Dilepton signal from an ultrahigh-temperature quark-gluon plasma

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(Submitted 2 March 1989)
Zh. Eksp. Teor. Fiz. **96**, 1161–1166 (October 1989)

Collisions between ultrarelativistic nuclei can result in an ultrarelativistic hot quark-gluon plasma with perfect-gas properties. The inevitable hydrodynamic expansion of this plasma and the accompanying cooling of matter are used to find the resulting momentum distribution of thermal dileptons ("heavy photons"). The dilepton distribution integrated over the masses is also obtained. Comparison with the inclusive distribution of individual charged leptons, obtained earlier by a similar method, is then used to analyze the diagnostic content of the electromagnetic signal under real conditions (taking expansion into account).

1. INTRODUCTION

The nature of high-temperature hadron matter is one of the most important questions in high-energy physics. It has attracted increasing attention in connection with the development of new experiments.[1,2] Modern chromodynamics (see, for example, Refs. 3 and 4) suggests that nuclear matter, which consists of colorless "elementary" particles (nucleons) at low temperatures, should undergo a transition to the quark-gluon plasma as the system becomes hotter.[5] It is widely believed that this process of quark deconfinement, which occurs as the temperature is raised, is a typical phase transition with $T_c \sim \Lambda \sim 100$ MeV (Λ is a dimensional parameter associated with the renormalization of the color charge[3,4]). From the experimental point of view, the problem is to find the properties of quark-gluon plasma that could be used to identify it reliably. One of them is the thermal production of charged leptons.

In principle, their distribution functions carry information about the nature of their source.[6–8] However, the plasma that will be created, probably as a result of collisions between ultrarelativistic nuclei, will experience hydrodynamic expansion into vacuum. The thermal emission of leptons must therefore be integrated over the volume of the expanding plasma and over the expansion time. This leads to a kind of smoothing of the distribution functions of the emitted particles. The last point emphasizes the importance of a correct choice of the measured distribution function of the electromagnetic reaction products.

The inclusive spectra of leptons and γ rays have been investigated in some detail in Ref. 9 in the limiting case of gas plasma for which

$$\ln (T/\Lambda) \gg 1, \qquad (1)$$

The shape of these distributions is determined by the factor

$$dW \propto \frac{d^3p}{\varepsilon}\frac{1}{p_\perp^4} = 2\pi \frac{dp_\perp}{p_\perp^3}\frac{d\theta}{\sin\theta}, \qquad (2)$$

where $p_\perp = \varepsilon \sin\theta$ is the transverse momentum of an individual emitted particle ($\hbar = c = 1$). Because this is a simple law, it is not very informative. Essentially, it merely confirms that the given medium would emit in accordance with the T^4 law if it were at rest in equilibrium.[10] However, strictly speaking, expression (2) is subject to the further assumption that the mass of the emitted electromagnetic particle must be small, i.e., $T \gg m_1$.

The question is: are there some other more complex and more informative product distribution functions in the limit defined by (1)? Thermal (direct) photons are inclusive by the very essence of elementary events that produce them in the perfect gas of quarks and gluons. However, fermions can only be created in pairs. The shape of the dilepton signal due to such pairs ("heavy photons") is therefore of particular interest. For a lepton-antilepton pair, the square of the invariant mass is hardly negligible in comparison with temperature. In fact, $M^2 \sim T^2$ holds even though each of the individual particles is separately ultrarelativistic. The appearance of a further dimensional parameter means that a more detailed characterization of the source becomes possible.

In this paper, we shall find the distribution functions for dileptons emitted by an expanding quark-gluon plasma, subject to the restriction defined by (1).

2. DILEPTON DISTRIBUTION FUNCTION

The distribution of dileptons in energy $\omega = \varepsilon_+ + \varepsilon_-$ and mass M was found in Ref. 9 for a uniform density plasma ($T = \text{const}$) at rest. This distribution takes the following form (per unit space-time volume)

$$dw(\omega, M^2) = \frac{e^4 n_f \overline{q^2}}{\pi^3}\frac{T}{\exp(\omega/T)-1}\ln\frac{\text{ch}[(\omega+k)/4T]}{\text{ch}[(\omega-k)/4T]}d\omega\, dM^2, \quad k = (\omega^2 - M^2)^{1/2} \qquad (3)$$

where n_f is the number of excited quark flavors with masses $m_q < T$ and $\overline{q^2}$ is the mean square electric charge, evaluated over these flavors. The absolute magnitude of the velocity of a pair is $u = k/\omega$ in the frame in which the emitting medium is at rest (the energy and momentum of the dilepton in the laboratory frame will be denoted by ε and p, respectively). Since the momentum 4-volume is invariant, the distribution (3) will now be written in the following form, which will be convenient for our purposes later:

$$dw = \frac{e^4 n_f \overline{q^2}}{2\pi^4 \tau(\exp\omega\tau - 1)}\ln\frac{\text{ch}[(1+u)\omega\tau/4]}{\text{ch}[(1-u)\omega\tau/4]}\frac{d^3p\, d\varepsilon}{\omega u}, \quad \tau = \frac{1}{T}. \qquad (4)$$

In order to be able to integrate over the space and time of expansion, let us consider the simple case of a head-on coalescence of two nuclei of equal size R. Prior to expansion, the coalescing system lies in a layer of half-thickness $l \ll R$, and the signal is emitted mostly during the one-dimensional

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stage of the process in which $t \ll R$. Khalatnikov's exact solution[11] of the one-dimensional hydrodynamic problem goes over into the simpler formulas (35) and (40) of Ref. 9 in the region in which most of the quarks are localized. These formulas enable us to express the usual 4-volume in the form

$$\pi R^2 dx\, dt = \pi R^2 dv\tau\, dt, \tag{5}$$

so that the integration with respect to time reduces to integration with respect to the reciprocal of temperature if we introduce the dimensionless variable $\xi = \omega\tau$:

$$\int dw = \int d^3p\, d\varepsilon \frac{R^2\, dv}{1-v^2} \frac{3}{\pi^4} e^4 \overline{n_f q^2} \frac{l^2 T_0^6}{\ln(R/l)} \frac{F(u^2)}{\omega^6}, \tag{6}$$

where T_0 is the temperature of the plasma prior to expansion. The function

$$F(u^2) = \frac{1}{u} \int_0^\infty \frac{\xi^4}{\exp \xi - 1} \ln \frac{\text{ch}[(1+u)\xi/4]}{\text{ch}[(1-u)\xi/4]} d\xi \tag{7}$$

must then be expressed in terms of the hydrodynamic velocity v and the laboratory characteristics of the pair. The Lorentz transformation enables us to transform from the longitudinal $p_\parallel$ to the transverse $p_\perp$ momentum of the dilepton, and the argument of the function can be found from the expression

$$1-u^2 = \frac{(1-v^2)(\varepsilon^2 - p_\parallel^2 - p_\perp^2)}{(\varepsilon - v p_\parallel)^2}. \tag{8}$$

Integration with respect to the remaining variable v then leads to the following expression for the number of pairs of leptons of a particular type per head-on collision between the ultrarelativistic nuclei:

$$d^4 W_p(p_\parallel, p_\perp, \varepsilon) = d^3p\, d\varepsilon \frac{3}{\pi^4} e^4 \overline{n_f q^2} \frac{l^2 R^2}{\ln(R/l)} T_0^6 \int_{-1}^{1} F(u^2) \frac{(1-v^2)^2}{(\varepsilon - v p_\parallel)^6} dv. \tag{9}$$

The condition $\ln(R/l) \gg 1$ is assumed throughout and actually corresponds to (1). More accurate estimates of l are given by Eq. 947) in Ref. 9.

The favorable behavior of the function $F(u^2)$ helps us to take the general expression (9) to a specific result. Figure 1 shows a plot of this function, obtained by numerical calculation. Analysis shows that, at least on the physical interval $0 < u^2 < 1$, the function given by (7) decreases very smoothly and has a relatively small slope. Its ultrarelativistic expansion for $u^2 \to 1$ has the form

$$F(u^2) \approx F_1 + |F_1'|(1-u^2), \quad F_1 = F(1) = 44{,}62, \quad F_1' = F'(1) = -7.53. \tag{10}$$

In practice, this expansion remains valid throughout the interval of interest here.

As an illustration, let us consider what would seem to be the least favorable end of the interval $u^2 \to 0$. The approximation defined by (10) then yields $F(0) \approx 52.14$. However, the integral (7) can be evaluated exactly in this limit, since it reduces to the one-parameter Riemann ζ-function: $F(0) = 51.57$. Bearing in mind the integral form of (9), we see that the use of the linear extrapolation (10) results in an error of at most 1%. If we use (8), we can reduce the integration with respect to v to the following two integrals:

$$\int_{-1}^{1} \frac{(1-v^2)^2}{(\varepsilon - v p_\parallel)^6} dv = \frac{16}{15} \frac{1}{(\varepsilon^2 - p_\parallel^2)^3},$$

$$\int_{-1}^{1} \frac{(1-v^2)^3}{(\varepsilon - v p_\parallel)^8} dv = \frac{32}{35} \frac{1}{(\varepsilon^2 - p_\parallel^2)^4}. \tag{11}$$

Moreover, a small modification of the analysis will enable us to take into account the weak inhomogeneity of the one-dimensional fluid flow in the y and z directions. When the transverse gradients are small, the conversed entropy

$$S = \int \eta\, dy\, dz \tag{12}$$

propagates along straight lines parallel to the x axis of the reaction (η is the transverse entropy density). The factor T_0^6 in (9) represents the fact that the final answer involves the mean square total energy

$$\overline{S^2} = \pi R^2 \int \eta^2\, dy\, dz. \tag{13}$$

The coefficient in this expression is most readily established by turning to the formula for the entropy of a uniformly heated volume V of the quark-gluon plasma, i.e.,

$$S = \pi^2 \lambda T_0^3 V, \quad \lambda = (32 + 21 n_f)/45. \tag{14}$$

In the special case of an initial configuration in the form of a plane disk of constant thickness $2l$, we must have $\overline{S^2} \to S^2$.

Transforming from the energy ε to the mass M of the dilepton, and using the above formulas and ideas, we finally, obtain

$$dW_p(p_\parallel, p_\perp, M) = \frac{8}{5\pi^9} \frac{e^4 \overline{n_f q^2}}{\lambda^2} \frac{\overline{S^2}}{R^2 \ln(R/l)} \times \left\{ F_1 + \frac{6}{7} |F_1'| \frac{M^2}{M_\perp^2} \right\} \frac{p_\perp\, dp_\perp\, dp_\parallel\, M\, dM}{M_\perp^6 (p_\parallel^2 + M_\perp^2)^{1/2}}, \tag{15}$$

where $M_\perp = (M^2 + p_\perp^2)^{1/2}$ is the transverse mass of the dilepton. We emphasize that, strictly speaking, none of the masses has separately disappeared. On the contrary the two terms in braces involve different masses. All in all, the ratio of numerical factors in front of them can be used as a diagnostic indicator that is probably specific to the gaseous quark-gluon plasma.

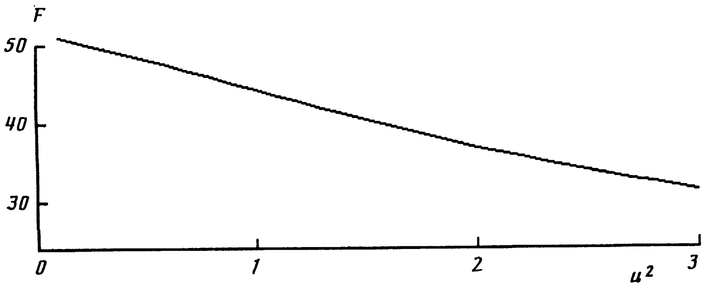

FIG. 1. The function $F(u^2)$ in (7).

Effectively, $M \sim p_\perp$ in the distribution given by (15). This is not unexpected: all the invariants of the one-dimensional flow are of the order of the running temperature. At the end of the one-dimensional stage, matter cools down near the origin $v = 0$ to a temperature

$$T_1 \sim T_0 \left[\frac{l}{R} \left(\ln \frac{R}{l} \right)^{-1/2} \right]^{1/3}. \tag{16}$$

In the laboratory frame, a reasonably well defined angular dependence is exclusively due to hotter regions in the ultra-relativistic flow, which provide the main contribution at small angles of dilepton emission ($\sin \theta \ll 1$). However, the massive nature of the emission ($M \neq 0$) then has little effect on the very approximate estimate of the angle, based on the relativistic transformation formulas. The final criterion for the validity of the theory is

$$M, p_\perp \gg \left[\frac{l}{R} \left(\ln \frac{R}{l} \right)^{-1/2} \frac{1}{\sin \theta} \right]^{1/3} T_0. \tag{17}$$

Depending on the formulation of the experiment, or the method used to analyze it, it may be useful to integrate the above distribution with respect to the mass M. It is also interesting to compare the results with the inclusive signal,[9] taking into account the differences between the notations used in the two cases [in (2) the notation refers to the characteristic of an individual inclusive lepton-antilepton pair]. However, before we can carry out the comparison with (2), it is desirable to have the corresponding dilepton formula in the two-parameter form. For similar reasons, the remaining variables are best taken to be the transverse momentum. $p_\perp$ and the angle θ at which the dilepton is emitted relative to the axis of the nuclear reaction.

Integrating the general formula (15) with respect to the mass, we readily obtain

$$dW_p(p_\perp, \theta) = \frac{2}{5\pi^9} \frac{e^4 n_f \overline{q^2}}{\lambda^2} \frac{\overline{S^2}}{R^2 \ln (R/l)} \Phi(\theta) \frac{dp_\perp}{p_\perp^3} \frac{d\theta}{\sin \theta}. \tag{18}$$

The singular part of this distribution has the same form as the inclusive signal. The "regular" factor or, more precisely, the factor that is always finite and continuous, depends only on the angle. Figure 2 shows a plot of the function

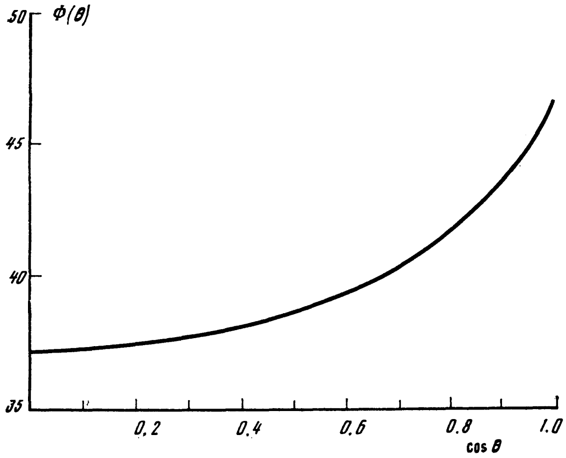

FIG. 2. The regular angular factor [expressions (18) and (19)].

$$\Phi(\theta) = \left(F_1 + \frac{6}{7} |F_1'| \right) \times \left[\frac{1}{\cos^2 \theta} - \frac{3}{2} \frac{\sin^2 \theta}{\cos^4 \theta} \left(1 + \frac{\sin^2 \theta}{\cos \theta} \ln \operatorname{tg} \frac{\theta}{2} \right) \right] - \frac{4}{7} |F_1'| \left[\frac{1}{\cos^2 \theta} - \frac{5}{4} \frac{\sin^2 \theta}{\cos^4 \theta} + \frac{15}{8} \frac{\sin^4 \theta}{\cos^6 \theta} \times \left(1 + \frac{\sin^2 \theta}{\cos \theta} \ln \operatorname{tg} \frac{\theta}{2} \right) \right]. \tag{19}$$

The regular angular factor tends to emphasize longitudinally emitted dileptons, for which $\theta = 0, \pi$, as compared with those emitted at right angles to the reaction axis. According to (19), we have $\Phi_\perp / \Phi_\parallel = 0.8$.

Figuratively speaking, we have here an echo of the criterion that is specific to quark-gluon plasma and has already been mentioned in connection with the more detailed formula (15). The inclusive distribution of the individual leptons does not carry this type of diagnostic information. Expression (2) formally corresponds to the degenerate case $\Phi = \Phi_0 = \text{const}$.

3. DISCUSSION

The practical difficulties of formulating an experiment that will identify the above plasma by its electromagnetic manifestations are reasonably well known. They were very briefly discussed in the concluding section of Ref. 9. The very crude estimates given in Ref. 9 predict an intensity of the order of one thermal dilepton per head-on collision between heavy nuclei. We shall not go any further into this question, and we will confine our attention to another aspect of the problem.

Is there any intuitive way to understand why the dilepton signal is analytically more informative than the inclusive signal? The immediate thought is that we should first consider the characteristics

$$p_\perp, \; M = \text{inv} \tag{20}$$

of the electromagnetic product, which have the property of relativistic invariance in the one-dimensional flow of the emitted matter. Since the number of emitted particles is itself invariant, their distribution function normalized to the invariant phase volume should also be invariant. This is exemplified by the differential equation to the right of the braces in (15). However, if this distribution function can be expressed exclusively in terms of the quantities in (20), then significantly different situations must arise for different signals.

The second invariant disappears in the case of the inclusive signal. For an individual lepton (antilepton), we must have $m_l \approx 0$ at such high temperatures. We are then left with only the first invariant $p_\perp$. However, even this invariant appears in the final result only so as to confirm the T^4 law for the emission of stationary matter in thermodynamic equilibrium. The dilepton signal looks quite different from this point of view. Here, we still have the second invariant $M \neq 0$, and there are no sensible *a priori* reasons why it should not appear independently. The presence of two independent parameters has not only confirmed the T^4 law, but has also generated additional information that is more specific for the plasma. Although, in the final analysis, the result is positive, the analysis of dilepton radiation presented above illustrates the difficulties that will be encountered in the identification of quark-gluon plasma on the basis of its electromagnetic signal.

I am indebted to K. A. Ter-Martirosyan for useful conversations and to A. M. Kamchatnov for help and discussions about the results.

[1]G. Baym, Prog. Theor. Phys. Supp. No. **91**, 251 (1987).
[2]C. Sutton, "CERN bids to create primordial quagma," New Scientist (London), **117**, 45 (1988).
[3]M. B. Voloshin and K. A. Ter-Martirosyan, *Theory of Gauge Interactions Between Elementary Particles* [in Russian], Energoatomizdat, M., 1984.
[4]M. Creutz, Quarks, Gluons and Lattices, Cambridge University Press, Cambridge, 1983, [Russ. transl., Mir, M., 1987].
[5]E. V. Shuryak, Zh. Eksp. Teor. Fiz. **74**, 408 (1978) [Sov. Phys. JETP, **47**, 212 (1978)].
[6]K. Kajantie and H. I. Miettinen, Z. Phys. C **9**, 341 (1981); **14**, 357 (1982)].
[7]G. Domokos and J. I. Goldman, Phys. Rev. D, **23**, 203 (1981).
[8]J. Kapusta, Phys. Lett. B, **136**, 201 (1984).
[9]V. G. Nosov and A. Kamchatnov, Zh. Eksp.Teor. Fiz. **94**, 36 (1988) [Sov. Phys. JETP, **67**, 19 (1988)].
[10]E. L. Feinberg, Izv. Akad. Nauk. SSSR. Ser. Fiz. **26**, 622 (1962).
[11]I. M. Khalatnikov, Zh. Eksp. Teor. Fiz. **27**, 529 (1954).

Translated by S. Chomet