Black-hole formation in the center of a nondissociative gravitational singularity

A. V. Gurevich and K. P. Zybin

Lebedev Physics Institute, USSR Academy of Sciences
(Submitted 11 September 1989)
Zh. Eksp. Teor. Fiz. **97**, 20–31 (January 1990)

The joint dynamics of baryon and nondissipative matter is considered. The conditions for the onset of a nondissipative gravitational singularity (NGS) are elucidated. It is shown that energy emission and the action of a gravitational field at an NGS center enhance the spherical baryon-matter contraction that leads ultimately to formation of a massive black hole. The black hole subsequently increases rapidly by absorbing both the baryon and the nondissipative matter.

The Friedmann model of a homogeneous and isotropic universe is valid only for scales on the order of the horizon radius. Small deviations from the homogeneous state are enlarged by the action of universal gravitation forces. The dynamics of these perturbations over scales substantially smaller than the horizon radius determines the formation of galaxies, of galactic clusters, etc. The principal role is played in this process by the hidden matter, which is nondissipative, i.e., it interacts only via gravitational forces.[1]

A linear relativistic theory that describes the initial growth of small fluctuations in an expanding universe was developed by E. M. Lifshitz.[2] We have previously investigated[3,4] the nonlinear stage of the gravitational instability of nondissipative matter. We have shown that an individual growing perturbation can undergo an appreciable three-dimensional contraction that leads to formation of a nondissipative gravitational instability (NGS)—a self-trapped cluster of matter that has at the center $r\to 0$ a power-law singularity of density $\rho\propto r^{-\alpha}$ and a gravitational potential Ψ such that

$$|d\Psi/dr|\propto r^{-\alpha+1}.$$

The theory referred to was developed only for nondissipative matter. Simultaneously with the nondissipative contraction, however, ordinary baryon matter is also compressed in the course of NGS formation. It is this which leads ultimately to galaxy formation. The presence of a singularity of the gravitational potential at the center of an NGS should determine in this case the formation of the galaxy center (or of the cluster center).

The present paper is in fact devoted to the formation of the center. We shall show that energy emission and the action of gravitational forces produce at the NGS center an enhanced baryon-matter spherical contraction that leads ultimately to formation of a massive black hole.

The black hole rapidly increases subsequently by absorbing both baryon and nondissipative matter.

1. NONDISSIPATIVE GRAVITATIONAL SINGULARITY

We consider the joint dynamics of nondissipative and baryon matter in an expanding universe. We are interested, as before,[3,4] in scales much smaller than the horizon radius, when the dynamics of the matter can be described in the Newtonian approximation. We recognize, in addition, that we are interested only in the nonlinear stage corresponding to not too early instants of time ($z\leqslant 10$), long after the radiation was separated from the matter.[5] The system of hydrodynamic equations describing under these conditions the process of joint dynamics of nondissipative and baryon matter is

$$\begin{gathered}\frac{\partial\rho_d}{\partial t}+\nabla(\rho_d\mathbf{U})=0,\quad \frac{\partial\mathbf{U}}{\partial t}+(\mathbf{U}\nabla)\mathbf{U}+\nabla\Psi=0,\\ \frac{\partial\rho}{\partial t}+\nabla(\rho\mathbf{V})=0,\quad \frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}+\nabla\Psi=-\frac{\nabla P}{\rho},\\ \frac{\partial T}{\partial t}+(\mathbf{V}\nabla)T+{}^2/_3T(\nabla\mathbf{V})=L,\qquad P=nT,\\ \Delta\Psi=\rho_d+\rho.\end{gathered}\tag{1}$$

Here ρ_d and $\mathbf{U}$ are the density and velocity of nondissipative matter, ρ, $\mathbf{V}$, and T are the density, velocity, and temperature of the baryon matter, L is the loss of its energy to radiation, P is its pressure, and n is the density of the baryon particles. Lastly, Ψ is the gravitational-field potential; we use for simplicity a system of units in which $4\pi G=1$ (G is the gravitational constant).

We recognize that the average density of the nondissipative matter exceeds that of the baryon matter by almost two orders. The system (1) can therefore be expanded in the parameter $\rho/\rho_d\ll 1$. In first-order approximation in this parameter, the nondissipative matter moves freely

$$\begin{gathered}\frac{\partial\rho_d}{\partial t}+\nabla(\rho_d\mathbf{U})=0,\quad \frac{\partial\mathbf{U}}{\partial t}+(\mathbf{U}\nabla)\mathbf{U}+\nabla\Psi=0,\\ \Delta\Psi=\rho_d,\end{gathered}\tag{2}$$

while the baryon matter moves in a specified gravitational potential Ψ determined by the motion (2) of the nondissipative matter:

$$\begin{gathered}\frac{\partial\rho}{\partial t}+\nabla(\rho\mathbf{V})=0,\quad \frac{\partial\mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}+\frac{1}{n}\nabla(nT)+\nabla\Psi=0,\\ \frac{\partial T}{\partial t}+(\mathbf{V}\nabla)T+{}^2/_3T(\nabla\mathbf{V})=L.\end{gathered}\tag{3}$$

The dynamics (2) of the contraction of nondissipative matter was considered earlier in Ref. 3. There the development of the arbitrary initial inhomogeneity prior to the onset of the singularity was investigated in the zerangular-momentum approximation. We need here a more accurate solution, to which the present section is devoted.

We consider thus the dynamics of nondissipative matter in the vicinity of the minimum of the initial potential Ψ. As shown in Ref. 4, the problem reduces in this case to a solution of Eqs. (2) with initial conditions

 0038-5646/90/010010-06$03.00

$$V(0, r)=0,\ \rho(r, 0)=\rho_0(1-\xi^2),\ \xi^2=x^2/a^2 + y^2/b^2+z^2/c^2, \tag{4}$$

where a, b, and c are the semi-axes of the initial ellipsoidal distribution. We rewrite the system (2) in the form

$$\begin{cases} \dfrac{\partial \mathbf{V}}{\partial t}+(\mathbf{V}\nabla)\mathbf{V}+\mathbf{F}-\tilde{\mathbf{F}}=0, \\ \operatorname{div}\mathbf{F}=\rho, \quad \operatorname{div}\tilde{\mathbf{F}}=0, \\ \dfrac{\partial\rho}{\partial t}+\nabla(\rho\mathbf{V})=0. \end{cases} \tag{5}$$

For convenience, the gradient $\nabla\Psi$ of the potential is represented here is a force difference $\mathbf{F}-\tilde{\mathbf{F}}$. In the spherically symmetric case we have $\tilde{\mathbf{F}}=0$. We obtain now for the system (5) a solution that does not differ greatly from the case of spherical symmetry when the force $\tilde{\mathbf{F}}\neq 0$. The velocities induced by the force $\tilde{\mathbf{F}}$, as with be shown below, remain small up to the onset of the singularity. Neglecting in first approximation the force $\tilde{\mathbf{F}}$ and introducing the notation

$$\mathbf{V}=\mathbf{r}U(\xi, t)/\xi,\ \mathbf{F}=\mathbf{r}B(\xi, t)/\xi,$$

we obtain from the system (5)

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial\xi}+B=0, \quad \frac{\partial B}{\partial t}+\frac{U}{\xi^2}\frac{\partial}{\partial\xi}(\xi^2 B)=0. \tag{6}$$

Equations (6) were considered earlier, and their solutions under initial conditions (4) were obtained in Ref. 3. It is shown there that after a time of the order of the Jeans time

$$t_0=3^{1/2}\cdot 2^{-1/2}\pi(4\pi G\rho_0)^{-1/2}$$

a density singularity $\rho\propto\xi^{-12/7}$ is produced at the point $\xi=0$.

We take now into account the presence of the force $\tilde{\mathbf{F}}$. Using the solutions of the system (6), we write for the velocity $\mathbf{V}$

$$\mathbf{V}=\mathbf{r}U(\xi, t)/\xi+\mathbf{u} \tag{7}$$

Substituting (7) in (5) and retaining only the terms linear in the deviation from nonsphericity, we get

$$\frac{\partial\boldsymbol{\omega}}{\partial t}=\operatorname{rot}\left[\frac{\mathbf{r}}{\xi}U\mathbf{u}\right]-\operatorname{rot}\tilde{\mathbf{F}}, \tag{8}$$

where

$$\boldsymbol{\omega}=\operatorname{rot}\mathbf{u}, \quad \operatorname{rot}\tilde{\mathbf{F}}=-[\mathbf{r}\nabla(B/\xi)].$$

The solution of the linear equation (8) with initial conditions (4) can be expressed in terms of the solution of the system (6), which takes the simple form

$$\boldsymbol{\omega}=-[\mathbf{r}\nabla(U/\xi)]. \tag{9}$$

The solution of the equation for the density perturbation, however, shows that the deviations from the solution (6) can appear only in second order in the nonsphericity parameter $[\mathbf{r}\times\nabla\xi]$. The corrections to the radial velocity U and to the potential Ψ are of the same order. As $t\to t_0$ the resultant solution has a singularity at the point $\xi=0$:

$$\rho=\frac{3}{7}\left(\frac{40}{9\pi}\right)^{6/7}\rho_0\xi^{-12/7}, \quad \Psi=\Psi_m+\frac{7}{2}\beta\xi^{2/7},$$
$$V_r=(2\beta)^{1/2}\xi^{1/7}, \quad \beta=\frac{4}{3}\pi G\rho_0 a^2\left(\frac{40}{9\pi}\right)^{6/7},$$
$$u_{x\perp}=\frac{3}{2}V_r\frac{x}{r}(\varepsilon_1 y^2+\varepsilon_2 z^2),$$
$$u_{y\perp}=\frac{3}{2}V_r\frac{y}{r}\varepsilon_1, \quad u_{z\perp}=\frac{3}{2}V_r\frac{z}{r}\varepsilon_2, \tag{10}$$

where a is the major semi-axis of the initial ellipsoid; the deviation of the latter from sphericity are described by the quantities $(a-b)/a=\varepsilon_1$ and $(a-c)/a=\varepsilon_2$.

The velocity $\mathbf{u}_\perp$ determines the angular momentum of the gas relative to the inhomogeneity center. We see that in the nonspherical case the velocity $\mathbf{u}_\perp$ differs from zero, and has at $t=t_0$ a singularity of the same order as that of the radial velocity V_r; $u_\perp$ is always smaller than V_r because of the small deviation of the initial distribution from spherical: $\varepsilon=(\varepsilon_1^2+\varepsilon_2^2-\varepsilon_1\varepsilon_2)^{1/2}\ll 1$.

We shall investigate hereafter the singularities in which self-capture of the matter is possible. Just such a singularity appears when the velocity $u_\perp$ is neglected in the three-dimensional problem, since the capture condition

$$V_r^2/2=\beta\xi^{2/7}<\Psi-\Psi_m={}^7/_2\beta\xi^{2/7}$$

is satisfied at each point:

With increase of $u_\perp$, i.e., of ε, the capture becomes weaker, and it can be shown that it becomes altogether impossible as $\varepsilon\to 1$. No gravitational singularity is produced at the center, but two diverging caustics appear with stable caustical singularities of the form $\rho\propto(x-x_c)^{-1/2}$. This structure agrees fully with that considered in Ya. B. Zel'dovich's papers[6] and with V. I. Arnol'ds general theory of singularities.[7] On the other hand, the onset of a three-dimensional pointlike singularity was not investigated in these references. Note that in an approximation linear in ε the boundary defining the region of existence of a three-dimensional singularity is specified by the condition $\varepsilon<\varepsilon_c$, where $\varepsilon_c=6/(7+85^{1/2})=0.3669$. After the onset of the primary singularity, multistream flows are produced in the capture region and their dynamics is described by a kinetic equation with a self-consistent gravitational field[3]

$$\frac{\partial f}{\partial t}+\mathbf{V}\nabla f-\nabla\Psi\frac{\partial f}{\partial\mathbf{V}}=0, \quad \Delta\Psi=\int f\,d\mathbf{V}. \tag{11}$$

If the nonsphericity is weak, we can confine ourselves in (11) to a dynamics averaged over the angular variables of the distributions. We obtain then from (11), in the variables m (angular momentum) and V_r (radial velocity):

$$\frac{\partial f}{\partial t}+V_r\frac{\partial f}{\partial r}+\left(\frac{m^2}{r^3}-\frac{\partial\Psi}{\partial r}\right)\frac{\partial f}{\partial V_r}=0,$$
$$\frac{\partial}{\partial r}r^2\frac{\partial\Psi}{\partial r}=2^{1/2}\int_0^\infty dm^2\int_{\Psi+m^2/2r^2}^0 f(E, m^2, r, t)\frac{dE}{(E-\Psi-m^2/2r^2)^{1/2}}, \tag{12}$$

where we have introduced the energy

$$E=V_r^2/2+\Psi+m^2/2r^2.$$

Equation (12) is solved in the adiabatic approximation in full analogy with Ref. 3. The adiabatic approximation can be used because the period of the oscillations of the captured fluxes in the potential well is much shorter than the time of variation, determined by the caustic-surface motion, of the well itself.

In the approximation considered the function f depends only on the squared angular momentum m^2 and on the adiabatic invariant I:

$$I=2^{1/2}\int_{r_{min}(E)}^{r_{max}(E)}\left[E-\Psi(r_1,t)-\frac{m^2}{2r_1^2}\right]^{1/2}dr_1. \qquad (13)$$

The form of the function f is determined by the condition on the boundary of the capture region (on the caustic), and is expressed in terms of the adiabatic invariant

$$I_0=2^{1/2}\int_{r_{min}(E)}^{r_{max}(E)}\left[E-\Psi_0 r_1^{2/7}-\frac{m_0^2 r_0^{16/7}(E)}{2r_1^2}\right]^{1/2}dr_1. \qquad (14)$$

The energy E in (13) and (14) is measured from the caustic. Here, according to the hydrodynamic solution, we have

$$m^2=m_0^2 r_0^{16/7},\quad E(r_0)=E,\quad E(r_0)={}^9/_7\Psi_0 r_0^{2/7}+o(\varepsilon),$$
$$m_0^2=0.0881\tilde{\rho}_0\varepsilon^2,\quad \tilde{\rho}_0=\rho_0 a^{12/7}. \qquad (15)$$

From the condition that the flux be continuous on the caustic we have

$$\rho_h(r_0)\,dr_0=f(I_0)\,dI_0. \qquad (16)$$

Starting from (14), (15), and (16) and taking the angular-momentum conservation law into account, we obtain the distribution function

$$f=f_0 I^{1/8}(E)\,\delta(m^2-l_0^2 I^2), \qquad (17)$$

where

$$f_0=\frac{49}{72}\frac{\tilde{\rho}_0}{\Psi_0}\left(\frac{7}{9\Psi_0}\right)^{7/2}C_1^{-9/8},\quad l_0^2=\left(\frac{7}{9\Psi_0}\right)^8\frac{m_0^2}{C_1^2},$$
$$C_1=2^{1/2}\int_{\varkappa_{min}}^{\varkappa_{max}}\left[1-\Psi_0\varkappa^{2/7}-\frac{m_0^2}{2\varkappa^2}\left(\frac{7}{9\Psi_0}\right)^8\right]^{1/2}d\varkappa.$$

Substituting (17) in (13) and (12) we get

$$\frac{\partial}{\partial r}r^2\frac{\partial\Psi}{\partial r}=2^{1/2}f_0\int_0^\infty dm^2\int_{\Psi+m^2/2r^2}^{0}\frac{I^{1/8}(E)\,\delta(m^2-l_0^2I^2)}{[E-\Psi-m^2/2r^2]^{1/2}}dE,$$
$$I=2^{1/2}\int_{r_{min}}^{r_{max}}\left[E-\Psi(r_1,t)-\frac{l_0^2}{2r_1^2}I^2\right]^{1/2}dr_1. \qquad (18)$$

As l_0^2 tends to zero, Eqs. (18) go over exactly into Eqs. (48) of Ref. 3, which were obtained in the absence of angular momentum. In this limit, the field potential is

$$\Psi=-({}^3/_2C_0\ln(r_0(t)/r))^{2/3}, \qquad (19)$$

where $r_0(t)$ is the location of the caustic that separates the regions of single-stream and multistream flows, and C_0 is a normalization constant determined in Ref. 3.

An analysis of the system (18), perfectly similar to the earlier one in Ref. 3, shows that in the radial region

$$0\leqslant r\leqslant r_\varepsilon,\qquad r_\varepsilon=0.0731\left(\frac{\chi_0(t)}{\Psi_0}\right)^{7/2}\varepsilon^{46/49},$$

there exists a power-law solution of the system (18) in the form

$$\Psi=-\chi_0(t)+\Psi_1 r^{2/7},\quad \Psi_1=5.742\Psi_0\varepsilon^{-32/49}. \qquad (20)$$

Outside the region indicated in (20), the angular momenta are unimportant, so that the solution (19) is valid. The potential-well depth $\chi_0(t)$ in (20) is obtained from the requirement that the potential be continuous at the point $r=r_\varepsilon$:

$$\chi_0(t)=\Psi_1 r_\varepsilon^{2/7}+({}^3/_2C_0\ln(r_0(t)/r_\varepsilon))^{2/3}.$$

With logarithmic accuracy, we have

$$\chi_0=\left\{\frac{3}{2}C_0\ln\left[\frac{r_0}{0.07}\left(\frac{3}{2}C_0\right)^{-7/3}\Psi_0^{-7/2}\varepsilon^{-46/49}\right]\right\}^{2/3}. \qquad (21)$$

Relations (19), (20), and (21) describe, in the adiabatic approximation, the onset of a nondissipative gravitational singularity in the presence of an angular momentum. The dependence of the density near the singularity on the radius is shown in Fig. 1. The dashed plot in Fig. 1 shows the density corresponding to (19) (upper curve) and to (20) (lower curve). It can be seen that the difference between these curves is small. Thus, in the power-law form $r^{-\alpha}$ we get for the value $r_\varepsilon=10^{-2}$ assumed in the figure, the values $\alpha=1.87$ and $\alpha=1.72$ for the upper and lower curves, respectively. Both curves agree satisfactorily with the observed distribution of nondissipative matter in galaxies, and also of galaxies in clusters.[8,9]

2. DYNAMICS OF BARYON GAS IN NGS

We consider now baryon matter. In first order the parameter ρ/ρ_d it moves in a specified gravitational field that is determined by the motion (3) of the nondissipative matter. Radiation plays here an important part. Indeed, in the absence of radiation the gravitational contraction would be accompanied by intense heating of the gas, $T\propto\rho^{2/3}$, and it can be seen from (20) that $T\to\infty$ at the singularity point. It is clear, however, that the heating alters the conditions of motion of the baryon gas, so that the dynamics of the gas and the

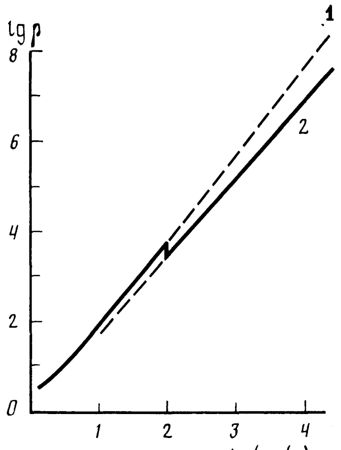

FIG. 1. Logarithm of density of a nondissipative gas in a stirred state vs the logarithm of the distance $r_\varepsilon=10^{-2}$. Curves 1 and 2 correspond to Eqs. (19) and (20). Both curves can be extrapolated by using the power law $r^{-\alpha}$ ($1-\alpha=1.87$, $2-\alpha=1.72$).

character of the resultant density distribution are determined to a considerable degree by the balance between the adiabatic heating and the energy loss to radiation. It is important that the energy loss depends substantially on the gas composition and on the degree of its ionization. Therefore Eqs. (3) should be supplemented by relations that determine the changes of the chemical composition and the ionization in the course of the dynamics of the baryon gas.

It is known that the baryon gas in an expanding universe consists at $z \leqslant 10$ of molecular hydrogen and helium (25%). Under these conditions, at temperatures $T = 10^2$–10^3 K energy is lost by radiation in the molecular-hydrogen lines, while electron deceleration loss becomes substantial at higher temperatures $T > 3 \cdot 10^3$ K. The main reaction that leads to fusion of the H_2 molecules as well as to their decay takes under the above conditions the form[10]

$$\begin{aligned} &e^- + H^+ \to H + \hbar\omega, \\ &e^- + H \to H^- + \hbar\omega, \\ &H^- + H \to H_2 + e^-, \\ &H + H_2 \to 3H. \end{aligned} \tag{22}$$

The equations that describe the behavior of the gas in accordance with the chain of Eqs. (22) are[10]

$$\begin{cases} \partial x/\partial t + (\mathbf{V}\nabla) x = nC[-\gamma_1 x^2 + \gamma_2 (X - x)/n], \\ \partial h_1/\partial t + (\mathbf{V}\nabla) h_1 = n(\beta_2 x - \beta_3 h_1), \\ \partial h_2/\partial t + (\mathbf{V}\nabla) h_2 = n(\beta_2 h_1 - \beta_4 h_2) \end{cases} \tag{23}$$

Here $x = n(e^-)/n$ is the degree of ionization of the matter, $h_1 = n(H^-)/n$ is the density of the negative hydrogen ions, and $h_2 = n(H_2)/n$ is the density of the H_2 molecules. The reaction constants β_2, β_3, β_4, γ_1, γ_2, and C are known. In particular, their dependences on the density and temperature are given in Ref. 10. The values of these parameters will be given in the next section.

The first equation of the system (23) takes into account both recombination and ionization processes. This equation was written under the assumption that the degree of ionization is low, and we shall corroborate this assumption below. The second equation describes creation of H^- ions by adhesion, as well as their annihilation by collisions with neutral hydrogen atoms. The third equation of (23) describes production of H_2 and of its decay by collisions.

The function L that determines the system energy loss through radiation has the form

$$L = -6{,}4 \cdot 10^{-22} \left(\frac{T}{10^3 \text{ K}} \right)^{3{,}770} h_2 - \frac{2^{9/2}}{9\pi^{3/2}} \frac{e^6}{m_e^{3/2} c^3 h} n x T^{1/2} \text{ erg/s}. \tag{24}$$

The first term of (24) describes the loss in the molecular-hydrogen lines[10] and the second the deceleration losses.[11]

Equations (3) and (23) describe together with (24) the nonlinear dynamics of dissipative baryon matter under the conditions of interest to us. We formulate now the initial conditions of the problem. Assuming that the nonlinear contraction of nondissipative matter starts at $z \leqslant 10$, we obtain the temperature T_0 and the density n_0 for a given red shift z_0 from the relations

$$T_0 = T(0)(1 + z_0)^2, \quad n_0 = n(0)(1 + z_0)^3. \tag{25}$$

Here $n(0) = 5 \cdot 10^{-7}$ cm^{-3} and $T(0) = 2.10^{-2}$ K are the density and temperature of the gas at $z = 0$, determined from the temperature of the relict radiation in our epoch. The value of z_0 in (25) is $\sim$3–5. In the derivation of (25) we used the proximity of the cosmological parameter Ω to unity, and also neglected the interaction of the gas with the radiation.[5]

The initial data for the degree of ionization should include the degree of ionization x_0 remaining after the hydrogen-recombination epoch. The value of this parameter was calculated earlier by a number of workers.[5,12] According to their results

$$x_0 = 10^{-4} - 10^{-5}. \tag{26}$$

We assume the densities h_2 and h_1 at $t = t_0$ to be zero.[10] Estimates show that during the period after the recombination of the hydrogen, the fusion of H_2 molecules is not very effective under the conditions of an expanding universe.

We proceed now to determine the initial distribution of the baryon matter in space. This distribution is a result of a prolonged joint evolution of small perturbations of the baryon and nondissipative matter. The joint evolution of the perturbations have been considered earlier by a number of workers.[13,14] For our purposes it is important that the spatial distributions of the baryons and of dark matter are similar in the region of scales corresponding to the maximum of the spectrum of the fluctuations of nondissipative matter.[13]

The set of equations (3), (23), and (24) with the initial conditions (6) and (7) permit thus an analysis of the dynamics of a baryon gas in a potential Ψ produced by nondissipative matter.

We consider next the behavior of baryon matter in the region of a nondissipative gravitational singularity. The latter, as shown above, evolves as a result of nonlinear dynamics of nondissipative matter near a individual initial minimum of the gravitational potential. At an instant $t = t_0$ during contraction of nondissipative matter, a primary singularity of the velocity, density, and potential (10) is produced. Prior to the onset of the singularity, the distributions of density ρ, of the velocity $\mathbf{U}$, and of the potential Ψ have, in the vicinity of the minimum of the potential and at small ε the form

$$\begin{aligned} &\rho = {}^2/_3 (1 - t/t_0)^{-2}, \quad \mathbf{U} = -{}^2/_3 r/(1 - t/t_0), \\ &\nabla\Psi = {}^2/_9 r (1 - t/t_0)^{-2}. \end{aligned} \tag{27}$$

Baryon matter also is compressed similarly, until the heating begins to influence its dynamics substantially.

3. INFLUENCE OF ENERGY LOSSES IN MOLECULAR-HYDROGEN LINES

Since the initial gas temperature T_0 and the degree of ionization x_0 are not large, during the period of the initial contraction the principle role is played by adiabatic heating of the gas and by radiation in molecular-hydrogen lines. This process is described by the system of equations (3), (23), and (24) with initial conditions (25) and (26). We shall consider the dynamics (27) of a baryon gas in the vicinity of the minimum of the potential produced by nondissipative matter, and the period (10) of formation of the primary NGS. It follows from (27) that the characteristic time of gas compression is of the order of t_0. Estimates of the characteristic times of the reactions show that equilibrium sets in most rapidly between the electrons and the H^- ions,[10] so that

$$h_1=\beta_2 x/\beta_1. \quad (28)$$

Substituting (27) and (28) in (23) in the region of the minimum of the potential, we obtain the following equations:

$$\frac{dx}{d\tau}=-\frac{\nu_x}{(1-\tau)^2}x^2T^{-0,644},$$

$$\frac{dh_2}{d\tau}=\frac{\nu_h}{(1-\tau)^2}xT^{1,756},$$

$$\frac{\partial T}{\partial \tau}-\frac{4}{3}\frac{T}{1-\tau}=-\nu_T h_2 T^{3,770}. \quad (29)$$

τ in (29) is the dimensionless time, i.e., $\tau = t/t_0$. Expressions for the reaction rates ν_x, ν_h, ν_T and their dependences on the temperature and density in the considered range of the parameters (25) and (26) were taken from Ref. 10. The dimensionless quantities ν_x, ν_h, ν_T are equal then to

$$\nu_x=0.11(1+z_0)^{-3},\ \nu_h=1.61\cdot10^{-11}(1+z_0)^{-3},$$
$$\nu_T=4.6\cdot10^8.$$

The temperature T in the system (29) was made dimensionless for 10^3 K—reduced to the dimensionless form $T\to T/10^3$ K, where 10^3 K is the characteristic temperature at which the losses in the lines are most substantial. From conditions (25) and (26) it follows that the losses are unimportant for some time after the start of the contraction. During this time the gas is heated adiabatically and its temperature increases with time as follows:

$$T=T_0(1-\tau)^{-4/3}, \quad (30)$$

where T_0 is the gas temperature at the instant corresponding to the red shift z_0 [see Eq. (25)].

The variations of the degree of ionization and the H_2-molecule density are given by

$$x=x_0\left\{1+\frac{x_0\nu_T T_0^{-0,644}}{0.14}[(1-\tau)^{-0,14}-1]\right\}^{-1},$$

$$h_2={}^1/_2\nu_h x_0 T_0^{1,756}\{(1-\tau)^{-2}-1\}. \quad (31)$$

Using (30) and (31), it is easy to calculate the temperatures at which the rate of adiabatic heating becomes equal to the rate of energy loss. Substituting (31) in (29) we obtain

$$T^*=1.6\cdot10^6(1+z_0)^{-1,733}\ \mathrm{K}. \quad (32)$$

Here T^* is the temperature at which the loss in the lines becomes significant. It is evidently very high. Recognizing that $z_0\sim3$–5 [see (25)] we find thus that $T^*\sim10^5$ K. This is too high, and Eqs. (29) and (31) are no longer valid at such high energies: the ionization losses become decisive much earlier. It follows thus from (30) and (32) that radiations in the H_2 lines can actually not halt the temperature rise due to the adiabatic heating of the contracting binary gas.

It should be noted that in preceding studies[10] of contraction of a baryon gas by its intrinsic gravitation (without allowance for nondissipative matter), the process due to energy loss in the H_2 lines turns out to be quite significant. The cause of the difference is that in our case the contraction of the baryon gas is due to the action of the gravitational field produced by nondissipative dark matter. This contraction is significantly stronger and ensures in fact the more intense heating of the baryons.

4. GROWTH OF BARYON-MATTER DENSITY IN THE VICINITY OF A SINGULARITY

By the instant when the gas temperature reaches several thousand degrees, the ionization degree increases noticeably, so that the main energy loss is now due to deceleration processes. Under these conditions the radiation in the lines and the molecule formation can be neglected. We obtain then from the system (3) and (23)

$$\lambda\left(\frac{\partial V}{\partial t}+(\mathbf{V}\nabla)V\right)+\frac{\partial\Psi}{\partial r}+\frac{1}{n}\frac{\partial}{\partial r}(nT)=0, \quad (33)$$

$$\frac{\partial n}{\partial t}+\nabla(n\mathbf{V})=0, \quad (34)$$

$$\lambda\left(\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial r}+\frac{2}{3}T(\nabla\mathbf{V})\right)=-\alpha T^{1/2}nx, \quad (35)$$

$$x_0\lambda\left(\frac{\partial x}{\partial t}+V\frac{\partial x}{\partial r}\right)=n\gamma_1\left\{-\frac{x^2x_0^2}{T^{1/2}}+\frac{\gamma_2}{\gamma_1}\frac{T}{n}e^{-\frac{E_0}{T}}\right\}. \quad (36)$$

For future convenience, we have made the following variables dimensionless:

$$t\to t/t_1,\ t_1=r_0/\tilde{T}^{1/2},$$
$$V\to V/V_0,\ x\to x/x_0,$$

where V_0 is the characteristic rate of gas accumulation and x_0 is the typical electron density. Here and hereafter the time t is measured from the instant t_0 of the singularity.[10] It will be shown below that $x_0\ll1$. The parameter $\lambda=V_0/\tilde{T}^{1/2}$, where $\tilde{T}=E_0/\ln[(\gamma_2/\gamma_1)(\alpha/\lambda)^2]$ is the characteristic value of the temperature. The definition of the numerical coefficient α can be easily understood by comparing (24) with (35). The constants γ_2, γ_1, E_0 were taken from Ref. 10: in dimensionless form their values for our numerical estimates are

$$\gamma_1=2.84\cdot10^7(\tilde{T}/1\ \mathrm{K})^{-1/2},$$
$$\gamma_2=6.86\cdot10^{21}(\tilde{T}/1\ \mathrm{K}),$$
$$E_0=158\,000/\tilde{T}\,[\mathrm{K}].$$

The system (33)–(36) is written in a spherically symmetric form, inasmuch as in the region of the maximum density the gas experiences a threefold contraction prior to the formation of the singularity [see (27)], whereas after the singularity is formed the potential takes rapidly a spherically symmetric stationary form (20) [see Ref. 3].

Owing to the stationary character of the potential Ψ, one can expect the rate of mass growth in the vicinity of the singularity to be small compared with the speed of sound, i.e., $\lambda<1$.

We expand (36) in powers of the parameter $x_0\lambda\ll1$. In the zeroth approximation we get

$$x^2=\frac{T^{3/2}}{n}\frac{\gamma_2}{\gamma_1}e^{-E_0/T}.$$

Substituting this expression in (35) we get

$$e^{\frac{E_0}{T}}=\frac{\gamma_2}{\gamma_1}\left(\frac{\alpha}{\lambda}\right)^2nT^{5/2}\left[\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial r}+\frac{2}{3}T(\nabla\mathbf{V})\right]. \quad (37)$$

It can be seen from (36) and (37) that the rate of energy emission has a strong exponential dependence on the gas temperature. Using this fact, we obtain as the first approximation (with logarithmic accuracy)

$$T=1. \tag{38}$$

Substituting (38) in (33), we arrive at the following system of equations:

$$\begin{cases} \dfrac{\partial \rho}{\partial t}+\dfrac{1}{r^2}\dfrac{\partial}{\partial r}(r^2\rho V)=0, \\ \lambda\left(\dfrac{\partial V}{\partial t}+V\dfrac{\partial V}{\partial r}\right)+\dfrac{\partial \Psi}{\partial r}+\dfrac{1}{\rho}\dfrac{\partial P}{\partial r}=0. \end{cases} \tag{39}$$

We seek the solution of the second equation of (39) in the form of a series in the parameter λ:

$$V=V_0+\lambda V_1+\dots$$

In the zeroth approximation we get

$$\ln\rho(r,\ t)+\Psi(r)=A_0(t), \tag{40}$$

where $A_0(t)$ varies slowly with time. It can be seen from the solution (40) that the baryon matter has a Boltzmann distribution and its bulk is concentrated in the vicinity of a radius r_0 that is small compared with the main scale a of the NGS:

$$r_0=a(7T/\Psi_0)^{7/2}. \tag{41}$$

To calculate the radius r_0 in which the incoming baryon gas accumulates, we must know the temperature T. Substituting the parameters in (38), we get

$$T\approx 4000-5000\ \text{K},\ x_0\approx 10^{-4}. \tag{42}$$

The constant A_0 can be obtained by integrating the first equation of the system (39), but for the analysis that follows it is more convenient to calculate directly the mass-change rate dM/dt in the vicinity of the NGS. From (39), recognizing that in the region $r\leqslant r_0$ the gas pressure is insignificant, and using the solutions (10) and (20), we obtain

$$\frac{dM}{dt}=13\frac{\rho_b(0)}{\rho(0)}\varepsilon^{-32/49}\frac{M_{\text{tot}}}{t_g}\left(\frac{t}{t_g}\right)^{17/6}. \tag{43}$$

Here M_{tot} is the total mass of the substance (including the dark matter) in the scale of a, and $t_g=(4\pi G\rho_0)^{-1/2}$ is the time of free fall of the matter to the center.

The solution (40) and (43) exists for a limited time. As the density of the baryon matter increases, a substantial role in the center of the NGS is assumed by photon scattering by electrons, and at the instant when the photon mean free path for Thomson scattering becomes comparable with the scale r_0, a "protostar" is produced—the radiation comes only from its surface, and Eqs. (33)–(39) are no longer valid. The mass accumulating at the NGS center by the instant of protostar formation is shown in Fig. 2 as a function of the scale of the initial density distribution. At the present time the scale a corresponds to $a_1=a(1+z_0)$, where $z_0\sim 2$–3. It can be seen that for a scale a of order 1 Mpc the mass of the produced protostar can amount to 10^3–$10^5\ M_\odot$. The dashed line in the figure is the limit of a below which the theory considered is not valid. The line corresponds to the minimum contraction needed to heat the gas to a temperature 4000 K. An important role is apparently assumed near this boundary by the cooling of the gas through radiation in the H_2 lines.

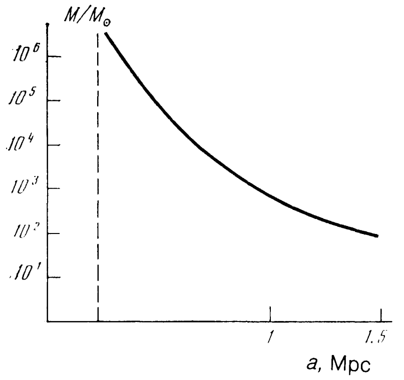

FIG. 2. Mass of a protostar produced in an NGS center vs the scale a of the initial distribution. The dashed line shows the limit of applicability of the considered theory.

The question of the stability of a protostar was considered earlier in the approximation of homogeneous self-contraction of substance in the absence of nondissipative matter (see Refs. 10 and 14). Under our condition the proton star formation is due to accumulation of gas in the inhomogeneous external field of the NGS. The time of gasdynamic accumulation of matter in an NGS field is shorter than the time of evolution of the Jeans instability, and this contributes to stabilization of the process. The question of the stability of a protostar calls therefore for a separate investigation.

The further evolution of the protostar depends on its mass. For masses of order 10^3–$10^5\ M_\odot$. The evolution is extremely rapid (practically within the Jeans time), and the result is a black hole.[15] Once produced, the black hole begins to grow by influx of both baryon and nondissipative matter. The question of the rate of influx of matter into a black hole under NGS conditions calls for a separate investigation. As a rough estimate we can use relation (43). This relation shows that the flux of matter begins to grow rapidly with time and by instant $t/t_g\sim 0.1\varepsilon^{1/4}$ it reaches 1–10 $M_\odot$ annually. Absorption of just this amount of matter by a black hole is well known to be able to ensure the observed radiation intensity of galactic centers and quasars.

The authors are grateful to V. L. Ginzburg for a helpful discussion and for interest in the work.

[1] S. F. Shandarin, A. P. Doroshkevich, and Ya. B. Zel'dovich, Usp. Fiz. Nauk **139**, 83 (1983) [Sov. Phys. Usp. **26**, 46 (1983)].
[2] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **16**, 587 (1946).
[3] A. V. Gurevich and K. P. Zybin, Zh. Eksp. Teor. Fiz. **94**(1), 3 (1988) [Sov. Phys. JETP **67**, 1 (1988)].
[4] A. V. Gurevich and K. P. Zybin, *ibid.* **94**(10), 5 (1988) [**67**, 1957 (1988)].
[5] Ya. B. Zel'dovich, V. G. Kurt, and R. A. Syunyaev, *ibid.* **55**, 278 (1968) [**28**, 246 (1969)].
[6] Ya. B. Zel'dovich, Astrofizika **6**, 319 (1970).
[7] V. I. Arnol'd, Trudy Sem. im. I. G. Petrovskogo No. 8, 21 (1982).
[8] N. A. Bahcall and R. M. Soneira, Astrophys. J. **270**, 30 (1985).
[9] D. Burstein and V. C. Rubin, *ibid.* **297**, 423 (1985).
[10] J. B. Hutchins, *ibid.* **205**, 103 (1976).
[11] V. L. Ginzburg, *Theoretical Physics and Astrophysics*, Pergamon, 1979.
[12] B. J. T. Jones and R. F. G. Wyse, Mon. Not. R. Astron. Soc. **205**, 983 (1983).
[13] A. G. Doroshkevich, Ya. B. Zel'dovich, R. A. Syunyaev, and M. Yu. Khlopov, Pis'ma Astron. Zh. **6**, 457 (1980) [Sov. Astron. Lett. **6**, 252 (1980)].
[14] I. G. Rozgacheva and R. A. Syunyaev, *ibid.* **21**, 370 (1980) [*sic*].
[15] J. R. Bond, W. D. Arnett, and B. J. Carr, Astrophys. J. **280**, 825 (1984).

Translated by J. G. Adashko