A simple method for calculation of nonexponential wave damping in a randomly inhomogeneous medium with a long-range noise correlation of the $1/r$ type

L. Ts. Adzhemyan, A. N. Vasil'ev, M. M. Perekalin, and Kh. Yu. Reittu

A. A. Zhdanov State University, Leningrad
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A simple derivation is presented of the formula for nonexponential damping of a wave propagating in a randomly inhomogeneous medium with a noise correlation of the $1/r$ type for $r\to\infty$ (Goldstone fluctuations in liquid crystals). The calculational method is based on the separation of the noise field into a hard and a soft component. The hard component is taken into account in the lower order of the perturbation theory, whereas the soft component is taken into account in the eikonal approximation. For real values of the parameters this accuracy is sufficient to describe the results of experiments on the propagation of light in nematic liquid crystals, the errors being less than a fraction of a percent.

1. INTRODUCTION

The propagation of harmonic oscillations $u(\mathbf{x}, t) = u(\mathbf{x})\exp(-i\omega t)$ in a randomly inhomogeneous medium is described by the Helmholtz equation for the amplitude $u(\mathbf{x})$, (for a more precise account, see Sec. 2) with a random component (noise) $\varphi(\mathbf{x})$ in the dielectric constant. One usually assumes the noise distribution to be Gaussian with zero mean $\langle\varphi\rangle = 0$ and prescribed correlation $D(\mathbf{x}, \mathbf{x}') \equiv \langle\varphi(\mathbf{x})\varphi(\mathbf{x}')\rangle$. Usually the noise is taken to be short-range: $D\propto\exp(-r/r_c)$ as $r\equiv|\mathbf{x}-\mathbf{x}'|\to 0$, and the parameter r_c is called the correlation length. For such correlations the mean field $\langle u(\mathbf{x})\rangle$ (the square of its modulus determines the intensity of the coherent component) always damps exponentially.

A long-range correlation $D\propto r^{-\sigma}$ as $r\to\infty$ is associated formally with $r_c=\infty$. The correlation of the density fluctuations of a liquid right at the critical point has such a form (there $D\propto r^{-1-\eta}$, where $\eta\approx 0.03$ is the Fisher index[1]), as does the correlation of the Goldstone fluctuations in nematic (and other) liquid crystals. For them $D\propto 1/k^2$ in the momentum representation, and $D\propto 1/r$ in the coordinate representation (space is always taken to be three-dimensional). For correlations $D\propto r^{-\sigma}$ with $\sigma>1$ the damping still remains exponential, but for $\sigma\leqslant 1$ it becomes faster.[1)] In particular, for $D\propto 1/r$ it was shown in Ref. 2 that the field of a point source at large distances has the form

$$\langle u\rangle=(4\pi r)^{-1}\exp\{imr-\xi mr[\ln(2mr)-1+C+i\pi/2]\},\qquad(1)$$

where ξ is a dimensionless small ($\sim 10^{-5}$) parameter which characterizes the strength of the interaction of the wave with the noise, $m=2\pi/\lambda$, λ is the wavelength in the medium without noise, and $C=0.577$ is the Euler constant. Relation (1) is approximate. In the exponent small corrections with higher powers of ξ, which are unimportant in the region $\xi mr\sim 1$, have been discarded.

Expression (1) for the scalar field was obtained in Ref. 2 with the help of the infrared (IR) representation of the propagator proposed there and later rigorously founded in Ref. 3. A generalization of Eq. (1) to the real case with vector and tensor objects (liquid crystals) is given in Ref. 4. The mathematical manipulations of Refs. 2 and 3 are quite complicated and cumbersome. Here we present another derivation of Eq. (1), which is suitable only for finding the asymptotic limit for $mr\gg 1$ (only it is usually needed), but is much shorter and based on simple physical considerations.

2. FORMULATION OF THE PROBLEM AND DIAGRAMMATIC TECHNIQUE

As in Refs. 2 and 3, we will consider the scalar case: we seek a solution of the wave equation

$$[\varepsilon(\mathbf{x})c^{-2}\partial_t^2-\Delta]u(\mathbf{x}, t)=J(\mathbf{x}, t)$$

with prescribed source $J(\mathbf{x}, t)$, dielectric constant $\varepsilon(\mathbf{x})=\varepsilon_0+a_0\varphi(\mathbf{x})$ with prescribed constants ε_0 and a_0, and a time-independent random noise field $\varphi(\mathbf{x})$. All of the quantities in the expression for ε are dimensionless, φ has the sense of relative fluctuations of the density of the medium, and a_0 is the electrooptical coupling constant. The correlation $\langle\varphi(\mathbf{x})\varphi(\mathbf{x}')\rangle$ will always be assumed to depend only on $r\equiv|\mathbf{x}-\mathbf{x}'|$, and we will use the notation $D(\mathbf{x}, \mathbf{x}')$ or $D(r)$ for it, and for its Fourier transform $D(\mathbf{k})$ or $D(k)$, where $k\equiv|\mathbf{k}|$. The Fourier transformation for functions of the type under consideration will always be defined by the relation

$$F(\mathbf{x}, \mathbf{x}')=(2\pi)^{-3}\int d\mathbf{k}\,F(k)\exp[i\mathbf{k}(\mathbf{x}-\mathbf{x}')].$$

The correlation

$$D(k)=\frac{D_0l_0}{k^2},\qquad D(r)=\int\frac{d\mathbf{k}}{(2\pi)^3}D(k)e^{i\mathbf{kr}}=\frac{D_0l_0}{4\pi r},\qquad(2)$$

where l_0 is the characteristic minimal dimension of the fluctuations and D_0 is a dimensionless coefficient which has been introduced for generality, corresponds to fluctuations of Goldstone type. In our problems $l_0\approx 5\cdot 10^{-8}$ cm is the interatomic distance; $D_0\sim 1$; in the dielectric constant we have $\varepsilon_0\gtrsim 1$ and $a_0\sim 0.3$; and the typical wavelength of light is $\lambda=5\cdot 10^{-5}$ cm.

For a harmonic source $J(\mathbf{x}, t)=J(\mathbf{x})\exp(-i\omega t)$ with $\omega>0$, with the substitution $u(\mathbf{x}, t)=u(\mathbf{x})\exp(-i\omega t)$, the initial wave equation reduces to the form

$$L_\varphi u=J,\quad L_\varphi\equiv L_0-v\varphi(\mathbf{x}),\quad L_0\equiv-\Delta-m^2,\qquad(3)$$

$$m=2\pi/\lambda+i0,\quad v=a_0m^2/\varepsilon_0,\quad \lambda=\lambda_0/\varepsilon_0^{1/2},\quad \lambda_0=2\pi c/\omega,\qquad(4)$$

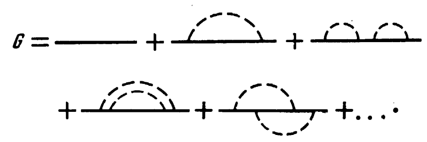
where λ_0 is the wavelength of light in vacuum, and λ is the wavelength of light in the homogeneous medium in the absence of noise. The term $+i0$ in m corresponds to the standard delay condition and is important only in the definition of the unperturbed Green's function (propagator) $G_0 = L_0^{-1}$ in Eq. (3):

$$G_0(\mathbf{p}) = 1/(p^2 - m^2),\quad G_0(\mathbf{x}, \mathbf{x}') \equiv G_0(r) = e^{imr}/4\pi r. \tag{5}$$

The solution of Eq. (3) has the form

$$u(\mathbf{x}) = \int d\mathbf{x}'\, G_\varphi(\mathbf{x}, \mathbf{x}')\, J(\mathbf{x}'),$$

or, symbolically, $u = G_\varphi J$, where $G_\varphi = L_\varphi^{-1}$ is the propagator in the random field φ. We are interested in $\langle u \rangle = GJ$, where $G \equiv \langle G_\varphi \rangle$ is the propagator averaged over the noise. In the theory of perturbations in $v\varphi$ in Eq. (3), it is represented in the form of an infinite sum of Feynman diagrams:



(6)

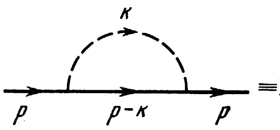
The propagator (6) satisfies the Dyson equation $G^{-1} = G_0^{-1} - \Sigma$, or in the momentum representation

$$G^{-1}(p) = p^2 - m^2 - \Sigma(p), \tag{7}$$

where Σ is the sum of all irreducible diagrams without external lines:

(8)

The contribution of the first diagram in detailed notation has the form



$$= \Sigma_1(p) = \frac{v^2}{(2\pi)^3} \int d\mathbf{k}\, D(\mathbf{k})\, G_0(\mathbf{p} - \mathbf{k}). \tag{9}$$

For the correlation (2) the quantity (see the equations in Sec. 4)

$$\xi = v^2 D_0 l_0 / 16\pi m^3 = a_0{}^2 D_0 l_0 / 8\varepsilon_0{}^2 \lambda \tag{10}$$

is a dimensionless parameter of the perturbation theory, which characterizes the strength of the interaction of the wave with the noise. For real values of the parameters $\varepsilon_0 \sim D_0 \sim 1$, $a_0^2 \sim 0.1$, $l_0/\lambda \sim 10^{-3}$, we have $\xi \sim 10^{-5}$, so that the interaction is quite weak.

Let us discuss briefly some properties of the solution for different correlations D, assuming throughout that the positive (by definition) function $D(k)$ does not vanish at infinity for finite values of k, and that $k \to \infty$ in such a way that in the diagram there are no ultraviolet (UV) divergences in the region of large momenta. For a weak interaction the asymptotic limit of $G(r)$ as $r \to \infty$ is determined in the momentum representation by the behavior of $G(p)$ in the neighborhood of the "mass surface" $p = m$. Infrared (IR) divergences from the region of small momenta k on the noise lines can appear on it in the diagrams $\Sigma(p)$ since for $p = m$ propagators of the type $G_0(\mathbf{p} - \mathbf{k}) = [(\mathbf{p} - \mathbf{k})^2 - m^2]^{-1}$ have a singularity of the form $1/k$ as $k \to 0$.

Depending on the behavior of $D(k)$ as $k \to 0$, we distinguish two cases, namely 1) the case of a short-range correlation for which $D(k)$ is regular in the vicinity of zero, and 2) the case of a long-range correlation with the singularity $D(k) \propto k^{-\alpha}$ as $k \to 0$, which corresponds to $D(r) \propto r^{-3+\alpha}$ as $r \to \infty$. The first case is the usual one, and the Ornstein-Zernike correlation ("massive noise"), for example, $D(k) \propto (k^2 + r_c^{-2})^{-1}$ with finite correlation length r_c and the regularized Komolgorov spectrum with $D(k) \propto (k^2 + r_c^{-2})^{-11/6}$, for which the external turbulence scale plays the role of r_c, are of this type.[5] About long-range correlations we have already spoken in the Introduction. In cases 1 and 2 with $\alpha \leqslant 1$, all the diagrams $\Sigma(p)$ are finite for $p = m$; for $1 < \alpha < 2$ the first diagram $\Sigma_1(p)$ is finite, but IR divergences are present in the subsequent diagrams; for $\alpha \geqslant 2$ the first diagram (9) also diverges; for $\alpha = 2$ [correlation (2)] this divergence is logarithmic, and for $\alpha > 2$ it follows a power law. If $\Sigma_1(p = m)$ is finite (cases 1 and 2 with $\alpha < 2$), then the right side of Eq. (7) has a zero for some, generally speaking, complex value of $p = m + \delta m$. The "mass shift" δm is calculated by perturbation theory, in cases 1 and 2 with $\alpha \leqslant 1$—in the form of a series of integer powers of the coupling constant ξ [defined by Eq. (10) for correlation (2)], and for $1 < \alpha < 2$—in the form of a series of fractional positive powers of ξ. In the latter case the IR divergences of the higher diagrams of Σ for $p = m$ are effectively regularized by the small finite mass shift of the first diagram. For short-range correlations the asymptotic behavior of the propagator (i.e., of the field of the point source) in the region $mr \gg 1$ differs from that of the free propagator (5) only in the mass shift and the normalization factor Z, and the imaginary part δm determines the damping

$$G(r)|_{mr \gg 1} \approx (Z/4\pi r) \exp[i(m + \delta m) r]. \tag{11}$$

For a weak interaction $|\delta m|$ is much less than m, therefore in the calculation of the damping it is sufficient to keep only the first diagram:

$$\delta m_1 = \Sigma_1(p = m)/2m. \tag{12}$$

For long-range correlations the asymptotic limit (11) is distorted, and this case is considered in the following section.

3. THE EIKONAL APPROXIMATION

In contrast with Refs. 2 and 3, here we are interested only in the asymptotic limit $mr \gg 1$, which corresponds to the asymptotic limit as $p \to m$ in the momentum representation. In a concrete diagram $\Sigma(p)$ a higher singularity in $p - m$ is generated by the region of small momenta $k_1, k_2, \ldots$ for every $D(k)$. The internal lines G_0 of the diagram correspond to the factors

$$G_0(\mathbf{p} - \Sigma \mathbf{k}_\alpha) = [(\mathbf{p} - \Sigma \mathbf{k}_\alpha)^2 - m^2]^{-1} = [p^2 - m^2 - 2\mathbf{p}\Sigma \mathbf{k}_\alpha + (\Sigma \mathbf{k}_\alpha)^2]^{-1}, \tag{13}$$

whose denominators have a zero of the order of k on the mass surface $p = m$ for all of the $\mathbf{k}_\alpha$. In the region of small k the contributions to expression (13) which are quadratic in k are unimportant in comparison with the linear ones, so discarding them in all of the internal lines has no effect on the leading singularity of the diagram in $p-m$. This corresponds to the eikonal approximation, in which, as is well known,[5,6] all the diagrams of the propagator are summed exactly for arbitrary noise correlation; in the coordinate representation the result has the form

$$G_{\text{eik}}(\mathbf{x},\mathbf{x}')=\left\langle \frac{1}{4\pi r}\exp[imr+i\psi(\mathbf{x},\mathbf{x}')]\right\rangle, \tag{14}$$

where ψ is the eikonal phase:

$$\psi(\mathbf{x},\mathbf{x}')=\frac{v}{2m}\int_{\mathbf{x}'}^{\mathbf{x}} dt\,\varphi(\mathbf{x}_t). \tag{15}$$

The integration in Eq. (15) extends over the ray joining the source point x' with the observation point $\mathbf{x}$; the scalar t is the pathlength along the ray; and $\mathbf{x}_t$ is the corresponding point on the ray. The quantity mr in Eq. (14) is the integral of the constant m over the same ray, and the total phase in Eq. (14) is obtained from this integral by making the substitution $m\to m+\delta m$, where $\delta m(\mathbf{x})=v\varphi(\mathbf{x})/2m$ is a variable small increment to the mass which arises from the addition $v\varphi(\mathbf{x})$ to m^2 in Eq. (3).

Averaging the exponent in Eq. (14) over the Gaussian noise gives

$$G_{\text{eik}}(\mathbf{x},\ \mathbf{x}')=(4\pi r)^{-1}\exp[imr-\beta(r)], \tag{16}$$

where

$$\beta(r)\equiv\frac{v^2}{8m^2}\iint_{\mathbf{x}'}^{\mathbf{x}} dt\,dt'\,D(\mathbf{x}_t,\mathbf{x}_{t'})=\frac{v^2}{4m^2}\int_0^r d\rho\,(r-\rho)D(\rho). \tag{17}$$

The second equality was obtained by taking into account the assumption $D(\mathbf{x}_t,\mathbf{x}_{t'})=D(\rho)$, where $\rho=|\mathbf{x}_t-\mathbf{x}_{t'}|$ is the distance between the points along the ray.

By construction, the eikonal approximation correctly reproduces the leading singularities of $G(p)$ as $p\to m$, corresponding to the leading singularities of

$$\ln G(r)=imr-\ln(4\pi r)-\beta(r)$$

in r as $r\to\infty$. The singularities in $\beta(r)$ are present only for long-range correlations $D(k)\propto k^{-\alpha}$ as $k\to 0$ (case 2 in Section 2). To estimate them, the function $D(\rho)$ in Eq. (17) can be replaced by its asymptotic limit $\sim\rho^{-3+\alpha}$ as $\rho\to\infty$, whereupon

$$\beta(r)\sim\int_0^r d\rho\,(r-\rho)\rho^{-3+\alpha} \tag{18}$$

(for integrals (18) which diverge at $\rho=0$ the UV cutoff is understood. From Eq. (18) we obtain for the different values of $\alpha>0$

$$\beta(r)=cr^{\alpha-1}+\tilde{c}r+\tilde{c},\quad \alpha<1,\quad 1<\alpha<2, \tag{19}$$

$$\beta(r)=c\ln r+\tilde{c}r+\tilde{c},\quad \alpha=1, \tag{20}$$

$$\beta(r)=cr\ln r+\tilde{c}r,\quad \alpha=2, \tag{21}$$

$$\beta(r)=cr^{\alpha-1},\quad \alpha>2. \tag{22}$$

Here all of the c and $\tilde{c}$ are constants, and the ones marked with the tilde are those which due to the divergence at $\rho=0$ depend on the indeterminate parameter of the UV cutoff. From Eqs. (19)–(22) it can be seen that the eikonal contribution to the damping becomes more important than the usual exponential contribution $\sim cr$ in $\ln G(r)$ for $\alpha\geqslant 2$; for $1<\alpha<2$ it gives the correction $r^{\alpha-1}$ in $\ln G(r)$, for $\alpha=1$— only the logarithm [which corresponds to varying the power of r in the coefficient of the exponential in Eq. (16) as in quantum electrodynamics[7]], and for $\alpha<1$ the correction $\sim r^{\alpha-1}$ disappears as $r\to\infty$.

From here on we will concentrate our attention on the case of the correlation (2), which is the most important in practice. The eikonal approximation uniquely determines the coefficient of $r\ln r$ in Eq. (21), but it does not determine the constant $\tilde{c}$, as a result of the presence in it of the UV divergence. This can also be seen in the momentum approximation: after discarding the terms $\sim k^2$ in Eq. (13), the lines of G_0 behave like $1/k$ not only at zero in k, but also at infinity, which along with $D\sim 1/k^2$ gives the logarithmic divergence in Σ_1. The well-known improvements of the eikonal approximation, e.g., the method of the parabolic equation, are presented in Ref. 5, retaining contributions $\sim\mathbf{k}_\perp^2$, where $\mathbf{k}_\perp$ is the part of $\mathbf{k}$ orthogonal to $\mathbf{p}$. This, obviously, eliminates the UV divergence in Σ_1 and leads to a unique determination of $\tilde{c}$. But the advantages in this are small since the value of $\tilde{c}$ thus obtained is equally incorrect—this constant is in fact determined by the entire region of integration over $\mathbf{k}$, not just by the small momenta; therefore any approximation which distorts the behavior of the factors (13) in the region of large k will give an incorrect value of $\tilde{c}$. To determine $\tilde{c}$ correctly, it is necessary to augment the eikonal approximation with an exact account of the region of large momenta k in the diagram Σ_1, which will be done in the next section.

4. CALCULATION OF DAMPING FOR THE CORRELATION $D\propto 1/r$

We will make use of the well-known (see, e.g., Refs. 6 and 8) idea of dividing the field φ and the corresponding correlation into a "hard" (φ_h) and a "soft" (φ_s) component by introducing a "dividing momentum" μ and taking those contributions with $k\equiv|\mathbf{k}|\geqslant\mu$ to be hard and those with $k<\mu$ to be soft. The soft correlation is defined by the integral in Eq. (2) with truncation of $k<\mu$, and calculation gives

$$D_s(r)=D(r)\left[1+\frac{2}{\pi}\operatorname{si}(\mu r)\right],\quad \operatorname{si}z=-\int_z^\infty\frac{dx}{x}\sin x, \tag{23}$$

where $D(r)$ is the usual correlation (2). The expression in square brackets in Eq. (23) has a root $\sim r$ as $r\to 0$, so that when we substitute (23) in Eq. (17) the UV divergence disappears, but then to make up for it there appears a dependence on the indeterminate parameter μ. To eliminate it and correctly determine $\tilde{c}$ in Eq. (21), it is necessary to also take into account part of the hard contributions, making use in the estimates of the smallness of the coupling constant (10). The selection principle consists in the following: the hard contributions are important in those graphs and subgraphs of Σ which in the eikonal approximation have surface UV divergences, i.e., divergences which can be detected by a simple estimate of the powers of k for $D\propto k^{-2}$, $G_0\propto k^{-1}$. It is easy to convince oneself that the only UV-divergent diagram of Σ is the simple loop (9), and it is "dangerous," both in itself and as a subgraph, for example, in the second diagram in Eq. (8). By keeping the line $D=D_s+D_h$ complete in all the simple loops Σ_1 we neglect the hard component in all the other lines D, keeping only the soft contribution D_s (we will discuss errors below). Then substituting $\Sigma_1 = \Sigma_{1s} + \Sigma_{1h}$ everywhere, we obtain for Σ a representation in the form of a sum of Σ_{1h} and all soft (i.e., with soft lines D_s) diagrams with any number of hard inserts Σ_{1h} in the internal lines of G_0.

In general for arbitrary p and r it is difficult to sum the obtained diagrams in the coordinate representation (this problem has in fact been solved in the IR representation[2,3]). But if we are only interested in the asymptotic limit $mr \gg 1$, then everything simplifies since then accounting for all the hard elements Σ_{1h} reduces, as in Eq. (11), to a simple mass shift $m \to m + \delta m_{1h}$ in the soft diagrams. Summing the latter in the eikonal approximation, we obtain

$$G(r) = (4\pi r)^{-1} \exp[i(m+\delta m_{1h})r - \beta_s(r)], \tag{24}$$

where β_s is expression (17) with the substitution $D \to D_s$, and we have neglected the small corrections of Z in Eq. (11) and of $\delta m_{1h} \ll m$ in the coefficient in Eq. (17).

Let us roughly estimate (with logarithmic accuracy) the errors in the above approximations. We are interested in the solution for those values of r for which damping is important, but the wave has not yet disappeared, which corresponds approximately to the condition $r\delta m_{1h} \sim r\xi m \sim 1$, i.e., in our case $mr \sim \xi^{-1} \sim 10^5$. In the first hard diagram of Σ the divergence as $\mu \to 0$ is logarithmic, and we have neglected it in estimating δm_{1h}. In the higher orders the IR divergences of the hard graphs of Σ are already power-law:

$$\delta m_{2h} \sim \xi^2 m^2 \mu^{-1}, \quad \delta m_{3h} \sim \xi^3 m^3 \mu^{-2}$$

etc. The requirement $\delta m_{2h} \ll \delta m_{1h}$ is therefore equivalent to $\xi m \mu^{-1} \ll 1$, which, taking $\xi mr \sim 1$ into account (see above), means that $\mu r \gg 1$. Thus, the errors that result from throwing out the hard contributions of the lines D outside Σ_1 are roughly estimated by the quantity $(\mu r)^{-1}$. On the other hand, the use of the eikonal approximation to sum the soft diagrams gives rise, first, to errors of the order of $(\mu r)^{-1}$ in the singular contributions to $\ln G(r)$ and, second, to errors of the order of μ/m in the regular terms of the type $\tilde{c}$ in Eq. (21). The optimal choice of μ is determined by the condition $\mu/m \sim (\mu r)^{-1}$, whence $(\mu r)^2 \sim mr \sim \xi^{-1}$ (see above), which corresponds to an error of

$$\mu/m \sim (\mu r)^{-1} \sim \xi^{1/2}. \tag{25}$$

Neglecting quantities of the same order in the calculations in Eq. (17), from Eqs. (23) and (17) we find

$$\beta_s(r) \approx mr\xi[\ln(\mu r) + C - 1]. \tag{26}$$

For the hard contribution in Eq. (12) of diagram (9) with correlation (2), we have

$$\delta m_{1h} = \frac{v^2 D_0 l_0}{2m(2\pi)^3} \int_{k \geq \mu} \frac{d\mathbf{k}}{\mathbf{k}^2[(\mathbf{p}-\mathbf{k})^2 - m^2]} \Bigg|_{p=m}.$$

Calculation accurate to corrections of the order of $\xi^{1/2}$ gives

$$\delta m_{1h} \approx m\xi[i \ln(2m/\mu) + \pi/2]. \tag{27}$$

Substituting Eqs. (26) and (27) into Eq. (24) leads to result (1), in which the dependence on μ cancels out. This is not a random phenomenon: expression (24), which is accurate to within contributions of the order of expression (25), which we have discarded, is the initial sum of the diagrams, in which there is no dependence on μ.

5. CONCLUSION

In the calculation of the damping we have made use of a very well-known idea—that of dividing the noise field into a soft and hard component, the first of which is taken into account in the eikonal approximation, and the second—in the theory of perturbations, in actuality—only the first graph (9), which at large distances is equivalent to the shift $m \to m + \delta m_{1h}$. The hard and soft contributions each depend individually on the dividing momentum μ, but in the sum of these contributions this dependence drops out. The relative error of the obtained result is of the order of $\xi^{1/2}$, where ξ is a dimensionless parameter which characterizes the strength of the interaction of the wave with the noise, so that the method is suitable, of course, only for a weak interaction (for light $\xi \sim 10^{-5}$, and therefore the errors consist of fractions of a percent). The method generalizes immediately to the real problem of the propagation of light in liquid crystals with vector and tensor objects. We have not presented any explicit formulas since they are cumbersome and have already been derived in Ref. 4 with the help of a much more complicated technique.[2,3] They can be obtained more simply by the above method, as in other similar problems. We only make the observation, however, that in addition to the phases in the eikonal approximation there are also small local corrections $\sim \varphi$ in the polarization vectors in the ordinary and the extraordinary modes of the light wave, but they are unimportant in the calculation of the attenuation.

The eikonal contribution correctly reproduces the leading singularity of $\ln G(r)$ as $r \to \infty$, generated by the singularity of $D(k)$ at small k, i.e., by the long-range nature of $D(r)$. For short-range correlations with finite correlation length r_c the attenuation is asymptotically everywhere exponential. But it is clear that at large values of r_c this asymptotic exponential regime will in actuality only be attained for very large values of r, i.e., where the solution is given by the same formula [Eq. (24)] as in the case of a long-range field. Analysis of expression (17) indicates that in estimating the degree of importance of r_c this quantity should be compared not with the wavelength $\lambda = 2\pi/m$, but with the characteristic damping length r_{att} defined by Eq. (17): for $r_c \ll r_{att}$ the correlation is effectively short-range, and for $r_c \gg r_{att}$ the correlation is effectively long-range. For example, for the turbulence spectrum of the atmosphere r_c is much smaller than the damping length of a laser beam ($r_c \sim 100$ cm [Ref. 5]), i.e., the correlation is effectively short-range since the power-law character of the Komolgorov spectrum ($\sim k^{-11/3}$) in the inertial range is hardly manifested (and all the more so for radio waves). Therefore real problems of a long-range character are only enumerated in the Introduction: a liquid right at the critical point T_c and any system with Goldstone fluctuations below T_c of liquid crystal type. The problem of calculating the angular distribution of the scattering intensity (the attenuation is determined by the total cross section) remains unsolved for such systems. The solution of this problem requires some kind of combination of the eikonal method with the standard equation of radiative transfer,[5] which would make it possible to solve this problem in the case of short-range correlations.

[1)] Such behavior corresponds in the momentum representation to the disappearance of the propagator pole,[2] and in analogy with the "confinement of quarks" it can be called the "confinement of light."

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