Torons and the breaking of chiral symmetry in QCD and SQCD

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A new class of self-dual toron solutions to $SU(2)$ gauge theories is considered. The solution is defined on a manifold with boundary, has topological charge $Q = \frac{1}{2}$ and action $S = (8\pi^2/g^2)Q$. The contribution of the corresponding configurations to the chiral condensate is calculated. It is shown that a nonvanishing value of the condensate is due to the quasi-zero modes in the continuum. The anomalous Konishi identity is automatically satisfied in supersymmetric QCD. A value for $\langle\bar{\psi}\psi\rangle$ is obtained for the magnitude of the condensate for QCD with $N_j = 2$.

1. INTRODUCTION

The purpose of the present paper is an analysis of the physical consequences of the existence of torons[1]—self-dual solutions with fractional topological charge. We recall that the integrality of the topological charge Q (Ref. 2) for the instanton was used due to the compactification of the physical space to a sphere, i.e., to the identification of all points at infinity. A choice of different boundary conditions may, in principle, lead to fractional topological charges. In particular, for $SU(N)$ gluodynamics defined on the hypertorus $T_1 \times T_1 \times T_1 \times T_1$ the introduction of so-called twisted boundary conditions[3] has allowed one to obtain solutions of the classical equations—torons[4]—with $Q = 1/N$ and with action $S = (8\pi^2/g^2)N^{-1}$.

In addition to the twisted boundary conditions there are other possible methods for describing solutions with fractional Q: analytic continuation into a complex space containing several Riemann surfaces, transition to a description in terms of orbifolds, or on manifolds with boundary. The latter approaches were applied in Refs. 1 and 5 and will be used in the present paper.

Although the toron solution, described for gauge theories in Ref. 1 and for σ-models in Ref. 5, is in principle formulated in different terms than in Ref. 4, we retain the term "toron" introduced in Ref. 4. By this we underline the fact that the solution minimizes the action and has topological charge $Q = 1/N$, i.e., exhibits all the properties characteristic of a toron in the sense of Ref. 4.

Some words about the interpretation of the toron solution, described in Ref. 1 and used in the present paper. In contrast with the instanton solution (Ref. 2) which is defined in a compact manifold without boundary (a sphere), the toron solution can be defined only on a manifold with boundary. In particular, the boundary for the description of the solution in Ref. 1 was the two edges of a cut in the complex plane $z = r + it$, $r = (x_i x_i)^{1/2}$. Although the fields may experience a jump across the cut, the physical quantities (of the type of the squared field strength $G_{\mu\nu}^2$) are single valued. By means of a conformal mapping a manifold with a cut can be mapped onto a disc of radius R (Ref. 5, c). The initial Euclidean space can be interpreted as the limit of this disc for $R \to \infty$. In such an interpretation the toron solution strongly reminds one of the solution of Ref. 4, with the size L of the four-dimensional box playing the role of a regulator.

One may proceed differently and effect a conformal mapping onto the exterior of a circle of radius $\Delta \to 0$. In this case the toron solution can be interpreted as a point defect (as the magnitude of the regulator $\Delta \to 0$). The toron action $S_{\rm cl} = (8\pi^2/g^2)Q$ does not depend on the magnitudes of the dimensional parameters R, Δ. The latter interpretation of the toron as a point defect regularized in such a manner that the self-duality equations are satisfied also for nonzero values of Δ, seems to be the preferable one. Only at the end of the computations is the magnitude of Δ set equal to zero. We note that although the classical action is finite, $G_{\mu\nu}^2$ has an integrable singularity $|\Delta - z|^{-1}$, Ref. 1, which is a reflection of the fractional character of Q.

Technically it is most convenient to work with an initial manifold with a cut in the z plane. In this case the global boundary conditions of Ref. 6, imposed on the modes, are satisfied by virtue of the natural requirement of single-valuedness of the physical gauge-invariant quantities (Ref. 5, c). We recall that global boundary conditions (Ref. 6) arise from the requirement that the Hamiltonian be hermitean in the analysis of the theory on a manifold with boundary, and they play a key role in the Atiyah-Patodi-Singer (APS) index theorem (Ref. 6) (not to be confused with the Atiyah-Singer index theorem, which is formulated for compact manifolds without boundary and having relation to counting the number of zero modes in the field of a standard instanton).

We note that there is a beautiful analogy[1)] between the toron defects under discussion and the description of dislocations in solid state theory (see, e.g., Ref. 7). In the latter case the object to be described in a displacement vector $\mathbf{u}(r)$ of a node from its position in an ideal crystal lattice. The existence of dislocations manifests itself in the fact that when $\mathbf{u}(r)$ is transported along any closed contour surrounding the dislocation line it acquires an increment $\mathbf{b}$ equal to the period of the lattice. Thus the displacement vector becomes a non-single-valued function of the coordinates; however, there is no physical nonuniqueness, since the increase by one period does not change the state of the system. In particular, the stress tensor is a single-valued function of the coordinates. Technically, for the description of the field of displacements one introduces fictitious δ-like singularities which produce the required jumps (Ref. 7). In the toron problem we prefer to use a description on two Riemann surfaces (Ref. 1), which frees us from the necessity of introducing fictitious forces. We note that dislocations are a linear structure defect

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defined by the Burgers vector **b**. In our case the analog of b is a vector orthogonal to the (r,t) plane, directed along a fifth, unphysical dimension. Therefore in physical space the toron looks like a point defect.

Several words on the motivation for considering configurations with $Q=1/2$. The clearest manifestation of the necessity to analyze such configurations is visible in supersymmetric theories. In particular, in supersymmetric gluodynamics the instanton guarantees a nonvanishing value only for the correlator $\langle\lambda^2(0),\lambda^2(x)\rangle$ (Refs. 8, 9), in agreement with the four zero modes of the gluino λ in the field of an instanton (see, e.g., the review, Ref. 10). In the field of a toron, with Q diminished by a factor of two, the number of gluino zero modes is also halved, ensuring a nonvanishing expectation value of the condensate $\langle\lambda^2\rangle$ proper (Ref. 1).

As will be shown in the present paper, a similar situation occurs in SQCD, i.e., in a theory with matter fields in the fundamental representation. In this case also the instanton is capable of ensuring nonvanishing values only for a few correlators, but not of the separate condensates $\langle\lambda^2\rangle$, $\langle\tilde{\varphi}\varphi\rangle$,... . The toron exhibits exactly the properties necessary to guarantee nonvanishing values of these condensates. However, compared to the case of supersymmetric gluodynamics there is a difference of principle, related to the existence of zero modes. The reason, as was explained in more detail in Ref. 1, is that the toron guarantees the existence of two gluino zero modes (fermions in the adjoint representation) and it is exactly these modes that give a nonvanishing contribution to $\langle\lambda^2\rangle$. The fermions of the fundamental representation behave essentially differently, since in the field of a toron they can have no zero modes at all (one can convince oneself of this from the form of the axial anomaly). However a trace of the fact that the toron is a topological anomaly manifests itself in the existence of quasi-zero modes embedded in the continuous spectrum. As we shall see in the following sections, these modes play a key role in the computation of the toron measure and of the chiral condensates.

The plan of the paper is the following. In Sec. 2 the spectrum of the Dirac operator is discussed for fermions in the fundamental representation with a small mass m. On the basis of the results obtained, the toron measure is defined for SQCD. In Sec. 3 we turn to the physically interesting case of QCD. It will show that the theory with N_f light flavors equal to the number N_c of colors is distinguished and in some aspects (cancellation of the non-zero modes between fermions and bosons) is reminiscent of supersymmetric models. The calculation of the toron measure and the quark condensate $\langle\bar{\psi}\psi\rangle$ for $N_f=N_c=2$ concludes Sec. 4.

2. THE FERMION DETERMINANT AND THE TORON MEASURE IN SQCD

We first recall some results derived for the toron measure in supersymmetric quantum gluodynamics[1]:

$$Z_{SYM}=C\frac{M_0^4\,d^4x_0}{g^4}\frac{d^2\varepsilon}{M_0}\exp\left(-\frac{4\pi^2}{g^2}\right)=C\Lambda^3\frac{1}{g^4}d^2\varepsilon\, d^4x_0,$$
$$\Lambda^3=M_0^3\exp\left(-\frac{4\pi^2}{g^2}\right). \tag{1}$$

Here the factor $g^{-4}M_0^4d^4x_0$ owes its existence to the four bosonic zero modes; the factor $M_0^{-1}d^2\varepsilon$ comes from the two gluino modes; the factor $\exp(-4\pi^2/g^2)$ is the contribution of the classical action of the toron. The expression (1) guarantees a nonvanishing value of the gluino condensate $\langle\lambda^2g^2\rangle$ and has the exactly renormalization-group-invariant form:

$$\langle g^2\lambda^2\rangle=2C\Lambda^3/g^2. \tag{2}$$

We now calculate the toron measure in SQCD. In this case the introduction of fermions and bosons belonging to the fundamental representation introduces the following additional multiplier (the contribution of the regulator has been omitted)[11]:

$$Z_{SQCD}=Z_{SYM}(d_F)^{N_f}(d_B)^{-N_f},$$
$$d_F=\mathrm{Det}\left[\frac{-i\hat{D}-im}{-i\hat{\partial}-im}\right],\quad d_B=\mathrm{Det}\left[\frac{-D^2+m^2}{-\partial^2+m^2}\right]. \tag{3}$$

Standard determinant manipulations allow one to express it in terms of the Green's function of the corresponding operator[12]:

$$m\frac{d}{dm}\ln d_F=\mathrm{Tr}\left(\frac{m^2}{-\hat{D}^2+m^2}-\frac{m^2}{-\hat{\partial}^2+m^2}\right)$$
$$=\mathrm{Tr}\left(\frac{m^2}{-D^2+m^2}-\frac{m^2}{-\partial^2+m^2}\right)-\mathrm{Tr}\frac{m^2\gamma_5}{(-\hat{D}^2+m^2)}. \tag{4}$$

Here the symbol Tr is to be understood in a generalized sense, as a trace over space-time, color, and Lorentz indices. In the last stage of the derivation of Eq. (4) we have used the relation $-\hat{D}^2(1+\gamma_5)/2=-D^2(1+\gamma_5)/2$, which is valid for any self-dual field.

It is well known (Ref. 13) that the last term in Eq. (4) is related to the index of the Dirac operator, does not depend on m at all, and is exactly equal to the topological charge Q of the external field. It is simplest to convince oneself of this by taking the derivative d/dm^2 Tr, proving the TR is independent of m^2, and then calculating the Tr under discussion for large m^2.

Regarding the first two terms in Eq. (4), their contribution is related to the nonzero modes and cancels exactly against the bosonic terms which define d_B. Summarizing: The additional contribution due to the matter fields, taking into account the regulator, is determined by the factor

$$m\frac{d}{dm}\ln Z_{SQCD}=QN_f,\qquad Z_{SQCD}\sim\left(\frac{m}{M_0}\right)^{QN_f}, \tag{5a}$$

$$-\mathrm{Tr}\frac{m^2\gamma_5}{-\hat{D}^2+m^2}=Q. \tag{5b}$$

In particular, for instantons with $Q=1$ Eq. (5) reproduces the well-known result (Ref. 11) of suppression of the $\sim m^1$ instanton transition with massless fermions. As is well known, this suppression is related in turn to the existence of zero modes. Thus, the expression (5b) for integer values, of Q is a projection operator onto the zero modes, each of which, when normalized to unity (in the sense of $\mathrm{Tr}\sim d^4x$) gives a contribution to Eq. (5b) exactly equal to one.

Although the formal expression for the measure is obtained in the form [$Q=1/2$ for $SU(2)$]

$$Z_{SQCD}=C\frac{1}{g^4}M_0^4\,d^4x_0\frac{d^2\varepsilon}{M_0}\left(\frac{m}{M_0}\right)^{N_f/2}\exp\left\{-\frac{4\pi^2}{g^2}\right\}, \tag{6}$$

a natural question arises: How can the relation (5b) be satisfied with a fractional right-hand side, if each nonzero mode contributes to the left-hand side only an integer-valued contribution? A formal answer is well known (Refs. 6, 14) and is related to the fact that our solution is defined on a manifold with boundary. Therefore the effects of the boundary, which are absent in the analysis of the instanton solution, play an important role in the case under consideration.

From a physical point of view the situation can be described as follows (Refs. 15, 16). The left-hand side of the expression (5b), being the divergence of the axial vector current, can be expanded in the standard manner with respect to the eigenfunctions of the Dirac operator $-i\hat{D}\varphi_\lambda \lambda \varphi_\lambda$, with eigenvalues λ. Following Ref. 15, we denote the corresponding spectral density by $C(\lambda,r)$. Here $r\to\infty$ and the dependence of $C(r)$ on r reminds one of the fact that an integral over an exact divergence has been taken, and its magnitude depends on the integrand for large r. Thus, the assertion that (5b) does not depend on m is equivalent to the assertion

$$\lim_{r\to\infty}\int d\lambda C(\lambda,r)\frac{m^2}{\lambda^2+m^2}=Q,\quad \lim_{r\to\infty}C(\lambda,r)=Q\delta(\lambda).\quad (7)$$

The relation (7) corresponds exactly to the results of Ref. 15, which means that for large fractional values of Q the quasi-zero modes in the continuum ("unbound resonance at $\lambda = 0$" in the terminology of Ref. 15) contribute to expression (5b). These quasi-zero modes have the same distinguishing property that for a system placed in a box of size r the eigenvalues of the mentioned modes tend to zero faster than r^{-1}.

As will be seen below, a spectrum for which the continuum starts without a gap from $\lambda = 0$ plays an exclusive role in the formation of chiral condensates. On the other hand, as can be seen from Eqs. (5), (7), just such a spectrum is an unremovable part of configurations with fractional charges Q. For integer values of Q the relation (5b) is saturated exclusively by the zero modes with $C(\lambda) = (+1)\delta(\lambda)$; the continuum in this case is separated from $\lambda = 0$ by a gap of nonzero width (for the instanton the size of the gap is $\sim\rho^{-1}$).

Independent of how one interprets Eq. (5)—whether in terms of the index theorem (Refs. 6, 14), the point of view of spectral density (Ref. 15), or in the language (Ref. 16) of Jost functions and Levinson's theorem for potential scattering—the main result of the present section is the expression (6). It is the expression (6) which shows that each fermion introduces a multiplicative factor $m^{1/2}$ and therefore that the formulation of the theory with an initially vanishing mass is not adequate. Previously the authors of Refs. 10 and 17 have insisted on a similar assertion. The chiral limit will always be understood in the sense of passing to the limit $m\to 0$ in the final expressions.

3. CHIRAL CONDENSATES IN SQCD

Having the explicit expression (6) for the toron measure In SQCD, it is not difficult to calculate the gluino condensate similar to the way this was done in gluodynamics [see Eq. (2) and Ref. 1]. For this it is necessary, as usual, to substitute the zero modes for the λ-fields, and to integrate over the collective coordinates. As a result of this we obtain

$$\langle g^2\lambda^2\rangle=(2C/g^2)\Lambda^{3-N_f/2}m^{N_f/2},\qquad (8)$$
$$\Lambda^{3-N_f/2}\equiv M_0{}^{3-N_f/2}\exp\{-4\pi^2/g^2\}.$$

As is easy to see, the expression has an exactly renormalization-group invariant form. Moreover the mass-dependence of the condensate ($\langle\lambda^2\rangle_{N_f=1}\sim m^{1/2}$, $\langle\lambda^2\rangle_{N_f=2}\sim m$) agrees with the instanton calculations (Refs. 10, 17, 18) and differs only by a numerical factor from the latter (see below). As regards theories with $N_f > N_c = 2$, the instantons in these cases do not generate any Green's functions allowing one to determine $\langle\lambda^2\rangle$. In this sense Eq. (8) is a new result.

More important, however, is the fact that the toron calculus allows one to find the condensates proper, rather than correlators of a specific form from which these condensates need to be extracted. In addition, we note that the dependence of the condensate on the mass m with a fractional exponent looks very unnatural in instanton calculations, where the mass can enter in the results only to an integer power (technically, a fractional power appears when one takes the square root of correlators of a certain form). In toron calculations, Eq. (6) such a mass dependence is an indelible trait of configurations with fractional Q. In this case the stable solutions are exactly configurations with $Q = 1/2$, Ref. 1, which guarantee, in turn, a mass-dependence of Z of the form $Z\propto m^{N_f/2}$ [see Eq. (5)]; such a dependence is predicted in the form of a theorem as a consequence of the supersymmetry and the Ward identities (Ref. 10). In the more general case, for the group $SU(N)$ it is natural to expect the existence of $Q = 1/N_c$ (see below). In this case a nontrivial dependence of $\langle\lambda^2\rangle$ on the mass ($\langle\gamma^2\rangle\propto(m)^{N_f/N_c}$) also finds its natural explanation.

Another important distinction from the instanton calculations consists in the following. As is known (Ref. 10), in SQCD with the gauge group $SU(N)$ there exist N degenerate vacua $|\Omega_k\rangle$, corresponding to the existence of a Z_N symmetry. The nonvanishing of the condensate (8) signals a spontaneous breaking of this Z_N symmetry. The labeling of the vacuum states $|\Omega_k\rangle$ is determined by the phase of the condensate

$$\langle\Omega_k|\lambda^2|\Omega_k\rangle=\exp(2\pi ik/N)\langle\Omega_0|\lambda^2|\Omega_0\rangle,\ k=0,\ 1,\dots,\ N-1.\qquad (9)$$

We also recall that the instanton calculations correspond to averaging over all N-states (Ref. 10), so that a nonvanishing result can be obtained only for Green's functions which are invariant under Z_N, e.g., $\langle\Pi_{i=1}^{N_f}\lambda^2(x_i)\rangle$. The result (9) is then obtained by extracting the Nth root of unity.

In toron calculations we deal directly with a separately taken vacuum state $|\Omega_k\rangle$. Indeed, a toron vacuum transition with $Q = 1/N$ changes the chiral charge Q_5 by two units (which follows, e.g., from the expression for the anomaly: $\partial_\mu a_\mu = 2NQ$). The state $|\Omega_k\rangle$ is a superposition of vacua with definite chiral charges $|Q_5 = 0\rangle$, $|Q_5 = 2\rangle$,..., $|Q_5 = 2(N-1)\rangle$ (Refs. 1, 19):

$$|\Omega_k\rangle=\frac{1}{N^{1/2}}\sum_{l=0}^{N-1}\exp\left\{i\frac{2\pi kl}{N}\right\}|Q_5=2l\rangle.\qquad (10)$$

Thus, a nonvanishing value of the transition matrix element in expression (8), $\langle Q_5 = 2|g^2\lambda^2|Q_5 = 0\rangle$, account being taken of Eq. (10), reproduces Eq. (9).

Yet another chiral condensate which is of interest is the scalar condensate $\langle \tilde{\varphi}\varphi \rangle$ (we use the notations of the review, Ref. 10). Its magnitude can be uniquely reconstructed from the anomalous Konishi identity (Ref. 20),

$$\langle g^2\lambda^2\rangle/32\pi^2=m\langle\tilde{\varphi}\varphi\rangle. \qquad (11)$$

We, however, prefer to calculate $\langle \tilde{\varphi}\varphi \rangle$ explicitly, reproducing the identity (11), and thus convincing ourselves of the self-consistency of the approach as a whole.

On the technical side, the calculation of $\langle \tilde{\varphi}\varphi \rangle$ is somewhat more involved than that of $\langle \lambda^2 \rangle$, Eq. (8). This can be seen at least from the fact that a nonvanishing result for $\langle \tilde{\varphi}\varphi \rangle$ is obtained in next-higher order with respect to g^2 [see Eq. (11)], and therefore requires taking into account a Yukawa interaction of the type $\sim g\psi\lambda\varphi$ (see Fig. 1, where the solid line represents the quark field ψ, the wavy line represents the gluino field λ, the dash-dotted line represents the scalar field φ, and the crosses represent interactions with external fields). Using the methods described in Refs. 10 and 17, taking into account Eq. (6), we arrive at the following expression for $\langle \tilde{\varphi}\varphi \rangle$

$$\langle\tilde{\varphi}\varphi(x)\rangle=C\frac{1}{g^4}\Lambda^{3-N_f/2}m^{N_f/2}\,d^2\varepsilon\,(-2g^2m)\int d^4x_0\,d^4y\,d^4y'$$
$$\times\mathrm{tr}\left(\frac{1}{D^2-m^2}\right)_{xy}\lambda_0(y)\left(\frac{1}{\hat{D}^2-m^2}\right)_{yy'}\lambda_0(y')\left(\frac{1}{D^2-m^2}\right)_{y'x}. \qquad (12)$$

Here λ_0 are the zero modes of the gluino, $(D^2-m^2)^{-1}$ are the appropriate massive propagators in the field of a toron situated at x_0; tr is to be understood as a trace over the Lorentz and color indices.

The masses of all the flavors are assumed equal to m; the analysis of the more general case can be achieved by means of the trivial substitution

$$m^{N_f/2}\to\prod_{i=1}^{N_f}m_i^{1/2}$$

and by labeling the condensates $\langle \tilde{\varphi}^i\varphi_i \rangle$ (no summation over i) and the masses m_i in Eq. (12) for each flavor of interest, labeled by i.

Of course, an explicit calculation of (12) for small mass m seems a completely hopeless problem, at least because for a toron (just as for the instanton) there does not exist a closed expression for the massive propagator. We overcome this difficulty following the logic of Refs. 10 and 17 and calculating the quantity $\langle \tilde{\varphi}\varphi \rangle$ for large masses $m \ll \Lambda$. The result obtained is valid for all values of m as a consequence of supersymmetry (Refs. 10, 17, 18). Taking this general remark into account it is clear that for $m\to\infty$ the propagators get replaced by free propagators, and the integral of the gluino zero modes with respect to d^4x_0 is replaced by unity (this corresponds to the normalization of the zero modes λ_0). As a result we obtain

$$\langle\tilde{\varphi}\varphi\rangle=2C\Lambda^{3-N_f/2}(m^{N_f/2}/m)/32\pi^2. \qquad (13)$$

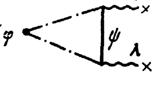

FIG. 1.

Comparing Eqs. (8) and (13) with Eq. (11), we convince ourselves that the Konishi relation is automatically satisfied, pointing to the self-consistency of the approach as a whole. We further note that if we could calculate the integral (12) for small m we would obtain m^{-2} as a result. Such a singular behavior for $m\to 0$ was expected, of course and due to the quasi-zero modes discussed in Sec. 2. The calculation described above demonstrates that the chiral limit, understood as the limit $m\to 0$, differs essentially from the situation when m is assumed equal to zero in the original Lagrangian. In particular, for $N_f=2$ the value of $\langle \tilde{\varphi}\varphi \rangle \sim \Lambda^2$ is a finite quantity, independent of the mass. However, starting from the measure for the massless theory [in this case Z_{SQCD} vanishes identically according to Eq. (6)], one might naively expect that all condensates also vanish. This does not happen, in spite of the fact that there do not exist genuine normalizable zero modes of the fundamental representation in a toron field.

In conclusion of the present section we discuss the possibility of extracting the constant C which occurs in Eq. (6) from the instanton formula (Ref. 11). For this we assume that two closely situated torons centers at the points x_1 and x_2 can be interpreted as an instanton of size $\rho = x_1 - x_2 \to 0$ situated at the point $x_0=\frac{1}{2}(x_1+x_2)$.

Such an interpretation agrees with the magnitude of the action and of the topological charge of each of the systems, as well as in the number of bosonic and fermionic zero modes. Indeed, in the field of an instanton there exist 8 bosonic and 4 fermionic gluino zero modes. These numbers agree with the four translational zero modes and the two gluino modes accompanying each of the two torons.[2)]

Thus, we write the instanton measure for SQCD[10]:

$$Z_{\mathrm{inst}}=\frac{1}{g^8}M_0{}^8\frac{d^4x_0\,d^4\rho}{2\pi^2}\frac{4}{\pi^2}(4\pi^2)^4\left(\frac{m}{M_0}\right)^{N_f}$$
$$\times d^2\varepsilon_1\,d^2\varepsilon_2\exp\left(-\frac{8\pi^2}{g^2}\right). \qquad (14)$$

In the expression (14) we have replaced the standard factor $\rho^3d\rho$ by $d^4\rho/(2\pi^2)$, keeping in mind that in the sequel we shall interpret $d^4x_0d^4\rho$ as the integration element with respect to the translations of each of the torons $d^4x_1d^4x_2$. As a result the expression (14) decomposes into the product of two factors, each of which can be interpreted as a toron measure (6). The fact that such a decomposition agrees with Eq. (6) confirms our original assumption [we recall that Eq. (6) has been obtained without any reference to instanton calculations]. Setting further

$$Z_{\mathrm{inst}}=Z_{\mathrm{tor}}(x_1)Z_{\mathrm{tor}}(x_2)/2!,$$

where 2! takes into account the identity of the particles, we obtain

$$C=2^5\pi^2. \qquad (15)$$

We can now compare the magnitude of the condensate $\langle \lambda^2 \rangle$, defined according to Eqs. (8), (15), with that resulting from the instanton calculations (Refs. 10, 17). The numerical difference turns out to be a factor $(4/5)^{1/2}$. We don't know at present how to interpret this result: Should these two numbers coincide, is there a risk of double counting, or, on the contrary, of not taking into account some contributions?

These questions are all the more justified since the calculations in the strong coupling approximation do not inspire full confidence in the possibility of neglecting the interaction (regarding both torons and instantons, see the discussions in Ref. 18), although each of the calculations by itself is self-consistent, since it guarantees the validity of the Konishi relation (11). In other words, although the dilute-gas approximation [Eq. (6)] yields a definite and reasonable result, there is no proof that the result is complete.

4. THE CHIRAL CONDENSATE IN QCD

Having acquired some experience in working with fields in the fundamental representation in supersymmetric theories, and having convinced ourselves of the reasonableness of the results we obtained based on the toron solution, we turn to a calculation of the toron measure in QCD.

Compared to the toron measure in SQCD, Eqs. (1) and (3) have in the case under discussion obvious distinctions: 1) absence of the factor $d^2\varepsilon/M$), related to gluino zero modes; 2) absence of the factor $(d_B)^{-N_f}$, related to the scalar fields; 3) the non-zero modes in QCD do not cancel and must be included in the analysis.

We start with a discussion of the nonzero gauge modes. As is well known,[21] their contribution can be reconstructed to logarithmic accuracy by means of a calculation of the Feynman diagram of Fig. 2 (where the dashed line represents the gluon field a_μ). The result is[21]

$$S=S_{cl}+\Delta S,$$

$$\Delta S_g=-\frac{2}{3}\frac{g^2}{16\pi^2}\ln M_0^2\int d^4x\left[\frac{1}{4}G^2_{\mu\nu}\right]_{cl}=-\frac{2}{3}(\ln M_0)Q, \tag{16}$$

$$\exp(-\Delta S_g)=\exp(-{}^2/_3\cdot{}^1/_2\ln M_0).$$

Similarly, the contribution of the non-zero modes related to the existence of fermions is determined by the graph in Fig. 3 and equals[21]

$$\Delta S_f=\frac{g^2N_f}{16\pi^2}\ln M_0^2\left(1-\frac{1}{3}\right)\int d^4x\left[\frac{1}{4}G^2_{\mu\nu}\right]_{cl}$$

$$=QN_f\left(1-\frac{1}{3}\right)\ln M_0. \tag{17}$$

We purposely separated the two contributions in Eq. (17). The first contribution in parentheses in Eq. (17) is related to the spin part of the interaction and leads to the factor $\exp(-QN_f\ln M_0)$. For integer values $Q=1$ this factor makes the contribution of the zero mode dimensionless: $(m/M_0)^{N_f}$. For fractional Q this factor is related to the quasi-zero modes [see Eqs. (4), (5)]. The second term in Eq. (17) differs only by a factor -2 from the contribution of a scalar particle, with the factor two corresponding to the two states of polarization and the minus sign, to the anticommutativity of the fermions. This correspondence agrees with the expression (4), where the first term corresponds to the contribution of the scalar determinant and the second term, which is related to spin, exactly reproduces the factor $\exp(-QN_f\ln M_0)$. We note that compared to the instanton calculations not only has the action decreased by a factor of two, but also the contribution of the zero modes has become two times smaller [$Q=1/2$ stands in front of $\ln M_0$ in Eqs. (16), (17)]. It is clear that this effect guarantees a correct renormalization-group invariant expression. In the language of eigenfunctions of the corresponding quadratic operators the effect under discussion was analyzed in detail in terms of the σ-model (Ref. 5a) and is related to a decrease of the number of admissible modes.

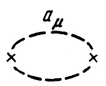

FIG. 2.

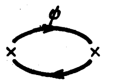

FIG. 3.

Finally, the last touch, completing the analysis of the non-zero modes, is related to the first term in Eq. (4) as $m\to0$. In SQCD this factor does not cause any worry since it cancels exactly against the appropriate bosonic determinant. In the case under discussion it does not vanish, but for $m\to0$ it is small and has the order $m^2\ln m$, and can be calculated to logarithmic accuracy from the expression of the massless Green's function in a toron field (see the Appendix).

Collecting all the factors we have

$$Z_{QCD}=K\frac{M_0^4}{g^4}d^4x_0\left(\frac{m}{M_0}\right)^{N_f/2}\exp\left(-\frac{4\pi^2}{g^2}\right)$$

$$\times\exp[-{}^1/_6(2-N_f)\ln M_0\Delta]. \tag{18}$$

The factor $M_0^4d^4x_0g^{-4}$ is due to the four translational zero modes; the factor $(m/M_0)^{N_f/2}$ is related to the quasi-zero modes [see Eq. (5)]; the contribution $\exp[-(2-N_f)\ln M_0\Delta/6$ corresponds to the nonzero gauge Eq. (16)] and fermion [Eq. (17)] modes; finally, $\exp(-4\pi^2/g^2)$ is the contribution of the classical action of the toron. It is easy to convince oneself that M_0 and $g(M_0)$ in Eq. (18) come together exactly in a renormalization-group invariant expression; in addition, we note that the logarithmic contribution related to the non-zero modes, $\ln(M_0\Delta)$ is dimensionless on account of the characteristic scale Δ which plays the role of a regulator in the description of the toron point defect (see the Introduction and Ref. 5a).

The most important trait distinguishing the expression (18) from analogous formulas for the supersymmetric theories, Eqs. (1) and (6), is the dependence of Z_{QCD} on the regulator parameter Δ. In particular, for $N_f=1$, $Z_{QCD}\sim d^4x\Delta^{-1/6}$. It is obvious that the growth of Z_{QCD} for $\Delta\to0$ signifies an increase in the toron density (even though each toron is small) and the impossibility of using the quasi-classical expression (18) which is valid for a dilute gas ($Z\ll1$). In the final count this means that the interaction is important when the density of torons increases. Such an interpretation finds its confirmation in the framework of the hypothesis that two closely situated torons at X_1 and X_2 transform into an instanton of size $\rho=X_1-X_2\to0$ (see the end of Sec. 3). Indeed, let us define the interaction energy of two torons similarly to the way this was done for two instantons in Ref. 22. For this we write the contribution to Z of two torons situated at the points X_1 and X_2:

$$Z_{2\text{tor}} \propto d^4x_1 d^4x_2 \Delta^{-(2-N_f)/6} \Delta^{-(2-N_f)/6} \exp(-W_{int}). \quad (19)$$

Here W_{int} takes into account the interaction of the torons and characterizes the difference compared to the dilute gas formula (18). According to our hypothesis, the interaction energy W_{int} can be determined by subtracting from the instanton contribution the contribution of two noninteracting torons. As regards the zero modes, just like in SQCD, the corresponding instanton factor $m^{N_f} d^4x_0 d^4\rho$ goes over exactly into the product for noninteraction torons $[d^4x_1 m^{N_f/2}][d^4x_2 m^{N_f/2}]$. The interaction energy W_{int} is determined specifically by the non-zero modes:

$$\exp(-W_{int}) = \exp\left\{-\left[\left(\frac{2-N_f}{3}\right)\ln M_0\rho - \left(\frac{2-N_f}{6}\right)\ln M_0\Delta - \left(\frac{2-N_f}{6}\right)\ln M_0\Delta\right]\right\} = \exp\left[-\left(\frac{2-N_f}{3}\right)\ln\left|\frac{x_1-x_2}{\Lambda}\right|\right\}. \quad (20)$$

Here the factor $\exp\{-[(2-N_f)/3]\ln M_0\rho\}$ is related to the nonzero instantonic modes (Ref. 11). Substituting Eq. (20) into (19) we convince ourselves that the dependence on the parameter Δ has disappeared; the place of Δ has been taken by the factor $|x_1 - x_2|$, which takes into account the interaction.

For $N_f = 1$

$$W_{int} = {}^1\!/_3 \ln|x_1 - x_2| \underset{x_1 \to x_2}{\to} -\infty.$$

This means, in turn, that there is a logarithmic attraction between the torons, i.e., they tend to approach each other increasing the toronic density. This agrees with the qualitative remark mentioned above. For $N_f \geqslant 3$ there is repulsion, which can be qualitatively explained as a consequence of Fermi-Dirac statistics. We note that the instanton interaction has the same qualitative properties including the dependence on N_f. In this case the interactions energy equals $W \sim (2 - N_f)\ln|x_1 - x_2|$ (Ref. 22).

The case $N_f = 2$ (or, in the more general situation, $N_f = N_c$) is distinguished. (This point of view has been expressed previously in Ref. 19.) In this case the toron maintains its individuality; the quantity $Z_{N_f=2}$ in Eq. (18) is finite and can be used for further calculations. From a technical point of view the distinguished nature of the case $N_f = 2$ is related to the cancellation of the non-zero modes, similar to that in supersymmetric theories (Secs. 2 and 3).

The general conclusion from the preceding analysis is the following. It is impossible to find the contribution of torons to physical quantities in the general case (excluding the case $N_f = 2$) within the approximation we have considered; it is necessary to take into account the interaction and to go beyond the quasi-classical approximation. It is most likely that torons as quasi-particles lose their individuality, similar to what happens to instantons in the σ-model.[23]

However for $N_f = 2$ the torons maintain their individuality and yield a nonvanishing contribution to physical quantities. We note that the theory with $N_f = N_c = 3$ which is realized in nature falls exactly into that distinguished class and therefore the analysis carried out above is of interest. Therefore we restrict our attention in the sequel to the case $N_f = N_c = 2$. In this case the toron measure has the form

$$Z_{N_f=2} = K\frac{M_0^3}{g^4} d^4x_0\, m \exp\left(-\frac{4\pi^2}{g^2}\right) = Km\Lambda^3_{1-loop} d^4x_0,$$
$$\Lambda^3_{1-loop} \equiv \frac{M_0^3}{g^4}\exp\left(-\frac{4\pi^2}{g^2}\right). \quad (21)$$

Now everything is prepared for the calculation of the chiral condensate in QCD with $N_f = 2$ in a toron field. As in SQCD, a fundamental role is played by the quasi-zero modes: They cancel the small mass m in Eq. (21) and guarantee a finite answer for $\langle\bar\psi\psi\rangle \propto \Lambda^3$.

By definition taking Eq. (21) into account, we have

$$\langle\bar\psi\psi\rangle_{\text{Minkowski}} = -i\langle\psi^+\psi\rangle_{\text{Euclid}} = -iKm\Lambda^3 \operatorname{tr}\int d^4x_0 \frac{-im}{-\tilde D^2 + m^2}. \quad (22)$$

Here ψ is any of the flavors u or d.

In Eq. (22) we have replaced $\psi\psi^+$ by the Green's function in a toron field. We note that the integral (22) we obtained has already been encountered earlier in the calculation of the fermion determinant (4). Although we do not know the Green's function of the massive particle, the integral over it for $m \to 0$ is known exactly! Indeed

$$\operatorname{tr}\int d^4x \frac{m^2}{-\tilde D^2 + m^2} = \operatorname{tr}\int d^4x \frac{m^2(1+\gamma_5)}{-D^2 + m^2} - \operatorname{tr}\int d^4x \frac{m^2\gamma_5}{-\tilde D^2 + m^2} = Q = \frac{1}{2}. \quad (23)$$

Here, as in Eq. (4), we have taken into account the fact that $\tilde D^2(1+\gamma_5) = D^2$. In addition we have used the fact that the first term in Eq. (23), which is not related to the spin, tends to zero as $m^2 \ln m$ (see Appendix). The second term does not depend on m and is exactly equal to Q. The antitoron yields exactly the same contribution. To summarize

$$\langle\bar\psi\psi\rangle_{\text{Minkowski}} = -K\Lambda^3_{1-loop}, \qquad \Lambda^3_{1-loop} \equiv (M_0^3/g^4)\exp(-4\pi^2/g^2). \quad (24)$$

Just as in the derivation of Eq. (50) we would like to stress here the exclusive importance of the quasi-zero modes (7) embedded in the continuum, which guarantee a nonvanishing value of the integral (23) and, in the ultimate count, of the condensate (24). We stress once again that although Eq. (24) does not depend on the quark mass, all calculations fundamentally assume a nonvanishing value of m and the chiral limit is to be understood only in the sense of taking the limit $m \to 0$.

Finally, following the same procedure as in the derivation of Eq. (15) for the constant C, one can determine the coefficient K. It equals

$$K = 2^5\pi^2(e^{5/6}/4)^{1/2}. \quad (25)$$

In some sense the mechanism for the appearance of a condensate (24) is reminiscent of the mechanism proposed in Ref. 14. In both cases a nonvanishing value of $\langle\bar\psi\psi\rangle$ is due to modes which are close to λ. There is also a distinction: In Ref. 24 this effect was achieved by the interaction of quasiparticles; in our case, as was remarked after Eq. (7), a nonvanishing density near $\lambda = 0$ is a nonremovable trait of configurations with fractional values of Q. This property of the spectrum is, in our opinion, determining the whole physics related to spontaneous breaking of the chiral symmetry.

5. CONCLUSION

The main results of the present paper are the following. A new mechanism was proposed for spontaneous breaking of chiral symmetry in gauge theories. It is based on self-dual configurations with fractional topological charge. An intrinsic feature of such solutions is the gapless spectrum of the Dirac operator. In the final analysis this guarantees the formation of condensates. The proposed method has proved itself in supersymmetric theories, where a nontrivial dependence of the condensate on the mass (fractional power) is a consequence of exact theorems. Our approach reproduces automatically this fractional power, as well as the Konishi relation, which bears witness (at least) to the self-consistency of the calculations.

As far as QCD is concerned, it turned out that for $N_f \neq 2$ [for the gauge group $SU(2)$] the torons tend to fuse and lose their individuality (in supersymmetric theories the torons preserve their individuality for arbitrary N_f). In this case quasi-classical calculations stop being meaningful and we are unable to determine the fate of the torons. The value $N_f = 2$ is distinguished and from a technical point of view resembles the supersymmetric models: The non-zero modes cancel between bosons and fermions.[3)] This makes it possible to calculate the toron density and the chiral condensate $\langle \bar{\psi}\psi \rangle$, the main results of this paper.

We recall that the motivation for considering configurations with fractional topological charge was related to the analysis of supersymmetric theories (see the Introduction and Refs. 1, 5). However, the necessity of configurations with fractional Q was noted considerably earlier in connection with the $U(1)$ problem of a discrete system of N_f-vacua, the puzzle of the θ-period, etc. (see the original papers Ref. 25 and the review, Ref. 26). Essentially the problem raised in Ref. 25 consisted in the strong restrictions imposed by the existence of a single isosinglet η'-meson on certain correlators. In particular, the correlator $\int d^4x \{G\tilde{G}, \bar{\psi}\gamma_5\psi\rangle$ must be of order unity (as $m \to 0$), and the topological susceptibility $\int d^4 \{G\tilde{G}(x), G\tilde{G}(0)\rangle$ must tend to zero as the first power of the mass (with a definite coefficient!). It is extremely difficult to guarantee such behavior by means of instantons (Ref. 26). In addition the Ward identities require that the dependence of the condensates on θ should be fractional, $\exp\{i\theta/N_f\}$. Such a behavior is also hard to obtain in the framework of the standard instanton approach (Ref. 26). At the same time all these problems are automatically solved without any difficulties by means of the scheme proposed in the present paper. An overview of such a wide range of questions from a unified point of view bears witness, in our opinion, of the correctness of this approach to violation of chiral symmetry.

In conclusion the author expresses his gratitude to A. I. Vaĭnshteĭn, D. I. D'yakonov, V. Yu. Petrov, and V. L. Chernyak for useful discussions.

APPENDIX

The purpose of this Appendix is the determination of the Green's function in a toron field and the calculation of the quantity

$$m^2 \int d^4x \, \mathrm{tr} \left(\frac{1}{-D^2+m^2} - \frac{1}{-\partial^2+m^2} \right) \tag{A1}$$

as $m \to 0$. As discussed in the text, the value of (A1) is necessary for the calculation of the toron measure (18) and the chiral condensate (23) in QCD.

We start from the following form of the toron solution (Ref. 1):

$$A_\mu{}^a = -\bar{\eta}_{\mu\nu}{}^a \partial_\nu \ln P, \quad P = \frac{1}{2r}(G+\bar{G}) = \frac{2(1-\bar{g}g)}{|i+g|^2(z+\bar{z})},$$
$$G = \frac{i-g}{i+g}, \quad g(z) = \left(\frac{1-z}{1+z}\right)^{\nu_2}, \quad z = r+it. \tag{A2}$$

Here, as in Ref. 1, all quantities are measured in units of Δ, and the main block of the solution is the function $G(z)$ analytic in z.

The form (A2) of the solution is such that one can apply all methods used for finding the Green's functions for an instanton in a singular gauge (Ref. 27). Following the latter paper, we look for the solution of the equation

$$-D_x{}^2\Delta(x, y) = \delta^4(x-y), \quad D_\mu = \partial_\mu + iA_\mu{}^a\tau^a/2 \tag{A3}$$

in the form

$$\Delta(x, y) = P^{-1/2}(x) \frac{F(x, y)}{4\pi^2(x-y)^2} P^{-1/2}(y), \tag{A4}$$

where $F(x, y = x) = P(x)$. The latter condition is related to the requirement that $\Delta(x \to y)$ should have the correct behavior for small distances:

$$\Delta(x \to y) \sim 1/4\pi^2(x-y)^2.$$

Following Ref. 27 it is easy to convince oneself that the equation (A3) for the function $F(x,y)$ can be written in the form

$$\sigma_\mu{}^+ \partial_\mu F(x, y) - \frac{2\sigma_\mu{}^+(x-y)_\mu}{(x-y)^2} [F(x, y) - P(x)] = 0,$$
$$\sigma_\mu{}^\pm = (\pm i, \boldsymbol{\sigma}). \tag{A5}$$

In order to solve this equation we introduce the notations

$$x_\mu = (t_1, \mathbf{n}_1 r_1), \quad z_1 = r_1 + it_1, \quad \bar{z}_1 = r_1 - it_1, \quad G_1 = G(z_1)$$
$$y_\mu = (t_2, \mathbf{n}_2 r_2), \quad z_2 = r_2 + it_2, \quad \bar{z}_2 = r_2 - it_2. \quad G_2 = G(z_2). \tag{A6}$$

After this explicit substitution one can verify that the function $F(x,y)$, defined by

$$F(x, y) = \frac{1}{4} \left\{ \frac{(1+\sigma\mathbf{n}_1)(1+\sigma\mathbf{n}_2)}{z_1+\bar{z}_2}(G_1+\bar{G}_2) - \frac{(1-\sigma\mathbf{n}_1)(1+\sigma\mathbf{n}_2)}{\bar{z}_2-\bar{z}_1}(\bar{G}_1-\bar{G}_2) - \frac{(1+\sigma\mathbf{n}_1)(1-\sigma\mathbf{n}_2)}{z_2-z_1}(G_1-G_2) + \frac{(1-\sigma\mathbf{n}_1)(1-\sigma\mathbf{n}_2)}{z_2+\bar{z}_1}(G_2+\bar{G}_1) \right\}, \tag{A7}$$

satisfies exactly the equation (A5) with the additional requirement $F(x,y) = P(x)$. The verification is essentially based on the properties of the projection operators $(1 \pm \sigma \cdot \mathbf{n})$ and the analyticity of $G(z)$. In the particular case of the function $G(z) = z + 1/z$, which corresponds to an instanton, Eq. (A7) goes over into the well known solution.

We can now calculate the integral (A1). Taking into account that the corresponding quantity is determined by large values of x, we substitute into Eq. (A1), to logarithmic accuracy, the expression for the massless Green's function (A7) and cut offer the integral at distances $x \lesssim m^{-1}$ (for similar calculations for the instanton see Ref. 12). As a result, taking into account the asymptotic behavior of the functions

$$G(z\to\infty)=2z/3+1/2z, \quad P(z\to\infty)={}^2/_3,$$
$$F(x, y)\to{}^2/_3+{}^1/_2(\sigma_\lambda-x_\lambda)(\sigma_\mu+y_\mu)/x^2y^2,$$

we have for $m\to 0$

$$m^2 \int_0^{1/m} d^4x \,\mathrm{tr}\left[\frac{1}{-D^2}-\frac{1}{4\pi^2(x-y)^2}\right] \approx \frac{3}{2} m^2 \ln m. \qquad (A8)$$

The result (A8) signifies that for small m one may neglect the corresponding contribution to the condensate (23), as well as to the expression of the toron density.

[1)] The author is grateful to A. Z. Patashinskiĭ for pointing out this analogy.

[2)] We note that this hypothesis is confirmed in the general case of an arbitrary Lie group G. As is known, the number of zero modes in the field of an instanton is determined by the quadratic Casimir operator $C(G)$ and equals $4C(G)$. In particular, for the group $SU(N)$ we have $C(G = SU(N)) = N$. On the other hand one may expect that the minimally admissible topological charge equals $Q = N^{-1}$. Thus, the $4N$ instanton zero modes are naturally interpreted as translations of the N-torons. In addition, in supersymmetric gluodynamics the existence of N vacua and $2N$ gluino zero modes (Refs. 10, 18) is also in agreement with the hypothesis that $Q = N^{-1}$ exists. The known form of the β function and the axial anomaly also confirm this hypothesis. What is most amazing is that all these facts are self-consistent for an arbitrary Lie group! The author is grateful to D. D'yakonov for a discussion of this question.

[3)] We note that the model with $N_f = N_c = 3$ realized in nature falls into this class and there is every reason to expect a situation analogous to $SU(2)$ with $N_f = 2$.

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