Exactly integrable models of resonance interaction of light with a thin film of three-particle levels

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(Submitted 20 November 1989)
Zh. Eksp. Teor. Fiz. **97**, 1530–1543 (May 1990)

We determine the conditions under which the inverse scattering transform method can be used to study the propagation of two-frequency surface-wave pulses that can pass through a thin film of resonant three-level particles. This method is used to examine, for various combinations of *TE* and *TM* waves, the propagation of a nonlinear surface wave in the simulton regime, and also to study the signals emitted by the film after a definite (for each carrier frequency) time interval following the passage of a refracted Fresnel pulse. The differences in the formation of these signals and the interaction of single-frequency pulses with a thin film of two-level particles are discussed.

1. INTRODUCTION

Much attention is being paid of late in the theory of surface optical waves to nonlinear phenomena. A theory of nonlinear surface waves on the interface between a linear and nonlinear (Kerr) dielectric, and of nonlinear directed waves in planar waveguides, was developed soon thereafter (see the review in Ref. 1). A theory was developed for self-induced transparency (SIT)[2–4] and for related phenomena.[4–7] The first experiments demonstrating the bistable behavior of nonlinear surface waves have been performed.[8,9]

In addition, phenomena accompanying the passage of an optical wave through a nonlinear interface have been attracting interest for a long time.[10–23] These investigations were performed mainly numerically,[13–15] until the publication of a paper[19] in which it was shown how the use of the inverse scattering method (ISM) can yield interesting results. The ISM, unfortunately, has a limited range of applications. For example, if allowance is made for the Lorentz field in the problem[20] of a thin film of resonant atoms on the interface of linear dielectrics, it is impossible to use the ISM to study the refraction by such an interface. This problem, however, is of great interest.

The effects investigated, the possibility of optical bistability,[21–23] and coherent transient processes of the photon-echo type[24] have all stimulated searches for new models (physical situations) of a nonlinear interface of two media. A simple generalization is a transition from a two-level model of atoms in a thin film to a three-level one, assuming an optical pulse having two carrier frequencies that are resonant with the corresponding transitions.

We generalize in the present paper the SIT theory[2,3] to include the case of double resonance. We derive generalized simplified Maxwell–Bloch equations that describe the propagation of a two-frequency ultrashort pulse along a nonlinear interface of two media. We obtain the condition under which SIT is possible and the corresponding pulses are solitons.

The passage of an ultrashort pulse through a nonlinear interface is investigated in greater detail. It is shown that by suitably choosing the incidence angle a situation can be created wherein the ISM method can be used to solve the relevant equations that determine the amplitudes of the transmitted wave. The solution obtained in this situation reveals the existence of an additional pulse that lags the incident pulse in time (a similar result was obtained in Ref. 19). The delay time depends on the problem parameters and on the area of the incident pulse. In contrast to Ref. 19, the delays of waves with different carrier frequencies are unequal, depending on the initial states of the three-level atoms. Using this result, the two-frequency pulse can be resolved in time into two single-frequency ones.

2. DESCRIPTION OF MODELS AND A NONLINEAR INTERFACE OF MEDIA

Let a thin film of atoms resonantly interacting with a two-frequency electromagnetic field be located on the interface of two dielectric media in the $x = 0$ plane. The dielectric media adjoining the film have dielectric constants ε_a for $x < 0$ and ε_b for $x > 0$. The z axis is chosen to be in the interface plane. The resonant atoms are described in the framework of the three-level-atom model corresponding to double resonance in the Λ or V configuration of the energy levels.[25–28] The electromagnetic-field pulses are assumed to be short compared with the polarization-relaxation times and the population difference, but much longer than the optical period, so that the method of slowly varying complex envelopes of the pulse can be used. These will hereafter be called ultrashort pulses (USP).

The thin-film thickness is assumed much smaller than the incident-radiation wavelengths. In such a film, the microscopic electromagnetic field acting on the atom differs, generally speaking, from the macroscopic field and depends on the location of the impurity atom. This dependence can be correctly obtained by taking into account the dipole–dipole interaction between the atoms, as was demonstrated, for example, in Ref. 21. It is also possible to neglect this dependence and assume the difference between the microscopic and macroscopic fields to be equal to the Lorentz field.[20] In a number of cases, however, the difference between the microscopic and macroscopic fields is immaterial. This applies to the analysis of surface waves propagating at sufficiently large distances, and also to reflection and refraction of waves in the case of a thin transition-metal film in a transverse dc quantizing magnetic field, when the film spectrum is discrete. We confine ourselves here to this model situation.

 0038-5646/90/050864-08$03.00

3. PROPAGATION OF ULTRASHORT PULSE ALONG AN INTERFACE

This case corresponds to a generalization of the results of Ref. 2 to double resonance in a three-level medium. Following Ref. 2, equations must be derived for the evolution of a two-frequency USP and of atoms of a thin film. Since the interface is planar, the system of Maxwell equation breaks up into two independent systems describes *TE* waves:

$$\mathbf{E}=(0, E_y, 0), \quad \mathbf{H}=(H_x, 0, H_z)$$

and *TM* waves:

$$\mathbf{E}=(E_x, 0, E_z), \quad \mathbf{H}=(0, H_y, 0).$$

It is convenient to express the strengths **E** and **H** of the fields and the polarization P of the three-level system in the form

$$\mathbf{E}(x,z,t)=\int\frac{d\beta}{2\pi}\frac{d\omega}{2\pi}\mathbf{e}(x,\beta,\omega)\exp[i(\beta z-\omega t)],$$
$$\mathbf{H}(x,z,t)=\int\frac{d\beta}{2\pi}\frac{d\omega}{2\pi}\mathbf{h}(x,\beta,\omega)\exp[i(\beta z-\omega t)],$$
$$\mathbf{P}(z,t)=\int\frac{d\beta}{2\pi}\frac{d\omega}{2\pi}\mathbf{p}(\beta,\omega)\exp[i(\beta z-\omega t)].$$

The fields **e** and **h** are determined by the Maxwell equations outside the film ($x\neq 0$) and by the continuity conditions at $x=0$. For *TE* waves we have

$$\begin{gathered} e_y=e, \quad \partial_x^2 e+(k^2\varepsilon_j-\beta^2)e=0, \\ h_x=-\frac{\beta}{k}e, \quad h_z=-\frac{i}{k}\partial_x e, \\ e(x=0+)=e(x=0-), \\ h_z(x=0+)-h_z(x=0-)=4\pi ikp_y(\beta,\omega), \\ \lim_{|x|\to\infty} e(x,\beta,\omega)=\lim_{|x|\to\infty} h_{x,z}(x,\beta,\omega)=0; \end{gathered} \tag{1}$$

and for *TM* waves

$$\begin{gathered} h_y=h, \quad \partial_x^2 h+(k^2\varepsilon_j-\beta^2)h=0, \\ e_x=(\beta/k\varepsilon_j)h, \quad e_z=(i/k\varepsilon_j)\partial_x h, \\ e_z(x=0+)=e_z(x=0-)=e(\beta,\omega), \\ h(x=0+)-h(x=0-)=4\pi ikp_z(\beta,\omega), \\ \lim_{|x|\to\infty} h(x,\beta,\omega)=\lim_{|x|\to\infty} e_{x,z}(x,\beta,\omega)=0. \end{gathered} \tag{2}$$

In these equations the subscript takes the values $j=i$ or b and we set $k=\omega/c$. We assume in addition that the normal component of the polarization vector of the thin-film atoms is zero.

Consider the case of *TM* waves. Outside the interface we have

$$h(x,\beta,\omega)=\begin{cases} A(\beta,\omega)\exp(-qx), & x>0, \\ B(\beta,\omega)\exp(px), & x<0, \end{cases} \tag{3}$$

where $q^2=\beta^2-k^2\varepsilon_b$, $p^2=\beta^2-k^2\varepsilon_a$ and $p>0$, $q>0$. It follows also from (2) that

$$e_z(x,\beta,\omega)=\begin{cases} -\dfrac{iq}{\varepsilon_b k}A\exp(-qx), & x>0, \\ \dfrac{ip}{\varepsilon_a k}B\exp(px), & x<0. \end{cases}$$

The coefficients $A(\beta,\omega)$ and $B(\beta,\omega)$ are determined from the condition for the continuity of $e_z(x,\beta,\omega)$:

$$A(\beta,\omega)=ik(\varepsilon_b/q)e(\beta,\omega), \quad B(\beta,\omega)=-ik(\varepsilon_a/p)e(\beta,\omega). \tag{4}$$

Using (3) and (4) and the boundary condition for $h(x,\beta,\omega)$ at $x=0$, we can obtain from (2) and equation for $e(\beta,\omega)$ and $p_z(\beta,\omega)$:

$$f_{TM}(\beta,\omega)e(\beta,\omega)=-4\pi p_z(\beta,\omega), \tag{5}$$

where

$$f_{TM}(\beta,\omega)=(\varepsilon_b q^{-1}+\varepsilon_a p^{-1})+4\pi\chi_s(\beta,\omega),$$

$\chi_s(\beta,\omega)$ is the nonresonant susceptibility of the thin film.

Up to now we have not considered the approximation of slowly varying USP envelopes or any information on the frequency makeup of the carrier wave, so that equating (5) to the analogous result of Ref. 2 is legitimate.

The case of *TE* waves can be considered similarly. The expression of type (5) will take here the form

$$f_{TE}(\beta,\omega)e(\beta,\omega)=-4\pi p_y(\beta,\omega), \tag{5'}$$

where

$$f_{TE}(\beta,\omega)=(q+p)/k^2+4\pi\chi_s(\beta,\omega).$$

Outside the interface we have

$$e(x,\beta,\omega)=\begin{cases} e(\beta,\omega)\exp(-qx), & x>0, \\ e(\beta,\omega)\exp(px), & x<0 \end{cases}$$

and

$$h_z(x,\beta,\omega)=\begin{cases} i(q/k)e(\beta,\omega)\exp(-qx), & x>0, \\ -i(p/k)e(\beta,\omega)\exp(px), & x<0, \end{cases}$$

$$h_x(x,\beta,\omega)=-(\beta/k)e(x,\beta,\omega).$$

If the electromagnetic wave is represented in the form

$$\mathbf{E}(x,z,t)=\vec{\mathscr{E}}(x,z,t)\exp[i(\beta_0 z-\omega_0 t)]+\text{c.c.},$$

the Fourier components for **E** and for the envelope $\vec{\mathscr{E}}$ are related by

$$\mathbf{e}(x,\beta_0+\beta,\omega_0+\omega)\approx\vec{\mathscr{E}}(x,\beta,\omega)$$

and we have $\beta\ll\beta_0$, $\omega\ll\omega_0$, this being a consequence of the slowly-varying-envelope approximation. Using this remark, we can obtain from (5) and (5′) similar relations for the Fourier components of the USP envelope. For a two-frequency USP we obtain

$$\mathbf{E}(x,z,t)=\vec{\mathscr{E}}_1(x,z,t)\exp[i(\beta_1 z-\omega_1 t)] +\vec{\mathscr{E}}_2(x,z,t)\exp[i(\beta_2 z-\omega_2 t)]+\text{c.c.}$$

Waves with carrier frequencies ω_1 and ω_2 can be both of *TM* or *TE* type, but can also differ with, for example $\vec{\mathscr{E}}_1$ and $\vec{\mathscr{E}}_2$ corresponding to *TE* and *TM* waves, respectively. For the Fourier components $\vec{\mathscr{E}}_\alpha(x=0,\beta,\omega)$, $\alpha=1,2$, Eq. (5) or (5′) are transformed into

$$\begin{gathered} f_A(\beta_1+\beta,\omega_1+\omega)\mathscr{E}_1(\beta,\omega)=-4\pi\mathscr{P}_1(\beta,\omega), \\ f_B(\beta_2+\beta,\omega_2+\omega)\mathscr{E}_2(\beta,\omega)=-4\pi\mathscr{P}_2(\beta,\omega). \end{gathered} \tag{6}$$

Here for $A = B = TE$ we have

$$\mathscr{E}_\alpha(\beta,\omega)=\mathscr{E}_{\alpha,y}(x=0,\beta,\omega),\ \mathscr{P}_\alpha(\beta,\omega)=\mathscr{P}_{\alpha,y}(\beta,\omega),$$

if $A = B = TM$

$$\mathscr{E}_\alpha(\beta,\omega)=\mathscr{E}_{\alpha,z}(x=0,\beta,\omega),\ \mathscr{P}_\alpha(\beta,\omega)=\mathscr{P}_{\alpha,z}(\beta,\omega),$$

and for $A = TE$, $B = TM$, we have

$$\mathscr{E}_1(\beta,\omega)=\mathscr{E}_{1,y}(x=0,\beta,\omega),\ \mathscr{E}_2(\beta,\omega)=\mathscr{E}_{2,z}(x=0,\beta,\omega),$$
$$\mathscr{P}_1(\beta,\omega)=\mathscr{P}_{1,y}(\beta,\omega),\ \mathscr{P}_2(\beta,\omega)=\mathscr{P}_{2,z}(\beta,\omega).$$

From (6) we can transform in the usual fashion (see, e.g., Ref. 29) to the evolution equations for the slowly varying envelopes $\mathscr{E}_\alpha(x=0,z,t)$ and $\mathscr{P}_\alpha(z,t)$

$$(\partial_z+v_{A1}^{-1}\partial_t)\mathscr{E}_1=4\pi i(\partial f_A/\partial\beta)_1^{-1}\mathscr{P}_1, \qquad (7)$$
$$(\partial_z+v_{B2}^{-1}\partial_t)\mathscr{E}_2=4\pi i(\partial f_B/\partial\beta)_2^{-1}\mathscr{P}_2,$$

where $v_{A\alpha} = (d\omega/d\beta)_{A\alpha}$ is the group velocity of a wave of type A, with carrier ω_α and propagation constant β_α (where $A = TE$ or TM). The values of β_α are determined from the dispersion relations for each type of wave:

$$f_A(\omega_\alpha,\beta_\alpha)=0. \qquad (8)$$

In (7) we have used the notation

$$(\partial f_A/\partial\beta)_\alpha=(\partial f_A/\partial\beta)_{\omega=\omega_\alpha,\beta=\beta_\alpha}.$$

The envelopes of the polarization wave of a thin film are determined in terms of the corresponding density-matrix elements of the three-level medium. Equations (7) must be supplemented by equations for this density matrix.

Let us consider, to be specific, a V configuration of energy levels. The wave with carrier frequency ω_1 is resonant for the $|1\rangle\to|2\rangle$, transition, and the one with carrier ω_2 for the $|1\rangle\to|3\rangle$ transition. Assume that the detuning $\delta\omega$ from exact resonance is the same for both transitions. The equations for the slowly varying element of the density matrix are of the form[26,30]

$$\partial_t u=i\delta\omega u+iq_1n_1+iq_2w^*,\ \partial_t v=i\delta\omega v+iq_2n_2+iq_1w,$$
$$\partial_t w=i(q_1^*v-q_2u^*), \qquad (9)$$
$$\partial_t n_1=2i(q_1^*u-q_1u^*)+i(q_2^*v-q_2v^*),$$
$$\partial_t n_2=i(q_1^*u-q_1u^*)+2i(q_2^*v-q_2v^*),$$

where $q_1 = -d_{21}\mathscr{E}_1/\hbar$, $q_2 = -d_{31}\mathscr{E}_2/\hbar$, $u = \sigma_{21}$, $v = \sigma_{31}$, $w = -\sigma_{32}$, $n_1 = \sigma_{11} - \sigma_{22}$, $n_2 = \sigma_{11} - \sigma_{33}$, d_{ab} is the matrix element of the dipole moment of the $|a\rangle\to|b\rangle$ transition, and σ_{ab} is a slowly varying density-matrix element $(a,b = 1,2,3)$. (For a Λ configuration of the energy levels q_1, q_2, u, v, w, n_1 and n_2 from (9) are connected with σ_{ab} by another rule.)[30]

The envelopes of the polarizations $\mathscr{P}_{1,2}$ are expressed in terms of u and v from (9) as follows:

$$\mathscr{P}_1=-d_{12}\langle u\rangle n_{at},\ \mathscr{P}_2=-d_{13}\langle v\rangle n_{at},$$

where the angle brackets denote summation over all possible detunings $\delta\omega$, and n_{at} is the surface density of the resonant atoms.

It is convenient to introduce new independent variables $T = t - z/v_{A1}$, $X = z/L_{A1}$ and rewrite (7) in dimensionless form

$$\partial_X q_1=i\langle u\rangle, \qquad (7')$$
$$(\partial_X+\mu\partial_T)q_2=i\gamma\langle v\rangle.$$

Here

$$\gamma=\frac{|d_{13}|^2(\partial f_A/\partial\beta)_1}{|d_{12}|^2(\partial f_B/\partial\beta)_2},\qquad \mu=L_{A1}(v_{B2}^{-1}-v_{A1}^{-1}),$$
$$L_{A1}^{-1}=4\pi|d_{12}|^2n_{at}[(\partial f_A/\partial\beta)_1)^{-1}.$$

The system (7') and (9) describes fully the propagation of a two-frequency USP along the interface between two media. It is known,[26,28] however, that these equations are fully integrable if the following conditions (simulton conditions) are satisfied:

$$v_{A1}=v_{B2},\ \gamma=1. \qquad (10)$$

In this case it is possible to use the ISM with a U–V pair of the following form[26,28,30]

$$U=i\begin{pmatrix}-\lambda & q_1 & q_2\\ q_1^* & \lambda & 0\\ q_2^* & 0 & \lambda\end{pmatrix}, \qquad (11a)$$

$$V=\left\langle\frac{i}{\delta\omega+2\lambda}\times\begin{pmatrix}-(n_1+n_2)/3 & u & v\\ u^* & -(n_2-2n_1)/3 & w\\ v^* & w^* & (2n_2-n_1)/3\end{pmatrix}\right\rangle. \qquad (11b)$$

If the populations of the states $|2\rangle$ and $|3\rangle$ are equal both for $t = -\infty$ and for $t = +\infty$ (all the atoms are in the ground state both before the arrival of the USP and after its passage), the single-soliton solution has the usual hyperbolic-secant form. If the initial and final states of the atoms are equal but the initial populations of the excited atoms are different, the single-soliton solution describes the transformation of a two-frequency pulse into a single-frequency one.[26,30] This process was investigated also in great detail in Ref. 31.

It should be noted that the requirement that the simulton condition be met is more stringent in the present problem than in the three-dimensional case,[25–28] although the dependence of the group velocity and of the parameter γ on the dielectric constants ε_a and ε_b does permit some degree of influence on simulton formation. It would be of interest to investigate the propagation of USP in the general case, without the need for satisfying (10). Since the propagation velocities v_{A1} and v_{B2} differ, the USP is expected to have spatially separated frequency components, each propagating independently and either dispersing or evolving into a single-frequency soliton (or solitons). An alternative can be the "wave trapping" investigated in Ref. 32 under the condition $v_{A1} = v_{B2}$, but with $\gamma\neq 1$.

4. REFRACTION OF ULTRASHORT PULSE BY AN INTERFACE

Consider now the case of an electromagnetic wave incident on the interface from the $x<0$ region. The reflected wave propagates back into this region, while the refracted wave propagates into the region $x>0$. This situation is described by Maxwell equations with boundary conditions that are the same as in (1) and (2) at $x=0$, but it is now necessary to change the boundary conditions as $|x|\to\infty$ to take into account waves that are incident and reflected as $|x|\to-\infty$, and refracted nonvanishing waves at $|x|\to+\infty$.

We begin with *TE* waves. The solution of the wave equation for the Fourier component of the intensity of the electric field $e(x,\beta,\omega)$ with allowance for the behavior as $|x|\to\infty$, outside the interface takes the form

$$e(x,\beta,\omega)=\begin{cases} A\exp(ipx)+B\exp(-ipx), & x<0,\\ C\exp(iqx), & x>0,\end{cases}$$

where

$$p^2=k^2\varepsilon_a-\beta^2>0,\ q^2=k^2\varepsilon_b-\beta^2>0,\ p>0,\ q>0.$$

In addition,

$$h_z(x,\beta,\omega)=\begin{cases} (p/k)[A\exp(ipx)-B\exp(-ipx)], & x<0,\\ (q/k)C\exp(iqx), & x>0.\end{cases}$$

The boundary conditions at $x=0$ yield the relations between the amplitudes of the incident (A), reflected (B), and refracted (C) waves and the film polarization p_y:

$$C=\frac{2p}{p+q}A+i\frac{4\pi k^2}{p+q}p_y(\beta,\omega),$$
$$B=\frac{p-q}{p+q}A+i\frac{4\pi k^2}{p+q}p_y(\beta,\omega). \quad (12)$$

It is possible to introduce in the usual manner the incidence (θ^{in}), reflection (θ^{r}) and refraction (θ^{tr}) angles:

$$p=k\varepsilon_a^{1/2}\cos\theta^{in},\ q=k\varepsilon_b^{1/2}\cos\theta^{tr},$$
$$\beta=k\varepsilon_a^{1/2}\sin\theta^{in}=k\varepsilon_b^{1/2}\sin\theta^{tr}. \quad (13)$$

We consider now *TM* waves. From (2) we obtain, taking into account the boundary conditions as $|x|\to\infty$,

$$h(x,\beta,\omega)=\begin{cases} A\exp(ipx)+B\exp(-ipx), & x<0,\\ C\exp(iqx), & x>0,\end{cases}$$

$$e_z(x,\beta,\omega)=\begin{cases} -(p/k\varepsilon_a)[A\exp(ipx)-B\exp(-ipx)], & x<0,\\ -(q/k\varepsilon_b)C\exp(iqx), & x>0,\end{cases}$$

where p and q are defined by the same equations as in the *TE*-wave case. Using the boundary conditions at $x=0$, we obtain the connections between the amplitudes A, B, and C of the magnetic field intensities $h(x,\beta,\omega)$:

$$C=\frac{2\varepsilon_b p}{\varepsilon_b p+\varepsilon_a q}A-\frac{4\pi ik\varepsilon_b p}{\varepsilon_b p+\varepsilon_a q}p_z(\beta,\omega),$$
$$B=\frac{\varepsilon_b p-\varepsilon_a q}{\varepsilon_b p+\varepsilon_a q}A+\frac{4\pi i\varepsilon_a q}{\varepsilon_b p+\varepsilon_a q}p_z(\beta,\omega). \quad (14)$$

In this case we need to have, in place of (14), the relations between the electric-field amplitudes $e_z(x=0,\beta,\omega)=e(\beta,\omega)$. Defining the electric-field incident-wave amplitude $e_z^{in}(0-,\beta,\omega)$ as

$$e_z^{in}(\beta,\omega)=-pA/k\varepsilon_a,$$

we obtain in place of (14)

$$e_z^{tr}(0,\beta,\omega)=\frac{2q\varepsilon_a}{\varepsilon_b p+\varepsilon_a q}e_z^{in}(\beta,\omega)+\frac{4\pi ipq}{\varepsilon_b p+\varepsilon_a q}p_z(\beta,\omega). \quad (15)$$

It is convenient to introduce the notation

$$R_{TE}(\beta,\omega)=\frac{2p}{p+q},\quad \varkappa_{TE}(\beta,\omega)=\frac{4\pi k^2}{p+q},$$

$$R_{TM}(\beta,\omega)=\frac{2\varepsilon_a q}{\varepsilon_b p+\varepsilon_a q},\quad \varkappa_{TM}(\beta,\omega)=\frac{4\pi pq}{\varepsilon_b p+\varepsilon_a q}$$

and combine the results (12) and (15) into a single expression

$$e_A^{tr}(\beta,\omega)=R_A(\beta,\omega)\,e_A^{in}+i\varkappa_A(\beta,\omega)p_A(\beta,\omega). \quad (16)$$

Here, for $A=TE$, we have

$$e_A^{tr}=e_y^{tr},\ e_A^{in}=e_y^{in},\ p_A=p_y$$

and for $A=TM$,

$$e_A^{tr}=e_z^{tr},\ e_A^{in}=e_z^{in},\ p_A=p_z.$$

Relation (16) is general: it is independent of the type of resonant atom and of the frequency makeup of the incident wave, and is valid for plane or nonplane waves. Although we shall consider here only the case $\varepsilon_a<\varepsilon_b$, when there is no total internal reflection, expression (16) can be extended also to include the case $\varepsilon_a>\varepsilon_b$. At an incident angle exceeding the critical angle at which total internal reflection takes place, it is necessary simply to replace q in (16) by $i(\beta^2-k^2\varepsilon_b)^{1/2}=iq'$.

Attention should also be called to the expression for the magnetic-field amplitude of the reflected wave, i.e., to the coefficient B in (14). If $p_z=0$ (there is no thin film), there is no y component of the magnetic field of the *TM* wave at a certain incidence angle determined from the condition $\varepsilon_b p=\varepsilon_a q$. This angle $\operatorname{tg}\theta^{in}=(\varepsilon_b/\varepsilon_a)^{1/2}$ is known as the Brewster angle. The presence of a thin film of resonant atoms, as follows from (14), destroys the Brewster effect.

Just as in the case of surface waves, it is possible here to proceed to consider the passage of a USP with a plane front through an interface, and obtain in place of (16) the connection between the pulse envelopes of the incident and refracted waves, on the one hand, and the polarization envelope that varies slowly with time. The carrier-wave frequencies ω_1 and ω_2 are chosen to meet the condition of double resonance with the three-level atoms of the film, while β_1 and β_2 are given by the incidence angles of the corresponding waves. Neglecting the derivatives of the transmission coefficients $R_A(\beta,\omega)$ and of the coupling constants $\varkappa_A(\beta,\omega)$, with respect to ω, we obtain equations that generalize the results of Ref. 17 and 19 to include the case of a two-frequency USP:

$$\mathscr{E}_{A1}^{tr}(t)=R_A(\beta_1,\omega_1)\mathscr{E}_{A1}^{in}(t)+i\varkappa_A(\beta_1,\omega_1)\mathscr{P}_{A1}(t),$$
$$\mathscr{E}_{B2}^{tr}(t)=R_B(\beta_2,\omega_2)\mathscr{E}_{B2}^{in}(t)+i\varkappa_B(\beta_2,\omega_2)\mathscr{P}_{B2}(t). \quad (17)$$

Various combinations of different polarizations of the fields of a two-frequency USP are possible: $A = B = TE$ or TM and $A = TE$, $B = TM$ or, conversely, $A = TM$ and $B = TE$.

A very important item in Ref. 19 was the remark that an expression such as (17) can be replaced by a differential equation with a singular right-hand side. This device can be applied here too. We introduce auxiliary functions $\mathscr{E}_{A\alpha}(x,t)$ satisfying the equations

$$\partial_x \mathscr{E}_{A\alpha} = 2i\varkappa_A(\beta_\alpha, \omega_\alpha)\delta(x)\mathscr{P}_{A\alpha}(t). \tag{18}$$

For $x = 0$ these functions are defined by the relation

$$\mathscr{E}_{A\alpha}(0, t) = {}^1/_2[\mathscr{E}_{A\alpha}(0+, t) + \mathscr{E}_{A\alpha}(0-, t)]. \tag{19}$$

Integrating (18) from $x = -\infty$ to $x = -\varepsilon$ ($\varepsilon \ll 1$), we obtain

$$\mathscr{E}_{A\alpha}(x=-\varepsilon, t) = \mathscr{E}_{A\alpha}(x=-\infty, t) \equiv \mathscr{E}_{A\alpha}^{(-)}(t).$$

If Eq. (18) will be integrated from $x = -\varepsilon$ to $x = \varepsilon$, we obtain

$$\mathscr{E}_{A\alpha}(x=\varepsilon, t) = \mathscr{E}_{A\alpha}(x=-\varepsilon, t) + 2i\varkappa_A(\beta_\alpha, \omega_\alpha)\mathscr{P}_{A\alpha}(t).$$

It follows from this, if (19) is used, that

$$\mathscr{E}_{A\alpha}(0, t) = \mathscr{E}_{A\alpha}(x=-\varepsilon, t) + i\varkappa_A(\beta_\alpha, \omega_\alpha)\mathscr{P}_{A\alpha}(t).$$

This expression coincides with (17) if we assume the identities

$$\mathscr{E}_{A\alpha}(0, t) \equiv \mathscr{E}_{A\alpha}{}^{tr}(t),$$
$$\mathscr{E}_{A\alpha}(x=-\varepsilon, t) \equiv R_A(\beta_\alpha, \omega_\alpha)\mathscr{E}_{A\alpha}{}^{in}(t).$$

Supplementing (17) or (18) with equations that determine the polarizations $\mathscr{P}_{A\alpha}(t)$, we can solve the initial problem of USP refraction by an interface, using the following algorithm: a) determine the initial conditions (18) for the specified value of $\mathscr{E}_{A\alpha}^{in}(t)$; b) solve (18) for the indicated initial condition for $\mathscr{E}_{A\alpha}(0-, t)$ and $\mathscr{P}_{A\alpha}$ yielding $\mathscr{E}_{A\alpha}(t)$ for $x > 0$ and consequently $\mathscr{E}_{A\alpha}(0+, t)$; c) determine the envelopes $\mathscr{E}_{A\alpha}^{tr}(t)$ of the transmitted ultrashort pulse in accordance with (19):

$$\mathscr{E}_{A\alpha}{}^{tr}(t) = {}^1/_2[R_A(\beta_\alpha, \omega_\alpha)\mathscr{E}_{A\alpha}{}^{in}(t) + \mathscr{E}_{A\alpha}(0+, t)]. \tag{19'}$$

For the equations that specify the evolution of the state three-level atoms, we should take (9) with $q_1 = d_{21}\mathscr{E}_{A1}^{tr}/\hbar$, $q_2 = d_{31}\mathscr{E}_{B2}^{tr}/\hbar$; the definitions of the remaining variables are the same as before.

It can be directly verified that if the condition

$$|d_{12}|^2\varkappa_A(\beta_1, \omega_1) = |d_{13}|^2\varkappa_B(\beta_2, \omega_2) \tag{20}$$

is met the system of equations (9) and (18) is the zero-curvature condition for the U–V pair (11), the only difference being that now V (Eq. 11b) should by multiplied by $\tau_s^{-1}\delta(x)$, where

$$\tau_s^{-1} = n_{at}|d_{12}|^2\varkappa_A(\beta_1, \omega_1)\hbar^{-1}.$$

The condition (20) is similar to the condition for the existence of a simulton regime of propagation of a two-frequency USP in a semi-infinite homogeneous medium.[25–28] Before we consider the solution of the system (9) and (18), let us dwell in greater detail on the "simulton" condition (20) which appears in the investigated problem. It is expedient to introduce two auxiliary functions $F_{TE}(\theta)$ and $F_{TM}(\theta)$ of the incidence angle $\theta_\alpha = \theta^{in}(\omega_\alpha)$ of each of the carrier waves of the two-frequency USP:

$$F_{TE}(\theta) = \cos\theta + (\Delta^2 + \cos^2\theta)^{1/2},$$

$$F_{TM}(\theta) = \frac{1}{\cos\theta} + \frac{1+\Delta^2}{(\Delta^2 + \cos^2\theta)^{1/2}},$$

where $\Delta^2 = (\varepsilon_b - \varepsilon_a)/\varepsilon_b > 0$. The condition (20) can be expressed in terms of the following functions:

for $A = TE$, $B = TE$,

$$F_{TE}(\theta_1) = |d_{13}{}^y/d_{12}{}^y|^2(\omega_1/\omega_2)F_{TE}(\theta_2);$$

for $A = TM$, $B = TM$

$$F_{TM}(\theta_1) = |d_{13}{}^z/d_{12}{}^z|^2(\omega_1/\omega_2)F_{TM}(\theta_2);$$

for $A = TE$, $B = TM$

$$F_{TE}(\theta_1) = |d_{13}{}^y/d_{12}{}^z|^2(\omega_1/\omega_2)F_{TM}(\theta_2).$$

We denote the factor preceding the function $F_B(\theta_2)$ by ξ. The question is then whether suitable incidence angles θ_1 and θ_2 can be found to satisfy the simulton condition (20), which reduces to finding the solution of the equation

$$F_A(\theta_1) = \xi F_B(\theta_2). \tag{21}$$

Figure 1 shows plots of the functions $F_{TE}(\theta)$ and $F_{TM}(\theta)$. Since the function $F_{TM}(\theta)$ increases without limit as $\theta \to \pi/2$, it can be seen that Eq. (21) has a solution for any parameter ξ for $A = B = TM$. By the same token, this is the most favorable case for satisfying the simulton condition. For $A = TE$ and $B = TM$, there exists at $\xi < 1$ a value $\theta_1 = \theta_2$ for which (21) is satisfied, and if θ_2 is given one can obtain $\theta_1 (\neq \theta_2)$ for which (21) is also satisfied. Equation (21) cannot be solved for $\xi > 1$. In this case it is necessary to interchange the wave types, i.e.,choose $A = TM$ and $B = TE$. There must then exist θ_1 and θ_2 pairs for which (21) holds.

The least favorable choice is $A = B = TE$. Given the dielectric constant difference $\varepsilon_a - \varepsilon_b$ there exists an interval of the values of the parameter ξ,

$$\xi_{min} \leqslant \xi \leqslant \xi_{max},$$

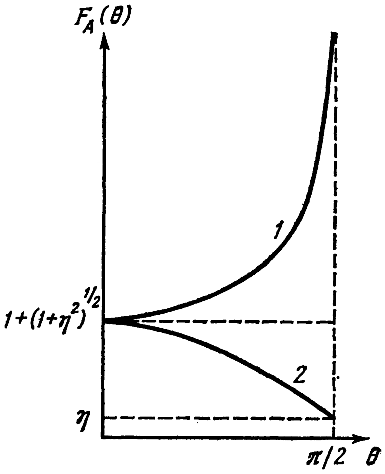

FIG. 1. Plots of auxiliary functions $F_A(\theta)$:1–$A = TM$, 2–$A = TE$.

for which, given θ_2, we can find a value of θ_1 that ensures satisfaction of the simulton condition. Here

$$\xi_{min}=\Delta/[1+(1+\Delta^2)^{1/2}],\quad \xi_{max}=\xi_{min}^{-1}.$$

It is curious that it is preferable in this case to use media for which $\varepsilon_b-\varepsilon_a \ll \varepsilon_b$. The solution of (21) for $A=B=TE$ can be written explicitly as

$$\cos\theta_1=\Delta\,\mathrm{sh}\,\ln[\xi F_{TE}(\theta_2)/\Delta].$$

Similarly, for $A=TE$ and $B=TM$ we have

$$\cos\theta_1=\Delta\,\mathrm{sh}\,\ln[\xi F_{TM}(\theta_2)/\Delta].$$

Consider now the solution of Eqs. (9) and (18) by the ISM, assuming beforehand that the three-level system returns to the initial state after passage of the USP. This means that at $t\to\pm\infty$ the matrix $V(\lambda,t)$ takes the form of a diagonal matrix $V^{(\pm)}(\lambda)$:

$$V^{(+)}=V^{(-)}$$

$$=\frac{i\delta(x)}{3\tau_s}\left\langle\frac{1}{\delta\omega+2\lambda}\,\mathrm{diag}\{-(n_{10}+n_{20}),2n_{10}-n_{20},2n_{20}-n_{10}\}\right\rangle,$$

where n_{10} and n_{20} are the differences of the populations of the three-level system as $t\to-\infty$.

From the given values of $\mathscr{E}^{in}_{Aa}(t)$ or their normalized values $q_a^{in}(t)$, solving the spectral problem (11a) for $U(\lambda)$, we can determine the scattering data, viz., the transition matrix $T^{in}(\lambda,x<0)$ and the eigenvalues (discrete spectrum $\{\lambda_n\}^{in}$. To find $T^{out}(\lambda)=T(\lambda,x>0)$ and $\{\lambda_n\}^{out}$ we can use the reasoning of Ref. 19 and generalize it to cover the case analyzed here. If we have $V^{(+)}=V^{(-)}$, then $T(\lambda,x)$ is defined by the equation

$$\partial_x T=[T,\ V^{(-)}].\qquad(22)$$

Since $V^{(-)}$ is diagonal, the diagonal elements of the matrix T are independent of x; consequently

$$T_{aa}^{out}(\lambda)=T_{aa}^{in}(\lambda).\qquad(23)$$

From this we have $\{\lambda_n\}^{out}=\{\lambda_n\}^{in}$. For the off-diagonal elements T_{ab} we have

$$\partial_x T_{ab}=(V_b^{(-)}-V_a^{(-)})T_{ab},\qquad(24)$$

if $V^{(-)}(x,\lambda)=\mathrm{diag}(V_1^{(-)},V_2^{(-)},V_3^{(-)})$. What makes the situation unusual is that $V_a^{(-)}$ contains a delta function. Following Ref. 19, the definition of $T_{ab}(\lambda,x)$ is completed at the point $x=0$:

$$T_{ab}(\lambda,\ 0)={}^1/_2\{T_{ab}(\lambda,\ 0+)+T_{ab}(\lambda,\ 0-)\}.\qquad(25)$$

We put

$$V_b^{(-)}-V_a^{(-)}=2\delta(x)\Gamma_{ab}.\qquad(26)$$

Integrating (24), taking (26) into account, with respect to x from $x=-\infty$ to $x=-\varepsilon(\varepsilon>0,\varepsilon\ll1)$, we obtain

$$T_{ab}(x=-\varepsilon)=T_{ab}(x=-\infty)=T_{ab}^{in}(\lambda).$$

Integrating (24) from $x=-\varepsilon$ to $x=\varepsilon$ we find

$$T_{ab}(\lambda,\ x=\varepsilon)=T_{ab}(\lambda,\ x=-\varepsilon)+2\Gamma_{ab}(\lambda)T_{ab}(\lambda,\ 0).$$

From this, taking (25) into account, it follows that

$$T_{ab}(\lambda,x=\varepsilon)=\frac{1+\Gamma_{ab}(\lambda)}{1-\Gamma_{ab}(\lambda)}T_{ab}^{in}(\lambda).\qquad(27)$$

Since the right-hand side of (27) does not depend on x, we have

$$T_{ab}^{out}(\lambda)=T_{ab}(\lambda,x=0+)=\frac{1+\Gamma_{ab}(\lambda)}{1-\Gamma_{ab}(\lambda)}T_{ab}^{in}(\lambda).\qquad(28)$$

The solutions of Eqs. (9) and (18) are obtained as solutions of the inverse problem using the scattering data $\{\lambda_n\}^{out}$, $T_a^{out}(\lambda)$ and Eq. (19'). Manakov's spectral problem, which is used here, is the result of the analysis of self-focusing of polarized radiation[33] and of the theory of self-induced transparency,[34,35] so that we can build on known results. To reconstruct the $q_a(t,x)$ that enter $U(\lambda,x,t)$ we need the quantity $r_a(\lambda,x)$ defined as

$$r_1(\lambda)=T_{12}(\lambda)/T_{11}'(\lambda),\quad r_2(\lambda)=T_{13}(\lambda)/T_{11}'(\lambda),$$

where $T_{11}'(\lambda)=dT_{11}/d\lambda$. Using the explicit form of $V^{(-)}$, we can find the necessary quantities Γ_{12} and Γ_{13}:

$$\Gamma_{12}(\lambda)=\frac{i}{2\tau_s}\left\langle\frac{n_{10}}{\delta\omega+2\lambda}\right\rangle,\quad \Gamma_{13}(\lambda)=\frac{i}{2\tau_s}\left\langle\frac{n_{20}}{\delta\omega+2\lambda}\right\rangle.\qquad(29)$$

In addition, at the discrete-spectrum points we can represent r_a^{in} in the form

$$r_a^{in}(\lambda_n)=2\eta_n l_a^{(n)}\exp[2\eta_n t_{0n}],\quad \alpha=1,\ 2,\qquad(30)$$

where

$$\lambda_n=\xi_n+i\eta_n,\quad t_{0n}=(2\eta_n)^{-1}\ln|r_0^{in}(\lambda_n)|,$$

$$l_a=r_a^{in}(\lambda_n)/r_0^{in}(\lambda_n),\quad r_0^{in}(\lambda_n)=\left[\sum_\alpha r_a^{in}(\lambda_n)r_a^{in*}(\lambda_n)\right]^{1/2}.$$

Denoting

$$\frac{i}{2\tau_s}\left\langle\frac{n_{a0}}{\delta\omega+2\lambda}\right\rangle=R_a^{(n)}+iI_a^{(n)},$$

we get

$$r_a^{out}(\lambda_n)=2\eta_n l_a^{(n)}\exp(2\eta_n t_{0n})\frac{1+R_a^{(n)}+iI_a^{(n)}}{1-R_a^{(n)}-iI_a^{(n)}}.\qquad(31)$$

If the inhomogeneously broadened line is symmetric in shape, and the USP incident on the interface is such that $\zeta_n=0$, we have $I_a^{(n)}=0$ and

$$r_a^{out}(\lambda_n)=2\eta_n l_a^{(n)}\exp[2\eta_n(t_{0n}+t_{an})],\qquad(32)$$

where

$$t_{an}=(2\eta_n)^{-1}\ln[(1+R_a^{(n)})/(1-R_a^{(n)})].\qquad(33)$$

It is useful to compare (30) and (32) with the analogous expressions from the theory of self-induced transparency.[34] This yields a rule for obtaining the needed results from those already known. Namely, wherever the path Z covered by the pulse occurs in an expression, the substitution $R_a Z\to 2\eta_n t_{an}$ yields an expression relevant to the problem considered here.

By way of example, let the incident USP have an envelope $\mathscr{E}^{in}_{Aa}(t)$ such that

$$(d_{12}/\hbar)R_A(\beta_\alpha,\omega_\alpha)\mathscr{E}_{A\alpha}{}^{in}(t)=2\eta l_\alpha \operatorname{sech}[2\eta(t-t_0)]. \quad (34)$$

The solution of the spectral problem is then known:

$$\lambda_1=i\eta,\quad T_{11}{}^{in}=\frac{\lambda-i\eta}{\lambda+i\eta},\quad T_{12}{}^{in}=T_{13}{}^{in}=T_{21}{}^{in}=T_{31}{}^{in}=0$$

(the remaining matrix elements are unnecessary); $r_\alpha^{in}(\lambda_1) = 2\eta l_\alpha \exp(2\eta t_0)$. The solution (18) with $x>0$, obtained by the ISM, can be written down right away, using the rule formulated above:

$$q_\alpha(t,x>0)=\frac{d_{12}}{\hbar}\mathscr{E}_{A\alpha}(t,x>0)=2\eta l_\alpha \exp[2\eta(t_0-t+t_{\alpha1})]D^{-1},$$

$$D=1+\exp[4\eta(t_0-t)]\{l_1^2\exp(4\eta t_{11})+l_2^2\exp(4\eta t_{21})\}. \quad (35)$$

For the case of a V configuration of the energy levels we have $R_1^{(1)}=R_2^{(1)}$, since $n_{10}=n_{20}$, if all the atoms are in the ground state at $t=-\infty$. Expression (35) simplifies to

$$q_\alpha(t,x>0)=2\eta l_\alpha \operatorname{sech}[2\eta(t_0-t+t_1)], \quad (36)$$

where

$$t_1=t_{11}=t_{21}=(2\eta)^{-1}\ln[(1+R_1^{(1)})/(1-R_1^{(1)})],$$

$$R_1^{(1)}=\frac{\eta n_{10}}{2\tau_s}\left\langle\frac{1}{4\eta^2+\delta\omega^2}\right\rangle.$$

The envelope of the refracted USP takes, according to (19), (34), and (36), the form

$$\mathscr{E}_{A\alpha}{}^{tr}(t)={}^1/_2 R_A(\beta_\alpha,\omega_\alpha)[\mathscr{E}_{A\alpha}{}^{in}(t)+\mathscr{E}_{A\alpha}{}^{in}(t-t_1)]. \quad (37)$$

Thus, just as in Ref. 19, the refracted USP consists of two subpulses—the first due to the jump of the dielectric constants (Fresnel subpulse) and the second due to the resonant atom and therefore delayed in time relative to the first. Note the logarithmic dependence of the delay time t_1 on the parameter $R_1^{(1)}$. The value of this parameter depends both on the peak value of the electric field intensity of the incident USP and on the incidence angle (via τ_s).

For a Λ configuration of the resonance levels, if all the atoms are initially in the ground state, we have $n_{10}=1$ and $n_{20}=0$. It follows then from (35) that

$$q_2(t,x>0)=2\eta l_2\frac{\exp[2\eta(t_0-t)]}{1+\exp[4\eta(t_0-t)]\{l_1^2\exp(4\eta t_{11})+l_2^2\}},$$

$$q_1(t,x>0)=2\eta l_1\frac{\exp 2\eta(t_0-t+t_{11})}{1+\exp[4\eta(t_0-t)]\{l_1^2\exp(4\eta t_{11})+l_2^2\}}.$$

These envelopes do not have the shape of a hyperbolic secant, and consequently do not duplicate the shape of the incident pulse. In addition, their time delays are different. This constitutes the difference from the case of two-level atoms.[19] In general, if the atoms of a thin film are pre-excited in the initial state, the delays of the frequency components of the USP are not equal.

5. CONCLUSION

The foregoing analysis of the propagation of a surface nonlinear wave is restricted to the simulton regime. As is made clear here, this condition is difficult to meet, for in addition to equality of the resonance absorption lengths (which is necessary in the three-dimensional case), the group velocities must be equal. For surface waves the propagation constant $\beta(\omega_\alpha)$ at the carrier frequency ω_α has a complicated dependence on ω_α, and equality of the derivatives of β with respect to ω at the points ω_1 and ω_2 would be surprising. Further investigation of the process of propagation of a surface two-frequency USP requires a numerical solution of the system (7') and (9).

In the analysis of the passage of USP through a nonlinear interface, attention was focused here also on cases that admit of an exact solution by the ISM. If the angle of incidence of each carrier wave is suitably chosen, the simulton condition (20) is easily satisfied. An interesting result is the appearance of a time-delayed pulse. Only in a particular case is the delay time the same for both frequencies of the carrier wave, while in general they are different. Attention should be called to the fact that the delayed pulses correspond in this paper to the discrete spectrum of the Manakov problem. At definite incidence angles [provided that they guarantee satisfaction of the simulton condition (20)], there may be no discrete spectrum at all, while at other incidence angles it appears. An investigation of the parameters of the delayed pulse with variation of the incidence angles, just as the solution of refraction of small-area USP (when there is no discrete spectrum at any incidence angle) calls for numerical simulation of this process.

The ISM can likewise not be used when the USP is incident on a nonlinear interface from a medium with a larger refractive index, and the incidence angle exceeds the total-internal-reflection angle. An investigation of the process of the reflection of USP in this case requires again numerical simulation.

In conclusion, we wish to point out also the need in many cases of taking the Lorentz field into account and of generalizing the results of Ref. 20.

[1] D. Mikhalke, R. G. Nazmitdinov, and V. K. Fedyanin, Elemantarn. Chastitsy, Atomy i Yadra **20**, 198 (1989) [Sov. J. Part, Nucl. **20**, 86 (1989)].
[2] V. M. Agranovich, V. I. Rupasov, and V. Ya. Chernyak, Pis'ma Zh. Eksp. Teor. Fiz. **33**, 196 (1981) [JETP Lett. **33**, (1981)] Fiz. Tverd. Tela (Leningrad) **24**, 2992 (1982) [Sov. Phys. Solid State **24**, 1693 (1982).
[3] H. E. Ponath and M. Schubert, Optica Acta **30**, 1139 (1983).
[4] Hung Xi-Yi, Lin Ji-teng, and T. F. George, Z. Phys. B **50**, 181 (1983).
[5] N. N. Akhmediev, Zh. Eksp. Teor. Fiz. **84**, 1907 (1983) [Sov. Phys. JETP **57**, 1111 (1983)].
[6] G. T. Adamashvili, *ibid.* **92**, 2202 (1987) [**65**, 1242 (1987)].
[7] G. T. Adamashvili, Phys. Lett. A **120**, 73 (1987).
[8] P. W. Smith and W. J. Tomlinson, IEEE J. **QE-20**, 30 (1984).
[9] H. Vach, C. T. Seaton, G. I. Stegeman, and I. C. Khoo, Opt. Lett. **9**, 238 (1984).
[10] A. E. Kaplan, Zh. Eksp. Teor. Fiz. **72**, 1710 (1977) [Sov. Phys. JETP **45**, 896 (1977)].
[11] N. N. Rozanov, Pis'ma v ZhTF **4**, 74 (1978) [Sov. Tekh. Phys. Lett. **4**, 30 (1978)]. Opt. Spektrosk. **47**, 606 (1979) [Opt. Spectrosc. (USSR) **47**, 335 (1979)].
[12] P. W. Smith, J. P. Hermann, W. J. Tomlinson, and P. J. Moloney, Appl. Phys. Lett. **35**, 846 (1979); IEEE J. **QE-17**, 340 (1981).
[13] D. Marcuse, Appl. Opt. **19**, 3130 (1980).
[14] J. Tomlinson, J. P. Gordon, P. W. Smith, and A. E. Kaplan, Appl. Opt. **21**, 2041 (1982).
[15] M. G. Benedict and I. Guemant, Acta Phys. et Chem. Szeged **30**, No. 3–4, 115 (1984); **31**, No. 3–4, 695 (1985).
[16] W. Chen and D. L. Mills, Phys. Rev. B **35**, 524 (1987).
[17] V. I. Rupasov and V. I. Yudson, Kvantovaya Elektron. (Moscow) **9**, 2179 (1982) [Sov. J. Quantum Electron. **12**, 1415 (1982)].
[18] N. N. Rozanov and A. V. Fedorov, Opt. Spektrosk. **59**, 1355 (1985)

[Opt. Spectrosc. (USSR) **59**, 810 (1985)].
[19] V. I. Rupasov and V. I. Yudson, Zh. Eksp. Teor. Fiz. **93**, 494 (1987) [Sov. Phys. JETP **66**, 282 (1987).
[20] G. Benedict, A. I. Zaitsev, V. A. Malyshev, and E. D. Trifonov, Opt. Spektrosk. **66**, 726 (1989) [Opt. Spectrosc. (USSR) **66**, 424 (1989)].
[21] Y. Ben-Arye, C. M. Bowden, and J. C. Englund, Phys. Rev. A **34**, 3917 (1986).
[22] S. M. Zakharov and E. A. Manykin, Poverkhnost', No. 2, 137 (1988).
[23] A. M. Basharov, Zh. Eksp. Teor. Fiz. **94**, 12 (1988) [Sov. Phys. JETP **67**, 1741 (1988)].
[24] S. M. Zakharov and E. A. Manykin, Opt. Spektrosk. **63**, 1069 (1987) [Opt. Spectrosc. (USSR) **63**, 630 (1987)].
[25] M. I. Konopnitski and J. H. Eberly, Phys. Rev. A **24**, 2567 (1981).
[26] A. I. Maĭmistov, Kvantovaya Elektron. (Moscow) **11**, 567 (1984) [Sov. J. Quantum Electron. **14**, 385 (1984).
[27] L. A. Bol'shov and V. V. Likhanskiĭ, *ibid.* **12**, 1339 (1985) [**15**, 889 (1985)].
[28] L. A. Bol'sov, V. V. Likhanskiĭ, and M. I. Persiantsev, Zh. Eksp. Teor. Fiz. **84**, 903 (1983) [Sov. Phys. JETP **57**, 524 (1983)].
[29] A. M. Basharov, A. I. Maĭmistov, and E. A. Manykin, *Photonics. Nonlinear Coherent Processes* [in Russian], Moscow Eng. Phys. Inst. (1986).
[30] A. I. Maĭmistov and Yu. M. Sklyarov, Opt. Spektrosk. **59**, 760 (1985) [Optl. Spectrosc. (USSR) **59**, 459 (1985)].
[31] L. A. Bol'sov, N. N. Elkin, V. V. Likhanskiĭ, and M. I. Persiantsev, Zh. Eksp. Teor. Fiz. **94**, 101 (1988) [Sov. Phys. JETP **87**, 2013 (1988)].
[32] L. A. Bol'sov, N. N. Elkin, V. V. Likhanskiĭ, and M. I. Persiantsev, *ibid.* **88**, 47 (1985) [**61**, 27 (1985)].
[33] S. V. Manakov, *ibid.* **65**, 505 (1973) [**38**, 248 (1974)].
[34] A. M. Basharov and A. I. Maĭmistov, *ibid.* **87**, 1594 (1984) [**60**, 913 (1984)].
[35] V. Ya. Chernyak and V. I. Rupasov, Phys. Lett. A **108**, 434 (1985).

Translated by J. G. Adashko