Collective pinning of soliton lattice in Josephson junctions

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(Submitted 14 September 1989)
Zh. Eksp. Teor. Fiz. **97**, 976–989 (March 1990)

The field dependence of the critical current through a large, nonuniform Josephson junction is found. In weak magnetic fields, in the region of a sparse soliton lattice, the pinning of an individual Josephson vortex is independent of the pinning of others, and the critical current does not depend on the field. As the field is increased, the pinning becomes collective, and the critical current begins to decrease sharply. When the distance between vortices reaches a value on the order of the Josephson penetration depth, a plateau appears on the field dependence of the critical current. An independence of the critical current from the field in strong fields is characteristic of disordered Josephson junctions of any size. The effect of thermal fluctuations in the lattice and of the self-magnetic field of the external current on the critical current is also studied. A plateau of the critical current in strong fields has been noted in numerous experimental studies of the field dependence of the critical current of ceramic high T_c superconducting samples.

1. INTRODUCTION

The properties of the vortex structures in Josephson junctions have recently attracted considerably more interest, primarily because of the active research on the new superconducting ceramics. It has now been established reliably that these ceramics are a set of grains weakly linked by Josephson junctions. Evidence for this model comes from the huge difference (several orders of magnitude) between the transport critical current and the current found from magnetic measurements; further evidence comes from the sharp decrease in the transport critical current with increasing magnetic field.[1–7] Dimos *et al.*[8,9] have reported direct evidence that the grain boundaries are Josephson junctions. Furthermore, there is the possibility that single crystals are also broken up into domains which are linked by Josephson junctions.

This model implies a certain hierarchy in the macroscopic magnetic properties of high T_c superconductivity. In particular, in fields below the critical field H_{c1} for penetration into grains or domains, the magnetic, transport, or microwave properties of superconducting ceramics or single crystals are governed by vortex structures which form at grain boundaries or twin boundaries. One can work from the typical fields which suppress the critical current or which increase the microwave absorption of dense ceramic high T_c superconductors ($\sim$10–50 Oe) to estimate the typical Josephson penetration depth: $\delta_J \sim 1\ \mu$m. This figure is about an order of magnitude smaller than the typical grain size $L \sim 10\ \mu$m. One would thus expect that the Josephson junctions which form at the grain boundaries in dense ceramics are "long."

Our purpose in the present study was to learn about the behavior of the critical currents through a long, nonuniform Josephson junction as a function of the field and the temperature. The field dependence of the critical current density through a Josephson junction has been studied in many places (there is a fairly comprehensive review of this work in, for example, the book by Barone and Paterno[10]). In a uniform junction with linear dimensions much smaller than the Josephson penetration depth δ_J (a "short" junction), the well-known "Fraunhofer" dependence holds:

$$j_c(H)=j_{c0}\frac{\sin(\pi\Phi/\Phi_0)}{\pi\Phi/\Phi_0}, \qquad (1)$$

where j_{c0} is the critical current density at $H=0$, Φ_0 is the quantum of magnetic flux, and Φ is the flux through the junction. In a short junction with random variations in the density of the Josephson critical current, $j_{c0}(\mathbf{r}) = j_{c0} + \delta j_{c0}(\mathbf{r})$, the current j_c does not vanish with increasing magnetic field; it instead reaches a constant value[10,12]

$$j_{c\,min}=[r_0^2\langle(\delta j_{c0})^2\rangle/S]^{1/2}, \qquad (2)$$

where r_0 is a typical size of the fluctuations, and S is the junction area. In a long, uniform junction, the existence of a nonvanishing critical current in a magnetic field is a consequence of the existence of a surface barrier for the penetration of vortices into the junction. This problem was studied numerically in Ref. 13.

In the case which we are discussing here, that of a long nonuniform Josephson junction, the critical current is determined by the pinning of the vortex structure in the junction. To estimate the critical current, we will use methods which have been used by Larkin and Ovchinnikov for a lattice of Abrikosov vortices.[14,15] An important feature of the curves of $j_c(H)$ which are found is the appearance of a plateau in strong fields, as in the case of a short disordered junction. In the case at hand, the current density j_c is independent of H in strong fields because various factors which determine the collective pinning of the soliton lattice cancel out. It is interesting to note that a saturation of the $j_c(H)$ curves in strong fields has been observed in several studies of ceramic high T_c superconductors.[4–7]

In a previous study by Feĭgel'man and one of the present authors,[16] it was shown that the short coherence length of the high T_c superconductors and the high superconducting transition temperature promote thermal fluctuations of the vortex structure to a governing role in shaping the field and temperature dependence of the critical current. Thermal fluctuations sharply reduce the effective pinning force. The method developed by Larkin and Ovchinnikov[14] has also been used to estimate the critical current; when thermal

 0038-5646/90/030547-07$03.00

fluctuations are taken into account, the critical current is reduced to an extent which depends on the ordinary Debye-Waller factor.[5] We will show that thermal fluctuations in the soliton lattice strongly influence the critical current of only small Josephson junctions. In an infinite junction, in contrast, thermal fluctuations have a completely negligible effect on the temperature dependence of the critical current.

2. FORMULATION OF THE PROBLEM

The energy of a planar, nonuniformly distributed Josephson junction is given by[17]

$$E=\int_0^{L_x} dx\int_0^{L_z} dz\left\{E_J\left[\frac{\delta_J^2}{2}(\nabla\theta)^2+1-\cos\theta\right]\right.$$
$$\left.+v(\mathbf{r})(1-\cos\theta)-\frac{\hbar}{2e}j\theta\right\},\qquad(3)$$

where $\theta=\theta_1-\theta_2$ is the gradient-invariant phase difference between superconductors which are in contact, $E_J=\hbar j_{c0}/2e$ (j_{c0} is the average critical current density at $H=0$), $\delta_J=(c\Phi_0/8\pi^2\Lambda j_{c0})^{1/2}$ is the Josephson penetration depth, $\Lambda=d+2\lambda$ (d is the thickness of the junction, and λ is the London penetration depth), $H_{c1}=2\Phi_0/\pi^2\Lambda\delta_J$ is the Josephson critical field, j is the current density through the junction and $v(\mathbf{r})$ is a random potential associated with spatial fluctuations of the critical current density j_{c0}. The statistical properties of the random potential are described by the correlation function

$$\langle v(\mathbf{r})v(\mathbf{r}')\rangle=\gamma\delta(\mathbf{r}-\mathbf{r}'),$$
$$\gamma=E_J^2r_0^2q,\quad q=\langle(\delta j_{c0})^2\rangle/j_{c0}^2,\qquad(4)$$

where q is the amplitude of the current fluctuations, and r_0 is a typical length scale of the fluctuations. We direct the z axis along the magnetic field $\mathbf{H}$, and we direct the x axis perpendicular to $\mathbf{H}$; the xz plane is the plane of contact, and $\mathbf{r}$ is a vector in this plane.

In the absence of this random potential, the equilibrium phase distribution $\theta_0(x)$ is given by[17]

$$\theta_0(x)=2\,\mathrm{am}\,(x/\delta_J,k)-\pi,\qquad(5)$$

where $\mathrm{am}(x)$ is the elliptic amplitude, and k is the elliptic modulus. With $0<k<1$, this formula describes a soliton lattice with a period $a=2kK(k)\delta_J$ [$K(k)$ is the complete elliptic integral of the first kind]. The relationship between the parameter k and the external magnetic field is not universal; it is determined by the particular geometry of the sample. If we choose a thin-plate sample, and if we direct the field parallel to the plate, we can ignore the demagnetizing factor, and the parameter k will be related to the external magnetic field by

$$E(k)=kH/H_{c1},\qquad(6)$$

where $E(k)$ is the complete elliptic integral of the second kind. In this simple case the lattice period a is related to the external magnetic field by

$$a=\begin{cases}2\delta_J\ln(4/k'), & H-H_{c1}\ll H_{c1},\\ (\pi^2H_{c1}/2H)\delta_J, & H\gg H_{c1},\end{cases}$$

where

$$\frac{k'^2}{4}\ln\frac{16e}{k'^2}=\frac{H-H_{c1}}{H_{c1}},$$

and $k'^2=1-k^2$ is the conjugate modulus. In this case the magnetic field penetrates into the junction as a chain of vortices which lie in the contact plane. We assume that the current is low and the disorder slight:

$$\gamma\ll E_J^2\delta_J^2,\qquad(7a)$$
$$j\ll j_{c0}.\qquad(7b)$$

In this case the phase distribution in the junction is slightly deformed and can be described by

$$\theta(\mathbf{r},t)=\theta_0(x-u(\mathbf{r},t)),\qquad(8)$$

where the length scale of the variation in $u(\mathbf{r},t)$ is far larger than the lattice period. In contrast with a lattice of Abrikosov vortices,[15] any monotonic phase distribution can be described by (8) if the function $u(\mathbf{r},t)$ is chosen appropriately. In the case of a weak perturbation of the phase, on the other hand, this function has the meaning of the displacement of the soliton lattice.

Substituting (8) into (3), we find the effective Hamiltonian

$$E=\int_{-u(0)}^{L_x-u(L_x)}d\tilde{x}\int_0^{L_z}dz\left[\frac{C(\tilde{x})}{2}(\nabla u)^2-\left(1+\frac{\partial u}{\partial\tilde{x}}\right)v(z,\tilde{x}+u)p(\tilde{x})\right.$$
$$\left.-\left(1+\frac{\partial u}{\partial\tilde{x}}\right)\beta(\tilde{x})j\right],\qquad(9)$$

where

$$\tilde{x}=x-u,\quad C(\tilde{x})=E_J\delta_J^2(\partial\theta_0/\partial\tilde{x})^2,\quad p(\tilde{x})=2\,\mathrm{cn}^2\,(\tilde{x}/\delta_J,k),$$
$$\beta(\tilde{x})=(\hbar/2e)\theta(\tilde{x}),$$

and $\mathrm{cn}(x)$ is the elliptic cosine.

3. CRITICAL CURRENT AT $T=0$. DIMENSIONALITY ESTIMATE

The critical current at absolute zero can be found on the basis of simple dimensionality considerations, as in the case of the collective pinning of a lattice of Abrikosov vortices.[15] We assume strong magnetic fields $H\gg H_{c1}$. The interaction with the random potential destroys the long-range order in the soliton lattice, while a short-range order survives within a correlation area $S_c=R_{cx}R_{cz}$, where R_{cx} and R_{cz} are correlation radii. The critical current in this case is determined by the equality of the Lorentz force and the pinning force acting on the correlation area:[15]

$$\frac{\Phi_0}{ca}j_c=f\left(\frac{n}{S_c}\right)^{1/2},\qquad(10)$$

where n is the density of pinning centers, f is the average force of the interaction between a pinning center and the soliton lattice, and c is the velocity of light. In strong magnetic fields we would have $f\sim U/a$, where U is a characteristic potential [U and n are related to the parameter γ in (4) by $\gamma=U^2n$]. The correlation radii R_{cx} and R_{cz} can be found from the balance condition involving the strain energy and the energy of the interaction with the random potential. The result is the estimate

$$R_{cx} \sim R_{cz} \sim 4Ca^2/\gamma^{1/2}. \tag{11}$$

Substituting this estimate into (10), we find the following expression for the critical current:

$$j_c \sim c\gamma/4\Phi_0 Ca^2. \tag{12}$$

Using values of the parameters $C = H^2\lambda/2\pi$ and $a = \Phi_0/2\lambda H$ valid in the strong-field limit, we can express j_c and $R_{cx,z}$ in terms of physical characteristics of the problem

$$j_c \approx j_{c0}\frac{q}{2}\left[\frac{r_0}{4\pi\delta_J}\right]^2, \quad H \gg H_{c1}, \tag{13a}$$

$$R_{cx} \sim R_{cz} \sim (2\pi\delta_J)^2/r_0 q^{1/2}, \quad H \gg H_{c1}. \tag{13b}$$

Note an important aspect of this result: In strong fields $H \gg H_{c1}$ the critical current does not vanish and instead reaches a constant value.

This estimate is valid if the spatial dimensions of the junction are greater than the correlation length R_c. If, on the other hand, the size of the junction in the direction of the magnetic field, L_z, is smaller than R_c (a "one-dimensional" junction), a similar analysis yields

$$j_c^{(1)} \sim (j_{c0}/2)(r_0^2 q/4\pi L_z\delta_J)^{2/3}, \tag{14a}$$

$$R_{cx}^{(1)} \sim 4\pi\delta_J(\pi\delta_J L_z/r_0^2 q)^{1/3}. \tag{14b}$$

From (13) and (14) we find the useful relation

$$R_c \sim \delta_J (j_{c0}/j_c)^{1/2}$$

between the correlation radius and quantities which can actually be measured.

In the case of a point contact (L_x, $L_z \ll R_c$), on the other hand, we find

$$j_c^{(0)} \sim j_{c0} r_0 q^{1/2}/(L_x L_z)^{1/2}. \tag{15}$$

This result was derived previously[10-12] for a short, disordered junction (L_x, $L_z \ll \delta_J$). We see that the random potential causes the correlation length $R_c \gg \delta_J$ to replace the Josephson depth δ_J as the length scale which distinguishes long Josephson junctions from short ones. Note also that the critical current j_c remains independent of the magnetic field in the strong-field limit in all three cases.

We now consider a sparse soliton lattice ($a \gg \delta_J$). In this case the pinning is determined by centers at the cores of Josephson vortices, and the typical force exerted by one pinning center on the lattice can be estimated as $f \sim U/\delta_J$. The density of these effective centers is $n_{\text{eff}} = (\delta_J/a)n$ and the critical current satisfies

$$\frac{\Phi_0}{ca} j_c \sim \frac{U}{\delta_J}\left(\frac{n\delta_J}{aS_c}\right)^{1/2}. \tag{16}$$

The correlation lengths R_{cx} and R_{cz} are determined by equating the rms displacement $\langle (u(\mathbf{r}) - u(0))^2 \rangle^{1/2}$ to the typical range of the random potential, which is δ_J in this case:

$$U\left(\frac{n\delta_J}{aR_{cx}R_{cz}}\right)^{1/2} \sim C_{11}\left(\frac{\delta_J}{R_{cx}}\right)^2 \sim C_{44}\left(\frac{\delta_J}{R_{cz}}\right)^2.$$

Hence,

$$R_{cz} \sim (a\delta_J C_{11}^{1/2} C_{44}^{3/2}/\gamma)^{1/2}, \tag{17a}$$

$$R_{cx} \sim (a\delta_J C_{11}^{3/2} C_{44}^{1/2}/\gamma)^{1/2}. \tag{17b}$$

Using

$$C_{11} = 32E_J\frac{a}{\delta_J}\exp\left(-\frac{a}{\delta_J}\right),$$

$$C_{44} = 8E_J\frac{\delta_J}{a}$$

as the values of the elastic moduli for a sparse lattice soliton vortices, we find

$$j_c \sim j_{c0}\frac{2q}{\pi}\left(\frac{r_0}{8\delta_J}\right)^2 \exp\left(\frac{a}{2\delta_J}\right). \tag{18}$$

The condition for collective pinning, $R_{cx} \gg a$, means

$$32\exp(-3a/2\delta_J) \gg q(r_0/8\delta_J)^2. \tag{19}$$

If condition (19) is not satisfied, the pinning of one vortex occurs in a process which is independent of the pinning of the others, and the critical current reaches a constant value

$$j_{c\,max} \sim j_{c0}[q(r_0/\delta_J)^2]^{2/3}. \tag{20}$$

Corresponding estimates can be made for a one-dimensional junction, with

$$L_z \ll R_{cz} \sim 2^{7/2}(\delta_J^2/r_0 q^{1/2})\exp(-a/4\delta_J).$$

In this case we find

$$R_{cx}^{(1)} \sim 8\delta_J\left[\frac{2L_z a^3 \exp(-2a/\delta_J)}{\delta_J^2 r_0^2 q}\right]^{1/3}, \tag{21a}$$

$$j_c^{(1)} \sim \frac{j_{c0}}{2}\left[\frac{r_0^2 q\exp(a/2\delta_J)}{2L_z\delta_J}\right]^{2/3}. \tag{21b}$$

Expressions (21) hold under the condition

$$\exp(2a/\delta_J) \ll 2^{10}L_z\delta_J/r_0^2 q.$$

When this condition is violated, the critical current reaches a constant value which corresponds to individual pinning:

$$j_{c\,max}^{(1)} \sim j_{c0}(qr_0^2/\delta_J L_z)^{1/2}. \tag{22}$$

Since the correlation radius R_{cz} falls off rapidly with increasing period a in the case of a sparse vortex lattice, there can be a situation in which a size crossover occurs as the magnetic field is varied. This situation arises if L_z is smaller than R_{cz} in strong magnetic fields [see (13b)] but larger than the minimum radius $R_{cz}^{\min}$ with $R_{cz} \sim a$:

$$R_{cz}^{min} \sim 2\delta_J(2\delta_J/r_0 q^{1/2})^{2/3} \ll L_z \ll 4\pi^2\delta_J^2/r_0 q^{1/2}$$

4. CURRENT-VOLTAGE CHARACTERISTIC ABOVE THE CRITICAL CURRENT

Another method for estimating the temperature and field dependence of the critical current is to calculate the current-voltage characteristics of a junction at high currents, through an expansion in the random potential.[14] This method yields a correct estimate of the critical current if the current-voltage characteristic of the junction has only a single characteristic current and can be written in the form

$$U(j)=R(H,T)jf(j/j_c(H,T)),$$

where $f(x)$ is a dimensionless function [$f(x)\to 1$ as $x\to\infty$]. At $t\neq 0$, the voltage U is nonzero at arbitrarily low currents for a junction of finite area. In this case we understand the critical current as the characteristic current at which the resistance switches to its value in the normal state [i.e., at which the function $f(x)$ reaches a value on the order of unity].

The voltage across the junction, U, is related to the average velocity of the lattice, V, by

$$U=(h/2ea)V.$$

The lattice velocity is determined by the current through the junction and can be found from the equation of motion of the lattice:

$$\eta(\tilde{x})\left(1+\frac{\partial u}{\partial\tilde{x}}\right)\dot{u}=\nabla C(\tilde{x})\nabla u+j\beta'(\tilde{x})-v(z,\tilde{x}+u)p'(\tilde{x})+f_T(t,\mathbf{r}),\qquad(23)$$

where

$$\eta(\tilde{x})=(\hbar/2e)^2(\partial\theta_0/\partial\tilde{x})^2/R$$

is the viscosity of the lattice, R is the resistance to the quasiparticle current through the junction, and $f_T(t,\mathbf{r})$ is the Langevin random force, which satisfies

$$\langle f_T(t,\mathbf{r})f_T(t',\mathbf{r}')\rangle=2\eta(\tilde{x})T\delta(\mathbf{r}-\mathbf{r}')\delta(t-t').$$

At $j\gg j_c$ the velocity of the lattice can be sought as an expansion in the random potential.[14] In this case the critical current is approximately equal to the current at which the correction to the velocity for the random potential reaches a value on the order of the velocity itself.

In the zeroth approximation in the random potential, the lattice velocity is determined by the ratio of the average pulling force $j\overline{\beta'(\tilde{x})}$ to the average viscosity $\overline{\eta(\tilde{x})}$:

$$V_0=j\overline{\beta'(\tilde{x})}/\overline{\eta(\tilde{x})}.\qquad(24)$$

This case corresponds to a linear current-voltage characteristic:

$$U=(\pi^2R/4KE)j.$$

The velocity correction of first order in the random potential is calculated in the standard way (see Ref. 14 and the Appendix to the present paper). The result is

$$\bar{\eta}V_1=\frac{\Upsilon}{L_zL_x}\sum_{q_zq_x}\sum_{n=-\infty}^{\infty}\int_0^{\infty}\frac{dt}{\bar{\eta}}i(k_0n)^3p_n^2\times\exp\left\{-(C_{44}q_z^2+C_{11}q_x^2)\frac{t}{\bar{\eta}}+ik_0Vnt-\frac{(k_0n)^2}{2}\langle[u_f(t)-u_f(0)]^2\rangle\right\},\qquad(25)$$

where $k_0=2\pi/a$ is a reciprocal-lattice vector,

$$C_{44}=\langle C(\tilde{x})\rangle=(4E/k^2K)E_J,$$

$$C_{11}=(\langle C^{-1}(\tilde{x}_2)\rangle)^{-1}=(4k'^2K/k^2E)E_J,$$

$$p_n=\left(\frac{\pi}{kK}\right)^2\frac{n}{2\,\mathrm{sh}(n\pi K'/K)}$$

is a Fourier component of the function $p(x)$, and $K'=K(k')$.

Let us find the velocity correction at absolute zero. For a junction with linear dimensions greater than the correlation lengths, the summation over q_x and q_z can be replaced by an integration. Calculating the correction to the lattice velocity, we find the following expression for the current-voltage characteristic of a junction at high currents:

$$U=(\pi^2R/4KE)(j-j_1).\qquad(26)$$

The j-independent correction j_1 agrees in order of magnitude with the critical current and is given by

$$j_1=\frac{\pi^6}{64}j_{c0}\frac{qr_0^2}{\delta_J^2}\sum_{n=1}^{\infty}\frac{n^5}{k'k^4K^6\,\mathrm{sh}^2(n\pi K'/K)}.\qquad(27)$$

In the case of a dense soliton lattice ($k\ll 1$, with a lattice period $a=\pi\delta_Jk$), only the first term, with $n=1$, is important in the sum over harmonics in (27). Using the asymptotic expressions for the elliptic functions at small k, $K\approx\pi/2$ and $K'\approx\ln(4/k)$, we find the following expression for the correction:

$$j_1\approx\frac{j_{c0}}{64}q\left(\frac{r_0}{\delta_J}\right)^2\qquad a\ll\delta_J.\qquad(28)$$

This expression agrees within a constant coefficient with estimate (13a). In the case of a sparse lattice [$k'\ll 1$, $a=2\delta_J\ln(4/k')$], the summation over harmonics can be replaced by an integration, and we find the following expression for j_1 [$\zeta(x)$ is the Riemann zeta function]:

$$j_1\approx\frac{15}{8\pi^6}\zeta(5)q\left(\frac{r_0}{\delta_J}\right)^2\exp\left(\frac{a}{2\delta_J}\right),\quad a\gg\delta_J.\qquad(29)$$

This result also agree within a numerical coefficient with the corresponding estimate in (14a).

For a one-dimensional junction ($L_z\ll R_{cz}$) corresponding calculations lead to

$$U=\frac{\pi^2R}{4KE}\left(j-\frac{j_1^{3/2}}{j^{1/2}}\right),\qquad(30)$$

where

$$j_1^{(1)}=\frac{1}{4}j_{c0}\left[\frac{\pi^5qr_0^2}{L_z\delta_J}\left(\frac{E}{2K}\right)^{1/2}\sum_{n=1}^{\infty}\frac{n^{9/2}}{k'k^4K^5\,\mathrm{sh}^2(n\pi K'/K)}\right]^{2/3}.\qquad(31)$$

In some simple limiting case we have

$$j_1^{(1)}=\frac{1}{8}j_{c0}\left(\frac{qr_0^2}{L_z\delta_J}\right)^{2/3},\quad a\ll\delta_J,\qquad(32a)$$

$$j_1^{(1)}=\frac{1}{2}j_{c0}\left[\frac{Aqr_0^2\exp(a/2\delta_J)}{(2\pi)^{1/2}L_z\delta_J}\right]^{2/3},\quad a\gg\delta_J,\qquad(32b)$$

where the constant A is $[9!!/(2\pi)^5]\zeta(9/2)\approx 1.0$. Figure 1

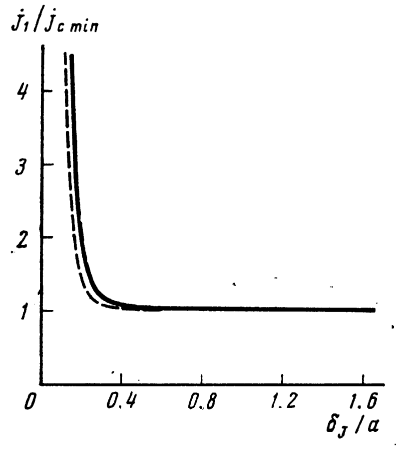

FIG. 1. The critical current j_1, which determines the correction to the current-voltage characteristic (see the text proper), versus the parameter δ_J/a in the region of collective pinning. Solid line—Two-dimensional junction; dashed line—one-dimensional junction.

shows the quantities j_1 and $j_1^{(1)}$, normalized to their minimum values, versus the parameter δ_J/a.

5. ROLE OF THERMAL FLUCTUATIONS

How do thermal fluctuations in the soliton lattice affect the critical current? We consider a dense soliton lattice in a two-dimensional junction ($L_x, L_z \gg R_c$). In this case the displacement correlation function, which determines the Debye-Waller factor in (25), is given by

$$\langle (u_f(t)-u_f(0))^2\rangle = \frac{T}{2\pi C}\ln\frac{Ctk_0^2}{\eta}. \qquad (33)$$

Incorporating the Debye-Waller factor results in a decrease in the correction to the current j_1 in (26) by an exponential factor:

$$j_1(T,j)=j_1\exp\left[-\frac{T}{2\pi E_J\delta_J^2}\ln\left(\frac{2eC}{\hbar j}\right)\right]. \qquad (34)$$

The characteristic temperature at which the fluctuations begin to cause an appreciable decrease of the critical current (the depinning temperature) is estimated from

$$T_p=\Phi_0^2\Big/(4\pi)^2\lambda\ln\frac{32B^2\lambda\delta_J}{\pi E_J q r_0^2}. \qquad (35)$$

An estimate shows that the value of T_p is typically on the order of 10^4 K, so fluctuations have a negligible effect on the temperature dependence of the critical current of a two-dimensional Josephson junction.

We turn now to the case of a dense soliton lattice in a one-dimensional ($L_z \ll R_{cz}$) Josephson junction. The displacement correlation function in this case is

$$\langle (u_f(t)-u_f(0))^2\rangle = \frac{2T}{L_z}\left(\frac{t}{\pi C\eta}\right)^{1/2}. \qquad (36)$$

Substituting this correlation function into (25), we find the following estimate of the depinning temperature of a dense soliton lattice in a one-dimensional junction:

$$T_p^{(1)}\sim\left[\frac{C\gamma a^2L_z^2}{(2\pi)^2}\right]^{1/3}\sim T_{p0}\left(\frac{L_z}{R_{cz}}\right)^{2/3}, \qquad (37)$$

where $T_{p0}=\Phi_0^2/(4\pi)^2\lambda$. The depinning temperature depends on the junction "thickness" L_z. Fluctuations have a strong effect on the temperature dependence only for thin junctions, with $L_z \ll R_{cz}(T_c/T_{p0})^{3/2}$.

At low temperatures $T\ll T_p^{(1)}$, one-dimensional junctions in a strong magnetic field have an exponentially small linear resistance:

$$R(T)=A_1R\exp(-A_2T_p^{(1)}/T),$$

where A_1 and A_2 are constants on the order of unity. As the temperature is increased, there is a decrease in the characteristic critical current, and there is also an increase in the linear response a low currents. At $T\sim T_p^{(1)}$, thermal fluctuations completely suppress the critical current, and the linear resistance of a thin junction reaches a value on the order of the normal resistance.

The case of a sparse soliton lattice in a thin junction is more complicated. When the temperature reaches a characteristic value

$$T_{p1}^{(1)}=E_J[qr_0^2\delta_J^2L_z^2\exp(-a/\delta_J)]^{1/3}, \qquad (38)$$

the fluctuational displacement of a soliton over the time scale $t_0=\eta\delta_J/\beta' j_c$ reaches a size on the order of the range of the random potential, δ_J, and thermal fluctuations begin to strongly influence the critical current. At $T>T_{p1}^{(1)}$, the summation over n in (28) is cut off from above by the Debye-Waller factor. A calculation of the correction to the lattice velocity in this temperature region yields the following expression for the current-voltage characteristic of the junction at high currents:

$$U=(\pi^2\delta_J/2a)R(j-j_1^{1/3}j^{2/3}), \qquad (39a)$$

where

$$j_1=A_1j_{c0}E_J^7T^{-7}(qr_0^2)^3\delta_J^4L_z^4\exp(-2a/\delta_J), \qquad (39b)$$

$$A_1=2^{22}[\Gamma(1/3)]^6/3^{9/2}\pi^2\approx1{,}12\cdot10^6.$$

When the temperature reaches the second characteristic value

$$T_{p2}^{(1)}=(a/2\pi\delta_J)^{1/3}T_{p1}^{(1)},$$

the fluctuations completely suppress the critical current, and the current-voltage characteristic of the junction becomes linear up to currents $j\sim j_{c0}$.

Let us examine the field dependence of the critical current through a one-dimensional junction in the region $a\gg\delta_J$ for various temperatures. If the temperature is below the characteristic depinning temperature for a single Josephson vortex,

$$T_p^{(v)}\sim E_J(qr_0^2\delta_JL_z)^{1/3},$$

the field dependence of the critical current is the same as at absolute zero: As the magnetic field is lowered, the critical current increases in accordance with (32b) and then reaches the plateau described by (22) (Fig. 2). In the temperature interval $T_p^{(v)}<T<T_p^{(1)}$ there is a qualitative change in the behavior: The critical current density reaches a maximum value

$$j_{c\,max}^{(1)}(T)\approx j_{c0}E_Jqr_0^2/T \qquad (40)$$

in a magnetic field B_{p1}, which corresponds to the following value of the soliton lattice period:

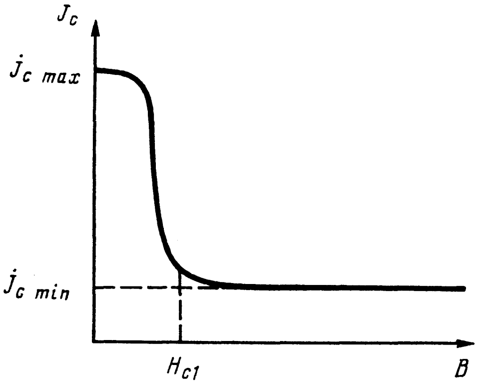

FIG. 2. Typical plot of the critical current versus the magnetic induction for disordered Josephson junctions with large dimensions.

$$a_{p1}=\delta_J \ln E_J q r_0^2 \delta_J^2 L_z^2/T^3. \tag{41}$$

With a further lowering of the field, the critical current falls off sharply, in accordance with (39b), and it vanishes when the lattice period reaches the value a_{p2} found from

$$\frac{2\pi\delta_J}{a_{p2}}\exp\left(\frac{a_{p2}}{\delta_J}\right)=\frac{E_J q r_0^2 \delta_J^2 L_z^2}{T^3}. \tag{42}$$

6. EFFECT OF THE SELF-FIELD OF THE CURRENT

In the sections above we essentially calculated the local critical current density as a function of the local magnetic induction in the junction, $j_c(B)$ [the magnetic induction B is related to the period of the soliton lattice by $B=(\pi^2\delta_J/2a)H_{c1}$.] Since the current flowing through the junction has a self-field, an irregularity appears in the distribution of the magnetic induction in the junction. For a junction of large dimensions, the values of the induction at the edges of the junction may differ substantially from the average value $\overline{B}$, which is determined by the external field and the geometry of the sample. When the maximum current is flowing, a current $j_c(B)$ flows at each point of the junction, where B is the local value of the induction at this point. On the other hand, the induction distribution itself depends on the current distribution and must be determined in a self-consistent way.

Let us consider a two-dimensional junction [$L_x \sim L_z > R_c$, where R_c is the correlation radius in strong fields; see (13b)]. At $B > H_{c1}$ the local critical current density does not depend on the field, so the average critical current density through a nonuniform junction of any dimensions must reach a plateau in sufficiently strong fields. In the case of a sparse vortex lattice, the self-field of the current can be ignored only for junctions whose dimensions lie below a certain characteristic value. To estimate this value we compare the irregularity in the magnetic field at the maximum value of the critical current,

$$\Delta B \sim (4\pi/c)(\delta_J/R_c)^{4/3} L j_{c0},$$

with the characteristic value B_{cr}, at which the critical current reaches its maximum,

$$B_{cr} \sim H_{c1}/\ln(R_c/\delta_J).$$

From the condition $\Delta B \ll B_{cr}$ we find

$$L \ll R_c\left(\frac{R_c}{\delta_J}\right)^{1/3} \Big/ \ln\frac{R_c}{\delta_J}. \tag{43}$$

For a junction with large dimensions, an additional analysis is required in order to extract the $j_c(B)$ dependence from experimental data.

For a one-dimensional junction ($L_z < R_c < L_x$) the limitations which stem from the self-field of the current are far less stringent.[10] At values of the induction below

$$B_{cr}^{(1)} \approx H_{c1}/\ln(R_c^{2/3} L_c^{1/3}/\delta_J),$$

and at absolute zero, the critical current in the junction is at its maximum value, (22). In this case the field irregularity in the junction is estimated from

$$\Delta B \sim \frac{1}{c} j_{c\,max}^{(1)} L_z \ln\frac{L_x}{L_z}.$$

The self-field of the current is unimportant under the condition $\Delta B \ll B_{cr}^{(1)}$, which is equivalent to the restriction

$$\frac{(\delta_J L_z)^{1/2}}{R_c}\ln\frac{L_x}{L_z}\ln\frac{R_c^{2/3}L_z^{1/3}}{\delta_J} \ll 1. \tag{44}$$

Since the coefficient of the logarithms is less than unity, restriction (44) is very weak.

7. DISCUSSION OF RESULTS

We have derived the temperature and field dependence of the critical current density j_c of Josephson junctions with irregularities. We have discussed junctions (respectively one-dimensional and two-dimensional) in which one or two geometric dimensions are greater than the correlation length R_c in (13b), which is a measure of the degree of disorder in the system. A temperature dependence arises from the change in the parameters of the junction and also because of thermal fluctuations of the positions of the Josephson vortices. We have shown that at realistic temperatures the thermal fluctuations do not affect the critical current in two-dimensional junctions, but they may cause a sharp decrease in the critical current in sufficiently thin one-dimensional junctions. For a two-dimensional junction, the behavior of the critical current is determined over essentially the entire temperature range by the temperature dependence of the parameters of the junction (j_{c0}, δ_J, Λ). In particular, the minimum critical current found from (13a) has the following temperature dependence near T_c:

$$j_{c\,min} \propto \begin{matrix} (T_c - T)^{3/2} & \text{for an SIS junction,} \\ (T_c - T)^{7/9} & \text{for an SNS junction.} \end{matrix}$$

The field dependence of the critical current density, $j_c(B)$, has two limiting values, $j_{c\,min}$ and $j_{c\,max}$, in respectively weak fields ($B < H_{c1}$) and strong fields ($B > H_{c1}$) (Fig. 2). The maximum value of the current density is set by the pinning of individual vortices [in the one-dimensional case, expression (22) reproduces the result derived by Mineev *et al.*[18] for one-soliton pinning]. The critical current reaches saturation in strong fields because various factors determining the average pinning force cancel out: The increase in the elastic moduli of the lattice, which is proportional to the square of the field, is canceled out by the increase in the characteristics force exerted on the lattice by an individual pinning center (the latter force is proportional to the field itself). The three characteristic values j_{c0}, $j_{c\,\min}$, and $j_{c\,\max}$ are related by a universal equation which is independent of the degree of disorder and which can be tested experimentally. For a two-dimensional junction this equation is

$$(j_{c\,max})^3/j_{c0}(j_{c\,min})^2=\text{const}, \tag{45a}$$

and for a one-dimensional junction it is

$$(j_{c\,max}^{(1)})^4/j_{c0}(j_{c\,min}^{(1)})^3=\text{const}. \tag{45b}$$

These results may prove useful in the analysis of experimental $j_c(H)$ curves for dense superconducting ceramics, in which the field penetrates into grain boundaries at $H>10$ Oe. It must be kept in mind that the critical current $j_c\sim10^2$ A/cm^2 in bulk samples in weak fields is due to the self-field of the current[6] and that the characteristic Josephson current density is larger by one or two orders of magnitude. In strong fields the critical current stems from a pinning of the vortex structure at grain boundaries. The saturation of the critical current in strong fields which has been observed in ceramics[4-7] is a characteristic property of disordered Josephson junctions. Strictly speaking, expression (13a) gives a correct estimate of the minimum critical current density only if the correlation radius in (13b) is much smaller than the grain size L. In the opposite limit $R_c\gg L$, it becomes necessary to solve the problem of the percolation of a current through a network of disordered Josephson currents–a problem which we are not taking up in the present paper.

We are deeply indebted to M. V. Feĭgel'man for stimulating discussions.

APPENDIX

Calculation of correction to the velocity of a soliton lattice

For the equation of motion of the lattice in (23) there is a spatial variation of the parameters. This variation becomes most important in the region $a\gg\delta_J$. For a soliton lattice which is moving at a constant velocity V, we write the displacement $u(t,\mathbf{r})$ as the expansion

$$u(t,\,\mathbf{r})=Vt+u_e(\tilde{x})+\tilde{u}(t,\,\mathbf{r})+u_f(t,\,\mathbf{r}), \tag{A1}$$

where $\tilde{u}(t,\mathbf{r})$ is a correction for the random potential, $u_f(t,\mathbf{r})$ is a fluctuation increment ($\langle\tilde{u}(t,\mathbf{r})\rangle=\langle u_f(t,\mathbf{r})\rangle=0$, and $u_e(\tilde{x})$ is the lattice strain. This strain is proportional to the current and arises because of the spatial variation of the pulling force. In the case of a sparse lattice, the condition of a slight strain, $du_e/dx\ll1$, is

$$j\ll j_{c0}(a/\delta_J)\exp(-a/\delta_J).$$

We make the assumption that this condition holds, and we ignore the component $u_e(\tilde{x})$ below. Transforming to a coordinate system in steady-state motion, $x_s=\tilde{x}+\tilde{u}+u_f$, and taking the time average of Eq. (23), we find the relation

$$\bar{\eta}V=\bar{\beta}'j-\langle v(z,\;x_s+Vt)\,p'(x_s-\tilde{u}-u_f)\rangle. \tag{A2}$$

The displacement $\tilde{u}(\mathbf{r},t)$ is determined in first order in $v(\mathbf{r})$ by the equation

$$\eta(x_s)\left(\tilde{u}+V\frac{\partial\tilde{u}}{\partial x_s}\right)-\nabla C(x_s)\nabla\tilde{u}=-p'(x_s-u_f)\,v(x_s+Vt,z). \tag{A3}$$

Under the condition $j\ll j_{c0}(a/\delta_J)^2\exp(-a/\delta_J)$ the term $\eta V(\partial\tilde{u}/\partial x_s)$ is small in comparison with the elastic term and can be ignored. Substituting the expression found for the displacement $\tilde{u}(\mathbf{r},t)$ from (A3) into (A2), and taking an average over the random potential, we find a correction to the lattice velocity:

$$\bar{\eta}V_1=-\frac{\gamma}{a}\int dx_s\,dt\,\langle p''(x_s-u_f(0,x_s,0))\times p'(x_s+Vt-u_f(t,x_s+Vt,0))\rangle_T\times G(t,x_s,x_s+Vt,0), \tag{A4}$$

where $\langle\ldots\rangle_T$ means a thermodynamic average, and $G(t,x_s,x_{s1},z)$ is a Green's function, determined by the equation

$$\eta(x_s)\dot{G}-\nabla C(x_s)\nabla G=\delta(t)\delta(x_s-x_{s1})\delta(z).$$

At distances $x_s-x_{s1}\gg a$ this Green's function is essentially independent of everything except the coordinate difference x_s-x_{s1}, and its Fourier component at small wave vectors and low frequencies is given by

$$G(\omega,\,q)=(-i\bar{\eta}\omega+C_{11}q_x^2+C_{44}q_z^2)^{-1},$$

where $C_{44}=\langle C(\tilde{x})\rangle$ and $C_{11}=(\langle C^{-1}(\tilde{x})\rangle)^{-1}$. Carrying out Fourier expansions in t, x_s, and z in (A4), we find (25).

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Translated by Dave Parsons