitude.[5] It is therefore impossible to use directly in this case an analysis based on the concept of the known published[13] linear instabilities.

On the other hand, effects connected with electrostriction that is nonlinear in the sound amplitude are usually weak in view of the presence of linear electrostriction, which prevents them from building up to noticeable magnitude as second-order effects. Particularly interesting are cases in which there is no linear electrostriction (e.g., the corresponding moduli are equal to zero because of crystallographic symmetry). The elastic nonlinearity plays no noticeable role here in view of the short length of the acoustic pulse.[11] It is this case which is investigated in the present paper.

2. FORMULATION OF PROBLEM AND EQUATIONS OF MOTION

Consider a uniaxial crystal of symmetry $3m$, with a coordinate system aligned with the crystallographic axes. We investigate ordinary and extraordinary electromagnetic waves (with frequencies ω_1 and ω_2 and wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$) interacting with a transverse acoustic wave (frequency and wave vector $\mathbf{K}$):

$$E_{y,z}=\mathscr{E}_{1,2}(x,t)\exp(i\theta_{1,2})+\text{c.c.},$$
$$\mathbf{u}=U(x,t)[\mathbf{e}_y+\alpha\mathbf{e}_z]\exp(i\theta_3)+\text{c.c.}, \tag{1}$$

where

$$\theta_{1,2}=\omega_{1,2}t-k_{1,2}x,\quad \theta_3=\Omega t-Kx,$$
$$\mathscr{E}_{1,2}\equiv\mathscr{E}_{1,2}(x,t),\quad U(x,t)\equiv u_y=U,$$

U are slowly varying wave amplitudes, and $\alpha=u_z/u_y=0.86$ (Ref. 14). Since the electrostriction constants are zero for waves of the specified polarizations ($a_{45}=a_{46}=0$) the first nonvanishing nonlinearity causing an interaction is nonlinear (quadratic) electrostriction, corresponding to terms of the following type in the free energy (we put here and elsewhere $u_z=\alpha u_y$)

$$-\frac{1}{8\pi}\left\{g_1\left(\frac{\partial U}{\partial x}\right)^2E_y^2+g_2\left(\frac{\partial U}{\partial x}\right)^2E_z^2+2g_3\left(\frac{\partial U}{\partial x}\right)^2E_yE_z\right\}, \tag{2}$$

where $g_{1,2,3}$ are the effective components, renormalized with allowance for the piezoeffect, of the quadratic electrostriction; the explicit expressions for these components are too unwieldy to present here.

The equations of motion, which consist of the Maxwell equations and the equations of elasticity theory with allowance for (2) take the form

$$\left(\frac{\partial^2}{\partial x^2}-\frac{\varepsilon_{1,3}}{c^2}\frac{\partial^2}{\partial t^2}\right)E_{y,z}=\frac{1}{c^2}\left[(g_3E_{z,y}+g_{1,2}E_{y,z})\left(\frac{\partial U}{\partial x}\right)^2\right],$$

$$\left(\frac{\partial^2}{\partial x^2}-\frac{1}{s^2}\frac{\partial^2}{\partial t^2}\right)U$$

$$=\frac{1}{4\pi\rho s^2}\frac{\partial}{\partial x}\left[(2g_3E_yE_z+g_1E_y^2+g_2E_z^2)\frac{\partial U}{\partial x}\right], \quad (3)$$

where $\varepsilon_{1,3}$ are the diagonal components of the dielectric tensor,[15] s is the speed of sound, and ρ is the density of the crystal.

Such an interaction is most pronounced when the resonant parametric-coupling conditions $\omega_1-\omega_2=2\Omega+\Delta\omega$, $k_1-k_2=2K+\Delta k$, where $\Delta\omega$ and Δk are small mismatches. The sound frequency here is

$$\Omega\approx s(n_1-n_3)\omega_1/2c$$

($n_{1,3}=\varepsilon_{1,3}^{1/2}$ are the refractive indices) and is usually in the ultraviolet.

Taking into account the weakness of the interaction, it is easy to obtain dynamic equations for the complex amplitudes of the waves, by substituting (1) in (3) and retaining the synchronous terms. Changing to dimensionless variables, we rewrite these equations in the form

$$\frac{\partial A_{1,2}}{\partial\xi}=\mp A_{2,1}A_3^2\sin\varphi,$$

$$\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\tau}+\Gamma\right)A_3=A_1A_2A_3\sin\varphi,$$

$$\frac{\partial\varphi_{1,2}}{\partial\xi}=A_3^2\frac{A_{2,1}}{A_{1,2}}\cos\varphi-2R_{1,2}A_3^2, \quad (4)$$

$$\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\tau}\right)\varphi_3=A_1A_2\cos\varphi-Q,$$

where

$$Q=R_1A_1^2+R_2A_2^2,\quad A_1=\mathscr{E}_1/\mathscr{E}_0,$$

$$A_2=(\alpha_1/\alpha_2)^{1/2}\mathscr{E}_2/\mathscr{E}_0,\quad A_3=(\alpha_1/\alpha_3)^{1/2}U/\mathscr{E}_0,$$

$$\xi=x/l_n,\quad \tau=st/l_n,\quad \alpha_{1,2}=K^2k_{1,2}g_3/2\varepsilon_{1,3},$$

$$\alpha_3=Kg_3/4\pi\rho s^2,\quad \overline{\Delta k}=\Delta kl_n,$$

$$\overline{\Delta\omega}=\Delta\omega l_n/s,\quad \Delta\theta=\overline{\Delta\omega}\tau+\overline{\Delta k}\xi,$$

$$R_{1,2}=(g_{1,2}/g_3)(\alpha_{1,2}/\alpha_{2,1}),\quad l_n=(\alpha_1/\alpha_2)^{1/2}/\alpha_3\mathscr{E}_0,$$

l_n is a characteristic scale connected with the specified nonlinearity, γ is a phenomenological sound-absorption coefficient,

$$\Gamma=\gamma l_n,\quad \varphi=-\varphi_1+\varphi_2+2\varphi_3+\Delta\theta,\quad \mathscr{E}_{1,2}=|\mathscr{E}_{1,2}|\exp(i\varphi_{1,2}),$$

$$U=|U|\exp(i\varphi_3),\quad |\mathscr{E}_{1,2}|,\ |U|\to\mathscr{E}_{1,2},\ U,$$

and $\mathscr{E}_0$ is the pump-wave input amplitude. In the derivation of (4) we discarded terms of type $(n_{1,3}/c)\partial\mathscr{E}_{1,2}/\partial t$, which are small, of order the parameter $s/c\sim10^{-5}\ll1$ are connected with transient propagation of electromagnetic wave. The attenuation of the laser emission in the transparency region was also assumed negligibly small.

3. SOLUTION OF EQUATIONS

Equations (4) can be easily solved in the constant-pumping approximation ($A_1\approx1, A_{2,3}\ll1$). We shall therefore focus on the nonlinear regime, when pump depletion is significant ($A_1\sim A_2\sim A_3$) and the amplitudes of all waves must be taken into account under equal conditions (the conditions under which such a regime sets in are discussed below).

It is easy to obtain from Eqs. (4) the relation

$$A_1A_2A_3\cos\varphi=\Delta RA_3^2(A_1^2-A_2^2)-\int_0^\xi\left\{\left(\frac{\partial A_3^2}{\partial\tau}+2\Gamma A_3^2\right)\right.$$

$$\left.\times\frac{\partial\tilde\varphi_3}{\partial\xi}-\frac{\partial A_3^2}{\partial\xi}\frac{\partial\tilde\varphi_3}{\partial\tau}\right\}d\xi+C_0(\tau), \quad (5)$$

where

$$\Delta R=(R_1-R_2)/2=(n_1n_3)^{1/2}(g_1/n_1-g_2/n_3)/2g_3,\quad R=(R_1+R_2)/2,$$

$$\tilde\varphi_3=\varphi_3+R\int^\tau g^2(\tau')d\tau'-\overline{\Delta k}(\tau-\xi)/2,$$

$$C_0(\tau)=A_3^2[A_1A_2\cos\varphi-\Delta R(A_1^2-A_2^2)]_{\xi=0},$$

and $g^2(\tau)=A_1^2+A_2^2$ depends only on the time. In an isotropic medium we have $n_1=n_3$ and $g_1=g_2$,[15] so the parameter ΔR is indicative of the degree of anisotropy of the crystal. Usually $|n_1-n_3|/n_1\lesssim0,1$, and although few experimental studies have been made of the magnitudes of $g_{1,2}$, it is easily seen that, at any rate for a weakly anisotropic crystal, the condition $|(g_1-g_2)/g_3|\ll1$ should be met and correspondingly $|\Delta R|\ll1$. We shall therefore neglect ΔR henceforth.[1)]

For $R=0$ the relation (5) is satisfied for $\varphi\equiv\pi/2$, $\tilde\varphi\equiv$const, if one of the waves has zero amplitude on the boundary; from (5), in particular, we obtain $\overline{\Delta\omega}=-\overline{\Delta k}$. The phases are expressed then in terms of the wave amplitudes in the form

$$\varphi_{1,2}=-2R\int^\xi A_3^2(\xi',\tau)d\xi'\mp R\int^\tau g^2(\tau')d\tau'\mp\pi/4,$$

$$\varphi_3=-R\int^\tau g^2(\tau')d\tau'+\overline{\Delta\omega}(\xi-\tau)/2. \quad (6)$$

Equations (6) have a simple physical meaning. In a cubically nonlinear medium the terms containing $g_{1,2}$ in (2) give rise to *dc* components of the modulation of the refractive indices by sound and of the elastic moduli by the field. This leads to renormalization of the phase velocities or to rotation of the wave phases. For a mismatch

$$\Delta\omega=-(g_1\Omega/4\pi\rho s^2)\mathscr{E}_0^2=-s\Delta k$$

the shift of the relative phase φ is cancelled out and the interaction is optimal. The equations for the moduli of the amplitudes in (4) then acquire the simple form:

$$\frac{\partial A_{1,2}}{\partial\xi}=\mp A_{2,1}A_3^2,\quad \left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\tau}+\Gamma\right)A_3=A_1A_2A_3. \quad (7)$$

At first, assuming the excitations to be delocalized, we investigate the simplest situation, when an electromagnetic pump wave of frequency ω_1 and sound $A_{1,3}(-\infty,\tau)=f_{1,3}(\tau)$, are applied to the boundary at $\xi=-\infty$, and sound $A_2(-\infty,\tau)=0$ is excited in the course of the interaction. The crystal in this case contains an initializing sound pulse $A_3(\xi,0)=A_{30}(\xi)$.

Introducing the variable

$$\Phi=\int_{-\infty}^{\xi}A_3^2(\xi',\tau)\,d\xi',$$

which is connected with $A_{1,2}$ by the relations

$$A_1=f_1\cos\Phi,\quad A_2=f_1\sin\Phi,$$

we obtain from (7) for Φ the closed equation

$$\left(\frac{\partial}{\partial\xi}+\frac{\partial}{\partial\tau}+\Gamma\right)\Phi=f_1^2(\tau)\sin^2\Phi+f_3^2(\tau),$$
$$\Phi(-\infty,0)=0,\qquad \Phi(\xi,0)=\Phi_0(\xi)=\int_{-\infty}^{\xi}A_{30}^2(\xi)\,d\xi. \tag{8}$$

Equation (8) is easily solved if the sound damping is negligibly small ($\Gamma=0$) and $f_3=0$. The corresponding solutions for the initial wave amplitudes in (7) are

$$A_1=(1-P^2)^{1/2},\quad A_2=P,\quad A_3=A_{30}(\xi-\tau)P/\sin[\Phi_0(\xi-\tau)], \tag{9}$$

where

$$P=\left\{1+\left[\mathrm{ctg}(\Phi_0(\xi-\tau))-\int^{\tau}f_1^2(\tau')\,d\tau'\right]^2\right\}^{-1/2},$$
$$\Phi_0(\xi-\tau)=\int_{-\infty}^{\xi-\tau}A_{30}^2(\xi)\,d\xi.$$

It is important that (9) contains explicitly the envelope $A_{30}(\xi)$, which has not been specified beforehand, of the initializer. This makes it possible to investigate the space–time evolution of the waves without confining oneself to the nonlinearity level, i.e., under conditions such that energy exchange between the field and the sound is intense.

4. DYNAMICS OF LOCALIZED EXCITATIONS

By way of illustration we consider in greater detail certain cases that describe quite fully the essential features of this process (in the case of small fluctuations, the linear regime, i.e., for

$$\int_{-\infty}^{\infty}A_3^2\,d\xi\ll 1$$

we have in (7)

$$A_1=f_1(\tau),\quad A_2=0,\quad A_3=A_{30}(\xi-\tau)\quad(\Gamma=0).)$$

1. $A_{30}(\xi)=[\alpha/(1+\beta^2\xi^2)]^{1/2}$. For $\alpha=\beta>0$ we obtain

$$A_3=\{\beta/[1+\beta^2(\xi-v\tau)^2]\}^{1/2},$$

i.e., a stationary solitary wave propagating at a velocity $v=1-1/\beta$. Here A_1 and A_2 have respectively the form of a step and of a solitary wave:

$$A_1=\beta(\xi-v\tau)P_1,\quad A_2=P_1,\quad P_1=[1+\beta^2(\xi-v\tau)^2]^{-1/2}.$$

Note that this solution can also be obtained directly from (7) for $\Gamma=0$, by putting $A_i=A_i(\xi-v\tau)$. For $0<\beta<1$ or $\beta\geqslant1$ we get respectively $v<0$ or $v\geqslant0$, i.e., the pulse moves, depending on β, in the positive or negative x direction. In the general case the character of the evolution is more complex and depends substantially on the relation between α and β. Figure 1a illustrates the case $\alpha<\beta(\alpha=1,\beta=3)$. It can be seen that in the course of time the sound pulse is appreciably amplified and strongly compressed. Its total energy

$$W(\tau)\equiv\Phi(\infty,\tau)=\int_{-\infty}^{\infty}A_3^2(\xi,\tau)\,d\xi$$

increases and tends to the value π. On the other hand, in the case $\alpha>\beta(\alpha=1,\beta=0.8)$, (Fig. 1b) the initial pulse breaks up into two shorter pulses that move in opposite directions, and after a time $\tau\sim4$ they are well separated in space. These pulses are subsequently independently amplified and compressed.

2. $A_{30}(\xi)=[\alpha/\mathrm{ch}(\beta\xi)]^{1/2}$. For $\alpha=\beta$ we obtain a solution in the form

$$A_3(\xi,\tau)=\{\beta\,\mathrm{ch}[\beta(\xi-\tau)]/[1+(\mathrm{sh}[\beta(\xi-\tau)]+\tau)^2]\}^{1/2}.$$

The position of the center of the pulse can be obtained from the condition $\sinh[\beta(\tau-\xi)\approx\tau]$, from which it follows that the pulse propagates at a velocity $v=1-1/\beta(1+\tau^2)^{1/2}$, and its amplitude increases a $A_3\propto(1+\tau^2)^{1/4}$. Since the energy of such a pulse is fixed at $W=\pi$, the pulse will compress in the course of time. Just as in the preceding case, for $\alpha>\beta$ the initializer breaks up into two or several pulses.

3. Of great interest is head-on collision of these solitary waves. This occurs for an initial state equal, e.g., to the sum of the initializers considered above (in case 1, the pulse moves in the opposite direction at $\alpha=\beta<1$). Figure 2 shows the evolution of two pulses whose initial distribution is of the form

$$A_{30}^2(\xi)=\beta_1/[1+\beta_1^2(\xi-15)^2]+\beta_2/\mathrm{ch}(\beta_2\xi)$$

for $\beta_1=0.4$, $\beta_2=1$. It can be seen that the evolution in time can be arbitrarily divided into three stages. In the first stage the pulses approach each other and their amplitudes change little. In the second stage the interaction sets in: the pulses,

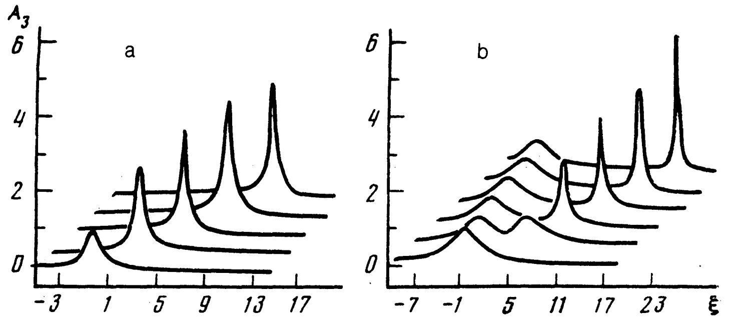

FIG. 1. Nonlinear evolution of initial acoustic pulse at $a-\alpha=1,\beta=3$; $b-\alpha=1,\beta=0.8$ for $\tau=0,2,4,\ldots$.

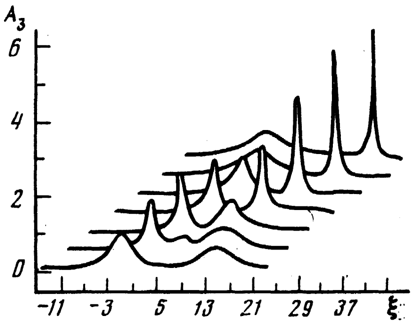

FIG. 2. Collisions of head-on acoustic pulses for $\tau = 0$; 2,5; 5; 7,5;... .

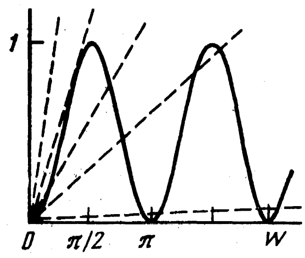

FIG. 3. Solution of the equation $\sin^2 W = 2\Gamma W$ for $\Gamma = 1$, 0.36, 0.2, 0.1, 0.005.

while not overlapping, exchange energy, and during the interaction the distance between them is constant. In the last stage the pulses diverge, so that the opposing pulse is not noticeably changed, but the forward pulse is strongly compressed. The result of the collision of the pulses thus demonstrates their quasisoliton behavior and is reminiscent of certain soliton-collision regimes.[16]

5. ALLOWANCE FOR ABSORPTION BY THE WAVES

For long times $\tau \to \infty$ account must be taken of the second absorption $\Gamma \neq 0$. However, instead of solving Eq. (7) directly, it is convenient to obtain from (8) a relation that defines the evolution of the total energy of the acoustic wave in the form

$$\frac{dW}{d\tau} = -2\Gamma W + f_1^2(\tau)\sin^2 W + f_3^2(\tau), \qquad W = \int_{-\infty}^{\infty} A_3^2(\xi, \tau)\, d\xi. \tag{10}$$

From the instant when the input signal $f_3 = 0$ is turned off, the change of W is determined by the balance between the absorption of the sound and its amplification by the nonlinear interaction with the field. At $f_1 = 1$, which we assume henceforth to be satisfied, we have $f_3 = 0$ and if absorption is neglected ($\Gamma = 0$) the solution of (10) takes the form

$$W = \mathrm{Arcctg}[\mathrm{ctg}\, W_0 - \tau]. \tag{11}$$

As $\tau \to \infty$ the energy W increases from $n\pi$ to $(n+1)\pi$, where $n = 0,1,2,\ldots$. In the case of an initial energy

$$W_0 \equiv \int_{-\infty}^{\infty} A_{30}^2(\xi)\, d\xi < \pi$$

the pulse will be amplified until its energy reaches π. This relation is similar in meaning to the area theorem for self-induced transparency.[16] Note that at $\Gamma = 0$ the right-hand side of Eq. (10) is non-negative, and consequently stationary states with $W = n\pi$ are unstable. As $\tau \to \infty$ allowance for sound absorption therefore becomes important. For $\Gamma \neq 0$, the solution of Eq. (10) is not expressed in terms of elementary functions, and it is best investigated qualitatively by graphical means.

The stationary values of W are determined by the intersection of the straight line $2\Gamma W$ with the function $\sin^2 W$ (Fig. 3). The picture of the intersection depends in this case substantially on the value of Γ.

For $\Gamma \geqslant 1$ there is only one stationary point $W = 0$; all the perturbations in the medium are then damped. For smaller Γ, however, starting with a threshold value $\Gamma_{\rm thr}$, two additional stationary points appear—one stable and the other unstable. This corresponds to formation of a new inhomogeneous state in the system—an undamped nonlinear pulse.

The condition for the onset of such a state can be easily established in the case $W \lesssim \pi/2$. The solution of (10) can then be written in the form

$$W = \pi/2 - \Gamma + \Delta - 2\Delta/(1 + a_0 e^{2\tau\Delta}), \quad \Delta = (\Gamma^2 - \pi\Gamma + 1)^{1/2}, \tag{12}$$

where a_0 is a constant connected with the initial energy. If at $\tau = 0$ the initializer had a sufficiently high energy: $W_0 > W_c \equiv \pi/2 - \Gamma - \Delta$, then as $\tau \to \infty$ the system goes over into a state with energy $W = W_\infty = \pi/2 - \Gamma + \Delta$, which does not depend on the details of the initial distribution. In particular, for $a_0 e^{2\tau\Delta} \ll 1$ we have

$$W = W_c + 2a_0 \Delta e^{2\tau\Delta},$$

and we see hence that the system is unstable for $\Delta^2 > 0$. The instability threshold corresponds to the equality $\Delta = 0$, from which we get the threshold value

$$\Gamma = \Gamma_{\rm thr} = \pi/2 - (\pi^2/4 - 1)^{1/2} \approx 0{,}36.$$

It is easily seen that $\Gamma_{\rm thr}$ is the first bifurcation point of Eq. (10).

Thus, when account is taken of sound absorption, an inhomogeneous state of the nonlinear medium we are studying sets in when two thresholds are exceeded (two types of supercriticality). The first determines, in the initial variables, the minimum amplitude of the pump wave

$$\mathscr{E}_0 > \mathscr{E}_{\rm thr} = \left[0{,}36 \frac{\gamma}{\alpha_3}\left(\frac{\alpha_1}{\alpha_2}\right)^{1/2}\right]^{1/2} = \left[0.36 \frac{4\pi\gamma\rho s^2}{K g_3}\left(\frac{n_3}{n_1}\right)^{1/2}\right]^{1/2} \tag{13}$$

The second threshold (the switchover threshold) is connected with the minimum energy of the initializing acoustic pulse and takes at $\Gamma \lesssim \Gamma_{\rm thr}$ the form

$$(1+\alpha^2)\int_{-\infty}^{\infty} \frac{1}{2}\rho\Omega^2 U^2\, dx \geqslant \frac{\rho s^2}{g_3}\frac{(n_1 n_3)^{1/2}}{\omega_1} W_c(\Gamma). \tag{14}$$

This condition is certainly not met as $U \to 0$, so that the corresponding instability is nonlinear.

When (13) and (14) are satisfied, instability appears in the crystal and gives rise to a nonlinear pulse. Far enough above threshold, $E_0 \gg E_{\rm thr}$, depending on W_0, more complicated states can set in with several solitary waves. Such states form nonstationary dissipative structures, since it is indeed

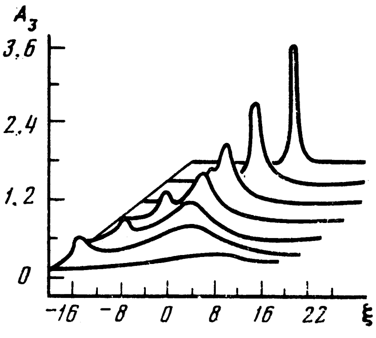

FIG. 4. Nonlinear interaction of initial acoustic pulse with an external signal in a dissipative medium for $\tau = 0,3,6,\ldots$.

the dissipation which stabilizes their energy.

Besides the global characteristics, interest attaches also to the direct form of the inhomogeneous state that is formed in the medium. However, an analytic solution of Eq. (8) is difficult in this case. Figure 4, obtained by numerical solution of (8), illustrates the nonlinear dynamics in a semibounded system with an external signal for initial and boundary conditions in the form

$$A_{30}^2(\xi)=0.1/[1+0.01(\xi-10)^2],$$

$$f_3^2=0.1\exp[-0.5(\tau-2)^2], \quad \Gamma=0.1.$$

During the initial stage, the pulses are spatially separated at the chosen parameters, and the initial pulse moves towards the boundary. At $\tau \sim 10$, both pulses merge into a single pulse, which then propagates along the system, increasing and becoming strongly compressed, while its energy stabilizes at $W = 0.76\pi$. Since $\Gamma \leqslant \Gamma_{thr}$ holds in this case, one solitary wave is formed in the medium. Similar states are formed also under multipulse initial conditions, when only one pulse "survives" the competition (the remaining pulses are damped). If the condition $W_0 \gg W_s$ and $\Gamma \ll \Gamma_{thr}$ obtain, a decay into two or more solitary waves takes place, and each wave is amplified and strongly compressed. Computer calculations have shown that this behavior continues up to the limits of applicability of the present system of equations.

6. CONCLUSION

Thus, in crystals with nonlinear electrostriction, soliton envelope waves can be formed in parallel acoustoelectromagnetic interaction. Collisions between these waves reveals them to have quasisoliton properties. The pulses are formed, amplified, and compressed when two thresholds, connected with the pump-wave amplitude and with the energy of the initializing acoustic pulse, are exceeded. The amplification and compression of the signals can be observed in crystals of trigonal symmetry (lithium niobate on barium titanate for $T < -90$ °C in the microwave band, when the anomalously large static values of the dielectric constant are still preserved.[12,17]

In addition, this nonlinear system, owing to its solvability and simplicity, can serve as a convenient model in the theoretical study of the dynamics of solitary waves formed as a result of some arbitrary instability.

[1)] Note that at $\Delta R \neq 0$ the correction to the solution for the relative phase is $\Delta\varphi \approx \Delta R(A_1^2 - A_2^2)$ and is of second order of smallness in the nonlinear regime. In strongly anisotropic materials the deviation of the phase from optimal can slow down the formation of solitary waves.

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