Spin diffusion and relaxation of two-domain structures in ^{3}He-B

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Equations are obtained which describe the relaxation of two-domain structures in the B-phase of ^{3}He as a consequence of spin diffusion through domain boundaries. The dependence of the velocity of domain wall motion on the isotropic part of the spin diffusion coefficient for weak magnetic fields is found for two possible types of two-domain structures, which differ in the orientation of the anisotropy axis of the order parameter in the static domains. The shapes of the domain walls are also obtained for certain values of a parameter which characterizes the ratio between the rate of the diffusion process and the rate of flow of the spin current.

The two-domain structures which arise in the superfluid B-phase of ^{3}He when it spins precess in a weakly inhomogeneous time-independent magnetic field has already been investigated in detail both theoretically[1] and experimentally.[2–4] The two-domain structure is a stationary solution to the equations of spin dynamics when the dissipation of the Zeeman energy is neglected. The presence of a gradient in the time independent magnetic field causes the spin precession to decompose in space into two domains—a dynamic domain and a static domain.

In the dynamic domain, the spin deviates from its equilibrium orientation by an angle β slightly larger than $\theta_0 = \arccos(-1/4)$, and precesses with a frequency ω_P which equals the Larmor frequency at the location of the boundary separating the domains; in the static domain the spin has its equilibrium direction. The dynamic domain is located in a region of relatively weak magnetic fields. The small deviation of the angle β from θ_0 is caused by a rather sizable local shift in the frequency of the free-induction signal in ^{3}He-B (for the expression for the frequency shift see Ref. 5), which compensates for the spatial variation of the Larmor frequency. The dissipation of energy leads to relaxation of the structure: the dimensions of the dynamic magnetic domain decrease, and the domain wall begins to move in the direction of smaller magnetic fields. The motion of the wall is caused by variation of the free-induction signal frequency.

The relaxation of the dynamic magnetic domain occurs because of two mechanisms: a surface relaxation mechanism involving spin diffusion through a domain wall of thickness on the order to

$$\lambda = \left(\frac{c_\parallel^{\,2}}{\omega_p \nabla \omega_L} \right)^{1/3}$$

($c_\parallel$ is the velocity of one of the two types of spin waves and $\nabla\omega_L$ is the gradient of the Larmor frequency for a linear dependence of the magnetic field **H** on the z-coordinate), and an "internal" bulk Leggett-Takagi relaxation mechanism,[6] which comes into play when the local Larmor frequency does not coincide with the precession frequency. The diffusive mechanism dissipates the Zeeman energy; this in turn leads to motion of the domain wall with a constant velocity W that depends on the spin diffusion coefficient D, and to a corresponding linear decrease of the frequency of the long-lived induction signal with time. This decrease is observable due to the in-phase character of the spin motion in the dynamic domain. In contrast, the contribution to the rate of variation of the precession frequency caused by the Leggett-Takagi relaxation mechanism is proportional to the cube of the size of the dynamic domain in the z-direction; this allows us to separate out the diffusion mechanism by extrapolating the experimental time dependence of the long-lived induction original frequency to a zero-size dynamic domain. Using this extrapolation procedure, we can use the rate of variation of the signal frequency, which is related to the velocity of domain motion through the equation $d\omega_P/dt = \nabla\omega_L W$, to obtain the spin diffusion coefficient D and the effective Leggett-Takagi relaxation time $\tau_{\rm eff}$ which determines the effectiveness of the bulk energy dissipation mechanism.[6] We will assume that the hydrodynamic approximation $\omega_P \tau_{\rm eff} \ll 1$ is applicable.

Experiments in which the spin diffusion coefficient is determined by varying the rate of change of the LLIS frequency were carried out in Ref. 3. In this paper a comparison was made with the results of a theoretical calculation,[1] based on the assumption that the relaxation caused by the spin diffusion through the domain boundary is slow, i.e., the parameter $\tilde{D} = D\omega_P/c_{11}^2$ is assumed to be small ($\tilde{D} \ll 1$). As a consequence of this, it was assumed in the calculation that the shape of the domain wall (i.e., the distribution of spins along the z-direction) is the same as in the static case. However, in these experiments[3] the parameter $\tilde{D}$ was not always small: for temperatures close to T_c it is of order unity. Therefore, a more precise determination of the spin diffusion coefficient from experiment requires that the equations of spin dynamics by solved again, this time without neglecting the diffusive terms.

The order parameter in ^{3}He-B is proportional to $R(\mathbf{n}, \theta)$, i.e., to the matrix which represents a rotation around the direction of the anisotropy axis **n** by an angle θ. It is more convenient to describe the motion of the order parameter by using the Euler angles.[1,7]

$$\hat{R}(\mathbf{n}, \theta) = \hat{R}(\alpha, \beta, \gamma) = \hat{R}_z(\alpha)\hat{R}_y(\beta)\hat{R}_z(\gamma),$$

where $R_x(\alpha)$ is the matrix for a rotation around the z-axis by an angle α, etc. (In what follows, the z-axis will be assumed to be the direction opposite the magnetic field **H**). The Leggett equation,[8] which describes the motion of the spin density **S** and the order parameter in a magnetic field, can be written in the form of Hamilton's equations in the variables $\alpha, \beta, \Phi = \alpha + \gamma$ and the combinations of the spin projections of **S** which are canonically conjugate to them:

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$P = S_z - S_\zeta, S_\beta$ and S_ζ (S_z is the projection of **S** along the z-axis, S_ζ is along the $\zeta = \hat{R}\mathbf{z}$ axis and S_β is along the line of nodes $[\mathbf{z}, \boldsymbol{\zeta}]$. Let us choose the normalization in such a way that $\chi/g^2 = 1$ (χ is the magnetic susceptibility per unit volume of ^{3}He-B, and g is the gyromagnetic ratio for ^{3}He-B). The density of the Leggett Hamiltonian in these variables is written in the following way:

$$\mathscr{H} = \frac{1}{1+\cos\beta}\left[S_\zeta^2 + PS_\zeta + \frac{P^2}{2(1-\cos\beta)}\right] - \omega_L(z)(P+S_\zeta) + \frac{1}{2}S_\beta^2 + U_D(\beta,\Phi) + F_\nabla. \quad (1)$$

The dipole energy U_D and the gradient energy F_∇ have the form

$$U_D(\beta, \Phi) = {}^2/_{15}\Omega^2[\cos\beta - {}^1/_2 + (1+\cos\beta)\cos\Phi]^2, \quad (2)$$

$$F_\nabla = {}^1/_2 c_\parallel^2[2(1-\cos\beta)\alpha'(\alpha'-\Phi') + \Phi'^2 + \beta'^2] - (c_\parallel^2 - c_\perp^2)[(1-\cos\beta)\alpha' - \Phi']^2, \quad (3)$$

where Ω is the frequency of the longitudinal resonance, $c_\parallel$ and $c_\perp$ are the velocities of the two types of waves, and the primes denote derivatives with respect to z. The constant magnetic field is assumed in this case to be weakly inhomogeneous, and the vector $\nabla\omega_L$ is directed along the z-axis. The Hamiltonian equations for the Hamiltonian $\mathscr{H}$ have the form

$$\frac{\partial\alpha}{\partial t} = \frac{\partial\mathscr{H}}{\partial P}, \quad \frac{\partial P}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\alpha},$$
$$\frac{\partial\beta}{\partial t} = \frac{\partial\mathscr{H}}{\partial S_\beta}, \quad \frac{\partial S_\beta}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\beta},$$
$$\frac{\partial\Phi}{\partial t} = \frac{\partial\mathscr{H}}{\partial S_\zeta}, \quad \frac{\partial S_\zeta}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\Phi}.$$

Here the sign δ denotes a variational derivative:

$$\frac{\delta\mathscr{H}}{\delta\alpha} = \frac{\partial\mathscr{H}}{\partial\alpha} - \frac{\partial}{\partial z}\left(\frac{\partial\mathscr{H}}{\partial\alpha'}\right) = \frac{\partial\mathscr{H}}{\partial\alpha} - \frac{\partial}{\partial z}\left(\frac{\partial F_\nabla}{\partial\alpha'}\right)$$

In these equations the diffusive and Leggett-Takagi relaxation terms are absent. We will assume that the size of the dynamic magnetic domain is sufficiently small that the bulk relaxation mechanism need not be included; as we have already mentioned, this contribution is important only when the dimensions of the dynamic domain and gradients of the magnetic field are sufficiently large, and decreases rapidly as the size of the domain shrinks. For this reason, we add to the equations only the terms connected with spin diffusion. Note that in ^{3}He the quantity D is a tensor of spin diffusion coefficients $D_{ik\zeta\eta}$. In the B-phase, symmetry considerations allow us to write this tensor in the form[9]

$$D_{ik\zeta\eta} = D\delta_{ik}\delta_{\zeta\eta} + D_1(A_{i\zeta}A_{k\eta} + A_{i\eta}A_{k\zeta}).$$

Here D and D_1 are respectively the isotropic and anisotropic parts of the tensor of spin diffusion coefficients, and $A_{i\zeta}$ is the instantaneous structure of the order parameter at a point z at time t. However, we know in advance that after expanding the rate of energy dissipation in space and time derivatives of **S** in the longitudinal geometry, the final result will contain only the isotropic part of the tensor of spin diffusion coefficients. Therefore, without any loss of generality we can immediately retain in the equations only the isotropic part of the tensor, i.e., D.

From this we see that in three of the six spin dynamic equations (specifically, in the equations for the rate of change of the spin density projections $\dot{P}$, $\dot{S}_\zeta$, and $\dot{S}_\beta$) it is necessary to add the projection of the term $D\Delta\mathbf{S}$ on the axes z, ζ, and the line of nodes. It is clear that as a zeroth approximation we need to substitute the stationary solution which describes the Larmor precession into the diffusion term. This solution has the form

$$S_x = \omega_p\sin\beta\cos\alpha,$$
$$S_y = \omega_p\sin\beta\sin\alpha, \quad (4)$$
$$S_z = \omega_p\cos\beta.$$

The variational equation then acquires the form

$$\frac{\partial P}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\alpha} + D\omega_p[\beta'^2(1-\cos\beta) - \beta''\sin\beta + \alpha'^2\sin^2\beta],$$
$$\frac{\partial S_\beta}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\beta} + D\omega_p[\alpha'' + 2\alpha'\beta'\operatorname{ctg}\beta],$$
$$\frac{\partial S_\zeta}{\partial t} = -\frac{\delta\mathscr{H}}{\delta\Phi} - D\omega_p[\alpha'^2\sin^2\beta + \beta'^2]. \quad (5)$$

Let us replace the variable α by the variable

$$\psi = \alpha + \int_0^t \omega_p(t)\,dt,$$

replace the Hamiltonian $\mathscr{H}$ by a new Hamiltonian $\tilde{\mathscr{H}} = \mathscr{H} + \omega_P P$, and divide it into two parts $\tilde{\mathscr{H}}_0 + V$, where $\tilde{\mathscr{H}}_0$ is the part of the Hamiltonian which does not depend on the coordinate z and spatial derivatives, while V is a perturbation having the form

$$V = F_\nabla - (P+S_\zeta)(\omega_L(z) - \omega_p).$$

The system of equations now appears as follows:

$$0 = \frac{\partial P}{\partial t} + \frac{\delta V}{\delta\psi} - D\omega_p[\beta'^2(1-\cos\beta) - \beta''\sin\beta + \psi'^2\sin^2\beta],$$
$$\frac{\partial\tilde{\mathscr{H}}^{(0)}}{\partial\beta} = -\frac{\partial S_\beta}{\partial t} - \frac{\delta V}{\delta\beta} + D\omega_p[\psi'' + 2\psi'\beta'\operatorname{ctg}\beta],$$
$$\frac{\partial\tilde{\mathscr{H}}^{(0)}}{\partial\Phi} = -\frac{\partial S_\zeta}{\partial t} - \frac{\delta V}{\delta\Phi} - D\omega_p[\psi'^2\sin^2\beta + \beta'^2],$$
$$\frac{\partial\tilde{\mathscr{H}}^{(0)}}{\partial P} = \frac{\partial\psi}{\partial t} - \frac{\partial V}{\partial P},$$
$$\frac{\partial\tilde{\mathscr{H}}^{(0)}}{\partial S_\beta} = \frac{\partial\beta}{\partial t},$$
$$\frac{\partial\tilde{\mathscr{H}}^{(0)}}{\partial S_\zeta} = \frac{\partial\Phi}{\partial t} - \frac{\partial V}{\partial S_\zeta}. \quad (6)$$

Use of the stationary solution as a zero-order approximation is legitimate only if the diffusion and gradient terms in the equations of motion are small compared to the magnetic and dipole terms. The condition for the gradient terms to be small is $\lambda \gg c_\parallel/\Omega$, while the condition for the diffusion terms to be small leads to the inequality $\tilde{D} \ll \Omega^2\lambda^2/c_\parallel^2 \sim 10^2$, which is clearly fulfilled in experiments.[3]

In order to find a solution in the form of a moving domain wall, let us take as the zeroth-order approximation the stationary periodic solution from Ref. 7:

$$P = \omega_p(\cos\beta - 1), \quad S_\beta = 0, \quad S_\zeta = \omega_p,$$
$$\cos\Phi = ({}^1/_2 - \cos\beta)/(1+\cos\beta).$$

The angles β and Φ can vary from zero to $\theta_0 = \arccos(-1/4)$. We can ignore the fact that the angle β can be larger than θ_0, because of the insignificance of this deviation ($\beta_{\max} - \theta_0 < 10^{-1}$ rad), which implies that the energy dissipation caused by the gradient of the angle β in the dynamic domain is negligibly small. Substituting the stationary solution into the right sides of these equations and applying the procedure described in Ref. 10, we can at once reduce the six equations to just two in the variables ψ and $u = \cos\beta$. In differentiating the projection of the spin with respect to time we can neglect the weak time dependence of ω_P. In contrast to the system obtained in Ref. 10, this system of equations, which describes the low-frequency dynamics of the order parameter, contains the diffusion terms.

$$\omega_p \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left\{2c^2(u)(1-u)\psi' - c^2(-1)(1-u)\frac{d\Phi}{du}u'\right\} + D\omega_p\left[u'' + (1-u^2)\psi'^2 + \frac{u'^2}{1-u^2}\right],$$

$$\omega_p\left(\frac{\partial\psi}{\partial t} + \omega_L(z) - \omega_p\right) = -\frac{d\Phi}{du}\left\{\frac{\partial}{\partial z}\left\{c^2(-1)\left[\frac{d\Phi}{du}u' - (1-u)\psi'\right]\right\} - D\omega_p\left[(1-u^2)\psi'^2 + \frac{u'^2}{1-u^2}\right]\right\}$$
$$+ [c^2(1) + 2c^2(u)]\psi'^2 + c^2(-1)\psi'\frac{d\Phi}{du}u' + \frac{c^2(1)}{(1-u^2)^2}[-uu'^2 - (1-u^2)u''] + D\omega_p\left[\psi'' - \frac{2uu'\psi'}{1-u^2}\right]. \quad (7)$$

Here

$$c^2(u) = uc_\parallel^2 + (1-u)c_\perp^2, \quad \left(\frac{d\Phi}{du}\right)^2 = \frac{3}{(1+4u)(1+u)^2}.$$

The two possible signs of $d\Phi/du$ indicate that in fact it is possible to form two types of two-domain structures and correspondingly domain walls which differ in the direction of the anisotropy axis **n** of the order parameter in the static domain: the vector **n** can be directed either opposite the magnetic field ($d\Phi/du > 0$) or along the field ($d\Phi/du < 0$), whereas **n**⊥**H** is the dynamic domain.

Due to the presence of dissipation caused by diffusion through the domain wall, there are no stationary solutions to the system of equations (7); therefore we need to seek solutions which describe a moving wall. If we take into account that the precession frequency of the two-domain structure, as in the earlier cases, equals the Larmor frequency at the location of the domain wall at a given instant of time, i.e.,

$$\omega_p(t) = \omega_0 + \nabla\omega_L(z_0 - Wt),$$
$$\omega_L(z) - \omega_p(t) = \nabla\omega_L(z - z_0 + Wt),$$
$$\alpha = \psi - (\omega_0 + \nabla\omega_L z_0)t + \nabla\omega_L W t^2/2 \quad (8)$$

(where ω_0 is the Larmor frequency at some point in the experimental vessel whose coordinate will be assumed to equal zero, z_0 is the coordinate of the domain wall at the initial instant of time, and W is a constant), this system admits solutions of the form $\psi = \psi(z - z_0 + Wt)$, so that all the functions in the equations now depend only on the combination $z - z_0 + Wt$. The coordinate z_1 of the domain wall is determined from the vanishing of this argument, i.e., from the condition $z_1 - z_0 + Wt = 0$. The sign in front of W corresponds for $W > 0$ to the relaxation of the wall in the direction of negative z. From the system of partial differential equations we pass to a system of ordinary differential equations by replacing $\partial\psi/\partial t$ by $W\partial\psi/\partial z$ and introducing the dimensionless coordinate $\xi = (z - z_0 + Wt)/\lambda$:

$$\frac{\partial}{\partial\xi}\left\{(1-u)\left[2\tilde{c}^2(u)\psi' - \tilde{c}^2(-1)\frac{d\Phi}{du}u' + \tilde{W}\right]\right\} + \tilde{D}\left[u'' + (1-u^2)\psi'^2 + \frac{u'^2}{1-u^2}\right] = 0,$$

$$\frac{d\Phi}{du}\frac{\partial}{\partial\xi}\left\{\tilde{c}^2(-1)u\frac{d\Phi}{du}u' + [2\tilde{c}^2(u) - \tilde{c}^2(-1)](1-u)\psi' + \tilde{W}(1-u) + \tilde{D}u'\right\}$$
$$+ \xi + \tilde{W}\psi' - [1 - 2\tilde{c}^2(u)]\psi'^2 - \tilde{c}^2(-1)\psi'\frac{d\Phi}{du}u' + \frac{[uu'^2 + (1-u^2)u'']}{(1-u^2)^2} - \tilde{D}\left[\psi'' - \frac{2uu'\psi'}{1-u^2}\right] = 0. \quad (9)$$

Here $\tilde{D} = D\omega_P/c_\parallel^2$ and $W = \tilde{W}\lambda\omega_P/c_\parallel^2$ are dimensionless parameters, and we introduce the notation $\tilde{c}^2(u) = c^2(u)/c_\parallel^2$; the dash in Eq. (9) denotes differentiation with respect to ξ. To the two branches of $\Phi(u)$ there correspond two solutions to this system of equations for a given value of $\tilde{D}$, or to two possible configurations of the spin with oppositely directed vectors **n** in the stationary domain (opposite and along **H**), moving with two different velocities $\tilde{W}(\tilde{D})$.

As boundary conditions let us take the conditions that the spin current equal zero at the boundaries of the chamber. This implies that the flux density S_z in the direction z, i.e.,

$$j_{zz} = \frac{c_\parallel^2}{\lambda}\{-u\tilde{c}^2(-1)\Phi' - (1-u)[2\tilde{c}^2(u) - \tilde{c}^2(-1)]\alpha' - \tilde{D}u'\}$$

must vanish as $\xi \to \pm\infty$. Using these boundary conditions for the spin current, we find from the system of equations the asymptotc forms for $u(\xi)$ and $\psi'(\xi)$, starting from the fact that $u(\xi)$ goes to one (or to $-1/4$) exponentially as $\xi \to +\infty$ $(-\infty)$. The system of differential equations we have obtained must be solved numerically, using the asymptotic forms we have found for $u(\xi)$ and $\psi'(\xi)$. Solving the system for the two branches of $\Phi(u)$ separately for various values of the parameter D, we find the two corresponding distributions $u(\xi)$ and $\psi'(\xi)$ which characterize the shape of the moving domain wall. In this case, for each branch of $\Phi(u)$ the value of $\tilde{W}(\tilde{D})$ is unambiguously determined. The function $\tilde{W}(\tilde{D})$ can also be found from the expression

$$\tilde{W} = \frac{4}{5}\tilde{D}\int_{-\infty}^{+\infty}[\psi'^2\sin^2\beta + \beta'^2]d\xi, \quad (10)$$

which is obtained by integrating the first equation of the system (9), and coincides with the corresponding expression in Ref. 1. Starting with Eq. (10), we can also estimate the insignificant variation in the shape of the domain wall caused by the gradual decrease in the precession frequency. For this we introduce a parameter which characterizes the shape of the wall:

$$\sigma = \int_{-\infty}^{\infty}[\psi'^2\sin^2\beta + \beta'^2]d\xi$$

and estimate its change over the dimensions of the vessel

$$\frac{\Delta\sigma}{\sigma}\sim\frac{\Delta\omega_p}{\omega_p}\sim 10^{-2}.$$

The results for the velocity $\tilde{W}$ and shape of the domain wall obtained by integrating the equations numerically are shown in Figs. 1–3. Figure 1 illustrates the computed dependence of the dimensionless velocity of domain wall motion $\tilde{W}$ on the dimensionless spin diffusion coefficient $\tilde{D}$. Curve *1* in this figure corresponds to a moving two-domain structure in which the anisotropy axis **n** within the static domain is directed antiparallel to the magnetic field **H** (i.e., the branch of the function $\Phi(u)$ with $d\Phi/du>0$). Curve *2* shows the same function for a structure in which the static domain is characterized by a vector **n** which is parallel to **H** (i.e., $d\Phi/du>0$). It is significant that the spin configuration for which the anisotropy axis in the static domain is directed parallel to the magnetic field dissipates faster than the configuration with **n** in the static domain directed antiparallel to the magnetic field. For large values of the parameter $\tilde{D}$ the difference in wall velocities becomes significant. For comparison, in Fig. 1, we show the straight line *3* obtained by assuming that $\tilde{D}$ is small ($\tilde{D}\ll 1$; see Ref. 1) and the corresponding function $\tilde{W}=4/5\tilde{D}\sigma$ for $\sigma\approx 1.1$.

In Ref. 1., as a zeroth approximation Fourier substituted into Eq. (10) the functions $u(\xi)$ and $\psi'(\xi)$ calculated by neglecting dissipation, i.e., for a stationary wall; thus,

$$u_0^1(\xi)=u_0^2(\xi),\quad (\psi_0^1(\xi))'=-(\psi_0^2(\xi))'$$

(the superscripts 1 and 2 distinguish the two possible two-domain structures) and $\tilde{W}^1(\tilde{D})$ coincides with $\tilde{W}^2(\tilde{D})$.

It is quite clear from Fig. 1 that traces 1–3 differ only slightly from one another in the range of values $\tilde{D}\leqslant 0.5$; therefore the value of the spin diffusion coefficient D does not deviate significantly from the number which is extracted from the experimental data on the basis of the calculation in Ref. 1, since most of these experiments were carried out for $\tilde{D}\lesssim 0.5$. However, we can now extend the temperature limits of experiments to determine the coefficient of spin diffusion. The lower limit remains as before (on the order of $0.4T_c$ to $0.5T_c$), since at lower temperatures the hydrodynamic approximation is inapplicable; however, by lifting the old restriction on $\tilde{D}$ we can approach closer to T_c, as the ratio $D\omega_P/c_\parallel^2$ cna be larger because $c_\parallel^2$ goes to zero as $T\to T_c$ ($c_\parallel\propto[1-T/T_c]^{1/2}$).

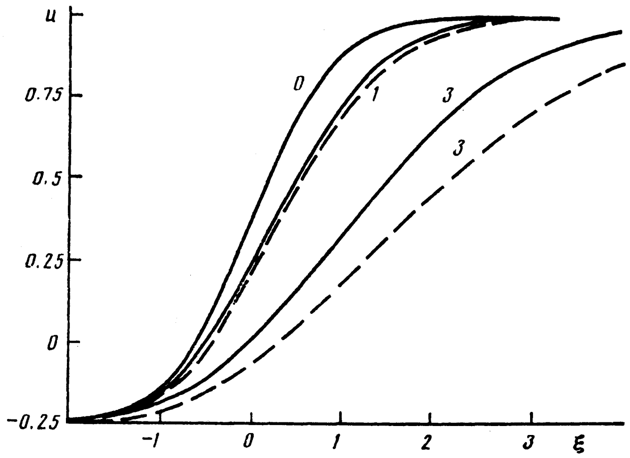

FIG. 2. Shape of the wall $u(\xi)$ for three values of the parameter (the figures next to the curves correspond to values of $\tilde{D}$) and two types of wall: $d\Phi/du>0$ (solid curves) and $d\Phi/du<0$ (dashed curves).

In Figs. 2 and 3 we show the shapes of the walls, i.e., the functions $u(\xi)$ and $\psi'(\xi)$, for several values of $\tilde{D}$ and for the two possible orientations of **n** in the static domain. From these figures it is clear that diffusion has a rather significant effect on the thickness and shape of the wall. For $\tilde{D}=0$ the shape $u(\xi)$ is the same for both walls; as $\tilde{D}$ increases both walls are smeared out and begin to differ in shape: the wall with the larger translational velocity $\tilde{W}^2$ also has the larger thickness. Our results are especially useful for investigating different types of two-domain structures, because we cannot even speak of one structure being energetically favored over another when dissipation is present as we can in a situation with spin influx.[11] The experimental data[2,3] lie essentially in the region where the difference of velocity of the two walls is not large. In principle, however, for sufficiently large $\tilde{D}(\tilde{D}-1)$ the difference in wall velocities can be used to observe the two types of wall.

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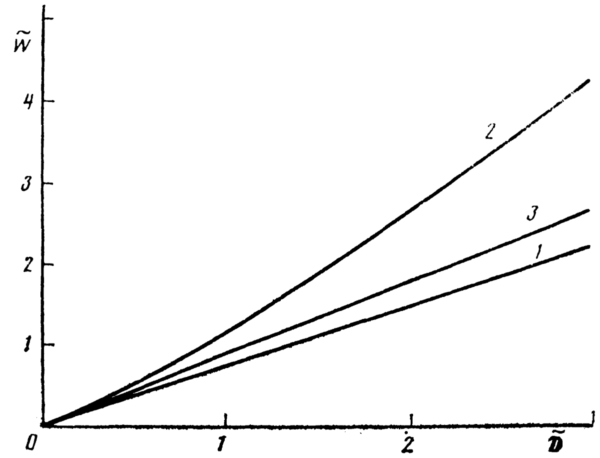

FIG. 1. The dependence of $\tilde{W}$ on $\tilde{D}$: 1—$d\Phi/du>0$.2—$d\Phi/du<0$, 3—the calculation of Ref. 1.

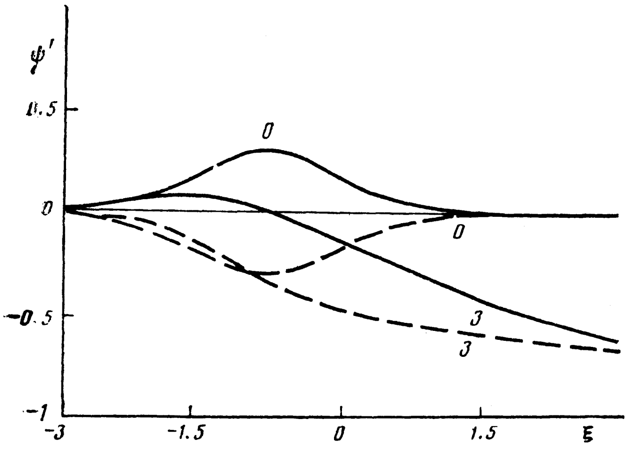

FIG. 3. The function $\psi'(\xi)$ for two values of the parameter (the figures next to the curves) and two types of wall: $d\Phi/du>0$ (solid curves) and $d\Phi/du<0$ (dashed curves).

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