Localization of 2*D* electrons by coherent surface scattering

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An investigation was made of the transport properties of narrow 2*D* conductors with an inhomogeneous boundary. It is shown that the conductance due to the scattering of electrons by a finite section, of length L, of a rough boundary decreases exponentially on increase in L. This provides the first proof of localization of electron states by coherent surface scattering. The localization length is determined. It is shown that mesoscopic fluctuations of the conductance and resistance are so strong that the values of these quantities are no longer self-averaging.

Coherent scattering effects have been investigated quite thoroughly for 1*D* systems. In particular, it has been shown that elastic scattering in one-dimensional disordered conductors results in full localization of electron states. One of the manifestations of such localization is that the average value of the conductance of a 1*D* conductor of finite dimensions L falls exponentially (on increase in L) in a localization length L_0, whereas the resistance rises exponentially. It is found that the conductance G and the resistance R are not self-averaging quantities. Mesoscopic fluctuations of these quantities are so strong that an increase in L causes the variances of the conductance and resistance to rise more rapidly than the squares of their average values.

We shall show that similar effects appear also in narrow 2*D* conductors because of coherent scattering of electrons by a randomly inhomogeneous surface. Two-dimensional systems of small transverse size are at present under active investigation both from the experimental[1,2] and theoretical[3-5] points of view. In particular, a theory of the transport properties of narrow 2*D* microcontacts is developed in Ref. 3 for the ballistic case. If a microcontact is sufficiently long and there is no bulk scattering, the resistance is mainly due to the surface scattering of electrons. It is important to note that the width of the experimentally investigated 2*D* microcontacts is of the order of de Broglie wavelength of electrons. Under these conditions the quantum interference between the electrons scattered by the surface is very important.

We shall demonstrate the mesoscopic behavior of the conductance and resistance due to the surface scattering by considering the following problem. A 2*D* electron gas is enclosed in a region shown in Fig. 1. The lower boundary $x = d$ is ideally flat, but the other boundary has a rough region: $x = \zeta(y)$ in the interval $-L \leqslant y \leqslant 0$. The width of the strip d is such that electrons fill only the lowest transverse quantization level:

$$1 < k_F d/\pi < 2. \tag{1}$$

Using the Kubo formalism, we can show that in this case the conductance is described by

$$G = (e^2/\pi\hbar)\,|T_L|^2, \tag{2}$$

where T_L is the transmission coefficient of an inhomogeneous part of the boundary of length L (Fig. 1). Therefore, the problem of finding the conductance reduces to determination of the modulus of the transmission coefficient and of its statistical characteristics.

We shall assume that the fluctuations $\zeta(y)$ of the shape of the boundary are small compared with the thickness of the strip:

$$\zeta \ll d, \tag{3}$$

and we shall expand the true boundary condition for the wave function $\psi = 0$ at the boundary in terms of a small parameter $k_x\zeta = \pi\zeta/d \ll 1$ ($2\pi/k_x$ is the electron wavelength at right-angles to the strip). We then obtain

$$\psi(d, y) = 0, \quad \psi(0, y) = \begin{cases} 0; & -\infty < y < -L, \quad 0 < y < \infty \\ -\zeta(y)\left.\dfrac{\partial\psi(x, y)}{\partial x}\right|_{x=0}, & -L \leqslant y \leqslant 0 \end{cases}. \tag{4}$$

The Schrödinger equation with the boundary conditions of Eq. (4) and behaving asymptotically in the limit $y \to \pm\infty$, as shown in Fig. 1, can be written down in the integral form

$$\psi(x, y) = \sin\left(\frac{\pi x}{d}\right)\exp(ik_y y) + \int_{-L}^{0} dy'\zeta(y')\left[\frac{\partial\psi(x', y')}{\partial x'}\frac{\partial G_0(x, x'; y-y')}{\partial x'}\right]_{x'=0}. \tag{5}$$

Here, $k_y = [k_F^2 - (\pi/d)^2]^{1/2}$ is the longitudinal wave number and G_0 is the Green function for the unperturbed problem, satisfying zero boundary conditions. Equation (5), subject to allowance for the "single-channel" nature of Eq. (1), yields the following integral equation for the function $\varphi(y) = (d^3/2\pi^2)^{1/2}[\partial\psi(x,y)/\partial x]_{x=0}$:

$$\varphi(y) = (d^3/2\pi^2)^{1/2}[\partial\psi(x, y)/\partial x]_{x=0}:$$

$$\varphi(y) = \exp(ik_y y) + \frac{\pi^2}{ik_y d^3}\int_{-L}^{0} dy'\zeta(y')\exp(ik_y|y-y'|)\varphi(y'). \tag{6}$$

We can easily see that Eq. (6) has the same structure as the equation for the wave function in a 1*D* system in which case the random potential $U(y)$ is

$$U(y) = \frac{(\pi\hbar/d)^2}{m}\frac{\zeta(y)}{d}, \tag{7}$$

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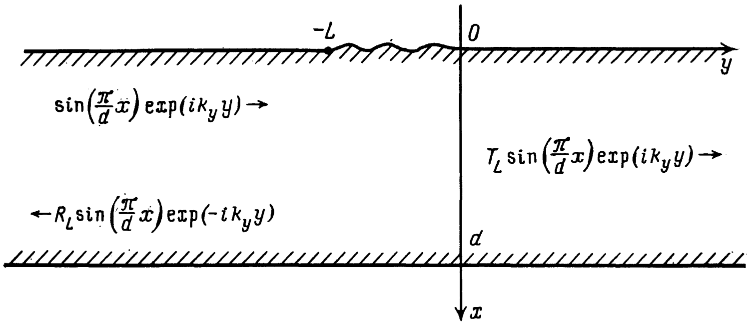

FIG. 1. Geometry of the problem. Asymptotes of the wave function scattered by an inhomogeneous part of the boundary.

where m is the electron mass. Using the relationship

$$\varphi(-L)=\exp(-ik_yL)+R_L\exp(ik_yL), \tag{8}$$

we can now obtain the equation for the reflection coefficient R_L based on the methods in the theory of invariant embedding (see, for example, Ref. 6):

$$\frac{dR_L}{dL}=-i\frac{\pi^2}{d^2}\frac{\zeta(-L)}{k_yd}[\exp(-ik_yL)+R_L\exp(ik_yL)]^2. \tag{9}$$

If we assume that a random process $\zeta(y)$ is Gaussian with zero average and a correlation length l,

$$\langle\zeta(y)\rangle=0,\quad \langle\zeta(y)\zeta(y')\rangle=\zeta^2w(|y-y'|/l), \tag{10}$$

we find that the probability density

$$P_L(u)=\langle\delta(u-u(L))\rangle,\qquad u(L)=\frac{1+|R_L|^2}{1-|R_L|^2} \tag{11}$$

is described by the Fokker–Planck equation

$$\frac{\partial P_L}{\partial L}=\frac{1}{L_0}\frac{\partial}{\partial u}(u^2-1)\frac{\partial P_L}{\partial u}. \tag{12}$$

The localization length L_0 is then given by the expression

$$L_0=2\pi d\left(\frac{k_yd}{\pi}\right)^3\left(\frac{d}{\pi\zeta}\right)^2[2k_ylW(2k_yl)]^{-1}. \tag{13}$$

Here, $W(2k_y l)$ is the dimensionless Fourier transform of the correlation function $w(s)$. The solution of Eq. (12) is well known (see, for example, Ref. 7). It can readily be used to find the average values of the conductance $\langle G\rangle$ and resistance $\langle R\rangle$:

$$\langle G\rangle=\frac{e^2}{\pi\hbar}\frac{4}{\pi^{1/2}}\left(\frac{L_0}{L}\right)^{3/2}\exp\left(-\frac{L}{4L_0}\right)\int_0^\infty\frac{x^2\,dx}{\operatorname{ch}x}\exp\left(-\frac{L_0}{L}x^2\right),$$

$$\langle R\rangle=\frac{\pi\hbar}{e^2}\cdot\frac{1}{2}[\exp(2L/L_0)+1]. \tag{14}$$

It is clear that the average resistance rises exponentially and the conductance falls on increase in L. This is a demonstration of full localization [in a distance L_0 given by Eq. (13)] of electron states in a $2D$ system with an inhomogeneous boundary. Moreover, the distribution function of Eq. (12) can be used readily to show that, as in the one-dimensional case, all the moments $\langle G^n\rangle$ and $\langle R^n\rangle$ are not self-averaging. This is proof of mesoscopic behavior of the conductance and resistance due to the surface scattering of $2D$ electrons.

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