Alternating distribution of the current density and a nonreciprocal current-voltage characteristic of a metal in a strong magnetic field

Yu. G. Leonov, N. M. Makarov, and V. A. Yampol'skiĭ

Scientific-Research Institute of Radiophysics and Electronics, Academy of Sciences of the Ukrainian SSR, Kharkov
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A theoretical investigation is reported of the spatial distribution of the current density in a plate of a compensated metal subjected to a strong external magnetic field parallel to the large faces of the sample. It is shown that when the current is sufficiently large, there are regions in a metal where the current density is directed opposite to the electric field. This unusual behavior of the current density is associated with a nonlinearity due to the influence of an inhomogeneous "intrinsic" (created by the current itself) magnetic field on the conductivity of the metal. Nonreciprocity of the current-voltage characteristics of such a plate is predicted: under the conditions assumed in the present treatment a reversal of the sign of the electric field changes the characteristics from sublinear to superlinear. Moreover, a magnetic moment is created. The relevant numerical estimates are given.

1. The conductivity of a compensated metal plate subjected to a strong external homogeneous magnetic field $\mathbf{h}_0$, parallel to the large surfaces of the plate, is governed by the magnetoresistance effect. If the condition

$$R \ll l, d \qquad (1)$$

is satisfied, the conductivity in a direction transverse to $\mathbf{h}_0$ is described by

$$\sigma = \sigma_0 (R/l)^2. \qquad (2)$$

Here, $R = cp_F/eh_0$ is the Larmor radius of the electron orbit; p_F and l are the Fermi momentum and the mean free path of electrons; d is the thickness of the plate; σ_0 is the conductivity of the bulk metal in the absence of the field $\mathbf{h}_0$; e is the absolute value of the elementary charge; c is the velocity of light. The conductivity in Eq. (2) differs from zero because of the diffusion of the centers of the Larmor orbits by bulk collisions of charges.

An electric current I flowing along the plate creates a magnetic field $H(x)$ which, together with the field h_0, determines the electron paths. Even when

$$H(x) \ll h_0, \qquad (3)$$

an inhomogeneity of the total magnetic field $h_0 + H(x)$ causes carriers to drift[1] and, consequently, gives rise to additional "gradient" terms in the expression for the conductivity of the metal.[2]

The competition between the two conduction mechanisms described above gives rise to a number of interesting nonlinear effects. For example, the "intrinsic" magnetic field, i.e., the field created by the current itself, may affect electron paths and the conductivity of metal so as to give rise to the pinch effect if the metal plate is sufficiently thin.[2]

We shall show that the spatial distribution of the current density in a metal plate subject to the conditions (1) and (3) is unusual. The current density near one of the faces of the sample oscillates with a period of the order of $2R$ and if the current I in the metal is sufficiently high, spatial regions may form where the electric current flows against the electric field $\mathbf{E}$. The current-voltage characteristic of such a plate becomes nonlinear. A nonreciprocity effect is observed: depending on the direction of $\mathbf{E}$, the current-voltage characteristic can be superlinear or sublinear. Moreover, under conditions assumed above a magnetic moment is excited in the sample.

2. We shall consider a plate of a compensated metal in which the total current is I. We shall assume that the current flows along the y axis, whereas the x axis is perpendicular to the large faces of the plate and $x = 0$ is the central plane of the sample. The intrinsic magnetic field of the current $\mathbf{H}(x)$ and the external field $\mathbf{h}_0$ are both parallel to the z axis. The dimensions of the sample along the x and z directions will be denoted by d and D, respectively (Fig. 1). The width D is assumed to be much greater than the thickness of the plate d or the mean free path of electrons l. Moreover, we shall assume that the conditions (1) and (3) are satisfied.

An analysis of the distribution of the current in the plate will be made for the case of an isotropic metal: we shall assume that the electron and hole Fermi surfaces are identical spheres. The mass and the mean free path are also assumed to be the same for electrons and holes. In this situation there is no Hall effect in the metal, i.e., the off-diagonal components of the conductivity tensor are identically equal to zero. In the case of an arbitrary dispersion law of electrons we can generally expect off-diagonal components of the conductivity for a compensated metal, which complicates calculations but does not affect the final result.

The magnetostatics equation for our geometry is

$$cH' = -4\pi j(x). \qquad (4)$$

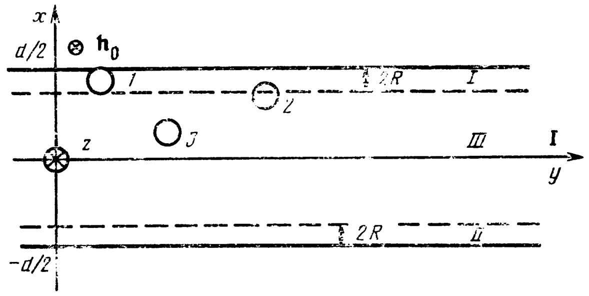

FIG. 1. Coordinate system and electron paths.

where $j(x)$ is the current density and a prime denotes a derivative with respect to the coordinate x. The boundary conditions to Eq. (4) are as follows:

$$H(-d/2)=-H(d/2)=H, \quad H=2\pi I/cD. \tag{5}$$

It follows from the Maxwell equation curl $\mathbf{E}=0$ that the electric field $E_y \equiv E$ inside a sample is spatially inhomogeneous.

We shall now consider the dynamics of charges in the total magnetic field $h_0 + H(x)$. Since in the selected model the motion of electrons and holes differs only in respect of the sign of the radius of curvature of the paths, we shall consider simply the dynamics of electrons. We must bear in mind that the contributions of electrons and holes to the diagonal components of the conductivity tensor are additive, whereas the off-diagonal components balance out.

We shall select the gauge of the vector potential of the total magnetic field in the form

$$\mathbf{A}=\{0; A(x); 0\}, \quad A(x)=h_0 x+\int_0^x dx' H(x'). \tag{6}$$

The integrals of motion of an electron in the field $h_0 + H(x)$ are the total energy, which is equal to the Fermi energy $\varepsilon_F = p_F^2/2m$, and the generalized momenta $p_z = mv_z$ and $p_y = mv_y - eA(x)/c$ (m is the mass of an electron, whereas v_z and v_y are the components of the electron velocity). The velocity component v_x is given by

$$v_x=\pm m^{-1}[p_\perp^2-(p_y+eA(x)/c)^2]^{1/2}, \quad p_\perp=(p_F^2-p_z^2)^{1/2}. \tag{7}$$

In the case under discussion it is natural to distinguish between two surface regions (I and II in Fig. 1) in the investigated metal plate:

$$d/2-2R<|x|<d/2. \tag{8}$$

The main contribution to the conductivity in these regions comes from "surface" electrons colliding with the boundary of the sample (electron paths represented by a circle 1 in Fig. 1). According to Refs. 3 and 4 the conductivity contribution of these electrons depends on the nature of their scattering by the surface and ranges from a value of the order of $\sigma_0 R/l$ for diffuse scattering to σ_0 for purely specular reflection. The surface regions contain also Larmor electrons (paths represented by circles of type 2 in Fig. 1), but their contributions to the conductivities of regions I and II are much less than the conductivity due to the surface electrons.

Well inside the metal (region III in Fig. 1), if

$$-d/2+2R<x<d/2-2R, \tag{9}$$

only the Larmor electrons are present (paths of type 2 and 3 in Fig. 1). In the momentum space $(p_y, p_\perp)$ these electrons occupy a region

$$-(e/c)A(x)-p_\perp<p_y<-(e/c)A(x)+p_\perp, \quad 0<p_\perp<p_F. \tag{10}$$

The general expression for the conductivity due to the Larmor electrons in an inhomogeneous magnetic field was first derived in Ref. 2; it has the form

$$j_L(x)=\frac{3}{2\pi}\frac{\sigma_0 E}{p_F^2 l}\left\{\int_{O_L}\frac{dp_\perp dp_y p_\perp v_y(x)}{[p_F^2-p_\perp^2]^{1/2}|v_x(x)|}\right.$$
$$\times\left[\int_{x_1}^{x} dx'\frac{v_y(x')}{|v_x(x')|}\operatorname{sh}(\nu\tau(x;x'))\right.$$
$$\left.\left.+\frac{\operatorname{ch}(\nu\tau(x_1;x))}{\operatorname{sh}(\nu T)}\int_{x_1}^{x_2} dx'\frac{v_y(x')}{|v_x(x')|}\operatorname{ch}(\nu\tau(x_2;x'))\right]\right\}. \tag{11}$$

Here, $\nu = p_F/ml$ is the frequency of electron collisions in the bulk of the metal; O_L is the region defined by Eq. (10) in the momentum space; $x_1 < x_2$ are the turning points of the electron described by the equation $v_x(x_{1,2}) = 0$;

$$\tau(x_1;x)=\int_{x_1}^{x}\frac{dx'}{|v_x(x')|}, \quad T=\tau(x_1;x_2). \tag{12}$$

We shall now expand Eq. (11) for the current density in terms of the small parameter $(\nu T)^2 \sim (R/l)^2 \ll 1$. The principal (gradient) term of the expansion is

$$j_g(x)=\frac{3}{2\pi}\frac{\sigma_0 mE}{p_F^3}\int_{O_L}\frac{dp_\perp dp_y p_\perp v_y(x)}{[p_F^2-p_\perp^2]^{1/2}|v_x(x)|}$$
$$\times\frac{1}{T}\int_{x_1}^{x_2} dx'\frac{v_y(x')}{|v_x(x')|}. \tag{13}$$

We shall analyze this term by making the following simple transformations in the density of the Larmor electron current. We shall integrate the integral with respect to x' by parts and then, altering the order of integration, we shall make the integral with respect to x' the outer one. Then, allowing for the condition (3), we obtain

$$j_g(x)=-\frac{3\sigma_0 E}{2\pi^2 h_0}\int_{x-2R}^{x+2R} dx' H'(x')\operatorname{sign}(x-x')Q\left(\left|\frac{x-x'}{R}\right|\right), \tag{14}$$

where

$$Q(t)=\int_{t/2}^{1} d\xi\int_{-\xi+t}^{\xi} d\varkappa\frac{\varkappa\xi[\xi^2-(\varkappa-t)^2]^{1/2}}{[1-\xi^2]^{1/2}[\xi^2-\varkappa^2]^{1/2}}. \tag{15}$$

The constant-sign function $Q(t)$ is plotted in Fig. 2. It is clear from Eq. (14) that the current density described by the principal term $j_g(x)$ exists only because of an inhomogeneity of the intrinsic magnetic field of the current.

The next term of the expansion of Eq. (11) in terms of the parameter $(\nu T)^2$ gives the familiar expression for the magnetoconductivity $\sigma_0 E(R/l)^2$.

It therefore follows that the Larmor electron current density can be represented by a sum of two terms represent-

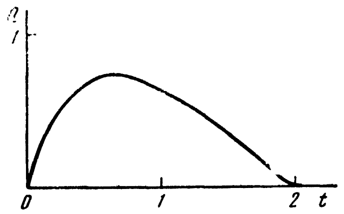

FIG. 2. Graphical representation of the function $Q(t)$ of Eq. (15).

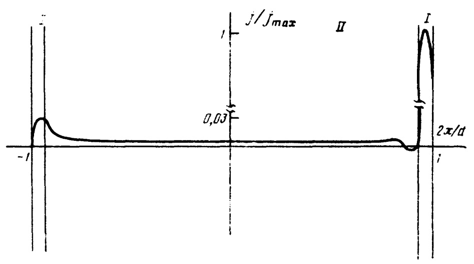

FIG. 3. Distribution of the current density in a plate characterized by specular reflection of one of the faces ($x = d/2$) and diffuse reflection by the other face; $j_{max} \approx 0.39\sigma_0 E$.

ing two conduction mechanisms described in Sec. 1: the drift of electrons in an inhomogeneous magnetic field $h_0 + H(x)$ and the diffusion of the centers of the Larmor orbits due to collisions in the bulk of the metal:

$$j_L(x) = j_g(x) + \sigma_0 (R/l)^2 E. \tag{16}$$

3. Substitution of the Larmor electron current density in Eqs. (14) and (16) into the magnetostatic equation (4) readily yields an integral equation for $j_g(x)$, which is valid in the bulk of the sample (region III in Fig. 1):

$$j_g(x) = \alpha \int_{x-2R}^{x+2R} \frac{dx'}{R} j_g(x') \operatorname{sign}(x-x') Q\left(\left| \frac{x-x'}{R} \right| \right) + \varphi(x),$$

$$\varphi(x) = \alpha \int_{x-2R}^{x+2R} \frac{dx'}{R} j_s(x') \operatorname{sign}(x-x') Q\left(\left| \frac{x-x'}{R} \right| \right),$$

$$\alpha = \frac{6}{\pi} \frac{\sigma_0 E R}{c h_0} \tag{17}$$

Here,

$$j_s(x) = j_{\rm I}(x)\theta(x-d/2+2R) + j_{\rm II}(x)\theta(-x-d/2+2R)$$

is the current density in the surface regions I and II which are $2R$ thick and are located near the upper and lower faces of the plate (Fig. 1). As pointed out in Sec. 2, the conductivity of these regions is primarily due to a group of the surface electrons. The current densities $j_{\rm I}(x)$ and $j_{\rm II}(x)$ depend strongly on the specularity of the boundaries of the sample. We cannot give here the expressions for $j_{\rm I}(x)$ and $j_{\rm II}(x)$ because the structure of the distribution of the current density in the bulk of the metal is sensitive not to the actual form of the functions $j_{\rm I}(x)$ and $j_{\rm II}(x)$, but to the characteristic order-of-magnitude of these functions.

The solution of Eq. (17) can be represented by the Neumann series:

$$j_g(x) = j_1(x) + j_2(x) + \ldots \quad j_1(x) = \varphi(x),$$

$$j_k(x) = \alpha \int_{x-2R}^{x+2R} \frac{dx'}{R} j_{k-1}(x') \operatorname{sign}(x-x') Q\left(\left| \frac{x-x'}{R} \right| \right). \tag{18}$$

The series (18) converges for moderately high values of the electric field when

$$\alpha \leqslant 1. \tag{19}$$

The inequality (19) is known to ensure the necessary low intensity of the intrinsic magnetic field of the surface electron current, compared with the field h_0 [see Eq. (3)].

We shall now consider the first term of the series (18), which is $j_1(x) = \varphi(x)$. It follows from the definition of $\varphi(x)$ given by the system of equations (17) that the current $j_1(x)$ differs from zero only in a region $d/2 - 4R < |x| < d/2 - 2R$. Then, in presence of the function $\operatorname{sign}(x - x')$ in the integrand the current $j_1(x)$ in the region $-d/2 + 2R < x < -d/2 + 4R$ (near the lower face in Fig. 1) is positive, whereas at the upper face when

$$d/2 - 4R < x < d/2 - 2R \tag{20}$$

it is negative. This means that in the case when $\varphi(x)$ exceeds the term $\sigma_0 E (R/l)^2$ in the expression for the current density (16), the current in the region defined by Eq. (20) is directed opposite to the electric field **E**.

If electrons are reflected specularly from the surface, $j_1(x)$ is of the order of $\alpha\sigma_0 E$, but if the reflection is diffuse, we have $-\alpha\sigma_0 E(R/l)$. Consequently, a region with a negative current exists if

$$(R/l)^2 < \alpha \leqslant 1 \tag{21}$$

in the case of specular reflection of electrons from the upper face of the plate and

$$R/l < \alpha \leqslant 1 \tag{22}$$

in the diffuse reflection case.

The appearance of the current density $j_1(x)$ is due to the surface current $j_s(x)$ of the Larmor electrons (paths of type 2 in Fig. 1). This effect is analogous with the anomalous penetration of a high-frequency current into a metal.[5] As in the case of anomalous penetration, the current layer of Eq. (20) excites a current in the next layer

$$d/2 - 6R < x < d/2 - 4R \tag{23}$$

and so on. The current density subsequently reverses its sign and decreases by a factor $\alpha^{-1} \gg 1$. The Neumann series (18) for $j_g(x)$ corresponds to a specific spatial distribution of the current density: if $\alpha \ll 1$, each term of the series $j_k(x)$ describes the current density in the next layer $d/2 - 2(k+1)R < |x| < d/2 - skR$. The oscillations of the current density resulting from this transport of carriers along paths exist against a background of the coordinate-independent current, which is due to the magnetoresistance effect. Therefore, well inside the metal where the amplitude of these oscillations becomes less than $\sigma_0 E(R/l)^2$, the regions with a negative current disappear.

We shall now estimate the number of the spatial regions with the negative current directed opposite to the electric field: $n \sim \ln(R/l)/\ln(0.3\alpha)$. Strictly speaking this estimate is valid if $\alpha \ll 1$. However, a numerical calculation shows that it is correct also for $\alpha = 1$. It is found that if $\alpha = 1$, the Neumann series of Eq. (18) converges well: the peaks of the current decay rapidly on increase in their number. The series (18) begins to diverge at $\alpha \approx 3$. The "reverse" processes of the transport of electrons from the next layer to the preceding one become important, which suppresses the correspondence between the k th term of the series (18) and the current density in the k th layer.

It therefore follows that the competition between two conduction mechanisms due to the Larmor electrons gives rise to a very distinctive distribution of the current density across the plate thickness. Near the upper face the current density oscillates with a period $2R$ and the amplitude of each subsequent oscillation decreases by a factor of $3\alpha^{-1}$ compared with the preceding one. At the lower face of the sample the current density varies monotonically from its maximum value in the surface layer II to $\sigma_0 E(R/l)^2$ well inside the metal.

Figure 3 shows graphically the distribution of the current density in a metal plate calculated on a computer for the case when the upper face of the sample is specularly reflecting and the lower is diffusely reflecting. The parameters R/l and R/d are each equal to 0.03; $\alpha = 0.01$.

Reversal of the direction of the electric $\mathbf{E}$ or magnetic $\mathbf{h}_0$ field destroys the oscillations of the current density at the upper face of the plate and creates them at the lower face. When the nature of the reflection of electrons by the upper and lower faces is different, this reversal of the sign of E or h_0 results in a change in the oscillation amplitude and, consequently, alters the number of regions carrying a negative current.

4. The characteristic features of the distribution of the current density across the thickness of our sample, investigated in the preceding section, affect directly the current-voltage characteristic of a metal plate. We can find the contribution I_g of the gradient conduction mechanism to the total current through the sample simply by integrating $j_g(x)$ of Eq. (17) over the whole region described by Eq. (9). We then obtain the following expression for the gradient current

$$I_g \approx \alpha (I_{II} - I_I) \propto E^2. \qquad (24)$$

Here, I_I and I_{II} are the surface electron currents at the upper and lower faces, respectively. It follows from Eq. (24) that the sign of the deviation of the current-voltage characteristic from Ohm's law is sensitive to the nature of the electron reflection by the faces of the metal plate.

If the degree of specularity of the face $x = d/2$ is higher than that of the face $x = -d/2$, the current-voltage characteristic is sublinear, whereas in the opposite case it is superlinear. For example, in the case when the $x = d/2$ face is specularly reflecting and the $x = -d/2$ face reflects diffusely, the current of Eq. (24) becomes

$$I_g = -\frac{2}{29} \frac{(\sigma_0 E R)^2}{c h_0} \operatorname{sign} E \operatorname{sign} h_0. \qquad (25)$$

We note a rather curious nonreciprocity of the current-voltage characteristic. Reversal of the sign of the electric field modifies the characteristic from sublinear to superlinear or vice versa. This is a consequence of a change in the structure of the distribution of the current density which occurs when the sign of E is reversed (see the end of Sec. 3).

According to Eq. (24), the deviation of the current-voltage characteristic from Ohm's law is small and quadratic in the electric field, if the parameter α is much less than unity. However, it is clear that an increase in the electric field when $\alpha \sim 1$ makes the nonlinearity of the characteristic very strong, so that it can be detected readily in experiment. Then, the distribution of the current density in the investigated sample is of alternating sign and the characteristic remains nonreciprocal.

We shall now give relevant numerical estimates of the nonlinearity of the current-voltage characteristic. In the case of a sample of tungsten with nonspecular faces and with $d = l = 3$ mm and $D = 1$ cm the parameter α becomes comparable with unity in an external magnetic $h_0 = 100$ Oe and the current-voltage characteristic becomes strongly nonlinear when the total current reaches $I \sim 100$ A. The Joule heat power released per unit surface of the sample is then of the order of 10^{-4} W/cm^2. The spatial distribution of the alternating-sign current density across the thickness of the sample appears at much lower currents, $I \sim 10$ A.

A direct consequence of an asymmetric spatial distribution of the current density is the excitation of a magnetic moment M in our sample, which can also be detected experimentally. A calculation shows that the average moment per unit volume is

$$M \approx (I_I - I_{II})/cD - \alpha (I_I + I_{II})/cD. \qquad (26)$$

The first term in Eq. (26) is a linear function of the electric field and is due to the difference between the degrees of specularity of the reflection of electrons by the upper and lower faces of the investigated plate. The second term in Eq. (26) depends quadratically on the electric field E and is related to spatial oscillations of the current density in the interior of the sample.

5. The above analysis was carried out assuming an infinite plate. It was postulated that the current I flowing in the plate induces only the z component of the magnetic field $H_z = H$, which in final analysis is the source of the investigated nonlinear effects. However, in any plate of finite thickness D we have not only the z component of the intrinsic magnetic field of the current, but also the normal component H_x, where H_x is of the same order of magnitude as H_z. In this section we shall discuss the possibility that the field H_x affects the phenomena of interest to us.

We note first of all that the quantity H_x is a much smoother function of the coordinates than the field H_z. In fact, it follows from the Maxwell equation $\operatorname{div} \mathbf{H} = 0$ that the characteristic scale of the field H_x along the x axis is governed by the size D. On the other hand, the field H_z varies over distances of the order of $2R \ll d \ll D$. Therefore, the contribution of the field H_x to the gradient conductivity of the Larmor electrons is extremely small.

Moreover, the influence of H_x on the magnetoresistance of our sample is negligible because $H_x \ll h_0$. Nevertheless, the presence of the field H_x has the effect that the Larmor electrons do in fact move in a total magnetic field which is tilted slightly relative to the surfaces of the sample. For this reason there is a drift of carriers at right-angles to the faces of the plate and this, in principle, can result in "smearing" of the variable-sign distribution of the current density. Obviously, such smearing can be ignored only if the drift of the centers of the Larmor orbits lH_x/h_0 along the x axis during the free electron time is considerably less than the characteristic scale $2R$ of the distribution

$$H_x/h_0 < R/l. \qquad (27)$$

The magnetic field $H_x \sim H$ occurring in Eq. (27) and governing the tilt of the total magnetic field relative to the faces of the sample is governed by the total current I. In the case of plates of thickness $d \lesssim l$ the current I is concentrated mainly in a surface layer of depth $2R$ and the conductivity of this layer depends on the nature of the interaction of the surface electrons with the faces of the plate. We can rewrite the inequality of Eq. (27) in a form containing explicitly the specularity parameter of the faces, so that the surface electron current becomes

$$I=\sigma_0 ERD[1+(1-\rho)l/R]^{-1}. \qquad (28)$$

Equation (28) is of interpolation nature and is qualitative. We can easily show that it gives the correct order of magnitude of the results in the case of purely specular reflection (when the specularity parameter ρ is unity) and in the case of diffuse scattering of electrons ($\rho = 0$). Equations (28) and(5) allow us to rewrite the inequality (27) in the form

$$\alpha<R/l+1-\rho, \qquad (29)$$

where the parameter α is given by Eq. (17).

In the situation when the reflection of surface electrons by the faces of the plate is not too close to specular,

$$1-\rho\sim 1, \qquad (30)$$

the inequality (29) is automaticallv satisfied because of the theoretical inequality (19). This means that the drift in the field H_x may play a role in the effects under discussion only if the reflection of the surface electrons is very nearly specular and the parameter α is sufficiently large.

A simple analysis shows that in the range

$$1-\rho<R/l, \quad R/l<\alpha<1 \qquad (31)$$

the presence of the field H_x reduces the surface electron conductivity σ_s compared with σ_0, which is the conductivity in the absence of H_x:

$$\sigma_s\sim\sigma_0(R/l\alpha)^{1/2}\ll\sigma_0. \qquad (32)$$

This reduction in σ_s is due to the fact that the surface electrons do not spend all their free time ν^{-1} near the face of a sample but move away from it due to the drift in the field H_x for a time $\nu^{-1}(R/l\alpha)^{1/2}\ll\nu^{-1}$. The parameter α, which occurs in the expression for σ_s given by Eq. (32), is proportional to the electric field E. Therefore, under the conditions described by Eq. (31) the current-voltage characteristic of the investigated metal plate becomes

$$V\propto I^2. \qquad (33)$$

It is important to stress that in the situation described by Eq. (31) the Larmor electrons ensure the transport of the current from the surface layer into the plate, in spite of the drift in the field H_x. Moreover, the presence of the field H_x does not prevent the appearance of the variable-sign distribution of the current density, similar to that described above and expected in the case when $\alpha<R/l$ or when $(1-\rho)\sim 1$. The occurrence of the drift under the conditions of Eq. (31) alters the characteristic spatial scale [it becomes $(Rl\alpha)^{1/2}$ instead of R] and the amplitude of the peaks. An analysis of the peak structure of the distribution of the current density in such situation is a separate topic.

It therefore follows that the presence of the normal component of the intrinsic magnetic field H_x may influence the distribution of the current density and other effects investigated above only if the faces of the sample are characterized by a very high degree of specularity, which can hardly be achieved in real experiments.

6. We shall conclude by noting that the variable-sign distribution of the current density appears also in one situation which is of major interest. We shall consider a plate of a compensated metal in the absence of an external magnetic field and we shall assume that it carries a large current so that the characteristic radius of curvature of the electron paths in the intrinsic magnetic field of the current is less than the mean free path l and the dimensions of the sample. According to Eq. (2), the pinch effect appears under these conditions: the current I is concentrated mainly in the central part of the plate of width $2R$ ($R=cp_F/eH$, $H=2\pi I/cD$). The Larmor electrons should transfer the current from this filament to the periphery exactly as in the problem discussed above the current is transferred from the surface layers into the metal. The spatial oscillations of the current density which appear in this case may result in an instability of the pinch-effect pattern and give rise to spontaneous oscillations of the voltage. Since such oscillations have been observed experimentally,[6] a theoretical analysis of a variable-sign distribution of the current density under the pinch-effect conditions would be highly desirable.

[1]V. T. Dolgopolov and P. N. Chuprov, Zh. Eksp. Teor. Fiz. **83**, 2287 (1982) [Sov. Phys. JETP **56**, 1325 (1982)].
[2]É. A. Kaner, Yu. G. Leonov, N. M. Makarov, and V. A. Yampol'skiĭ, Zh. Eksp. Teor. Fiz. **93**, 2020 (1987) [Sov. Phys. JETP **66**, 1153 (1987)].
[3]I. M. Lifshitz, M. Ya. Azbel', and M. I. Kaganov, *Electron Theory of Metals*, Consultants Bureau, New York (1973).
[4]A. I. Kopeliovich, Zh. Eksp. Teor. Fiz. **78**, 987 (1980) [Sov. Phys. JETP **51**, 498 (1980)].
[5]É. A. Kaner and V. F. Gantmakher, Usp. Fiz. Nauk **94**, 193 (1968) [Sov. Phys. Usp. **11**, 81 (1968)].
[6]S. I. Zakharchenko, S. V. Kravchenko, and L. M. Fisher, Zh. Eksp. Teor. Fiz. **91**, 660 (1986) [Sov. Phys. JETP **64**, 390 (1986)].

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