Electromagnetic generation of spikes of anomalous sound in metals in a magnetic field

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A new type of sound wave is predicted in metals irradiated with microwave radiation in a magnetic field. Anomalous sound, consisting of a standing wave in a semibounded sample, is excited in addition to the conventional wave propagating with the sound speed s_0. Anomalous sound appears in the form of a system of acoustic spikes which are separated in space by distances that are multiples of the electron cyclotron diameter. The amplitude of these spikes is $(q\delta)^2 \gg 1$ times greater than the amplitude of conventional sound ($q = \omega/s_0$ is the wave vector of the sound wave, where ω is the frequency of the sound wave, and δ is the thickness of the skin layer). This effect is determined by the ballistic transfer of the energy of the electromagnetic wave from the skin layer into the metal by individual groups of electrons. A unique "angular resonance effect" is predicted: the amplitude of the anomalous sound spikes grows rapidly as a function of the angle of inclination Φ of the magnetic field relative to the surface. This resonance effect is associated with the activation of an additional mechanism for separating effective electrons. The lineshape of the anomalous sound spikes is calculated and the conditions under which they can be observed experimentally are determined.

1. INTRODUCTION

Electromagnetic generation of sound in metals is caused by the interaction of conduction electrons with an electromagnetic wave and vibrations of the lattice. The transformation of an electromagnetic wave into a sound wave which is usually studied (see, for example, Refs. 1–4) occurs in the skin layer. In the process of a sound wave, consisting of a characteristic acoustic mode of the metal with a fixed wave vector $q = \omega/s_0$ and propagating with the velocity of sound s_0 (ω is the frequency of the wave), is excited. In an external magnetic field **H** under conditions of strong spatial dispersion at high frequencies, for

$$ql \gg qR \gg 1, \quad \omega \gg \nu \tag{1}$$

(l is the electron mean free path, R is the electron cyclotron radius, and ν is the electron relaxation rate), the picture of electromagnetic excitation of sound becomes much more complicated because acoustic oscillations of a new type are generated. The velocity of these acoustic oscillations is different from the sound speed s_0; it can be of the order of the electron Fermi velocity $v \gg s_0$ or it can be equal to zero, depending on the orientation of the vector **H**, i.e., a standing sound wave is excited in an unbounded metal. These oscillations are called anomalous sound. The excitation of anomalous sound is determined by the ballistic transfer of the energy of the electromagnetic wave from the skin layer into the metal by individual groups of electrons. The phenomenon of electromagnetic excitation of anomalous sound in metals was first studied in Ref. 5, where it is shown theoretically and experimentally that in a field **H** oriented perpendicular to the surface of the sample anomalous sound, determined by electrons moving with the maximum drift velocity $v_H^{\text{ext}} = v$ along the **H** vector, is excited together with the normal acoustic mode. In this case the phase velocity of the anomalous sound is equal to the Fermi velocity of the electrons $v \gg s_0$, and the distance over which it decays is determined by the mean free path of the electrons in the given group. Under the conditions (1) the amplitude of the anomalous sound is of the same order of magnitude as the amplitude of the normal acoustic mode. This circumstance is associated with the fact that under the condition

$$q\delta \gg 1, \tag{2}$$

(δ is the thickness of the skin layer) the normal sound generated in the skin layer is determined by the interaction of the electromagnetic field with a small set of spatial harmonics of acoustic oscillations (near the value of the wave vector q). The amplitude of the normal sound u_n is determined by the strength of the electric field in the skin layer E_{sk}, multiplied by the phase volume $\sim (q\delta)^{-2}$, in which efficient interaction of the electromagnetic and sound waves through the conduction electrons occurs, i.e., $u_n \sim E_{\text{sk}} (q\delta)^{-2}$.

Anomalous sound is excited by an anomalously penetrating electromagnetic field, formed by the ineffective[1)] electrons which drift into the metal with the extremal velocity v_H^{ext}. The magnitude of this field is $E_{\text{ap}} \sim E_{\text{sk}} (\delta/l)^2$. However the waves interact over the electron mean free path $l \gg \delta$, as a result of which the amplitude of the anomalous sound u_a is of the same order of magnitude as u_n:

$$u_a \sim E_{\text{ap}} (l/q\delta^2)^2 \approx E_{\text{sk}} (q\delta)^{-2}.$$

The dependences of the amplitude of the anomalous sound on the magnetic field H are significantly different. When the inequalities (1) hold the amplitude of u_n is a monotonic function of H, while the amplitude $u_a(H)$ oscillates with a constant period and exhibits diamagnetic resonance for $\omega \approx \Omega$ (Ω is the electron cyclotron frequency). The interaction of these two acoustic signals causes the lines $u(H) = u_n(H) + u_a(H)$ to be inverted when the frequency is changed by the among $\Delta\Omega \approx 4\pi s_0/d$ (d is the thickness of the metal plate) and the lineshape $u(H)$ to be restored periodically with period $2\Delta\omega$. All these effects have been ob-

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served in tungsten[5] and have made it possible to determine the velocity, momenta, and cyclotron masses of different electron groups.

In this work the electromagnetic generation of sound in a metal in a magnetic field making small angles Φ with the surface of the sample and in a parallel field ($\Phi = 0$) is studied. An electromagnetic field is generated in the bulk of the metal by the effective electrons, which repeatedly return into the skin layer and slowly drift into the volume of the sample.[6] The spatial distribution of the electric field $E(\eta)$ has the form of spikes separated by distance η, which are multiples of the cyclotron diameter $2R$, from the surface of metal. The amplitude of the field $E(\eta)$ in the spikes depends on the existence of an additional mechanism of selection of effective electrons. This mechanism operates when the vector $\mathbf{H}$ makes an angle Φ with the surface satisfying the inequality

$$\Phi \gg (\delta R)^{1/2}/l. \tag{3}$$

The spread in the diameters of the orbits of electrons which fall within both the skin layer and a spike is much less than the skin depth δ (the spatial size of a spike), thanks to which the amplitude of the field $E(\eta)$ in the spikes decays as a power of the spike number. The condition (3) together with the inequality $\Phi \ll \delta/R$ determines the region of magnetic fields where this effect should be observed. When the condition (3) for the angle of inclination is reversed and we also have $\Phi = 0$ there is no additional mechanism for selecting effective electrons. In this case the amplitude of the spikes $E(\eta)$ decays exponentially.

In the situation under study the electromagnetic field excites in the skin layer the normal sound $u_n(\eta)$, which propagates with velocity s_0, and a system of spikes of anomalous sound $u_a(\eta)$ at distances which are multiples of $2R$. In the intervals between the spikes the anomalous sound field is weak, of order $(\delta/R)^2 \ll 1$. At small angles $0 \leqslant \Phi \leqslant (\delta R)^{1/2}/l$ the amplitude of the anomalous sound spikes decays exponentially as a function of the spike number s. For this reason only the first few spikes, whose amplitude is greater than the amplitude of the normal sound, can actually be observed. For larger angles of inclination in a narrow range of angles $(R\delta)^{1/2}/l \ll \Phi \ll \delta/R$ the amplitude of the anomalous sound spikes u_a is sharply higher. It decays slowly, according to a power law, to ward the interior of the metal (as the number s increases). As the angle is further increased, i.e., for $\Phi \gg \delta/R$) the relative fraction of effective electrons returning repeatedly into the skin layer, decreases and the amplitude of the spikes decays rapidly. Thus a unique "angular resonance effect" should be observed; in this effect the amplitude of the anomalous-sound spikes $u_a(\Phi)$ grows rapidly and is $(q\delta)^2 \gg 1$ times greater than the amplitude of the normal-sound signal. In other words, in a semiinfinite metal a standing sound wave is generated.

2. FORMULATION OF THE PROBLEM

The propagation of electromagnetic and sound waves in a metal and their transformation into one another are described by a system consisting of Maxwell's equations, the linearized kinetic equation for the conduction electrons, and the equations describing the lattice vibrations (see, for example, Ref. 7). The boundary conditions for this system of equations are as follows: the tangential components of the alternating electric and magnetic fields are continuous at the surface of the metal $\eta = 0$ and the elastic stresses on these surfaces vanish. We fix the $\xi\eta\zeta$ coordinate system on the surface of the metal, $\eta = 0$: the η axis is parallel to $\mathbf{n}$, where $\mathbf{n}$ is the normal to the surface; the ξ axis is parallel to $\mathbf{E}(0)$, where $\mathbf{E}(0)$ is the electric field vector of the wave in free space; and, the ζ axis is oriented along the projection of the vector $\mathbf{H}$ on the $\eta = 0$ plane. The angle between the vectors $\mathbf{H}$ and ζ is equal to Φ.

The boundary conditions must be supplemented by a description of the character of the scattering of the electrons by the surface $\eta = 0$. It is well known that the distribution of the electromagnetic field in the skin layer is determined by the electrons scattered by the boundary of the metal and therefore by the character of their reflection—specular or diffuse. In the interior of the metal ($\eta \gtrsim 2R \gg \delta$), however, the field is formed by electrons which drift away from the surface and do not collide with it. It is obvious that the distribution of the field in the skin layer can change only the spatial dependence $\mathbf{E}(\eta)$ within spikes whose width is of the order of $\delta \ll 2R$ and will not affect the amplitude of the spikes and the field between them. For this reason, in solving the kinetic equation we shall assume that the metal is unbounded.

In the absence of strong coupling of the sound and the normal electromagnetic modes in the metal or when the waves are coupled weakly, the coefficient of transformation of the electromagnetic wave into sound is of the order of $\sim m/M \sim (s_0/v)^2$ (m is the electron mass and M is the ion mass) and is small. In this case, in the leading order approximation the equations for the electromagnetic and sound fields decouple; the coupling of the waves is described by the perturbation theory. The interaction of electromagnetic and sound waves via the conduction electrons results in small corrections of the order of $(q\delta)^{-4}$ (2) in the dispersion of the velocity of the normal sound and for this reason has virtually no effect on the distribution of the electromagnetic field in the metal.

When a sound wave propagates along a high-order symmetry axis of a crystal the longitudinal and two transverse modes are independent. This displacement field of the longitudinal sound wave $u(\eta,H)$ has the form $u(\eta,t) \propto \exp(-i\omega t)$

$$u(\eta) = \pi^{-1} \int_0^\infty dk \cos(k\eta) u(k),$$

$$u(k) = -2E'(0) \times \frac{kQ_{\eta\xi}(k)}{[k^2 - 4i\pi\omega c^{-2}\sigma_{\xi\xi}(k)]\{k^2 s_0^2 [1 - iM_{\eta\eta}(k)] - \omega^2\}}.$$

Here the tensors $\hat{\sigma}(k)$ and $\hat{Q}(k)$ are the Fourier transforms of the conductivity tensor and the deformation conductivity tensor[7]; the tensor $\hat{M}(k)$ describes the electronic renormalization of the elastic moduli. In symbolic form these tensors are as follows:

$$\sigma_{\alpha\beta}(k)=\langle v_\alpha v_\beta\rangle,\quad Q_{\alpha\beta}(k)=\rho_0^{-1}\langle\Lambda_{\alpha\eta}v_\beta\rangle,$$

$$M_{\alpha\beta}(k)=\frac{i\omega}{\rho_0}\langle\Lambda_{\alpha\eta}\Lambda_{\eta\beta}\rangle. \qquad (5)$$

The angular brackets denote averaging of the enclosed quantity over the Fermi surface with the Fourier transform of the nonequilibrium distribution function, found from the kinetic equation taking into account the spatial and temporal dispersion; the tensor $\hat{\Lambda}(\mathbf{p})$ is the deformation potential tensor; $\mathbf{v}$ and $\mathbf{p}$ are the electron velocity and momentum, and ρ_0 is the mass density of the metal.

3. DISTRIBUTION OF THE SOUND FIELD IN THE METAL

In a magnetic field $\mathbf{H}$ oriented parallel to the surface of the metal ($\Phi=0$) or at a small angle Φ the effective electrons from a neighborhood of the central section on the Fermi surface ($p_H\approx 0$) make the main contribution to the quantities (5) and, therefore, to the sound field (4). It is precisely these electrons, whose drift velocity is low, that repeatedly return into the skin layer and interact efficiently with the field of the wave.

Under the conditions of strong spatial dispersion

$$kl\gg kR\gg 1 \qquad (6)$$

the effective electrons form the skin layer and a system of weakly decaying electromagnetic and acoustic spikes, and they determine the characteristic values of the wave numbers k.

We shall study three regions of the values of the angle of inclination Φ.

1. The strongest generation of sound in the volume of the metal occurs in an oblique magnetic field in the case when the electromagnetic and acoustic fields are determined not by all the effective electrons, but only by a small fraction of them for which the spread in the diameters of the orbits $\Delta D\approx R(kl\Phi)^{-2}$ is small compared with the thickness of the skin layer $\delta\sim k^{-1}$. This requirement leads to the following restriction on the quantity Φ:

$$\frac{|\gamma_1|}{(kR)^{1/2}},\ \frac{1}{kl}\ll\Phi\ll\frac{1}{kR},\quad \gamma=\frac{\nu-i\omega}{\Omega},$$
$$\gamma=-in+\gamma_1,\quad |\gamma_1|\ll 1\quad (n=0,1,2,\dots). \qquad (7)$$

The first inequality in (7) ensures that the electrons return repeatedly into the skin layer and thereby singles out a group of effective electrons in the central section with $p_H\approx 0$. The condition $kl\Phi\gg 1$ means that the electrons leave the skin layer within the collision time. The inequality $\Phi\gg|\gamma_1|/(kR)^{1/2}$ determines the above-mentioned additional selection of effective electrons. Near the cyclotron resonance ($\omega=n\Omega$) the quantity $|\gamma_1|\ll 1$; for low frequency, when $\nu\ll\omega\ll\Omega(n=0)$, we have $|\gamma|\ll 1$.

The components of the tensors (5) for a metal having a quadratic and isotropic dispersion law have the following form when the condition (6) and (7) hold:

$$\sigma_{\xi\xi}=\frac{2\pi^2\sigma_H}{|kR\Phi|}[J_n'(kR)]^2\approx\frac{2\pi\sigma_H}{(kR)^2\Phi}[1-(-1)^n\sin(2kR)],$$
$$\sigma_H=(3/4\pi)(Ne^2/m\Omega); \qquad (8)$$

$$Q_{\eta\xi}=\frac{\pi^2\sigma_H p_F}{6e\rho_0|kR\Phi|}[J_n^2(kR)]'\approx\frac{\pi\sigma_H p_F}{3e\rho_0(kR)^2\Phi}\cos(2kR), \qquad (9)$$

$$M_{\eta\eta}=\frac{\pi^2\sigma_H p_F^2 i\omega}{18e^2\rho_0|kR\Phi|}J_n^2(kR)$$
$$\approx\frac{\pi\sigma_H p_F^2 i\omega}{18e^2\rho_0(kR)^2\Phi}[1+(-1)^n\sin(2kR)], \qquad (10)$$

where $J_n(kR)$ are Bessel functions, the prime indicates a derivative with respect to the argument p_F is the Fermi momentum of the electrons, and N is the electron density. The remaining components of the tensors (5) are small, by virtue of (6) and (7).

It is obvious that the functions $\sigma_{\xi\xi}(k)$ and $Q_{\eta\xi}(K)$ are oscillating functions of the variable k. It is important that the conductivity $\sigma_{\xi\xi}(k)$ has minima at points k_s $\sin(2k_sR)=(-1)^n$ near the real axis of the complex k plane. As shown in Ref. 6, this property of the function $\sigma_{\xi\xi}(k)$ is responsible for the appearance of spikes in the electromagnetic field far from the surface of the sample.

Using the asymptotic expansions (8)–(10) we obtain the following expressions for the displacement field of the longitudinal sound (4):

$$u(\eta)=u_0\int_0^\infty dx\,\frac{(-1)^n x}{x^2-\tilde{x}_0^2}\,\frac{\cos(k_0\eta x)\cos(Zx)}{x^4-2i[1-(-1)^n\sin(Zx)]}. \qquad (11)$$

Here

$$u_0=-E'(0)p_Fc^2/6\pi^2e\rho_0 s_0^2\omega$$

is the amplitude of the sound,

$$k_0=(4\pi^2\omega\sigma_H/c^2R^2\Phi)^{1/4}\sim\delta^{-1}$$

is the characteristic value of the wave number, $Z=2k_0R$, and

$$\tilde{x}_0=(q/k_0)\{1+i(Nm\Omega/48\rho_0\omega\Phi)$$
$$\times[1+(-1)^n\sin(2qR)+Z^{-1}]=x_0'+ix_0''$$

is the dimensionless complex wave number of the characteristic sound taking into account the electronic damping[2)] $x_0'=\mathrm{Re}\,\tilde{x}_0$, $x_0''=\mathrm{Im}\,\tilde{x}_0$). The damping of the sound is a small quantity, so that the following condition is satisfied:

$$x_0'\gg Z^{-1}\gg x_0''; \qquad (12)$$

Z^{-1} is the characteristic scale of oscillations of the integrand in (11). Because of the condition (12) the "acoustic" pole $x=\tilde{x}_0$ lies near the real axis.

The displacement field of the longitudinal sound (11) consists of two terms. The first term u_n represents the normal sounding propagating in the metal with velocity s_0. The normal sound u_n is determined by the pole of the integrand in (11) and $x=\tilde{x}_0$. The second term u_a is determined by the presence of oscillating functions in the kinetic coefficients(8) and (9) and represents anomalous sound, whose spatial structure in the given geometry is that of spikes of the acoustic field spaced by distances which are multiples of the electron cyclotron diameter $2R$. We emphasize that anomalous sound spikes are generated for the same reason that the spikes of the electromagnetic field are generated in the bulk of the metal (anomalous penetration of the electromagnetic field into the metal).[6] Thus

$$u(\eta,H,\omega)=u_n+u_a\ . \qquad (13)$$

The expression for the normal sound has the form

$$u_n = u_0(-1)^n \frac{\pi i}{x_0^{'4}} \begin{cases} \cos\frac{\omega\eta}{s_0}\exp\left(\frac{2i\omega R}{s_0} - 2Rk_0x_0''\right), & \eta<2R, \\ \cos\frac{2\omega R}{s_0}\exp\left(\frac{i\omega\eta}{s_0} - k_0\eta x_0''\right), & \eta\geqslant 2R. \end{cases} \quad (14)$$

Anomalous sound consists of a system of spikes in the interior of the metal and is expressed as a sum of terms u_s:

$$u_a = \sum_s u_s(\eta, H, \omega), \quad s=[\eta/2R]. \quad (15)$$

The asymptotic expression (for $s \gg 1$) for the shape of the spikes in the interior of the metal is described by the following formula:

$$u_s = \frac{u_0}{2^{1/2}\tilde{x}_0^2} \begin{cases} \frac{(-1)^{(s-1)/2}\exp(i\pi/4)}{s^{1/2}}\psi(|\eta-2Rs|), & s=2k+1, \\ \frac{(-1)^{s/2}\pi}{\Gamma(1/4)s^{3/4}}k_0(2Rs-\eta)\psi(|\eta-2Rs|), & s=2k. \end{cases} \quad (16)$$

The function $\psi(x)$ has a sharp maximum at $x=0$.

$$\psi(x) = \begin{cases} 1, & x\ll 1, \\ x^{-2}, & x\gg 1. \end{cases} \quad (17)$$

One can see from Eqs. (15)–(17) that the spikes of anomalous sound are a standing wave, arising in a semiinfinite metal. The amplitude of the spikes decays slowly as the spike number s increases: as $s^{-1/2}$ for even s and as $s^{-3/4}$ for odd s. The amplitude of the spikes scales as $\sim|\tilde{x}_0|^{-2}$ and is much greater than the amplitude of normal sound, which is of order $|\tilde{x}_0|^{-4}$ (2). In the intervals between spikes the acoustic displacement field is determined by the normal sound (14), since in our case the inequalities (2) and (12) are satisfied. The lineshape of the even and odd spikes is different. Even spikes are symmetric with a maximum at the center, $\eta = 2Rs$, while odd spikes are antisymmetric.

Figure 1 shows the calculated dependence of the shape of the spikes of anomalous sound (4), (8), and (9) $\mathrm{Re}\, u(x)/u_0$, $x=\eta/2Rs$ for $s\leqslant 10$ near the cyclotron resonance $n=2$. For the remaining cyclotron resonances $n=1,3,4,\ldots$, as well as for the case $\omega\ll\Omega\,(n=0)$ the picture of the spikes is analogous to that shown in Fig. 1. Since in an experiment, as a rule, the absolute value of the signal is measured, Fig. 2 shows the lineshapes $|u(x)/u_0|$ of the first four spikes. The absolute value of the signal for the even spikes has a minimum at the center of the line and two symmetric maxima on the wings of the line, while odd spikes are characterized by a maximum at the center of the line. Such is the picture of the distribution of the acoustic field for $\Phi \gg |\gamma_1|/(kR)^{1/2}$.

2. We shall now study a different limiting case, when the angle of inclination Φ is less than (7):

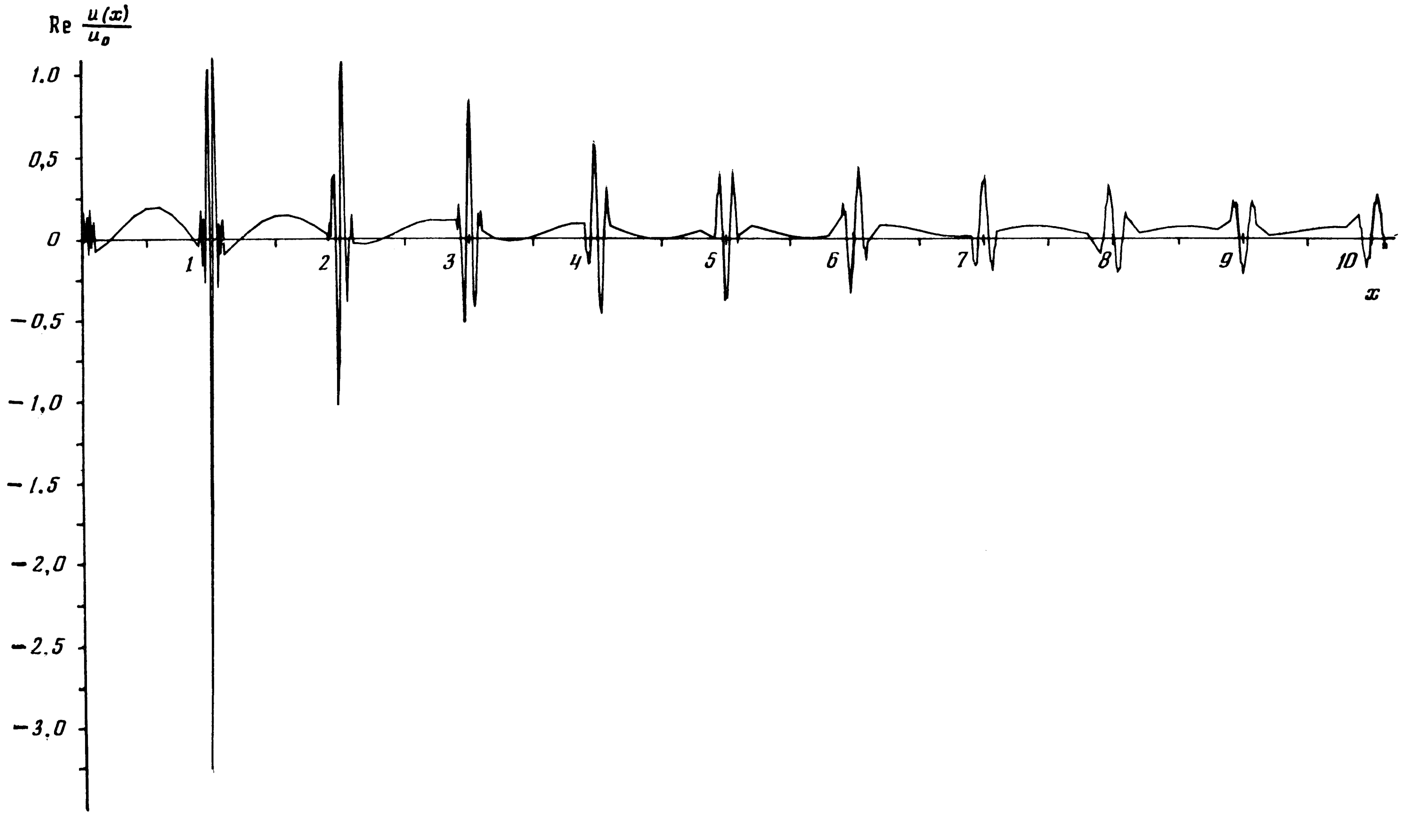

FIG. 1. The computed dependence $\mathrm{Re}[u(x)/u_0]$ of anomalous sound on the dimensionless parameter $x=\eta/2R$ with $n=2$, $Z=100$, and $x_0=10$.

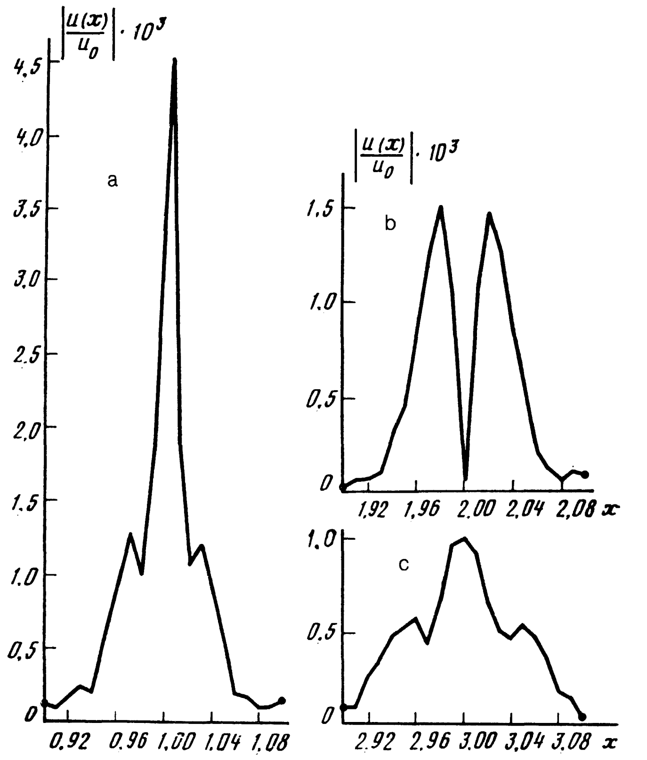

FIG. 2. The lineshapes $|u(x)/u_0|$ of anomalous sound spikes: a—first spike ($x\approx 1$); b—second spike ($x\approx 1$); c—third spike ($x\approx 2$).

$$\frac{1}{kl} \ll \Phi \ll \frac{|\gamma_1|}{(kR)^{1/2}},\ \frac{1}{kR}. \tag{18}$$

In this case there is no additional segregation of effective electrons, since the spread in the diameters of the orbits for electrons in a spike of the field will be much greater than the thickness of the skin layer. The amplitude of the oscillating terms in the conductivity tensor $\hat{\sigma}$, the deformation conductivity tensor $\hat{Q}$, and the tensor $\hat{M}$ (5) decreases. The components of these tensors have the following form:

$$\sigma_{\xi\xi} = \frac{2\pi\sigma_H}{(kR)^2\Phi}\left[1 - \frac{(-1)^n}{\pi^{1/2}w}\sin\left(2kR - \frac{\pi}{4}\right)\right], \tag{19}$$

$$Q_{\eta\xi} = \frac{(-1)^n\pi\sigma_H p_F \cos(2kR-\pi/4)}{3e\rho_0(kR)^2\Phi\pi^{1/2}w}, \tag{20}$$

$$M_{\eta\eta} = i\frac{\pi\sigma_H\omega p_F^2}{18\rho_0 e^2(kR)^2\Phi}\left[1 + \frac{(-1)^n}{\pi^{1/2}w}\sin\left(2kR - \frac{\pi}{4}\right)\right], \tag{21}$$

where

$$w = |\gamma_1|/\Phi(kR)^{1/2}.$$

The displacement field for longitudinal sound in the case (18) is given by the formula

$$u_\eta(\eta) = u_0\frac{(-1)^n}{w_0} \times \int_0^\infty \frac{dx\, x^{3/4}}{x^2 - \tilde{x}_1^2}\, \frac{\cos(Zx-\pi/4)\cos(k_0\eta x)}{x^4 - 2i + 2i[(-1)^n/w_0]\sin(Zx-\pi/4)}. \tag{22}$$

Here

$$w_0 = |\gamma_1|/\Phi(\pi k_0 R)^{1/2}$$

$$\tilde{x}_1 = \frac{q}{k_0}\left\{1 + i\frac{Nm\Omega}{48\rho_0\omega}\left[1 + (-1)^n\frac{q}{k_0 w}\sin\left(2qR - \frac{\pi}{4}\right)\right]\right\} = x_1' + ix_1''.$$

The imaginary part of the dimensionless wave number $\tilde{x}_1$ oscillates just as in the preceding case, but the amplitude of the oscillations is ω_0 times smaller.

The sound field in the metal is the sum of the normal sound and the spikes of anomalous sound (13). The expression for the normal sound has the form

$$u_n = (-1)^n u_\alpha F(\eta, 2R, x_\alpha''),$$
$$u_\alpha = u_1 = i\pi u_0/w_0\tilde{x}_1^{1/2},$$

$$F(\eta, 2R, x_1'') = \begin{cases} \cos\dfrac{\omega\eta}{s_0}\exp\left[\dfrac{2i\omega R}{s_0} - 2Rk_0x_1'' - \dfrac{i\pi}{4}\right], & \eta<2R, \\ \cos\left(\dfrac{2\omega R}{s_0} - \dfrac{\pi}{4}\right)\exp\left[\dfrac{i\omega\eta}{s_0} - k_0x_1''\eta\right], & \eta>2R. \end{cases} \tag{23}$$

The anomalous sound will also consist of a system of spikes, but the amplitude of these spikes will decrease as a function of the spike number. Near a spike u_s is described by the formula

$$u_s = u_0\frac{(-1)^{s(n+1)-1}\exp[i\pi(s+2)/2]}{(2^{3/4}w_0)^s\tilde{x}_1^2} \times\left[A_s\sin\frac{3\pi s}{4} + (Zs-k_0\eta)B_s\cos\frac{3\pi s}{4}\right], \tag{24}$$

where $|Zs - k_0\eta| \ll 1$, and the coefficients A_s and B_s are complex and depend on the spike number s. These dependences are very complicated, so we present the values for several numbers:

$$A_2 = 6.7013 + i\cdot 13.2018,\quad A_3 = -1.9114 - i\cdot 0.9935,$$
$$B_4 = -1.7437 + i\cdot 3.8433.$$

In the intervals between the spikes, when $|Zs - k_0\eta| \gg 1$, the anomalous sound field is small:

$$u_a \approx u_0\Gamma[(s+3)/2]/(2w_0)^s|Zs - k_0\eta|^{(s+3)/2}, \tag{25}$$

and decreases as s increases even more rapidly than does the spike amplitude (24). [In the expression (25) $\Gamma(x)$ is the gamma function.]

Analysis of the formulas (23) and (24) shows that the amplitude of the spikes of anomalous sound is much larger than the amplitude of the normal sound wave only for the first spikes whose number satisfies $s \lesssim 1 + \frac{4}{3}\ln|x_1|/\ln w_0$. The spikes of anomalous sound have the form of isolated peaks (23), and each fourth spike splits, i.e., it consists of two narrow maxima with opposite sign.

3. In the case of a parallel magnetic field ($\Phi = 0$) for metals having a quadratic and isotropic carrier dispersion law there is also no additional mechanism for segregating effective electrons. As a result of this situation the oscillating terms in the kinetic coefficients (5) are also small corrections:

$$\sigma_{\xi\xi}=\frac{\pi\sigma_H}{\gamma_1 kR}\left[1-\frac{2\cdot(-1)^n}{(\pi kR)^{1/2}}\sin\left(2kR-\frac{\pi}{4}\right)\right],$$

$$Q_{\eta\xi}=\frac{(-1)^n\pi\sigma_H p_F\cos(2kR-\pi/4)}{3e\rho_0\gamma_1 kR(\pi kR)^{1/2}}, \qquad (26)$$

$$M_{\eta\eta}=i\frac{\pi\sigma_H p_F^2\omega}{18\gamma_1 kRe^2\rho_0}\left[1+\frac{(-1)^n}{(\pi kR)^{1/2}}\sin\left(2kR-\frac{\pi}{4}\right)\right].$$

The displacement field $u(\eta)$ of longitudinal sound with $\Phi=0$ is determined by the following formula:

$$u(\eta)=u_0(-1)^n$$

$$\times\int_0^\infty\frac{dx\,x}{x^2-\tilde{x}_2^2}\frac{\cos(k_1\eta x)\cos(Z_1x-\pi/4)}{x^{1/2}(x^3-2i)+(-1)^n\cdot 2i\varepsilon\sin(Z_1x-\pi/4)}. \qquad (27)$$

Here the following notation has been introduced:

$$k_1=(2\pi^2\sigma_H\omega/c^2\gamma_1 R)^{1/3}\sim\delta_1^{-1}$$

is the characteristic wave number for $\Phi=0$, $\varepsilon=2/(\pi k_1R)^{1/2}$;

$$\tilde{x}_2=\frac{q}{k_1}\left\{1+i\frac{Nmv}{48\rho_0 s_0\gamma_1}\left[1+(-1)^n\frac{\varepsilon}{2}\sin\left(2qR-\frac{\pi}{4}\right)\right]\right\}$$

$$=x_2'+ix_2''.$$

The imaginary part of the dimensionless wave number oscillates with a small amplitude, as in the preceding case.

The normal sound $u_n(\eta,H,\omega)$ for $\Phi=0$ is described by Eq. (23), in which the quantity u_α is equal to

$$u_\alpha=u_2=i\pi u_0/\tilde{x}_2^{7/2}(\pi k_1R)^{1/2}, \qquad (28)$$

while the function $f(\eta,2R,x_2'')$ differs only by the attenuation factor x_2''. It should be noted that expressions (23) and (28) in the case of high frequencies $(\omega\sim\Omega)$ determine the electromagnetic generation of normal sound in magnetic fields corresponding to a neighborhood of the cyclotron resonances, i.e., when the condition $|\omega-n\Omega|\ll\nu$ holds. The generation of normal sound in the geometry under study $(\Phi=0)$ has been studied in a wide range of magnetic fields H and frequencies ω.[8-10]

The amplitude of the anomalous sound spikes, as in the preceding case, decays rapidly as a function of the spike number. In a neighborhood of a spike

$$u_s=u_0(0{,}52)^{s-1}\frac{\varepsilon^s}{\tilde{x}_2^2}\left[C_s\sin\frac{\pi s}{4}+(Z_1s-k_1\eta)D_s\cos\frac{\pi s}{4}\right]; \qquad (29)$$

for $|Z_1s-k_1\eta|\ll 1$ the complex quantities C_s and D_s are complicated functions of the number s. For the first several values of s we have

$$C_1=(-1)^n\cdot 0.2620(1+i),\quad C_2=0.4552+i\cdot 0.1994,$$
$$C_3=(-1)^n(0.5793-i\cdot 0.7306),\ D_4=0.4040+i\cdot 0.2337.$$

In the intervals between spikes, for $|Z_1s-k_1\eta|\gg 1$, the anomalous sound field is weak:

$$u_a\sim u_0/\tilde{x}_2^2|Z_1s-k_1\eta|^2,$$

and is virtually independent of the spike number.

Thus in the interval of small angles of inclination Φ (18) and for $\Phi=0$ the amplitude of the spikes of the anomalous-sound field $u_a(\eta,H,\Phi)$ is significantly smaller than the amplitude of the spikes $u_a(\eta,H,\Phi)$ (16) even for the first few spike numbers. As the spike number s increases this difference increases, since u_s (16) decays algebraically while the functions u_s in Eq. (24) and Eq. (29) decay exponentially.

The weak damping of spikes of the sound field is due to the presence of minima of the conductivity $\sigma_{\xi\xi}(k)$. In the case of small angles $0<\Phi<(R\delta)^{1/2}/l$ the depth of the minima of the functions $\sigma_{\xi\xi}(k)$ (19) and (26) is small and is proportional to the parameters $1/w_0\ll 1$ and $(\pi kR)^{1/2}\ll 1$, respectively. In the oblique field (7) in a neighborhood of the cyclotron resonances $|\omega-n\Omega|\lesssim\nu$ or for $\omega\ll\Omega$ the conductivity $\sigma_{\xi\xi}(k)$ has the form (8) and can vanish (to lowest order). The different intensity of the amplitude of the spikes $u_a(\eta,H,\Phi)$ is connected with this difference in the characteristics of the functions $\sigma_{\xi\xi}(k)$ (8) and (19), (26). In metals whose Fermi surface is complicated all electron groups make an additive contribution to the conductivity. As a result, in the low-frequency case $\omega\ll\Omega(|\gamma|\ll 1)$ the conditions (7) can hold for the effective electrons in all groups, while the conductivity is proportional to the sum of terms of the type (8), i.e.,

$$\sigma_{\xi\xi}(k)=\sum_j A_j[1-\sin(2kR_j)].$$

If the quantities A_j are of the same order of magnitude and the diameters $2R_j$ are incommensurate, then the minimum values of $\sigma_{\xi\xi}(k)$ cannot vanish for any real k. For this reason $u_a(\eta,H)$ will decay exponentially, just like in the interval of small angles Φ. At high frequencies $(\omega\sim\Omega)$ in a fixed magnetic field the cyclotron-resonance condition $\omega=n\Omega$ selects a definite group of electrons for which the inequalities (7) are satisfied. Then it is precisely this group of electrons that makes the main contribution to the conductivity; the conductivity to lowest order is determined by Eq. (8) and all results are analogous to the case of a metal with one group of carriers.

4. CONCLUSIONS

Electromagnetic generation of sound should be observed in a metal plate whose thickness d is of the order of the electron mean free path. Varying the magnitude of the magnetic field H can give rise to spikes of anomalous sound on the second surface of the metal for $d=2Rs$ (s is the spike number). The emergence of an anomalous sound spike corresponds to fixed values of the magnetic field $H_s=2scp_F/ed$. The amplitude of the anomalous-sound spikes varies sharply (resonantly) as a function of the angle of inclination Φ of the magnetic field **H** with respect to the surface. The amplitude of a spike is maximum in a narrow interval of angles Φ,

$$|\gamma_1|/(k_0R)^{1/2}<\Phi<1/k_0R \qquad (30)$$

and decreases rapidly as $\Phi\to 0$ and for $\Phi>1/k_0R$. This unique angular resonance effect arises owing to the appearance [when the conditions (30) hold] of an additional mechanism for segregating effective electrons. This effect is sharpest in a geometry when the electric field vector of the incident wave $\mathbf{E}(0)$ is perpendicular to the magnetic field vector $\mathbf{H}$.

Generation of normal sound, whose amplitude is

$(q\delta)^2 \gg 1$ times smaller than the amplitude of the spikes u_a [see Eqs. (14), (16), (23), (24), and (29)] should be observed in the intervals between the spikes ($H_{s-1} < H < H_s$). In addition, as one can see from Eqs. (14), (23), and (29), the function $u_n(H)$ undergoes resonant oscillations with the constant period $\Delta H = ep_F q/\pi c$.

We note that the physical reason for the electromagnetic excitation of anomalous sound in metals is analogous to that of the transfer of acoustic pulses with Fermi velocity v ("precursors"[11,12]) by conduction electrons. Acoustic precursors were first observed by Fil' *et al.*[11] and Bogachek *et al.*[12] in a geometry such that the magnetic field **H** is parallel to the surface of the sample ($\Phi = 0$) and perpendicular to the wave vector of the sound **q**. We emphasize that the amplitude of the precursors must also undergo an angular resonance effect when the condition (30) is satisfied.

[1)] According to Pippard, the effective electrons are those electrons which move under small angles relative to the surface of the metal and form the skin layer.

[2)] We neglect here the renormalization of the sound velocity, since it is a higher-order infinitesimal in the parameter $Z^{-1} \ll 1$ than the attenuation.

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