

# Parametric excitation of spin waves by a surface magnetostatic wave

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We have investigated experimentally the process of first-order parametric excitation of spin waves in tangentially-magnetized single-crystal films of yttrium iron garnet under the action of a 3.3 GHz surface magnetostatic wave (SMW). As a result of this process, reverse bulk magnetostatic waves (RBMW) which propagate along the direction of constant magnetic field are excited in these films. Those RBMW which possess the lowest excitation threshold are exchange spin waves with frequencies close to half the SMW frequency and wave vectors which significantly exceed the value of the inverse thickness of the film. Increasing the SMW power leads to additional excitation of nondegenerate pairs of RBMW with indices which differ by unity and frequencies which differ from half the SMW frequency by some tens of megahertz; their wave vectors are on the order of  $10^4 \text{ cm}^{-1}$ .

## INTRODUCTION

Parametric excitation of spin waves by a spatially homogeneous pump has been investigated in detail in many ferro- and antiferromagnetic materials.<sup>1,2</sup> Due to the new availability of high-quality single-crystal films of yttrium iron garnet (YIG), it is now possible to investigate a scenario in which the role of the pump is played by either a surface or internal magnetostatic wave. In the design of spin-wave electronic devices there are many advantages to using tangentially-magnetized ferrite films and surface magnetostatic waves (SMW) which propagate perpendicular to the direction of the internal constant magnetic field  $\mathbf{H}_i$  as the active waves.<sup>3</sup> Like all the other waves or oscillations in the ferrite, SMW give rise to parametric excitation of new waves or vibrations in the sample when a certain threshold amplitude is reached. This threshold is especially low at the point where the pump frequency  $\omega_p = 2\pi f_p$ , which equals half the SMW frequency, first enters the spin wave spectrum (see Fig. 1). In this case a first-order parametric instability is possible,<sup>4</sup> in which spin waves are excited with frequencies  $\omega'$  and  $\omega''$  such that  $\omega' + \omega'' = \omega_p$ ; the case where  $\omega' = \omega'' = \omega_p/2$  is referred to as degenerate parametric instability. For single-crystal YIG films the SMW power at which this process begins is  $\sim 1 \mu\text{W}$ , which is very important in limiting the range of working powers of SMW devices.

We will analyze all the possible first-order parametric interactions of SMW with spin waves illustrated in Fig. 1. In this figure we show the dispersion curves for surface magnetostatic waves, which propagate perpendicular to  $\mathbf{H}_i$ , and reverse bulk magnetostatic spin waves (RBMW), which propagate along  $\mathbf{H}_i$ . This figure was constructed by using the equations of Ref. 5, taking into account exchange effects by including the effective field of the exchange interaction in the expression for the internal field  $H_i$ .<sup>6</sup> Out of all the multitude of RBMW we choose only those waves which propagate along  $\mathbf{H}_i$ ; this is because the momenta (or wave vectors) of the parametrically interacting waves must be conserved:  $\mathbf{k}_p = \mathbf{k}' + \mathbf{k}''$ , where  $\mathbf{k}'$ ,  $\mathbf{k}''$  are the wave vectors of the RBMW and  $\mathbf{k}_p$  is the wave vector of the pump wave, i.e., the SMW. It is clear from Fig. 1 that  $k_p \ll k'$ ,  $k''$  (in practice  $k_p \sim 10^2$  to  $10^3 \text{ cm}^{-1}$ , while  $k', k'' \sim 10^4$  to  $10^5 \text{ cm}^{-1}$ ). This implies first of all that  $k' \approx k''$ , and secondly that  $\mathbf{k}'$  and  $\mathbf{k}''$  are directed approximately parallel and antiparallel to the

constant magnetic field: the polar angle  $\theta$  between  $\mathbf{k}'$ ,  $\mathbf{k}''$ , and  $\mathbf{H}_i$  has a very small value ( $\sin\theta = k'/k'' \sim 10^{-2}$ ); we will neglect this angle in what follows and assume that the parametric RBMW propagate along the constant magnetic field. Let us note that the process of parametric excitation of spin waves via SMW investigated here occurs only in a limited region of constant magnetic field. The maximum field  $H_{\max}$  is determined by the condition that the frequency  $\omega_p/2$  enter the spin wave spectrum, while the minimum field  $H_{\min}$  is determined by equating  $\omega_p$  to the maximum SMW frequency.

Figure 1 was constructed for a specific value of field  $H_i = 498 \text{ Oe}$ , a film thickness  $d = 16 \mu\text{m}$ , and a saturation magnetization  $4\pi M_0 = 1750 \text{ G}$ . In this figure we also show the location of the frequencies  $f_p = 3.3 \text{ GHz}$  and  $f_p/2 = 1.65 \text{ GHz}$  in the spin wave spectrum. It is clear from the figure that under these conditions it is possible in principle to have degenerate parametric excitation of RBMW with indices  $n = 1, 2, \dots, 11$  at the frequency  $f_p/2$ . In this case the excitation is possible in two wave vector regions: the large- $k$  region  $k', k'' \geq 10^5 \text{ cm}^{-1}$ , where the exchange interaction is

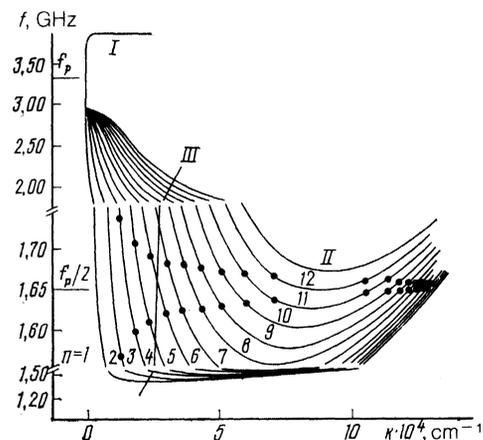


FIG. 1. Spectrum of waves of a tangentially magnetized YIG film of thickness  $16 \mu\text{m}$ ,  $H_i = 498 \text{ Oe}$ : I—SMW propagating perpendicular to  $\mathbf{H}_i$ ; II—RBMW, propagating along  $\mathbf{H}_i$ ; III—transverse elastic waves. For convenience, the part of the spectrum near half the value of the pump frequency is shown using an expanded scale. The numbers are the RBMW indices  $n$ , while the curves are the dispersion relations for the first twelve waves.

significant and the waves are practically planar, and the small- $k$  region, where the magnetic-dipole energy predominates and the boundary conditions at the surface of the film are important. As the field  $H_i$  grows (or as the frequency  $f_p$  decreases) the wavevector of the exchange waves decreases, while that of the magnetic-dipole wave increases. It is also possible to have nondegenerate instability in these two regions due to RBMW which are adjacent with respect to the index  $n$ , i.e.  $(n, n + 1)$ , with frequencies  $f' = f_n, f'' = f_{n+1}$ ,  $f_n + f_{n+1} = f_p$  (the points on Fig. 1); according to this figure, this nondegenerate excitation involves pairs of RBMW with subscripts  $(1,2), (2,3), \dots, (10,11)$ . The deviation of the frequencies of the excited waves from  $f_p/2$ :

$$\Delta f = f_{n+1} - f_p/2 = f_p/2 - f_n$$

decreases for magnetic-dipole waves with increasing field or with decreasing frequency, and as the wave vectors  $k_n \approx k_{n+1}$  increase. The opposite dependence is predicted for exchange waves; in addition,  $\Delta f$  is smaller for these waves than for magnetic-dipole waves. According to Fig. 1, the nondegenerate RBMW instability also occurs for waves with indices  $(n, n + 2), (n, n + 3)$ , etc.

The problem for this paper is to investigate first-order parametric processes in YIG films which take place under the action of SMW pumping. Specifically, we will determine the frequencies and wave vectors of the parametrically excited waves for various values of the pump power and external constant magnetic field  $H_0$ . The pump frequency will be kept constant at 3.3 GHz.

## EXPERIMENT

Parametric excitation of spin waves manifests itself experimentally in two ways. First of all, a pump pulse reflected from the sample or transmitted through a microwave structure containing the sample is distorted<sup>7</sup>; secondly, electromagnetic radiation is emitted by the sample at the frequencies of the parametrically excited waves  $f_n \approx f_p/2$ .<sup>8</sup> Whereas the pulse distortion, which has the form of a "step," i.e., a sudden dropoff of the initially flat top to a new and lower constant level,<sup>7</sup> is practically independent of the value of the wave vector of the exciting wave (this stepwise decrease is determined only by that part of the pump power which is dissipated in maintaining the parametric process), the radiation from the film depends significantly on the value of this vector. Appreciable conversion coefficients of magnetostatic waves into electromagnetic waves is observed only for values of the wave vectors  $k', k'' \ll \pi/w$ , where  $w$  is the width of the antenna for magnetostatic waves.<sup>9</sup> Detection of waves with larger wave vectors is difficult even when high-sensitivity microwave amplifiers are used. Therefore, the only evidence for parametric excitation of exchange RBMW with  $k \gg \pi/w$  is the presence of a step on the pump pulse. However, evidence for the presence of magnetic-dipole waves, which possess smaller values of wave vector, can also be inferred from the presence of electromagnetic radiation from the film at the frequency  $f_n$ .

Figure 2 shows a block diagram of the experimental setup we used to observe both the step on the pump pulse and electromagnetic radiation from the film. The signal from a pump oscillator 1, which can be used both in pulsed (for observing the step) and CW modes, arrives at an input an-

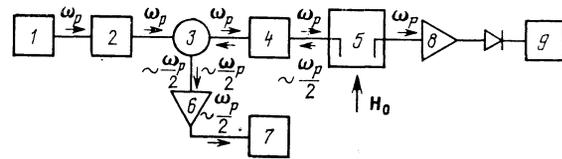


FIG. 2. Block diagram for the experimental setup: 1—pump oscillator with frequency 3.3 GHz; 2—gate; 3—wide-band circulator; 4—impedance transformer; 5—ferrite film with input and output antennas in a magnetic field; 6—amplifier at a frequency of  $1.65 \pm 0.25$  GHz; 7—spectrum analyzer; 8—amplifier at the pump frequency; 9—oscilloscope.

tenna and excites SMW in a ferrite film. We investigated various films with thicknesses from 5 to 50  $\mu\text{m}$  and ferromagnetic resonance line widths  $\Delta H_0 = 0.5$  to 0.7 Oe; the measurement was carried out at room temperature. The SMW which propagate along the film excite a signal at the pump frequency in the output antenna, which is recorded by the oscilloscope 9. The width of the antennas was  $w = 10 \mu\text{m}$ , the length of the antennas was  $l = 3\text{mm}$ , and the spacing between them was 1 mm. For SMW amplitudes above threshold spin waves are excited in the film; their creation gives rise to the appearance of the step on the pulse recorded at the oscilloscope 9, and then to the appearance of signals with frequencies  $\omega', \omega'' \approx \omega_p/2$  at the input and output antennas. It was found that for any mutual orientation of the antennas, including the one in which they are perpendicular to one another, the signal at the input antenna was always larger than that at the output. We explain this by noting that because the SMW are attenuated during propagation their amplitudes are largest near the input antenna, and so it is here that the parametric excitation of RBMW first begins; however, the RBMW themselves have short mean free paths, which seldom exceed several tens of microns. In view of this, the RBMW signal was taken off the input antenna and routed through the circulator 3 and microwave amplifier 6, from which it is sent to the spectrum analyzer 7. The impedance transformer 4 serves to maximize the transfer of the electromagnetic pump energy in the SMW and to compensate for reflection at the frequency  $\omega_p$  from the input antenna; this eliminates the effect of the pump on the operation of the amplifier cascade 6. The amplifier was a low-noise microwave amplifier made with Schottky-barrier field-effect transistors whose noise temperature was less than 100 K; its amplification coefficient was 40 dB. When used with the Sch-27 spectrum analyzer, this amplifier ensured a sensitivity of the apparatus of better than  $-155 \text{ dB}\cdot\text{W}$ .

We established the following experimental regularities for all the YIG films we studied. As the pump power  $P$  increased, the step always appeared first. The threshold power for appearance of the step, i.e., the threshold  $P_{\text{th}}$  for the initiation of parametric instability under the action of the SMW, increased smoothly as  $H_0$  increased; it was measured to be  $\approx 3 \mu\text{W}$  at minimum fields and  $\approx 100 \mu\text{W}$  for  $H_0 \approx H_{\text{max}}$ . The threshold amplitudes of the microwave magnetic field  $h_{\text{th}}$  corresponding to these threshold powers, and the AC magnetization  $m_{\text{th}}$ , in this case changed only imperceptibly: as was shown in Ref. 10, for all values of the constant magnetic field these thresholds corresponded approximately to the threshold for first-order paramagnetic instability against uniform magnetization oscillations.<sup>4</sup> The

strong variation in the threshold power is caused by the change in the SMW group velocity.<sup>10</sup> The experimental values of  $h_{th}$  and  $m_{th}$  can be determined with the help of the methods proposed in Ref. 10. In our case, that of a short antenna ( $l \ll c/f_p$ ),  $h_{th}$  and  $m_{th}$  are practically unchanged along the film up to the antenna; their values are  $h_{th} \approx 1$  Oe and  $m_{th} \approx 0.1$  G. As the ratio of the pump power to its threshold value, i.e., the "supercriticality ratio"  $\zeta = P/P_{th}$ , continues to increase, the "step" is followed by the appearance of radiation from the sample. This radiation has a frequency which departs considerably from  $f_p/2$ , i.e.,  $|\Delta f| \geq 30$  MHz, over almost the entire range of constant magnetic fields from  $H_{min}$  to  $H_{max}$ , except for a narrow interval of width 20 to 30 Oe near the field  $H_{min}$ . It appears at a certain threshold value of the supercriticality ratio  $\zeta$ ; this value depends significantly on the value of  $H_0$  and varies from 7 to 25 dB. Energy is radiated simultaneously at the frequencies  $f_p/2 + \Delta f$  and  $f_p/2 - \Delta f$ , although the amplitudes at these frequencies may differ significantly (by a factor of 10 or more). As the pump power increases, following the initial radiated signals with  $\Delta f = \Delta f_1$ , which have the smallest threshold, radiation appears at other frequencies with  $\Delta f = \Delta f_1, \Delta f_{II}, \Delta f_{III}, \dots$

Radiation near the maximum field had a frequency close to  $f_p/2$ , i.e.,  $|\Delta f| < 1.5$  MHz. It exists over a narrow range of constant magnetic fields  $H_{max} - 30 \text{ Oe} \leq H_0 \leq H_{max}$ , and appears most often without a threshold; its time of appearance practically coincides with that of the step ( $\zeta = 1$ ) for fields close to  $H_{max}$ , and increases considerably (by up to  $-10$  to  $20$  dB) as the field decreases.

What we have described above is well illustrated in the experimental results shown in Fig. 3 for one of our YIG films ( $d = 16 \mu\text{m}$ ). In this figure we show the dependence on the external constant magnetic field of the following quantities: 1) the value of the frequency deviation  $|\Delta f|$  of the radiation emitted by the sample from half the pump frequency; 2) the supercriticality ratio  $\zeta$  at which this radiation first appears. The theoretical dependences  $|\Delta f(H_0)|$  shown here are for nondegenerate excitation of RBMW with indices  $(n, n+1)$  for  $n = 1, 2, \dots, 7$ ; in constructing them we took into account the crystalline anisotropy field, which gives  $H_i = H_0 + 40$  Oe for the film under study. The good agreement between

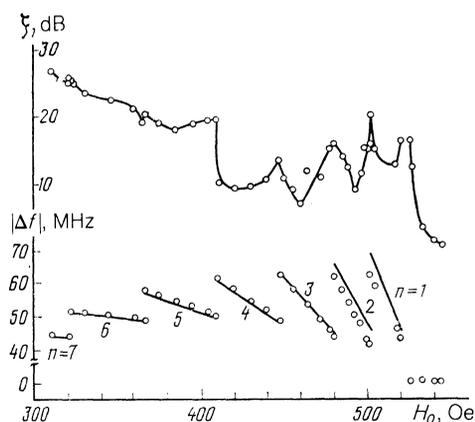


FIG. 3. Dependence of  $|\Delta f|$  (the deviation of the radiation frequency from half the pump frequency  $f_p/2$ ) and the supercriticality ratio  $\zeta$  at which this radiation appears on the value of the external magnetic field  $H_0$ . The solid traces in the lower part of the figure show the theoretical dependence of  $|\Delta f(H_0)|$  for nondegenerate excitation of RBMW with indices  $(n, n+1)$  for  $n = 1, 2, \dots, 7$ .

the experimental points and the theoretical curves indicates that nondegenerate parametric excitation of pairs of RBMW with indices which differ by unity does in fact occur in fields  $H_0 < 520$  Oe (see Fig. 3). As the constant field increases, at a certain field the indices of the interacting pairs decrease discontinuously by unity from (7,8) at the minimum field to (1,2) for  $H_0 = 503$  Oe. In the course of this increase in the constant field, the quantity  $|\Delta f|$  is found to increase at the jump discontinuity fields, while between jumps it decreases smoothly with increasing  $H_0$ ; for this case the absolute value of  $|\Delta f|$  lies in the range 44 to 63 MHz. According to Fig. 3, radiation with a significantly smaller value of  $|\Delta f|$  (for this film with  $|\Delta f| < 1.1$  MHz) is observed in fields  $H_0 \geq 525$  Oe; the supercriticality ratio at which this radiation is observed decreases rapidly from  $\zeta = 17$  dB at  $H_0 = 525$  Oe to  $\zeta = 0.5$  dB at  $H_0 = 544$  Oe.

As we have already noted, after the initial appearance of radiation which is characterized by the quantities  $\Delta f = \Delta f_1$  and  $\zeta = \zeta_1$  (the data shown in Fig. 1 are for this case), additional radiation appears with  $\Delta f = \Delta f_{II}, \Delta f_{III}, \dots$  for supercriticality ratios  $\zeta = \zeta_{II}, \zeta_{III}, \dots$ . This is especially simple to observe near those constant magnetic fields at which the jumps in  $\Delta f$  occur. For example, according to Fig. 3 the first instability arises at  $H_0 = 450$  Oe for waves with indices (3,4) with  $\Delta f = \Delta f_1 = 62$  MHz and  $\zeta_1 = 11$  dB. For  $\zeta = \zeta_{II} = 13$  dB additional radiation appears with  $\Delta f = \Delta f_{II} = 47$  MHz, which corresponds to the waves (4,5). For fields in the middle of the region of smooth variation of  $\Delta f(H_0)$  as  $\zeta$  increases we commonly observe the excitation of secondary waves with larger values of  $\Delta f$  than for the primary waves, i.e., waves with smaller indices. For example, when  $H_0 = 330$  Oe we observe the following dynamic development of the parametric instability in the RBMW spectrum under the action of the SMW:  $\zeta_1 = 23.5$  dB,  $\Delta f_1 = 52$  MHz, waves excited with indices (6,7);  $\zeta_{II} = 29$  dB,  $\Delta f_{II} = 61$  MHz, (5,6);  $\zeta_{III} = 34$  dB,  $\Delta f_{III} = 70$  MHz, (4,5).

## DISCUSSION OF RESULTS

The experimental results described above can be understood if we adopt the following picture of the development of parametric instability in the spin-wave spectrum under the action of the SMW. First, exchange spin waves are excited there; this is manifested in the appearance of the step in the pump pulse. The wave vectors of these waves are large,  $k > 10^5 \text{ cm}^{-1}$ ; as a result of this, they radiate energy very weakly into the antenna. In order to find the wave vectors  $k$  and frequencies  $\Delta f$  of the exciting waves we can use the dispersion relations, or plots of them similar to the one shown in Fig. 1. For example, for  $H_i = 498$  Oe ( $H_0 = 458$  Oe), according to Fig. 1 we have  $1.05 \cdot 10^5 \text{ cm}^{-1} < k < 1.3 \cdot 10^5 \text{ cm}^{-1}$ ;  $0 < \Delta f < 8$  MHz. In order to observe the radiation of exchange waves it is necessary to increase the constant magnetic field; in this case, as we showed above, the wave vector of the exchange waves decreases. According to Fig. 3, we observe the appearance of radiation almost simultaneously with that of the step in the pump pulse at fields  $H_0 = 544$  Oe, which corresponds to waves with  $k \approx 5 \cdot 10^4 \text{ cm}^{-1}$ ,  $|\Delta f| < 1$  MHz. In order to observe waves with larger  $k$  it was necessary to increase their amplitude by increasing the supercriticality ratio of the pump. In order to observe radiation from waves with  $k \approx 8 \cdot 10^4 \text{ cm}^{-1}$  ( $H_0 = 525$  Oe) it was necessary

to increase the pump power to 50 times the threshold for appearance of the step in the pump pulse. For waves with large  $k$  (i.e., for  $H_0 < 525$  Oe) no radiation is observed in this experimental setup.

As the supercriticality ratio increases, along with the growth in amplitude of the exchange waves parametric excitation of a second group of waves occurs. According to the theory of nonlinear self-consistent spin-wave interactions (the  $S$ -theory; see Ref. 1) this second group of waves will have wave vectors which differ considerably (in the present case they can only be smaller) from the wave vectors of the primary exchange spin waves. This implies that magnetic-dipole RBMW will be excited with  $k \ll 10^5 \text{ cm}^{-1}$ . Exactly which of these waves are excited depends on the value of the constant magnetic field; however, the excitations are always nondegenerate. For example, for  $H_0 = 458$  Oe (see Figs. 1 and 3) for  $\zeta = 7$  dB we excite RBMW with indices (3,4),  $f_3 = 1593$  MHz,  $f_4 = 1707$  MHz,  $|\Delta f| = 57$  MHz,  $k = 1.8 \cdot 10^4 \text{ cm}^{-1}$ . As  $H_0$  increases from 458 to 482 Oe, the wave vector of the RBMW with indices (3,4) increases up to  $2.3 \cdot 10^4 \text{ cm}^{-1}$ . At this point a jump occurs; at the field  $H_0 = 482$  Oe it first becomes possible to excite RBMW with indices (2,3) and wave vectors with  $k \approx 1.5 \cdot 10^4 \text{ cm}^{-1}$ . As the field increases further, once again there is an increase in  $k$ , again only up to a certain limit. In the general case, waves are excited in the film with wave vectors  $1.5 \cdot 10^4 \text{ cm}^{-1} < k < 2.5 \cdot 10^4 \text{ cm}^{-1}$  for arbitrary fields; as the magnetic field varies, each time the RBMW wave vector reaches one of the limiting values (from below by decreasing the field, from above by increasing it) a new pair of RBMW is excited, and the situation repeats itself in large part.

As the supercriticality ratio increases, the range of observed values of wave vectors of parametrically-excited RBMW can increase somewhat; for  $\zeta = 35$  dB we have  $10^4 \text{ cm}^{-1} \leq k \leq 2.5 \cdot 10^4 \text{ cm}^{-1}$ .

The picture we have proposed to describe the development of parametric instability in the spin-wave spectrum under the action of SMW explains all the experimental facts; however, this picture raises the following questions: 1) why do the exchange spin waves have the lowest excitation threshold? 2) Why is there no degenerate instability for the magnetic-dipole RBMW? 3) Why are the wave vectors of the excited RBMW only found within the limits  $1.5 \cdot 10^4 \text{ cm}^{-1} < k < 2.5 \cdot 10^4 \text{ cm}^{-1}$ ?

To answer the first two questions we must calculate the threshold for degenerate and nondegenerate excitation of RBMW under the action of SMW and compare them with the excitation threshold for plane spin waves.<sup>11</sup> For this we can use the general expressions presented in Ref. 11 for the instability threshold associated with parametric generation of magnetostatic waves under the action of SMW; however, in our case the oscillations which excite the spin waves are not plane waves, as in Ref. 11, but RBMW, whose magnetostatic potential was given in Ref. 5. The results of these calculations show that for  $k_p \ll k$  the threshold for degenerate parametric instability of RBMW propagating along the field is infinite; this is in complete agreement with the experiments, which in all cases indicate that there is no degenerate excitation of RBMW.

The threshold magnetization amplitude for surface magnetostatic waves during nondegenerate parametric instability of RBMW with indices  $(n, n+1)$  is

$$M_{n,n+1} = (\Delta H_n \Delta H_{n+1})^{1/2} (n\pi - 2\theta) [(n+1)\pi - 2\theta] \times \left[ 1 + \frac{\sin 2\theta}{n\pi - 2\theta} \right] \left[ 1 + \frac{\sin 2\theta}{(n+1)\pi - 2\theta} \right] \left\{ \pi \left( 1 + \frac{2\omega_H}{\omega_p} \right) \times \sin 2\theta [1 + \exp(-k_p d)] \right. \\ \left. \times \left[ 2n+1 - \frac{4\theta}{\pi} + \frac{\cos 2\theta}{3-4\theta/\pi} \right] \right\}^{-1}, \quad (1)$$

where  $\Delta H_n$  is the width of the resonance RBMW line with index  $n$ ,

$$\sin^2 \theta = [(\omega_p/2)^2 - (\omega_H + Dk^2)^2] [(\omega_H + Dk^2)\omega_M]^{-1}, \quad (2)$$

$\omega_H = \gamma H_i$ ,  $D$  is the exchange constant,  $\omega_M = 4\pi M_0 \gamma$ , and  $\gamma$  is the gyromagnetic ratio. In deriving (1) we have assumed that the mean free path of SMW is much larger than that of the RBMW, which allows us to neglect the dependence of the SMW amplitude  $M_{n,n+1}$  on the coordinates in the direction of propagation.

A numerical comparison of (1) with the threshold excitation amplitude for plane exchange waves whose mean free path is considerably smaller than the film thickness  $d$  shows that the threshold amplitude for excitation of magnetic-dipole RBMW given by (1) is 1.5 to 2 times (i.e., up to 6 dB) larger than the excitation threshold of the exchange waves for equal line widths; this is somewhat smaller than the experimental results, which show a difference of 7 dB or more. This can be related to the fact that, first of all, the resonance line width for RBMW due to scattering by surface inhomogeneities can be larger than for the plane exchange waves; the smaller the indices of the former, the larger this line width will be. Secondly, because of the increasing group velocity and thus the larger mean free path of the RBMW with decreasing constant magnetic field, Eq. (1), on which our comparison was based, may become invalid in the small-field region. It is well-known<sup>12</sup> that the threshold for parametric excitation grows as the group velocity of the waves increases. Therefore the threshold for RBMW excitation by means of SMW is smallest in the middle of the region of constant magnetic fields between  $H_{\min}$  and  $H_{\max}$ , which also is observed in experiment (see Fig. 3).

In order to answer the third question—why are the wave vectors of the excited RBMW found within the limits  $1.5 \cdot 10^4 \text{ cm}^{-1} < k < 2.5 \cdot 10^4 \text{ cm}^{-1}$ —we show in Fig. 1 the dispersion relation for transverse elastic waves. It is clear that in the frequency region of interest to us the wave vector of these waves is  $k_1 \approx 2.6 \cdot 10^4 \text{ cm}^{-1}$ . Thus, for those values of the excited RBMW wave vector which are less than the upper bound the relation  $k \leq k_1$  obtains. This relation is fully understandable by virtue of the fact that as the RBMW wave vector approaches  $k_1$  the magnetoelastic coupling becomes more important, which increases the threshold for parametric excitation.<sup>13</sup>

The lower value of wave vector of the excited RBMW is apparently due to the increase in group velocity of the RBMW as their wave vectors decrease. As we noted above, increasing the group velocity increases the threshold for parametric excitation. Increasing the pump power can bring about a decrease in the lower bound on the wave vectors of the excited RBMW; this is also observed in experiment. Apparently, it can also bring about an increase in the upper

bound on the wave vectors for the excited RBMW; however, for the pump powers available to us ( $P \leq 10$  mW) this did not occur.

We note in conclusion that the nondegenerate parametric excitation of waves with frequencies which differ by  $|\Delta f|$  from half the pump frequency which we have observed in this work should lead to self-oscillations of the magnetization at a frequency of  $2\Delta f$ .<sup>14</sup> Apparently, the author of Ref. 15 observed these very self-oscillations in films; they were also investigated in Ref. 16.

## CONCLUSIONS

In tangentially magnetized YIG films a SMW which propagates perpendicular to the constant magnetic field with wave powers of a few microwatts can lead to parametric excitation of RBMW propagating along the field direction as a result of first-order parametric processes.

The first result of increasing the SMW power is the excitation of exchange spin waves with wave vectors which significantly exceed the inverse thickness of the film and with frequencies close to half the SMW frequency. As the power is further increased, the threshold is reached for nondegenerate parametric excitation of magnetic-dipole RBMW with indices  $(n, n + 1)$ , whose frequencies differ by some tens of megahertz from half the RBMW frequency and whose wave vectors  $k < k_1$ , where  $k_1$  is the wave vector of a transverse elastic wave at the RBMW frequency. The specific value of  $k$  depends on the value of constant magnetic field and is determined by a compromise between two factors, both of which increase the threshold for parametric excitation: the influence of the magnetoelastic interaction for waves with wave vectors  $k$  close to  $k_1$ , and the increase in group velocity and

mean free path of the RBMW as their wave vectors increase. As a result of this compromise, for each value of constant magnetic field an RBMW is excited with specific indices  $(n, n + 1)$  whose wave vector ensures that the total effect of both these factors is minimal. In our experiments the various values of field corresponded to excitation of RBMW with  $n = 1, 2, \dots, 7$ . As the SMW power increases new RBMW are successively excited with values of  $n$  which differ by one from the  $n$  of previously excited waves.

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